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Training in-service teachers: study of questions and the organization of teaching

Formation des enseignants en service: étude des questions et organisation de l'enseignement

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Abstract

We analyse the study and research carried out by N = 31 teachers in service during an on-line course in Didactics of Mathematics, about a question that could generate a Study and research path (SRP). The teachers studied the question individually and in groups. Then, they had to organize a possible teaching, adapted to an institution well known to them .The written texts produced by the teachers are analysed by means of two types of techniques: one qualitative and one based on lexicometric statistical methods. In the long term, the aim of this research is to understand the potential and difficulties of in-service teachers to organize teaching based on questions.

Keywords: Teachers training; Study and research paths

Résumé

Nous analysons l'étude et la recherche menées par N = 31 enseignants en service lors d'un cours en ligne sur la didactique des mathématiques, autour d'une question qui pourrait

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générer un PER. Les enseignants étudient la question individuellement et en groupe, puis il est proposé d'organiser un possible enseignement adapté à une institution bien connue par les enseignants, sur la base de la question étudiée. Les textes écrits produits par les enseignants sont analysés en utilisant deux types de techniques: une qualitative et l'autre basée sur des méthodes statistiques lexico métriques. Dans le long terme, cette recherche essaie de comprendre le potentiel et les difficultés des enseignants pendant l'organisation d'un enseignement basé sur des questions.

Mots clés: Formation des enseignants; Parcours d'études et de recherche.

Training in-service teachers: study of questions and the organization of teaching l'enseignement

The importance that the training of the mathematics teacher includes knowledge that exceeds the mathematical contents that a teacher should teach, has been emphasized by numerous authors (Cirade, 2006; Chevallard & Cirade, 2009; Chevallard, 2013; Gómez, 2007; Llinares, Valls & Roig, 2008; Godino, 2009; Ribeiro, Monteiro & Carrillo, 2010; Font, 2011). The notion of Pedagogical Content Knowledge (PCK) developed by Shulman (1987), which specifically in mathematics, originates Mathematical Knowledge for Teaching (MKT), has been recognized as an essential mathematical knowledge in teacher training (Ball, 2000; Ball, Lubienski & Mewborn, 2001; Hill, Ball & Schilling, 2008). In particular, Ruiz, Sierra, Bosch & Gascón (2014) have described the difficulties and the challenges to define the praxeological equipment of mathematics teachers.

In a university context of teacher training, the theoretical framework of the Anthropological Theory of the Didactic (ATD) is adopted while a course of Didactics of Mathematics is developed, where the fundamental notions of ATD are studied. It is expected that the course could help in-service teachers to conceive and organize mathematics teaching involving some gestures proper to the Paradigm of Questioning the World (PQW) (CHEVALLARD, 2013). Specifically, this paper analyses the study performed by these teachers related to an issue that could generate a SRP: First, the teachers studied the question individually and in groups. Then, they should analyse and to propose a possible teaching adapted to a specific institution, based on the studied question. The work aims to describe the potential and difficulties of the teachers while they plan teaching according to certain gestures of the PQW. In the long term the research aims to propose possible aids to the study for in-service teachers training. This leads to the question: which mathematical praxeologies and which organization of teaching are proposed by these teachers concerning the studied question?

The investigations developed within the framework of the ATD (Romo, Barquero & Bosch, 2016; Ruiz, Sierra, Bosch & Gascón, 2014; Otero, Llanos, Gazzola, Arlego, 2016) have highlighted the relevance of disposing of relatively tested SRP for the teacher training. Here, we select a question developed by the IREM of Poitiers (Bellenoué et al. 2014) that is a part of a book intended for teachers at the French secondary level. The text is inspired by Chevallard's ideas about the functionality of mathematics in the education systems. Below we briefly present the generative question Q_0 and the approach adopted in this work.

The SRP

The question Q_0: how does a parabolic antenna work? has been proposed to the students of the first year of the French secondary level (15-16 years old). A possible praxeological model of reference refers to the problem of the construction of the tangents to a curve from the analytical geometry. In addition to re-discovering some properties of synthetic and analytical geometry, the reflection of light on different surfaces from physical and wave Optics could be studied. The explanations of how various transmission and light reception equipment being essentials for the current communications operate; and also in other uses, in domains like architecture, automobiles design, solar energy, etc, are also involved. According to the proposal of Bellenoué et al. (2014) the main mathematical know-how to deal with in secondary school would be the following:

> Déterminer une équation de cercle à partir d'éléments caractéristiques ; déterminer une équation de droite ; déterminer la position relative de deux droites ; déterminer la forme canonique d'un trinôme du second degré ; résoudre une équation de second degré ; déterminer algébriquement les coordonnées des points d'intersection de deux courbes ; démontrer qu'une droite est tangente à un cercle, à une parabole, à une hyperbole. (p. 47).

Also, Q_0 allows to study about the historical analysis of the problem of the tangents to a curve, and the development of mathematical knowledge linked to this problem. The questions concerning the reflection of light in different surfaces, lead to the study of the conics, and of the tangents to those curves. Experiments of the reflection on different surfaces could be carried out, considering several kinds of mirrors: cylindrical, parabolic or hyperbolic. In the text above questions like: Why a surface could be considered a mirror? What types of antennas exist? What are antennas for? Which mathematical and extra-mathematical knowledge could be necessary to study the problem? are answered. The development of possible answers could include tools of the synthetic or analytical geometric framework in R^2 or R^3, among others. On the other hand, if the curves were unknown, the insufficiency of the geometric-analytical framework to determine the tangent would require studying differential calculus.

The question Q_0 has been selected because it allows to study an important part of the mathematic knowledge involved in the teachers training and at the same time, some relevant praxeologies of the secondary school syllabus in Argentine could be covered.

Sample and methodology

This work involves in-service mathematics teachers who attend the second year of the Bachelors in Mathematics Education (BME) at a National University in Argentina. This course, allows teachers of mathematics graduates of Institutes of Teacher Education, which are non-university institutions of teacher training, to complement their mathematical and didactic training. The BME curriculum consists of eight four-month courses spread over two years: three are Mathematics courses and the others corresponding to Didactics of Mathematics, Information and communications technologies (ICTs), Epistemology, Methodology and Cognitive Psychology. The instruction is provided completely online by means of Moodle platform. The in-service

teachers (N = 31) are taking the Didactics of Mathematics course according to the BME curriculum. The course is in charge of three teachers (one teacher per ten students). Moodle allows that all the interactions between the teachers and the students are registered and stored.

In the mentioned course, fundamental notions of ATD are studied and some examples of various SRPs available and widely disseminated in the literature are analysed. During the interactions on the platform, some students said: “it does not seem possible to manage a SRP in the real classrooms”; “for us it would be impossible to develop something like a SRP”; “it would not be possible to complete the program”; “developing a SRP is much more complicated than the problem solving”; “What role do praxeologies play?”; “How do the mathematical issues of the program intervene?”, etc. Due this the last month of the course was devoted to study a question that could originate a SRP as Q_0The students grouped in six teams carried out the following tasks:

T1: Study Q_0 and prepare a possible individual written answer.

T2: Analyse and discuss the individual answer with the group and to propose a possible group written answer to Q_0.

T3: Propose in each group a possible organization of teaching by adapting the T1 and T2 answers to a given institution.

The research has an exploratory, descriptive and ethnographic character. Analysing the answers to T1, T2 and T3, we seek to identify and describe the difficulties and the most relevant drawbacks found by these in-service teachers studying Q_0 while they were planning teaching based on questions, and their abilities to teach in line with the PQW. It is not proposed to be staging in classroom, because the teachers cannot introduce such a device in a period of one week, nor the course team could help them

properly. The teachers responsible for the course interact with the students and make returns of each task.

In this paper, the analysis of the written group answers to T2 and T3 is presented. The analysis considers the questions pointed out by the teachers and the responses tagged. Two types of analysis techniques are used: one qualitative and another based on lexicometric statistical methods (Lebart, Morineau & Fenelon, 1985; Moscoloni, 2011) that contribute to triangulate the results. For reasons of space, only some results related to the questions formulated in the section 4 are proposed here. As an example, the questions and the answers to the tasks T2 and T3 proposed by the groups A and B are presented in the Table 1.

Questions

1. Which derived questions from Q_0 did the groups of teachers formulate and how these questions influenced the organization of the study?

2. Which answers did they enter into the medium? Where did these answers come from?

3. How was teaching from Q_0 organized by the teachers, especially concerning the constitution of the didactic medium?

4. Which praxeologies did the teachers study while they answered Q_0 for themselves? Which of these contents did they propose to study while they planned and organised teaching?

Some results and preliminary conclusions

Regarding the questions 1 and 2, in the beginning the groups (A, C and F) realised a search oriented by physics questions about: antennas and its types, electromagnetic waves, the reflection and refraction of waves, the wave model and the ray model of light. These groups mentioned the quadric surfaces and quickly focused their study on the cross

sections. In addition, they questioned if the wave optics model or the geometric optics model would be appropriate to treat the reflection phenomena. Finally, they made the right decision adopting the geometric optics model without well justify. This also leaded to the questions: It would be necessary to study quadrics or conics? Should be the question answered in R^3 or R^2? The second option was selected really, but some of the groups directly considered conics, and other ones studied the quadrics surfaces to analyse their cross sections later and make sense to conics study. Only the group A considered and resolved the problem of the existence and determination of the tangent to the parable, and they did it in the synthetic geometrical framework. They entered to the medium the demonstration of a theorem to establish if a line that cuts to the parable in a point would be or not the tangent in this point. Also, they visualised the tangents by means of the geometric view of GeoGebra® (https://www.geogebra.org/m/n6ZvMT6R). Thus, they used this theorem to justify that any incident beam of light (modelled by a straight line), being parallel to the axis of the parable will be reflected passing by the focus (modelled by another straight line).

The groups C and F, though they studied the physics questions, directly assumed the existence of a tangent at each point. They found on the internet the "reflective property of the parable" which is a didactic invention whose demonstration was entered to the medium without questioning https://www.geogebra.org/m/ktwEJK6D.

Some results of the groups A and B are synthesized in the Table 1. The two first columns summarizes the derived questions and the already made answers to T2, and the two latest summarizes the decisions made by these in-service teachers answering to T3.

Table 1

Questions and answers of the groups A and B about T2 and T3

Q_{iT2}	$A^{\Diamond}_{iT2}$	Q_{iT3} - Tasks	$A^{\Diamond}_{iT3}$
Group A			
$Q_1=Q_0$ Q_2: What is and what does an antenna serve for? Q_3: What types of antennas are there? Q_4: What are parabolic antennas used for? Q_5: According physics. Which characteristics have the electromagnetic waves? Q_6: How do the electromagnetic waves propagate? Q_7: How are the electromagnetic waves reflected? Q_8: Why is the paraboloid the most appropriate surface to obtain the major directivity? Q_9: Which characteristics do the paraboloids have? Q_{10}: How to define a parable and which characteristics does it have? Q_{11}: Given the directrix and focus. How to determine the points of the parable?	Functioning and types of antennas. Incident radiation and electromagnetic waves. Validity of geometric Optics model. Reflection and Refraction in geometric Optics. "Directivity of the paraboloids" The transition from 3D to 2D is justified by means of GeoGebra®. Analytic and synthetic Geometry techniques to study the parable. The existence of the tangent to the parable at a point is justified in the synthetic frame. The reflected rays always pass through the focus.	$Q_1=Q_0$ Q_7: How are waves reflected? If antennas are 3D objects. Why the information found refers to parables? Q_{10}: Given a straight line d and an external point F. How to determine the points placed at the same distance from the straight line and the point? How these points would be placed on the flat? Q_{11}: Demonstration that a parable having directrix d and focus F is symmetrical to the perpendicular line to d passing through F (focal axis). Given a parable and its directrix, to find the focus of the parable. Given a parable and its focus, find the straight line of the parable. Determine the tangent to a parable in any point. Proof: the incident beams of light parallels to the focal axis of the parabolic antenna will be reflected passing by the focus.	Three stages are proposed: Reflection of rays on the flat. Study of parable as geometric locus (symmetry, focus and directrix, by means of GeoGebra®). Applet of GeoGebra® showing the reflection of the rays parallels to the focal axis of the parable.
Group B			
Q_0 Q_1: Mathematically, What is a quadric surface? Q_2: How are the quadrics classified? Q_3: Which are the quadrics equations?	Classification of Quadrics: parabolic, elliptical and hyperbolic.	Q_1: Which is the mathematic definition of a paraboloid? Q_2	Quadrics. Paraboloid.

Q_4: Which are the equations of a paraboloid? Q_5: What does an antenna serve for? Q_6: What is a wave in physics? Q_7: Which are the characteristics of the electromagnetic waves? Q_8: How does a parabolic antenna function? Q_9: How do the parabolic shape of the antenna influence the reception of the signals emitted by the satellites? How rebounding and concentrating the waves in the universal Low Noise Block (LNB)? Q_{10}: How to define a parable? Q_{11}: Which is the equation of a parable? Q_{12}: Which are the characteristics of a reflective surface? Q_{13}: How to demonstrate "the reflective property of the parable"?	Traces and cross sections. Ellipse, Hyperbola Parable Parabolic Antennas. Waves The reflective surface accomplish the Snell's law. The "reflective property of the parable" justifies the reflection of rays towards the focus. (https://www.geogebra.org/m/xxeRSH7H)	Q_3 Q_4 Write in GeoGebra® the equations $-\frac{x^2}{9}+\frac{y^2}{4}=2z$, $\frac{x^2}{9}+\frac{y^2}{4}=2z$ and plot the paraboloids. Find the equations of the following hyperbolas, the coordinates of the vertices and the equations of the asymptotes: a. Focus (-8,0) y (8,0) and constant 6. b. Focus in (-6,0) y (6,0) and one vertice in (-1,0). Find the focus of the hyperbola: $\frac{x^2}{25}-\frac{y^2}{7}=1$ Q_5, Q_6, Q_7, Q_8 Q_9 Q_{12} Q'$_{13}$ Demonstrate the "reflective property of the ellipse and the hyperbola"	Ellipse, Hyperbola, Parable. Signal, satellite. Low Noise Block (LNB), reflection.

The groups B, D and E were different, because their questions and initial searches were mathematical. The most sophisticated group (B) begun with the question: "Mathematically, what is a quadric surface?" They entered to the medium the analytical classification of elliptical and hyperbolic paraboloids, establishing their traces. Thus, the cross-sections: ellipse, hyperbola and parable and their characteristics were studied. Just then, the group asked about the antenna types and the reflection of light. They modelled the "reflective surfaces" by means of the quadric surfaces, they also employed the "reflective property of the parable". The other two groups (D, E) also begun with mathematics questions but these were confined to the parabolic surfaces and its

applications. For instance: solar kitchens, parables, and the reflection phenomena. Synthesizing, in the written answers to Q_0 we identify that:

1) The search was guided by questions related to Physics or Mathematics, driven by two words: antenna and paraboloid.

2) The individual response of most teachers, improved considerably in the collective task.

3) Most of the questions were of type essential like what is...? and less were of the type how? or Why? The former kind promoted closed answers and definitions.

4) The answers entered to the medium are mostly obtained on internet, especially in video format, without questioning, except by the group A that had proposed questions around the tangent to the parable.

5) Only the group A formulated the problem of tangents to a curve and resolved it for the parable in the synthetic geometric framework, while the rest of groups did not question about it. This would prevent out the possibility of treating the more general problem of tangents to a known curve or not, and with it, a possible praxeological extension from analytical geometry to differential calculus.

The lexicometric analysis and the factorial analysis of correspondences allows to locate the groups focused in physics and those focused in mathematics on the factorial plane in opposition. Words with the lowest contribution to the inertia refer to parables and equations.

Regarding the questions 3 and 4 concerning the organization of a hypothetical teaching from Q_0 it is identified that:

1) The organization proposed in T1 and T2, was reproduced in T3: the group that had beginning by physics or mathematics, retained this condition and the answers, not so the questions.

2) There was a successive reduction of the mathematical and physical issues during the transition from T2 to T3.

3) The questions were replaced by type of tasks like to determine, to calculate, to plot, and to demonstrate, which were directly answered in the proposal. This reduction is functional to the monumentalism phenomena.

4) The questions were transformed into tasks, activities, stages, situations, responding to them, while questions could rest hidden.

5) The group D changed Q_0 by: how to build a solar kitchen?

6) The groups C, D, E and F proposed hands-on experiences and tasks.

7) The final reduction of the medium could be related to the curriculum proposed in the institution of destination: in the final version of the proposal, it has been decided to study only the contents relative to the parables in the geometric and functional frameworks.

8) The management of instruction was organized to avoid “losing” control of the medium.

9) The teachers assumed that the students will question, but also, that they should finally guide them. Thus they proposed tasks directed to control the medium M.

10) The groups more reticent to “loss of control” firstly “showed the works (Ok) to the students” and then, they proposed questions related these works O. Example: Let's observe all these paraboloids, could you say which of them have parables as cross sections?

In short, we conclude that teachers would have avoided losing control of the didactic medium. This strongly restricted the organization of teaching proposed, causing the loss of the research component that had survived until they had to think about how to organize teaching. This reduction could had aroused when these teachers had to organize

teaching based on questioning. This could be due to the conditions emerging at the level of society rather than that of pedagogy, such as the more or less explicit opinion that teachers should control the didactic medium and be completely responsible for him. If the loss of control is considered dangerous by society, especially in institutions, is reasonable that teachers try to keep it, even for ecological reasons.

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