

MPC for tracking with maximum domain of attraction

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Abstract

This paper presents a novel set-based model predictive control for tracking, with the largest domain of attraction. The formulation - which consists of a single optimization problem - shows a dual behavior: one operating inside the maximal controllable set to the feasible equilibrium set, and the other operating at the N -controllable set to the same equilibrium set. Based on some finite-time convergence results, global stability of the resulting closed-loop is proved, while recursive feasibility is ensured for any change of the set point. The properties and advantages of the controller have been tested on a simulation model.

1 Introduction

Model Predictive Control (MPC) is a strategy widely used in industries, due to its ability to deal with multivariable processes including both, state and input constraints.

A theoretical framework has been developed in the last two decades, showing that MPC is a control technique capable to provide asymptotic stability, constraint satisfaction and robustness, based on the solution of an on-line

tractable optimization problem for both, linear and non-linear systems ([30, 27, 7]).

Lyapunov theory ([18]) is a suitable framework to prove asymptotic stability of a system controlled by an MPC [27]. Different stabilizing formulations appeared in literature: *MPC with terminal equality constraint* [26], where the stability is guaranteed by imposing a terminal constraint; *MPC with terminal cost* [5, 23, 15], where stability is achieved by incorporating into the cost function a term that penalizes the terminal state; or *MPC with terminal inequality constraint* [28], which replaces the terminal equality constraint by an inequality one that forces the terminal state to be in a positive invariant terminal set containing the origin in its interior.

The stabilizing terminal constraint implicitly imposes hard restrictions on the state, since only those states that can be steered in a given number of steps to the terminal region will be properly stabilized. These states determine the so-called closed-loop domain of attraction, whose characterization is crucial because it represents a domain of validity determined by the controller itself, and not by the system dynamic and constraints. In this context, any effort to modify the classical MPC formulation to have a larger domain of attraction is remarkably beneficial, as it was stated in many seminal works ([17],[19],[22]).

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The most obvious way to enlarge the MPC domain of attraction is by enlarging the prediction/control horizon. Although this strategy is valid, it has two major drawbacks. Firstly, a large increase in the computational cost. Secondly, it is not unusual that during the operation of the plant the operating point changes due to potential changes in the parameters that are involved in the cost function. The stabilizing design of the MPC may not be valid at the new set point, and then, the feasibility of the controller may be lost. Consequently, the controller may fail to track the desired set point ([1, 4, 9, 20, 29, 31]).

Another approach, tending to enlarge the MPC domain of attraction in a rather theoretical form, was presented in [22]. The idea was to substitute the terminal constraint by a kind of contractive terminal constraint, which forces the terminal state to pass from one control invariant set to another. The proposed method reaches the maximum domain of attraction (the so-called *maximal controllable set to the equilibrium*, which is determined by the system and the constraints), but the computation of the control invariant sets, which may be computationally prohibitive, must be carried out on line every time a change in the set point occurs.

Regarding the aforementioned loss of feasibility under set point changes, different solutions known as MPC for Tracking (MPCT), were presented in [25, 9, 24, 1]. These strategies solve the problem of recursive feasibility by penalizing the distance from the predicted trajectories to some extra artificial optimization variables, which are forced to be a feasible equilibrium. This way, not only the recursive feasibility is ensured for any change of set points, but also the domain of attraction is enlarged (although it does not necessarily reach the maximum domain of attraction, for a given prediction horizon).

A different strategy was presented in [13], where a mode decomposition of the system is exploited by means of the proper use of slack variables. This way, by controlling separately the stable and unstable modes and by properly penalizing the slack variables, an enlarged domain of attraction is obtained. In [12], following a similar line, a sequence of disjoint nested control invariant sets is used to force the system (by minimizing a sequence of generalized Minkowsky functionals, associated to the sequence of sets) to reach a set of state than can be steered to the set point in a number of steps equal to (or smaller than) the control horizon. One drawback of the later strat-

egy is, again, the computation of the proposed sequence of invariant sets. In addition, none of the two aforementioned strategies reach the maximal domain of attraction, represented by the *maximal controllable set of the system*.

In this work we develop a novel MPC that combines the good properties of the strategies presented in [22] and [25], i.e. it enlarges the domain of attraction up to the maximum controllable set for any fixed prediction horizon without loss of feasibility under changes of the set point. The method consists in a decomposition of the maximum domain of attraction into a disjoint union of embedded layers defined by controllable sets of the system. The proposed controller shows a dual behavior, but into an unified formulation. The first one steers the system through those layers to reach a proper neighborhood of the equilibrium set. The second one, analogous to the classical MPCT, guarantees the asymptotic stability and recursive feasibility under changes of the set point.

This paper is organized as follows. We set up our notation in Section 1.1. In Section 2 we present general definitions and necessary results to formulate the proposed MPC. For the sake of completeness we include in Section 3 a brief recall of the MPCT. Section 4 is devoted to describing in detail the proposed MPC. The proof of the main results are address in Section 5. Finally, numerical simulations and conclusions can be found in Section 6 and 7, respectively.

1.1 Notation

We denote with \mathbb{N} the sets of integers, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ and $I_i := \{0, 1, \dots, i\}$. The euclidean distance between two points $x, y \in \mathbb{R}^d$ by $\|x - y\| := [(x - y)^t(x - y)]^{1/2}$. If P is a positive-definite matrix on $\mathbb{R}^{d \times d}$ then we define the quadratic form $\|x - y\|_P^2 := (x - y)^t P (x - y)$.

Let $\mathcal{X} \subseteq \mathbb{R}^d$. The *open ball with center in $x \in \mathcal{X}$ and radius $\varepsilon > 0$ relative to \mathcal{X}* is given by $\mathcal{B}_{\mathcal{X}}(x, \varepsilon) := \{y \in \mathcal{X} : \|x - y\| < \varepsilon\}$. Given $x \in \Omega \subseteq \mathcal{X}$, we say that x is an *interior point of Ω relative to \mathcal{X}* if there exist $\varepsilon > 0$ such that the open ball $\mathcal{B}_{\mathcal{X}}(x, \varepsilon) \subseteq \Omega$. The *interior of Ω relative to \mathcal{X}* is the set of all interior points and it is denoted by $\text{int}_{\mathcal{X}} \Omega$. In case $\mathcal{X} = \mathbb{R}^d$ we omit the subscript in the latter definition, i.e. $\text{int} \Omega := \text{int}_{\mathbb{R}^d} \Omega$. Finally, let Ω_1 and Ω_2 two sets in \mathbb{R}^d , we denote the difference between Ω_1 and Ω_2 by $\Omega_1 \setminus \Omega_2 := \{x \in \Omega_1 : x \notin \Omega_2\}$.

2 Problem statement

In this section, some preliminaries and novel concepts necessary to develop the main contribution of the work will be presented.

2.1 Model description

Consider a system described by a linear discrete-time invariant model

$$x(i+1) = Ax(i) + Bu(i) \quad (2.1)$$

where $x(i) \in \mathcal{X} \subset \mathbb{R}^n$ is the system state at time i and $u(i) \in \mathcal{U} \subset \mathbb{R}^m$ is the current control at time i , where \mathcal{X} and \mathcal{U} are compact convex sets containing the origin. As usual, we shall assume that the pair (A, B) is controllable, the state is measured at each sampling time.

The set of steady states and inputs of the system (2.1) is given by

$$\mathcal{Z}_s := \{(x_s, u_s) \in \mathcal{X} \times \mathcal{U} : x_s = Ax_s + Bu_s\}.$$

Thus, the equilibrium state and input sets are defined as

$$\mathcal{X}_s := \text{proj}_{\mathcal{X}} \mathcal{Z}_s \quad \text{and} \quad \mathcal{U}_s := \text{proj}_{\mathcal{U}} \mathcal{Z}_s.$$

2.2 Definitions and properties

The following definitions introduce the main sets necessary for the formulation of the MPC proposed in Section 4. The significance of such definitions comes from the fact that, to achieve the main properties (i.e., to preserve the feasibility under any modification of the reference and to exhibit the maximal domain of attraction that the system allows), the proposed MPC uses a set-based cost function and constraints.

Definition 2.1 (Control Invariant Set). *A set $\Omega \subset \mathcal{X}$ is a Control Invariant Set (CIS) of system (2.1) if for all $x \in \Omega$, there exists $u \in \mathcal{U}$ such that $Ax + Bu \in \Omega$. Associated to Ω is the corresponding input set*

$$\Psi(\Omega) := \{u \in \mathcal{U} : \exists x \in \Omega \text{ such that } Ax + Bu \in \Omega\}.$$

Note that the set of steady states \mathcal{X}_s is a CIS with its corresponding input set $\Psi(\mathcal{X}_s) = \mathcal{U}_s$.

Definition 2.2 (*i*-Step Controllable Set). *Given $i \in \mathbb{N}$ and two sets $\Omega \subseteq \mathcal{X}$ and $\Psi \subseteq \mathcal{U}$, the i -step controllable set to Ω corresponding to the input set Ψ , of system (2.1), is given by*

$$S_i(\Omega, \Psi) := \{x_0 \in \mathcal{X} : \forall j \in I_{i-1}, \exists u_j \in \Psi \text{ such that } x_{j+1} = Ax_j + Bu_j \in \mathcal{X} \text{ and } x_i \in \Omega\}.$$

For convenience, we define $S_0(\Omega, \Psi) := \Omega$ and $S_\infty(\Omega, \Psi) := \bigcup_{i=0}^{\infty} S_i(\Omega, \Psi)$, i.e. the set of admissible states which can be steered to the set Ω by a finite sequence of admissible controls in Ψ .

According to the latter definition, the maximal domain of attraction that the constrained system allows, for any set point x^* in the equilibrium set \mathcal{X}_s , is given by $S_\infty(\mathcal{X}_s, \mathcal{U})$. Note that this set does not depend on the control strategy used to ensure the stability of the set point, but only on the system dynamic and the input and state constraints.

Now, we are going to define a special type of invariant sets, which are a central concept of this work.

Definition 2.3 (Contractive CIS). *Let $\Omega \subset S_\infty := S_\infty(\Omega, \mathcal{U})$ be a CIS. Then Ω is a contractive CIS if for all $x \in \Omega$, there exists $u \in \mathcal{U}$ such that $Ax + Bu \in \text{int}_{S_\infty} \Omega$.*

Note that the above definition is similar to the definition of a γ -Control Invariant Set¹ (see [6, 2, 14]). Indeed, if Ω is a γ -control invariant set then for all $x \in \Omega$, there exists $u \in \mathcal{U}$ such that $Ax + Bu \in \text{int} \Omega \subseteq \text{int}_{S_\infty} \Omega$. Hence every γ -control invariant set is a contractive CIS. However the inverse result is not necessarily true. The importance of considering the weakened concept of interior relative to the set S_∞ will be addressed in Remark 2.6 and Remark 5.2.

The following result shows a geometric property of the contractive CIS, analogous to the geometric properties of CIS and γ -CIS presented in [8, Theorem 3.1] and [3, Property 1] respectively.

Lemma 2.4 (Geometric property of contractive CIS). *Let $\Omega \subset S_\infty$ be a compact and convex contractive CIS of system (2.1). Then, $\Omega \subseteq \text{int}_{S_\infty} S_1(\Omega, \Psi(\Omega))$.*

¹ Ω is a γ -Control Invariant Set (γ -CIS) if for $x \in \Omega$ there exists $u \in \mathcal{U}$ such that $Ax + Bu \in \gamma\Omega$, for some $\gamma < 1$.

Proof. It is easy to see that $\Omega \subseteq S_1(\Omega, \Psi(\Omega))$ by the invariance property of the set. It remains to show that every point of Ω is an interior point of $S_1(\Omega, \Psi(\Omega))$ relative to S_∞ . Let $x \in \Omega$, since Ω is a contractive CIS, there exists $u \in \Psi(\Omega)$ such that $Ax + Bu \in \text{int}_{S_\infty} \Omega$. Then, there exists $\varepsilon > 0$ such that $\mathcal{B}_{S_\infty}(Ax + Bu, \varepsilon) \subseteq \Omega$. Since $Ax + Bu$ is a continuous function at x from S_∞ to S_∞ , then there exists $\delta > 0$ such that for all $\tilde{x} \in \mathcal{B}_{S_\infty}(x, \delta)$ we have

$$A\tilde{x} + Bu \in \mathcal{B}_{S_\infty}(Ax + Bu, \varepsilon) \subseteq \Omega.$$

Hence $\tilde{x} \in S_1(\Omega, \Psi(\Omega))$. Therefore $\mathcal{B}_{S_\infty}(x, \delta) \subseteq S_1(\Omega, \Psi(\Omega))$, i.e. $\Omega \subseteq \text{int}_{S_\infty}(S_1(\Omega, \Psi(\Omega)))$. \square

The next lemma shows that the contractive invariance property is inheritable for the controllable sets.

Lemma 2.5. *Let $\Omega \subset \mathcal{X}$ be a compact and convex contractive CIS of system (2.1). Then for every $i \in \mathbb{N}$, the set $S_i(\Omega, \mathcal{U})$ is a convex and compact contractive CIS of system (2.1).*

Proof. Since Ω is under the assumptions of Lemma 2.4 and $\Psi(\Omega) \subseteq \mathcal{U}$ then

$$\Omega \subseteq \text{int}_{S_\infty} S_1(\Omega, \Psi(\Omega)) \subseteq \text{int}_{S_\infty} S_1(\Omega, \mathcal{U}).$$

Hence $S_1(\Omega, \mathcal{U})$ is a contractive CIS of system (2.1). By [16] we know that $S_1(\Omega, \mathcal{U})$ is compact and convex. Therefore $S_1(\Omega, \mathcal{U})$ is also under the assumptions of Lemma 2.4. The result follows by induction. \square

Remark 2.6. *The resemblance between definitions of contractive CIS and γ -CIS could make us wonder the reason to introduce this new type of set. Observe that in Lemma 2.4 we prove a geometric property of the contractive CIS analogous to Property 1 in [3] for γ -CIS. However in this last property it is required that $\Omega \subseteq \text{int} \mathcal{X}$. This requirement represents an obstacle in the proof of Lemma 2.5 when we apply recursively Lemma 2.4 to the sets $S_i(\Omega, \mathcal{U})$, because it is usual that for i large enough the sets $S_i(\Omega, \mathcal{U})$ collapse in the boundary of the set \mathcal{X} (see Figure 2). Therefore they will not fulfill the hypothesis $S_i(\Omega, \mathcal{U}) \subseteq \text{int} \mathcal{X}$.*

Now, we are going to define a class of sets that allows a disjoint decomposition of the state space necessary in the formulation of the propose MPC.

Definition 2.7 (*k*-Layer Set). *Let $\Omega \subset \mathcal{X}$ be a control invariant set, $\Psi \subseteq \mathcal{U}$ an input set and $N \in \mathbb{N}$. For any $k \in \mathbb{N}_0$ we define the *k*-Layer Set by $L_{kN}(\Omega, \Psi) := S_{(k+1)N}(\Omega, \Psi) \setminus S_{kN}(\Omega, \Psi)$.*

Remark 2.8. *In the above definition we ask for Ω to be an invariant set. This implies that the *i*-Step Controllable Sets $S_i(\Omega, \Psi)$, $i \in \mathbb{N}_0$, are nested (see Proposition 7 in [21]). Hence the *k*-Layer Sets are disjoint (see Figure 1) and even more*

$$S_\infty(\Omega, \Psi) = S_N(\Omega, \Psi) \cup \bigcup_{k=1}^{\infty} L_{kN}(\Omega, \Psi). \quad (2.2)$$

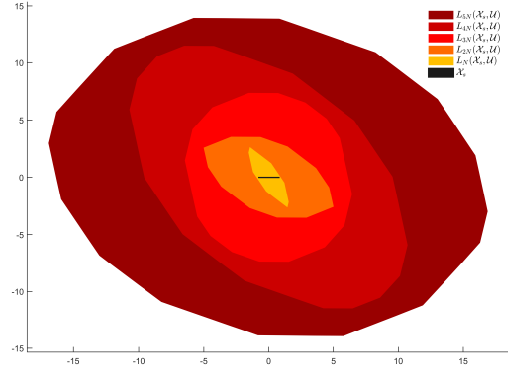


Figure 1: The first five layers for the harmonic oscillator system.

3 MPC for Tracking (MPCT)

The MPC for tracking ([25, 11, 10, 24, 9, 1]) attempts to track any admissible target steady state by means of admissible evolutions, with the following ingredients: (i) an artificial reachable set point in \mathcal{X}_s , added as decision variable, (ii) a stage cost that minimizes the deviation of the predicted trajectory from this artificial steady state, and (iii) an extra cost, the offset cost function, that penalizes the deviation between the artificial set point and the real (desired) set point.

The MPCT cost function has two terms. The first one is a quadratic cost of the expected tracking error with respect

to the artificial steady state and input (x_s, u_s) . The second one is the offset cost function $V_O(x_s, x^*)$, that penalizes the deviation from the artificial steady state x_s to the set point x^* . This cost function is given by

$$V_N(x, x^*; \mathbf{u}, x_s, u_s) = \sum_{j=0}^{N-1} \|x_j - x_s\|_Q^2 + \|u_j - u_s\|_R^2 + V_O(x_s, x^*), \quad (3.1)$$

where N is the control horizon, \mathbf{u} is a sequence of N future control inputs, i.e. $\mathbf{u} = \{u_0, \dots, u_{N-1}\}$, x_k is the predicted state of the system at time k given by $x_{k+1} = Ax_k + Bu_k$, with $x_0 = x$. The offset cost function is required to be a convex and positive-definite function (see [25]). In our case, for simplicity, we will consider it as a quadratic form $V_O(x_s, x^*) = \|x_s - x^*\|_T^2$.

In the case of terminal equality constraint, the MPCT optimization problem $P_N^T(x, x^*)$ is given by

$$\min_{\mathbf{u}, x_s, u_s} V_N(x, x^*; \mathbf{u}, x_s, u_s) \quad (3.2)$$

$$\text{s.t. } x_0 = x, \quad (3.3)$$

$$x_{j+1} = Ax_j + Bu_j, \quad j \in I_{N-1}, \quad (3.4)$$

$$x_j \in \mathcal{X}, \quad u_j \in \mathcal{U}, \quad j \in I_{N-1}, \quad (3.5)$$

$$(x_s, u_s) \in \mathcal{Z}_s, \quad (3.6)$$

$$x_N = x_s. \quad (3.7)$$

Constraints (3.3)–(3.5) force to the predicted trajectory to be consistent with the dynamic model equations while the state and input constraints are fulfilled. Constraint (3.6) ensures that the artificial variable (x_s, u_s) is an admissible equilibrium point. The terminal equality constraint (3.7) forces the terminal state to be the artificial state. These constraints ensure recursive feasibility, stability and enlargement of the domain of attraction of the resulting controller to $S_N(\mathcal{X}_s, \mathcal{U})$ ², avoiding the loss of feasibility in presence of changes in the set point.

4 The proposed MPC

For a fixed control horizon $N \in \mathbb{N}$, we are going to present a novel stable MPC for tracking that has, for any

²In the traditional MPC the domain of attraction is $S_N(x^*, \mathcal{U})$, which is significantly smaller than $S_N(\mathcal{X}_s, \mathcal{U})$.

set point in the equilibrium set, $x^* \in \mathcal{X}_s$, the maximal domain of attraction, i.e., a domain of attraction given by the maximum controllable set S_∞ . From now on, we simplify the notation of $S_i(\mathcal{X}_s, \mathcal{U})$ and $L_i(\mathcal{X}_s, \mathcal{U})$, denoting them by S_i and L_i , for any $i = 0, 1, \dots, \infty$.

Let $x \in S_\infty$. The following cost function is proposed

$$V_N(x; \mathbf{u}, \mathbf{x}^a, \mathbf{u}^a, x_s) = \sum_{j=0}^{N-1} \|x_j - x_j^a\|_Q^2 + \|u_j - u_j^a\|_R^2 + V_O(x_s, x^*), \quad (4.1)$$

where, as before, $\mathbf{u} := \{u_0, \dots, u_{N-1}\}$ represents a sequence of N future control inputs and x_j is the predicted state of the system at time j given by $x_{j+1} = Ax_j + Bu_j$, with $x_0 = x$. The sequence of N state and control auxiliary variables $\mathbf{x}^a := \{x_0^a, \dots, x_{N-1}^a\}$ and $\mathbf{u}^a := \{u_0^a, \dots, u_{N-1}^a\}$, have the purpose of representing the distance of the states x_j and inputs u_j to some sets that we will define later. Finally, x_s represents an artificial variable in \mathcal{X}_s and $V_O(x_s, x^*) = \|x_s - x^*\|_T^2$.

The controller is derived from the solution of the optimization problem $P_N(x, x^*)$ given by

$$\min_{\mathbf{u}, \mathbf{x}^a, \mathbf{u}^a, x_s} V_N(x; \mathbf{u}, \mathbf{x}^a, \mathbf{u}^a, x_s) \quad (4.2)$$

$$\text{s.t. } x_0 = x, \quad (4.3)$$

$$x_{j+1} = Ax_j + Bu_j, \quad (4.4)$$

$$x_j \in \mathcal{X}, \quad j \in I_{N-1}, \quad (4.5)$$

$$u_j \in \mathcal{U}, \quad j \in I_{N-1}, \quad (4.6)$$

$$x_j^a \in \Omega_x, \quad j \in I_{N-1}, \quad (4.7)$$

$$u_j^a \in \Psi(\Omega_x), \quad j \in I_{N-1}, \quad (4.8)$$

$$x_N \in \Omega_x, \quad (4.9)$$

$$x_s \in \mathcal{X}_s, \quad (4.10)$$

where Ω_x is a target set depending on the position of the initial state x , defined by

$$\Omega_x = \begin{cases} S_{kN}, & x \in L_{kN}, \text{ with } k \geq 1 \\ \{x_s\}, & x \in S_N, \end{cases} \quad (4.11)$$

and the set $\Psi(\Omega_x)$ is the corresponding input set to Ω_x . Note that when the system is inside S_N we have that $\Omega_x = \{x_s\}$, therefore the corresponding input set is $\Psi(\Omega_x) = \{u_s\}$, with u_s such that $x_s = Ax_s + Bu_s$.

Considering the receding horizon policy, the control law is given by $\kappa_{MPC}(x) = \dot{u}_0$, where \dot{u}_0 is the first element of the optimal input sequence $\dot{\mathbf{u}} = \{\dot{u}_0, \dots, \dot{u}_{N-1}\}$. The optimal cost is defined by

$$\dot{V}_N(x) := V_N(x; \dot{\mathbf{u}}, \dot{\mathbf{x}}^a, \dot{\mathbf{u}}^a, \dot{x}_s)$$

where $\dot{\mathbf{u}}, \dot{\mathbf{x}}^a, \dot{\mathbf{u}}^a, \dot{x}_s$ are the optimal solutions of Problem $P_N(x, x^*)$.

The formulation of problem $P_N(x, x^*)$ depends on the previous offline computation of the controllable sets S_{kN} , for all $k \in \mathbb{N}$. Since \mathcal{X} is a compact set, the sequence S_k converges (up to a given tolerance) in finite steps to the set S_∞ . These sets are computed only once and they are independent of any change on the set point, because these controllable sets depend on the entire equilibrium set \mathcal{X}_s and not on x^* .

The following properties show that Problem $P_N(x, x^*)$ presents a dual behavior depending on the position of the initial state.

Property 4.1. *When the current state x belongs to $S_\infty \setminus S_N$, there exists $k \geq 1$ such that $x \in L_{kN}$, and the artificial variable x_s does not depend on any other optimization variable. Therefore, by optimality, it will be equal to $x^* \in \mathcal{X}_s$ and $\|x_s - x^*\| = 0$. Hence the problem to be solved is equivalent to*

$$\min_{\mathbf{u}} \sum_{j=0}^{N-1} d_Q(x_j, \Omega_x) + d_R(u_j, \Psi(\Omega_x)) \quad (4.12)$$

$$\begin{aligned} \text{s.t.} \quad & x_0 = x, \\ & x_{j+1} = Ax_j + Bu_j, \quad j \in I_{N-1}, \\ & x_j \in \mathcal{X}, \quad j \in I_{N-1}, \\ & u_j \in \mathcal{U}, \quad j \in I_{N-1}, \\ & x_N \in \Omega_x, \end{aligned}$$

with $\Omega_x = S_{kN}$, $d_Q(x_j, \Omega_x) := \min\{\|x_j - x_j^a\|_Q^2 : x_j^a \in \Omega_x\}$ is a distance from the predicted state x_j to the target set Ω_x and $d_R(u_j, \Psi(\Omega_x)) := \min\{\|u_j - u_j^a\|_R^2 : u_j^a \in \Psi(\Omega_x)\}$ is a distance from the control u_j to the set $\Psi(\Omega_x)$.

In other words, while the current state does not reach the N -controllable set, the controller tries to minimize the distance of the predicted trajectory to the next layer.

Property 4.2. *When the controlled system reaches the set S_N , the target set is $\Omega_x = \{x_s\}$. Then by constraint (4.9)*

$x_N = x_s$, by (4.7) $x_0^a = x_1^a = \dots = x_{N-1}^a = x_s$, and by (4.8) $u_0^a = u_1^a = \dots = u_{N-1}^a = u_s$, with u_s such that $x_s = Ax_s + Bu_s$. This way, the MPCT described in Section 3 is recovered.

5 Asymptotic stability

From now on we consider the following assumption.

Assumption 5.1. *The set S_N is a contractive CIS.*

Remark 5.2. *Note that the above assumption is not so restrictive. For example it is sufficient to have $S_N \subseteq \text{int}_{S_\infty} S_{N+1}$, even when it is not true that $S_N \subseteq \text{int} S_{N+1}$ (see Figure 2 in the example presented in Section 6). Indeed, if $S_N \subseteq \text{int}_{S_\infty} S_{N+1}$ then S_N is a contractive CIS, and by Lemma 2.5 all S_{kN} are also contractive CIS, for any $k \geq 1$.*

5.1 Preliminaries results

First of all note that the recursive feasibility is an immediate consequence of the nested property of the controllable sets S_i . However, the proof of attractivity is more subtle since the optimal cost is not a Lyapunov function in the entire domain of attraction. In order to prove that the real trajectory produced by the proposed strategy reaches in a finite number of steps the set S_N , we need first to suppose the opposite, i.e we need to proceed by contradiction. The following lemma goes in this direction.

Lemma 5.3. *Let $x \in L_{kN}$ for some $k \geq 1$. Let $\{x(i)\}_{i=0}^\infty$ be the sequence given by the closed-loop system $x(i+1) = Ax(i) + B\kappa_{MPC}(x(i))$, with $x(0) = x$. If $x(i) \notin S_{kN}$ for all $i \in \mathbb{N}$ then*

$$d_Q(x(i), S_{kN}) \rightarrow 0, \quad \text{when } i \rightarrow \infty. \quad (5.1)$$

Proof. Suppose the solution of Problem $P_N(x(i), x^*)$ is given by $\dot{\mathbf{u}} = \{\dot{u}_0, \dots, \dot{u}_{N-1}\}$, $\dot{\mathbf{u}}^a = \{\dot{u}_0^a, \dots, \dot{u}_{N-1}^a\}$, $\dot{\mathbf{x}}^a = \{\dot{x}_0^a, \dots, \dot{x}_{N-1}^a\}$, $\dot{x}_s = x^*$ and the corresponding optimal state sequence is given by $\dot{\mathbf{x}} = \{\dot{x}_0, \dots, \dot{x}_N\}$, where $\dot{x}_0 = x(i)$ and $\dot{x}_N \in S_{kN}$. The optimal cost is given by

$$\dot{V}_N(x(i)) = \sum_{j=0}^{N-1} \|\dot{x}_j - \dot{x}_j^a\|_Q^2 + \|\dot{u}_j - \dot{u}_j^a\|_R^2.$$

Since S_{kN} is an invariant set, then there exists $\hat{u} \in \Psi(S_{kN})$ such that $\hat{x} = A\hat{x}_N + B\hat{u} \in S_{kN}$. Then a feasible solution to problem $P_N(x(i+1), x^*)$ is $\hat{\mathbf{u}} = \{\hat{u}_1, \dots, \hat{u}_{N-1}, \hat{u}\}$, $\hat{\mathbf{u}}^a = \{\hat{u}_1^a, \dots, \hat{u}_{N-1}^a, \hat{u}\}$, $\hat{\mathbf{x}}^a = \{\hat{x}_1^a, \dots, \hat{x}_{N-1}^a, \hat{x}_N\}$ and $\hat{x}_s = x^*$. The state sequence associated to the feasible input sequence $\hat{\mathbf{u}}$ is given by $\hat{\mathbf{x}} = \{\hat{x}_1, \dots, \hat{x}_N, \hat{x}\}$. Since $\hat{x}_1 = x(i+1) \notin S_{kN}$ then it is easy to see that $\hat{x}_1 \in L_{kN}$ ³. Therefore the feasible cost corresponding to $\hat{\mathbf{u}}$, $\hat{\mathbf{u}}^a$, $\hat{\mathbf{x}}^a$ and \hat{x}_s , is given by

$$\begin{aligned} V_N(x(i+1); \hat{\mathbf{u}}, \hat{\mathbf{x}}^a, \hat{\mathbf{u}}^a, \hat{x}_s) &= \sum_{j=1}^{N-1} \{ \|\hat{x}_j - \hat{x}_j^a\|_Q^2 \\ &+ \|\hat{u}_j - \hat{u}_j^a\|_R^2 \} \\ &+ \underbrace{\|\hat{x}_N - \hat{x}_N\|_Q^2 + \|\hat{u} - \hat{u}\|_R^2}_{=0} \end{aligned}$$

which means that

$$\begin{aligned} V_N(x(i+1); \hat{\mathbf{u}}, \hat{\mathbf{x}}^a, \hat{\mathbf{u}}^a, \hat{x}_s) - \dot{V}_N(x(i)) &= -\|\dot{x}_0 - \dot{x}_0^a\|_Q^2 \\ &- \|\dot{u}_0 - \dot{u}_0^a\|_R^2 \\ &\leq -\|\dot{x}_0 - \dot{x}_0^a\|_Q^2 \\ &= -d_Q(x(i), S_{kN}) \end{aligned} \quad (5.2)$$

where (5.2) is immediate from Property 4.1. Hence the optimal cost $\dot{V}_N(x(i+1))$ satisfies

$$\begin{aligned} \dot{V}_N(x(i+1)) - \dot{V}_N(x(i)) &\leq V_N(x(i+1); \hat{\mathbf{u}}, \hat{\mathbf{x}}^a, \hat{\mathbf{u}}^a, \hat{x}_s) \\ &- \dot{V}_N(x(i)) \\ &= -d_Q(x(i), S_{kN}), \end{aligned} \quad (5.3)$$

which implies that $\{\dot{V}_N(x(i))\}_{i=0}^\infty$ is a positive decreasing sequence. Thus, $\dot{V}_N(x(\cdot))$ converges and so

$$\dot{V}_N(x(i+1)) - \dot{V}_N(x(i)) \rightarrow 0$$

when $i \rightarrow \infty$. Therefore, by (5.3), $d_Q(x(i), S_{kN}) \rightarrow 0$ when $i \rightarrow \infty$. \square

5.2 Main results

The following Lemma shows that, when we are under the assumptions of Property 4.1, the closed-loop system steers the current state from one layer to the next one.

³Note that $\hat{x}_1 \notin S_{(k+2)N}$, otherwise \hat{x}_N it could not reach the set S_{kN} in $N-1$ steps.

Lemma 5.4 (Stepping through the layers). *Let $x \in L_{kN}$ for $k \geq 1$. System (2.1) controlled by the implicit law $\kappa_{MPC}(\cdot)$, provided by problem $P_N(x, x^*)$, reaches the next layer $L_{(k-1)N}$.*

Proof. We proceed by contradiction. Suppose that the sequence $\{x(i)\}_{i=0}^\infty$ given by the closed-loop system $x(i+1) = Ax(i) + B\kappa_{MPC}(x(i))$, with $x(0) = x$, does not reach the next layer $L_{(k-1)N}$. Then $x(i) \notin S_{kN}$, for any $i \in \mathbb{N}$. Therefore, by Lemma 5.3, $d_Q(x(i), S_{kN}) \rightarrow 0$, when $i \rightarrow \infty$.

Since by Lemma 2.5 S_{kN} is a compact and convex contractive CIS, then by Lemma 2.4 $S_{kN} \subset \text{int}_{S_\infty} S_1(S_{kN}, \Psi(S_{kN}))$. Hence there exists $i_0 \in \mathbb{N}$ such that $x_0 := x(i_0) \in S_1(S_{kN}, \Psi(S_{kN}))$. Therefore there exists $u_0 \in \Psi(S_{kN})$ such that

$$x_1 = Ax_0 + Bu_0 \in S_{kN}.$$

From the contractive invariance of S_{kN} there exist $u_1, \dots, u_{N-1} \in \Psi(S_{kN})$ such that $x_{j+1} = Ax_j + Bu_j \in \text{int}_{S_\infty} S_{kN} \subset S_{kN}$, for $j = 1, \dots, N-1$. Since we are under the assumptions of Property 4.1, the cost function for this control sequence is

$$\begin{aligned} d_Q(x_0, S_{kN}) &+ \underbrace{d_R(u_0, \Psi(S_{kN}))}_{=0} \\ &+ \underbrace{\sum_{j=1}^{N-1} d_Q(x_j, S_{kN}) + d_R(u_j, \Psi(S_{kN}))}_{=0} \\ &= d_Q(x_0, S_{kN}), \end{aligned}$$

while any control action that leaves x_1 outside S_{kN} produces a cost greater than $d_Q(x_0, S_{kN})$. Thus, $x(i_0+1) = Ax(i_0) + B\kappa_{MPC}(x(i_0)) \in S_{kN}$, which contradicts the fact that $x(i) \notin S_{kN}$, for all $i \in \mathbb{N}$. \square

Now we have all the ingredients to present and prove the main result of this work.

Theorem 5.5 (Attractivity in the maximal controllable set). *Let $x \in S_\infty$. Let $\{x(i)\}_{i=0}^\infty$ be the sequence given by the closed-loop system $x(i+1) = Ax(i) + B\kappa_{MPC}(x(i))$, with $x(0) = x$. Then*

$$d_Q(x(i), x^*) \rightarrow 0, \quad \text{when } i \rightarrow \infty. \quad (5.4)$$

Proof. Since $x \in S_\infty$ then, by (2.2), $x \in S_N$ or there exists $k_0 \geq 1$ such that $x \in L_{k_0 N}$. In the first case ($x \in S_N$), the problem $P_N(x, x^*)$ become the tracking problem (see Property 4.2). So, the recursive feasibility and the attractivity of $\{x^*\}$ can be proved by means of the same arguments as in [25]. In the second case, the recursive feasibility can be easily obtained by induction, noticing that any state in a kN -Layer belongs to the set $S_{(k+1)N} = S_N(S_{kN})$, and so, there exists a feasible trajectory which drives the closed-loop to S_{kN} in N steps. On the other hand, since $x \in L_{k_0 N}$ we can apply recursively Lemma 5.4 until the current state reaches the set S_N , which means that we are again under the conditions of the first case and, therefore, the attractivity is proved. \square

Corollary 5.6 (Asymptotic stability). *The set point $\{x^*\}$ is asymptotically stable for the closed-loop system controlled by $\kappa_{MPC}(\cdot)$, for all $x \in S_\infty$.*

Proof. Since $x^* \in \mathcal{X}_s \subseteq S_N$, our strategy inherits the local stability for any $x \in S_N$ from the MPCT, using the same Lyapunov function (see [25]). Then, the asymptotic stability is a straightforward consequence of Theorem 5.5. \square

6 Illustrative Example

In this section some simulations results will be presented to evaluate the proposed control strategy. First, a detailed description of the system is made - accounting for the sets associated to it - and then, the closed-loop simulations are shown. Finally a performance comparison with other strategies is made.

6.1 System description and dynamic simulations

In order to show the benefits and the properties of the proposed controller, we consider a constrained sampled double integrator:

$$x(i+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(i) + \begin{bmatrix} 0 & 0.5 \\ 1 & 0.5 \end{bmatrix} u(i) \quad (6.1)$$

with the following constraints:

$$\mathcal{X} = \{x \in \mathbb{R}^2 : -5 \leq x_1 \leq 5; -1 \leq x_2 \leq 1\},$$

$$\mathcal{U} = \{u \in \mathbb{R}^2 : \|u\|_\infty \leq 0.05\}.$$

Figure 2 shows the equilibrium set \mathcal{X}_s , the controllable set S_{N+1} , and a sequence of sets S_{kN} , $k \in \mathbb{N}$, with a prediction horizon $N = 3$. Observe that $S_N \subset \text{int}_{S_\infty} S_{N+1}$, which implies that S_N is a contractive CIS (see Remark 5.2). So, the propose MPC will be tested under the Assumption 5.1. Note also that the maximal domain of attraction of system (6.1) is reached for $k = 7$, i.e $S_\infty = S_{7N}$.

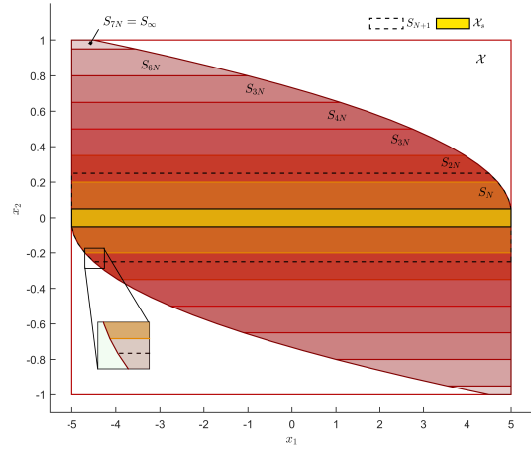


Figure 2: Sequence of controllable sets S_{kN} for $k = 1, \dots, 7$ and $N = 3$. Zoom window shows that the set S_N belongs to the interior of S_{N+1} relative to S_∞ .

To test the dynamic performance of the closed-loop system controlled by the proposed MPC, it is considered a starting point in the farthest layer from the equilibrium set, $x_0 = (-4.9; 0.96)$. Besides, a setpoint change has been considered. Before the closed-loop system reaches the initial setpoint $x_i^* = (-4; 0)$, the operating point switches to $x_f^* = (3.5; 0)$, at time $k = 70$. The state space evolution in Figure 3 clearly shows the capability of the proposed controller to drive the closed-loop system toward the desired setpoint, without loss of feasibility.

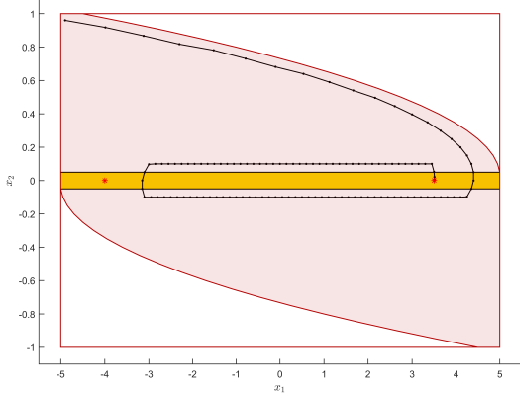


Figure 3: Closed-loop system evolution starting from $x_0 = (-4.9; 0.96)$. For time $k \leq 70$ the setpoint is $x_i^* = (-4; 0)$ and it is $x_f^* = (3.5; 0)$ for $k > 70$.

6.2 Performance comparison with other strategies

In this section, the performance of the proposed MPC is compared with two other strategies. The comparison will be performed first with the MPCT proposed in [25], which solves the problem of loss of feasibility under changes in the setpoint and enlarges the domain of attraction of the controller. Then, in a second stage, the proposed MPC is compared with the MPC presented in [22], which also exhibits the maximal domain of attraction that the system allows for any prediction horizon.

In order to properly quantify the performances we proposed the following controller's index

$$\Phi = \frac{1}{T_{sim}} \sum_{k=1}^{T_{sim}} \|x(k) - x^*\|_{\infty} + \|u(k) - u^*\|_{\infty}, \quad (6.2)$$

where T_{sim} represents the total simulation time. Index Φ penalizes the distance - given by the infinite norm - between the states and inputs of the closed-loop system with respect to the given setpoint.

a) The first strategy with which we will compare the performance of the proposed controller is the MPC for tracking with a terminal cost function and terminal inequality constraint, proposed in [25]. The terminal cost

function of this controller is given by $V_f(x - x_a) = \|x - x_a\|_P^2$, which is a Lyapunov function for the system under the local controller. The terminal constraint is given by $(x(N), x_a, u_a) \in \Omega_t^a$, where Ω_t^a is the so-called invariant set for tracking, and it is also associated to the local controller. The local (terminal) controller has been chosen as the Linear Quadratic Regulator (LQR) with $Q = 0.5I_n$ and $R = 2I_m$, and it is given by

$$K_{LQR} = \begin{bmatrix} 0.0509 & -0.3910 \\ -0.4335 & -0.7736 \end{bmatrix}. \quad (6.3)$$

Let Ω_t be the projection of Ω_t^a onto \mathcal{X} , then the domain of attraction is given by the set of states that can be admissible steered to the set Ω_t in N steps, i.e. $S_N(\Omega_t, \mathcal{U})$.

Figure 4 compares the domain of attraction of the MPCT, $S_N(\Omega_t, \mathcal{U})$, with the domain of attraction of the proposed MPC, S_{∞} , with prediction horizon $N = 3$ in both case. As it can be seen, the domain of attraction for the proposed MPC is significantly larger than the MPCT for the selected horizon. Even more, to reach the maximal domain of attraction with the MPCT, we would need a prediction horizon of $N = 18$, i.e. $S_{\infty} = S_{18}(\Omega_t, \mathcal{U})$, which would produce a non negligible increase in the computational cost.

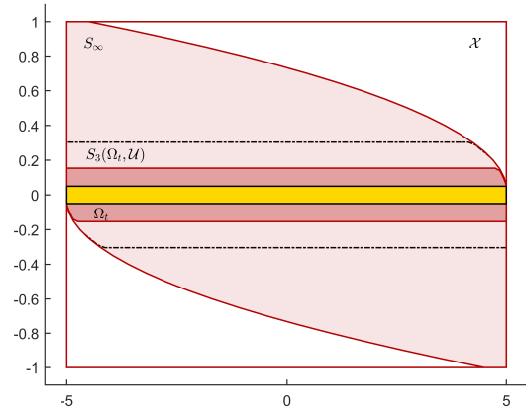


Figure 4: $S_3(\Omega_t, \mathcal{U})$: Domain of attraction of the MPCT, with prediction horizon $N = 3$. S_{∞} : Domain of attraction of the proposed MPC, with the same prediction horizon.

The numerical comparison between the performance of both controllers was made by using Index (6.2). To carry out this comparison, several initial random points were taken within the set S_∞ . Each initial point is steered to the given setpoint by both controllers. The proposed MPC design in this experiment used a prediction horizon $N = 3$. The MPCT is not able to control every point of S_∞ with $N = 3$, so it is designing with horizon $N = 18$. The average of the Index in every case is shown in Table 1.

	Average of Φ
Proposed MPC	2.0480
MPC proposed in [25]	2.0053

Table 1: Performance of the proposed MPC and the MPCT

As expected, the performance of the proposed controller is not better than the one of the MPCT. In fact, the better performance of the MPCT is justify by the larger prediction horizon ($N = 18$). Anyway, it should be noted that the performance difference is not significant, and seems to be a reasonable price to pay to obtain a meaningful prediction horizon reduction ($N = 3$).

b) The second strategy selected to compare the performance of the proposed controller is the MPC presented in [22]. The simulations will be made with the second order unstable linear system presented in the aforementioned work, i.e.

$$x(i+1) = \begin{bmatrix} 1.2775 & -1.3499 \\ 1 & 0 \end{bmatrix} x(i) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (6.4)$$

with $\mathcal{X} = \{x \in \mathbb{R}^2 : \|x\|_\infty \leq 5\}$ and $\mathcal{U} = \{u \in \mathbb{R} : \|u\|_\infty < 1\}$. The controllers are designed with equal parameters: $N = 5$, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 10$. Figure 5 presents the evolution of the closed-loop system controlled by the proposed MPC for the initial point $x_0 = (-4.17, -2)$.

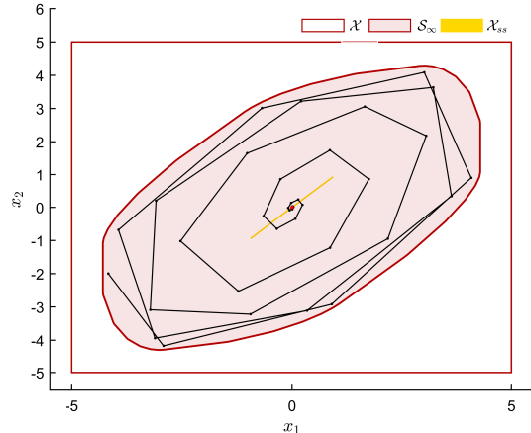


Figure 5: Closed-loop system evolution starting from $x_0 = (-4.17; -2)$ and the setpoint $x^* = (0; 0)$.

Once again, several initial random points inside S_∞ are considered to be controlled and steered to the setpoint $x^* = (0, 0)$ by both controllers. Table 2 shows that both controller show a similar performance according to Index 6.2.

	Average of Φ
Proposed MPC	0.6280
MPC proposed in [22]	0.6282

Table 2: Performance of the proposed MPC and the MPC presented in [22]

Summarizing, in spite of overall strengths of previous strategies to enlarged the domain of attraction [25, 22], these did not achieve a single formulation that does not lose feasibility under changes in the setpoint and reaches the maximum domain of attraction that the system allows for any prediction horizon. The present work proposed a MPC that solves this weakness without having differences in the performance. Even more, the proposed method avoids the use of the Invariant Set of Tracking [25] -which presents difficult computation in certain cases- and stores considerably less controllable sets than the strategy proposed by [22].

7 Conclusions

A novel set-based MPC for tracking was presented, which achieves the maximal domain of attraction that the constrained system under control allows for. The formulation consider a fixed (arbitrary) prediction/control horizon and, opposite to other existing strategies, have proved to be recursively feasible and asymptotically stable under any possible change of the set point. Furthermore, it preserves the optimizing behavior (i.e., it does not only pass from one state space region to the next, but also minimizes a cost function in the path) for every initial condition in the domain of attraction.

These benefits are achieved by solving a rather simple on-line, set-based, optimization problem, which depends on the off-line computation of a sequence of fixed controllable sets (in contrast to what is made, for instance, in [22], where the sets depend on the set point). The resulting controller have been successfully compared with other methods, by means of several simulating examples. Future works include more challenging application examples and a detailed robust analysis/extension

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