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Acceleration of Monte Carlo modeling of light transport in turbid media; an approach based on hybrid, theoretical and numerical, calculations

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ABSTRACT

A novel method to accelerate Monte Carlo (MC) simulations of photon migration in turbid media is presented. It is specifically suited for transillumination studies in slab geometries including some deep inhomogeneity. Propagation up to the inhomogeneity, at a given depth S_1, is replaced by theoretical calculations using well established models. Then, photon propagation is continued inside the complete slab using MC rules until detection or absorption occurs. We report improvements in speed in factors up to approximately 10; the deeper the inhomogeneity, the larger the improvement. Examples are given showing that information remains unchanged with respect to pure MC simulations.

1. Introduction

Propagation of near infrared (NIR) light in highly scattering biological media has been a very active research field during the past decades. The relative low absorption of tissue in the NIR region, allowed scientists to envisage applications of this non-invasive radiation to detect breast cancer and alteration in cerebral blood flow, to mention but a few [1]. Light propagation inside turbid media can be completely described in terms of several parameters, namely the absorption coefficient, μ_a, the scattering coefficient, μ_s and the anisotropy factor $g = \langle \cos\theta \rangle$, which is the average value of the cosine of the scattering angles [2]. In terms of these, the total interaction coefficient, $\mu_t = \mu_s + \mu_a$ is defined, so that its inverse, μ_t^{-1}, is the mean free path between two consecutive interactions. Another coefficient, μ_s', called the reduced scattering coefficient is defined as

$$\mu_s' = \mu_s(1 - g),$$

where $(\mu_s')^{-1}$ represents the transport mean free path [2].

The usual theoretical approach is made in terms of the scalar radiative transfer equation (RTE) [2], which is a description of energy balance inside a volume of the scattering medium. However, since the RTE is very complicated, in many cases the diffusion approximation (DA) [2] is preferred and renders good solutions. In particular, for an isotropic source

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and for $\mu_s' \gg \mu_a$, the time independent DA is

$$D\nabla^2 U(r) + \mu_a U(r) = E(r), \tag{1}$$

being $U(r)$ the average diffuse intensity, $E(r)$ the source term and $D = \frac{1}{3}(\mu_s' + \mu_a)$.

On the other hand, for a given geometry, i.e. infinite, semi-infinite or a slab, and with the use of the adequate boundary conditions, analytical solutions of DA for homogeneous media can be obtained with relative ease [3,4]. Additionally, in particular for the case of optical mammography, it is very important to solve the problem considering immersed inhomogeneities. The solution becomes more complicated in this case and it is still matter of active research [5].

Numerical Monte Carlo (MC) simulation, in which photons are tracked individually while they propagate inside the medium, is a very flexible, powerful tool capable of dealing with relative complex situations, and that has been extensively used as a validation gold standard for many diffusion problems. Its main drawback lies in its essence: results are built up stochastically accumulating photon by photon at a desired space or time point. Occasionally, many photons have to be launched before one succeeds and thus, very long calculation times may be required to achieve a result with reasonable good statistics [6–8]. Noise decreases with the square root of positive events (detected photons) and so, improving signal to noise ratio by a factor of three, requires increasing by a factor of almost 10 the number of launched photons, thus increasing calculation time by the same factor [9]. Because of this, several acceleration techniques have been proposed in the last years [10–12].

It should also be clearly stated that MC, as well as present analytical approaches, cannot deal with the real final objective; that is, the solution of the inverse problem, i.e. to find the inhomogeneity which produces a given solution. But it is precisely the flexibility of MC what allows to obtain a solution for almost any imaginable geometry and configuration, and is the reason why finding algorithms to improve the time performance of MC codes is of great interest.

At this point it is important to clarify that the term "photon" used along this text bears no resemblance to the photon concept defined in terms of quantum electrodynamics. According to this interpretation, the photon is a quantum of a single mode of the electromagnetic field. As such, it fills the volume of quantization, and thus it cannot be localized in space. Instead, the concept of "photon" is used here in a phenomenological sense and what is meant by "photon" is a discrete packet of energy which is launched in MC to solve the scalar RTE. A clear and detailed discussion about this subject can be found in Refs. [13,14]. In the present approach, since MC is applied to radiances rather than to electric fields, coherent backscattering is not taken into account. Possible cases in which it may be of relevance are described in [13].

In this paper, we propose a novel approach to reduce calculation time in MC simulations in turbid media slabs containing inhomogeneities. The basic idea is to avoid tracking photon by photon inside the homogeneous initial portion of the slab, until the inhomogeneity is reached, and replacing this MC procedure by either a theoretical or another (already done and stored) calculation. The method proves to be particularly good in cases with deep inhomogeneities, where the initial homogeneous portion is significant in comparison with the whole slab. Even if the principles of the proposed method are applicable to both, time resolved or continuous wave (CW) approaches, the results presented here are specialized for the CW case.

In the next section, we briefly describe the general principles of MC simulations for light propagation in turbid media. Section 3 is dedicated to bring a detailed description of the idea presented in this work, which leads to speeding up of the MC simulations. We then present some results for both, homogeneous slabs and slabs including an inhomogeneity, pointing out the reduction factor in computation time. At the end, a section is devoted to the discussion and conclusions.

2. Basic principles of MC simulation in turbid media

MC simulations are carried out by launching at the entrance face of a slab of turbid medium of thickness S, an infinitely narrow photon beam consisting of N_0 photons perpendicularly incident on the slab. Incidence direction is assumed to be z and x and y are the directions perpendicular to the incident beam. Photons are launched one at a time starting at position $(0, 0, 0)$. For the collection procedure, we distinguish two cases: (i) diffuse transmission or transmittance, in which collection is made at the opposite face of the slab, at points $(x, y, z = S)$ and (ii) diffuse reflectance in which photons are collected at $(x, y, z = 0)$. A particular case of diffuse transmission is called *transilluminance*, in which the input photon beam is launched at several x positions across the entrance surface of the slab, being $(x, 0, z)$ the light incidence point, and the detector is placed at the other face of the slab $(z = S)$ in the prolongation of the incidence direction, z.

The basic flow diagram adopted in this work for the MC code, follows the original idea from Wang et al. [15], and we shall give here only the general rules that describe the code. We have also compared the results obtained with this code with experimental and theoretical models in a previous paper [16].

The propagation of each photon is determined by a set of random numbers $\{R_j,\ j = 1, 2, 3, 4\}$, uniformly distributed between 0 and 1, and by the optical properties of the medium; that is, its scattering coefficient (μ_s), its absorption coefficient (μ_a), and the anisotropy factor g.

Between successive scattering events, a propagation step of length given by

$$l = -\frac{\ln(R_1)}{\mu_t} \tag{2}$$

is assumed. The new direction of propagation is determined by two additional random numbers, R_2 and R_3, which produce the new propagation angles. The azimuthal angle ϕ is considered to be uniformly distributed and it can take values between 0 and 2π; thus it results

$$\phi = 2\pi R_2. \tag{3}$$

The deflection angle θ has to be sampled according to certain probability distribution for its cosine, called the "phase function". The usual choice for it is the well-known function proposed by Henyey and Greenstein for stellar atmospheres [17]. In accordance with this proposal we take

$$\cos\theta = \begin{cases} \dfrac{1}{2g}\left[1+g^2-\left(\dfrac{1-g^2}{1-g+2gR_3}\right)^2\right] & \text{if } g\neq 0, \\ 2R_3-1 & \text{if } g=0, \end{cases} \tag{4}$$

where g for the case of biological tissue is, in most cases, a value between $g = 0.7$ and 0.9 [1].

At each new position of the photon it is decided if absorption takes place or if scattering process goes on, by R_4. Absorption occurs if

$$R_4 < \frac{\mu_a}{\mu_t} \tag{5}$$

and the photon is killed; otherwise, scattering proceeds [15].

Fresnel reflections are taken into account only at the input and output faces of the slab, since its lateral dimensions were assumed to be large enough so as to avoid border effects; photon propagation was allowed up to distances from the optical axis as large as 20 times the slab thickness. The real part of the refractive index of the inhomogeneities was considered to have negligible difference with respect to that of the bulk; consequently, Fresnel reflections between the bulk and the inhomogeneity were dismissed.

3. The proposed acceleration method

Consider the situation shown in the scheme of Fig. 1, in which we have represented the plane $x{-}z$ of a slab of a given turbid medium of thickness S. Illumination direction is assumed to be coincident with z, and lateral dimensions of the slab are taken to be large in comparison to its thickness, that is $L_x \gg S$ and $L_y \gg S$. The slab, which in the following will be called "the host", is characterized by the optical parameters n_0, μ_{a0}, μ'_{s0}. An inhomogeneity of parameters n, μ_a, μ'_s and of typical dimension r_{inh} is centered at coordinates (x_0, y_0, z_0) inside the host, such that the minimum absolute distance from any point in the inhomogeneity to the entrance face of the slab, $z = 0$, is S_1. In the above paragraph, n_0 and n refers to the real part of the refractive index.

In a normal MC simulation for a slab, homogeneous or containing inhomogeneities, a photon is launched at the entrance of the slab and it is tracked until it is absorbed or it emerges through a given desired collection area at $z = S$, namely the detector. Fresnel laws must be properly taken into account if any index mismatch occurs. This tracking has to be done for each one of the desired N_0 photons, and they may or may not interact with the inhomogeneity (if any is present) during their "trip" inside the medium. But the crucial point is that all the photons without exception will evolve within an

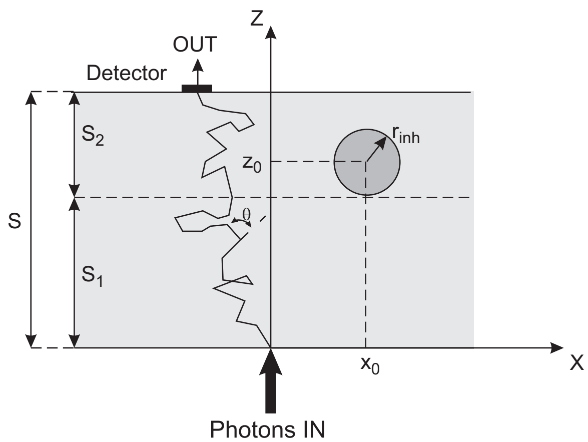

Fig. 1. Sketch showing the slab geometry considered. A slab of thickness S is divided into an homogeneous portion of thickness S_1 and another portion, S_2, containing the inclusion. Photons are launched at $z = 0$ and collected at $z = S$.

homogeneous medium of thickness S_1, until they reach for the very first time the frontier $z = S_1$ at a point of coordinates (x, y, S_1).

The basic idea for achieving acceleration in our proposal is to consider propagation within this first layer of the host, that is up to $z = S_1$, as already solved and to use this information as an input to proceed to track photons inside the whole host of thickness S, but starting at $z = S_1$.

Two points need to be emphasized here:

(i) At $z = S_1$ not all photons remain close to the z-axis. Because of diffusion, they are now spread around this axis, following a certain distribution $N(x,y)|_{S_1}$. This matrix contains the number of photons that have reached the plane $z = S_1$ with no interaction with the inhomogeneity, and must not be mistaken for $U(r)$ when the inhomogeneity is present; this means that, until this point is reached, no photon will have any chance of interaction with the inhomogeneity.

(ii) Despite the fact that now photons are launched starting from inside the host at depth S_1, this does not preclude that they may be backscattered into the first homogeneous layer again. Thus, MC simulation is started at a given depth S_1 but it is carried out considering the complete medium, that is the slab of thickness $S = S_1 + S_2$. This is a major point in our proposal and should not be misunderstood considering that photons starting at $z = S_1$ are only tracked forward with no chance of re-entering the first homogeneous layer.

There are two ways for obtaining $N(x,y)|_{S_1}$, namely (i) to run MC simulations for the homogeneous portion, for several values of S_1 and for several sets of optical parameters, building up a data base to be kept until it is required in a particular problem, or (ii) to run some theoretical calculation using the DA for the exact situation every time it is required. Clearly, the second choice is much more convenient because of several reasons: it lasts seconds, so it can be calculated after the particular problem has been defined, avoiding having many stored files considering hypothetical cases which may be not useful at all.

To test the validity of this assumption we show in Fig. 2a the absolute number of photons per pixel along the x direction obtained with both, MC and theory, at $(y = 0,\ z = S_1 = 1\,\text{cm})$ when 10^7 photons are incident upon the entrance plane.

The main assumption in the present approach, that must be clearly stated, is:

Since in MC simulations the photon can only "know" about the presence of the inhomogeneity after arriving at plane $z = S_1$, theoretical calculations must be made inside a homogeneous, semi-infinite medium, and considering a mathematical, index matched frontier inside the bulk at $z = S_1$, to build up the desired matrix of distribution of photons, $N(x,y)|_{S_1}$. Clearly, this value of $N(x,y)|_{S_1}$ *will not be* the final, stationary distribution of photons at $z = S_1$ after the complete slab is considered, since photons starting at that plane may be backscattered again into the first zone, following the probability law defined by the corresponding anisotropy factor.

As already stated, once the matrix at $z = S_1$, $N(x,y)|_{S_1}$ is known, it is used as the input data for running the MC simulation considering the complete slab of thickness S. It is now important to take into account Fresnel reflections at both physical ends of the slab (at $z = 0$ and at $z = S$).

Each bin centered at (x, y) in this matrix contains a certain number of photons, $N(x, y)$ which are to be launched starting at $(x, y, z = S_1)$. Initial direction cosines at $z = S_1$ are obtained within this step of the simulation, following the corresponding distribution given by a random law for the azimuthal angle ϕ and by the anisotropy factor of the particular problem for the deflection angle, θ (see Eqs. (3) and (4), respectively). Thus, direction cosines do not need to be stored in the matrix, since there would be three numbers for each photon, resulting in a huge amount of data to be stored in the auxiliary file. Additionally, if theory is used to obtain $N(x,y)|_{S_1}$, there is no information about the angle distribution. This is an important assumption, since now the scattering angle is obtained with respect to the z-axis and not relative to the incident photon direction, which can lead to a systematic error in the MC calculations. However, it is understandable from the very stochastic nature of the process which ensures isotropy after about $1/\mu_s'\,\text{cm}$ [18] and it is verified later by the numerical experiments. In fact, if this error were significative, the estimated acceleration factor (AF) defined at the end of this section would be larger than expected, since photons should be starting at $z = S_1$ with an additional forward component, thus reducing travel time within S_2. As shown later, this is not the case, thus supporting this assumption, at least for the thicknesses and optical parameters (typical for biological tissues) considered in this work.

Photons are tracked inside the whole slab, of thickness $S = S_1 + S_2$, until they are absorbed or collected at $z = S$ within the detector area.

It should be mentioned that the matrix $N(x,y)|_{S_1}$ obtained by MC contains, in each bin, the number of photons that have actually *passed through* the plane $z = S_1$, which is not exactly the number of photons *at* $z = S_1$, as it is the case in theoretical calculations. This error may be considerable if the thickness of the region including the inhomogeneity is comparable to the inverse of the scattering mean free path. For the case of typical biological tissues, this is of about $1/\mu_s \approx 0.02\,\text{cm}$ so we have calculated, using theory, the photon distribution at both, $z = S_1 = 1\,\text{cm}$ and at $z = S_1 \pm 1/\mu_s$, showing a discrepancy of the order of 6% at the axis $(x = y = 0)$ and of less than 4% in the total number of photons present at plane $z = S_1$. This discrepancy becomes less important if the initial homogeneous portion of the slab is increased in comparison to $1/\mu_s$, decreasing following an exponential-like law, which reaches 3% at the axis for $S_1 \approx 3\,\text{cm}$.

Taking into account that photon propagation time, up to a distance L inside an homogeneous medium, is proportional to L [18], it is possible to obtain a first estimation of the AF for the case shown in Fig. 1 and for the optical parameters of both,

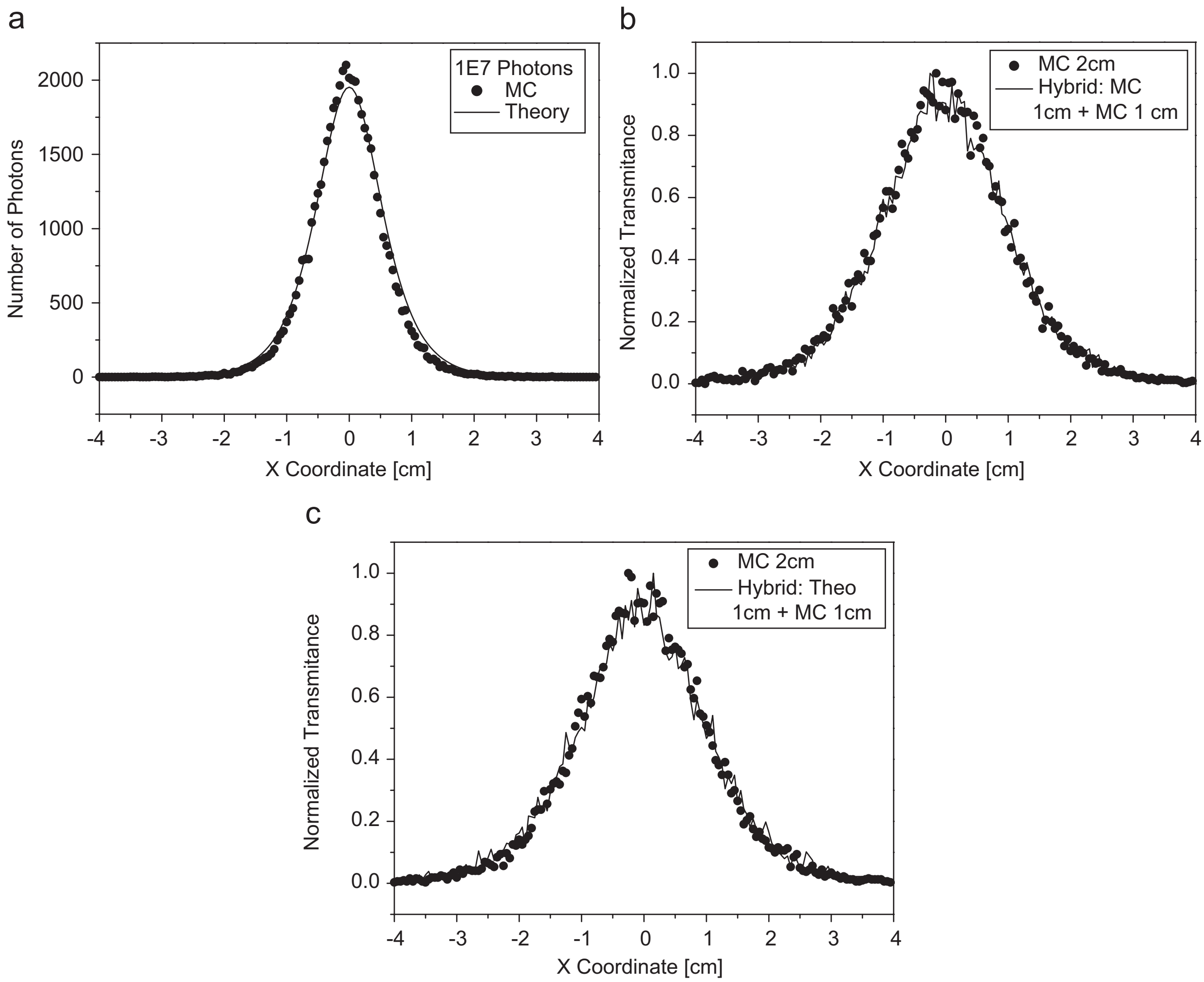

Fig. 2. (a) Comparison of the number of photons at $(x, y = 0, z = S_1 = 1\,\text{cm})$ obtained with MC and with theory when 10^7 photons are incident at $(0, 0, 0)$. Note that these are raw data and no normalization process was carried out. (b) Comparison of light transmittance at $(x, y = 0, z = S)$ for a 2 cm slab obtained by pure MC simulation over the 2 cm (dots) and by splitting the MC run into two steps as detailed in the text (line). (c) Same as (b) but using a theoretical calculation for computing the photon distribution $N(x, y)|_{S_1}$ at plane $z = S_1$.

host and inclusion, having values which are usual for biological tissues [19]. It results

$$AF \approx \frac{S}{S_2} = \frac{S}{S - S_1}. \tag{6}$$

The actual value for *AF* will depend on the relative values of the optical properties of the bulk and the inclusion, but it is clear that deeper inhomogeneities will result in larger values for *AF*.

4. Results and discussion

To test the performance of the proposed acceleration method, we carried out two kinds of numerical experiments. In all cases we used a slab of thickness $S = 2\,\text{cm}$, with optical properties similar to biological tissue, namely $\mu_a = 0.03\,\text{cm}^{-1}$, $\mu_s = 50\,\text{cm}^{-1}$, $g = 0.8$ and therefore, $\mu_s' = 10\,\text{cm}^{-1}$. The slab was considered either as a unique piece of $S = 2\,\text{cm}$ or two pieces, of thicknesses S_1 and $S_2 = S - S_1$. Refer to Fig. 1 for the arrangement. In all numerical simulations, photon propagation was allowed up to distances from optical axis as large as 20 times the slab thickness. However, for the parameters used, a relevant number of photons is counted up to lateral distances of about $L_x = L_y = \pm 4\,\text{cm}$. Thus, for collecting data, we defined 160 bins in each transverse direction, each one having an area of $[0.05\,\text{cm}]^2$. All calculations were carried out in a PC with a 2.66 GHz clock (sequential processing).

In the first experiment, we compare the light transmittance, that is the light distribution at $(x, y, z = S)$ obtained with a pure MC simulation over a slab of 2 cm, in which all photons are launched at $(0, 0, 0)$ with the same quantity when the slab is considered as being composed of two pieces of 1 cm each. In the later, the MC starts running at $z = S_1 = 1$ cm and continues up to $z = S$. The required input data, that is the photons distribution at plane $z = S_1$, which we have already defined as the matrix $N(x,y)|_{S_1}$, can be obtained either by MC or by analytical results. We present results considering both options.

This experiment has a double purpose; first, to check internal consistency and normal functioning of the MC when it starts at some depth $z = S_1$, using as input the file containing the matrix with the number of photons at each grid position, and second, to verify the *AF*, defined in Eq. (6).

Fig. 2(b) compares results for light transmittance using MC for a homogeneous slab of thickness $S = 2$ cm with those obtained splitting the calculation into two steps and calculating both by MC. The first step, within a homogeneous slab of thickness $S_1 = 1$ cm, to generate the matrix $N(x,y)|_{S_1}$, and the second considering the complete slab of thickness S but starting the simulation at S_1 and using $N(x,y)|_{S_1}$ as the initial photon distribution. This experiment shows the self-consistence of the method and will allow to deal with the more complex and interesting situation, when inhomogeneities are present. Dots in this figure were obtained considering the slab of 2 cm and launching 10^7 photons at $(0,0,0)$, and collecting those which survive at the opposite face of the slab, at coordinates $(x,y,z = S)$. Total calculation time for this MC experiment considering the slab of 2 cm was $\Delta t^{MC} = 41$ min. The solid line is the same output, but obtained splitting the MC simulation into two steps as mentioned above. Calculation time for the experiment starting at $z = 1$ cm was $\Delta t^{MC}_{z=1\ \mathrm{cm}} = 21$ min. Since the matrix $N(x,y)|_{S_1}$ can be considered as previously known, belonging to a data base, the total time for the complete simulation is thus, also $\Delta t^{Total} = \Delta t^{MC}_{z=1\ \mathrm{cm}} = 21$ min.

As expected, both results coincide very well. Note, however, that they are *not exactly* the same due to the statistical nature of MC. The resulting *AF* can be calculated straightforward and compared with Eq. (6):

$$AF = \frac{\Delta t^{MC}}{\Delta t^{MC}_{z=1\ \mathrm{cm}}} = \frac{41}{21} = 1.95 \simeq 2 = \frac{S}{S_2}. \tag{7}$$

Fig. 2c, corresponds to a similar situation as in Fig. 2b, but now the data contained in the matrix $N(x,y)|_{S_1}$ were obtained by a theoretical solution of the DA. In our case we followed the approach from Contini et al. [4] for obtaining $T(\rho)ds$, i.e. the probability of one photon emitted by the source of exiting the slab from the surface element ds at a certain distance ρ from the origin. Using this model, and multiplying for the desired number of input photons at the origin, we constructed the theoretical version of $N(x,y)|_{S_1}$. Run-time for this step is no more than a few seconds and can be neglected. Using this matrix as the input for MC we obtained the solid line of Fig. 2c. The computed time for this experiment starting at $z = 1$ cm was $\Delta t^{MC}_{z=1\ \mathrm{cm}} = 24$ min; this is, of course, very similar to the previous 21 min, since in both cases MC starts at $S_1 = 1$ cm. We can now compute *AF* again as

$$AF = \frac{\Delta t^{MC}}{\Delta t^{MC}_{z=1\ \mathrm{cm}}} = \frac{41}{24} = 1.71 \simeq 2 = \frac{S}{S_2}. \tag{8}$$

At this point it is worthwhile to notice that in both cases presented here, the actual *AF* is less than the predicted one. A possible explanation to this behavior could be the following: in MC simulations computation time increases each time the photon reaches a new point, since the corresponding new interaction needs to be evaluated. Thus, when the travel time to a given depth S is evaluated, what is actually accounted is the time to pass this depth up to a little larger depth, $S + \Delta S$ and therefore the travel time is overestimated. The thinner the considered slab, the more noticeable is this effect. In our examples the numerator involves MC calculations over 2 cm, while in the denominators MC runs only over 1 cm slabs; time overestimation is less important in the numerators than in the denominators, leading to a underestimation of the *AF*.

It is also interesting to notice that the calculation time for the denominators of Eqs. (7) and (8) is not exactly the same, even if both slabs are 1 cm deep. This is due to the fact that the starting photon distributions at plane $z = S_1$, $N(x,y)|_{S_1}$ are statistically equivalent but not *exactly identical*.

In the second type of simulations we carried out a transillumination experiment in which some inhomogeneity is deeply immersed in the slab. In the transillumination geometry, photons are launched at $(x,0,0)$, and then collected at the opposite face of the slab at the projection of the incidence direction of light (optical axis), that is at $(x,0,z = S)$, by a detector of an area of 1×1 bins and for several values of x. As an example of inclusion, we have taken a cylinder of 0.3 cm diameter having the same reduced scattering coefficient and the same anisotropy factor as the bulk, that is $\mu'_s|_{Cyl} = 10\ \mathrm{cm}^{-1}$ and $g|_{Cyl} = 0.8$, but with a higher absorption coefficient, namely $\mu_a|_{Cyl} = 0.9\ \mathrm{cm}^{-1}$ ($\mu_a|_{Bulk} = 0.03\ \mathrm{cm}^{-1}$). This high value was selected intentionally in order to clearly show the variation in light transmittance for the different positions of the inclusion relative to the optical axis. Note that in real cases of tumors immersed in biological tissue, it absorption coefficients may be up to 10 times larger than that of the surrounding medium [19].

The inclusion was placed with its center at a depth of 1.85 cm. That is, the cylinder surface is tangent to the plane $z = S = 2$ cm. We have then divided the problem in an homogeneous slab of $S_1 = 1.7$ cm plus a slab of 0.3 cm containing the inhomogeneity. We construct the matrix $N(x,y)|_{S_1}$ by using the theoretical model described above, and then run the MC code starting at $S_1 = 1.7$ cm and for several transverse positions of the inhomogeneity relative to the incident photon beam, scanning the x coordinate of this last one from $x = -1.5$ cm to 1.5 cm in 0.05 cm steps. Results are shown in Fig. 3, in which the light intensity collected at the detector is represented as a function of the center of the inhomogeneity relative to the incident photon beam. Dots are the result of running a pure MC code over a slab of $S = 2$ cm, and starting at $z = 0$, where

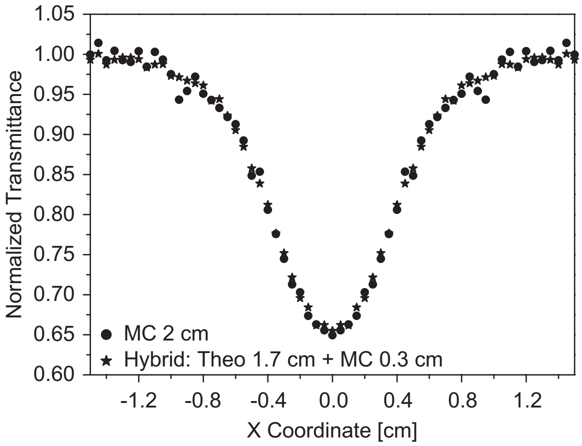

Fig. 3. Transillumination experiment for a slab of 2 cm with an inclusion of 0.3 cm diameter, centered at $z = 1.85$ cm. Comparison of a pure MC calculation (dots) with a hybrid calculation using theory up to $S_1 = 1.7$ cm and then continued by MC (stars). Because of symmetry, calculations were carried out only for the right half ($0 \leqslant x \leqslant 1.5$ cm) and the complete data set is obtained by mirroring this portion with respect to the origin for better visualization.

10^7 photons are launched. Stars are obtained by running a MC code starting at $z = S_1 = 1.7$ cm using $N(x, y)|_{S_1}$ as input. Clearly, both results are statistically identical.

Computation time for the pure MC simulation over the slab of $S = 2$ cm was measured to be $\Delta t^{MC}_{z=2\,\mathrm{cm}} = 77.5$ h, while simulation for the hybrid simulation took only 9.5 h. The resulting accelerating factor is then

$$AF = \frac{77.5}{9.5} = 8.16,$$

whereas the expected AF for a slab of 0.3 cm containing the inhomogeneity is $AF = S/S_2 = 2/0.3 = 6.67$. Note that we have chosen for the inclusion an absorption coefficient that is 30 times larger than that of the bulk. Thus, specially for positions of the center of the inhomogeneity close to the optical axis, photons will be killed comparatively faster after they enter the inhomogeneity. This explains why actual AF results greater than the expected limit for homogeneous slabs.

Our approach can be easily implemented with minor modifications of the original codes, requiring only to read the auxiliary file containing the photons distribution at $z = S_1$.

Moreover, it is worth noticing that the present approach can be used in conjunction with any other acceleration method, thus multiplying the individual gain factors.

5. Conclusions

We have proposed a simple method for reducing computation time in MC simulations, specially for transillumination geometries in slabs of thickness S containing deep inhomogeneities. It is based in splitting the simulation into two pieces, one for the homogeneous portion of the slab, of thickness S_1, which can be theoretically calculated in a negligible lapse, and another one where pure MC simulations are carried out, starting from the bidimensional photons distribution at a depth S_1 and considering the whole slab with the inclusion. As a result, computation time can be reduced in a factor close to $S/S - S_1$, depending on the optical properties and on the depth of the inclusion present in the second step of the calculation.

The proposal has been tested for homogeneous and inhomogeneous cases, producing no alterations with respect to pure MC simulations but with a noticeable AF.

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