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25 Years of Self-organized Criticality: Space and Laboratory Plasmas

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Abstract Studies of complexity in extended dissipative dynamical systems, in nature and in laboratory, require multiple approaches and the framework of self-organized criticality (SOC) has been used extensively in the studies of such nonequilibrium systems. Plasmas are inherently nonlinear and many ubiquitous features such as multiscale behavior, intermittency and turbulence have been analyzed using SOC concepts. The role of SOC in advancing our understanding of space and laboratory plasmas as nonequilibrium systems is reviewed in this article. The main emphasis is on how SOC and related approaches have provided new insights and models of nonequilibrium plasma phenomena. Among the natural plasmas the magnetosphere, driven by the solar wind, is a prominent example and extensive data from ground-based and space-borne instruments have been used to study phenomena of direct relevance to space weather, viz. geomagnetic storms and substorms. During geomagnetically active periods the magnetosphere is far from equilibrium, due to its internal dynamics and being driven by the turbulent solar wind, and substorms are prominent features of the complex driven system. Studies using solar wind and magnetospheric data have shown both

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global and multiscale features of substorms. While the global behavior exhibits system-wide changes, the multiscale behavior shows scaling features. Along with the studies based on observational data, analogue models of the magnetosphere have advanced the understanding of space plasmas as well as the role of SOC in natural systems. In laboratory systems, SOC has been used in modeling the plasma behavior in fusion experiments, mainly in tokamaks and stellarators. Tokamaks are the dominant plasma confinement system and modeling based on SOC have provided a complementary approach to the understanding of plasma behavior under fusion conditions. These studies have provided insights into key features of toroidally confined plasmas, e.g., the existence of critical temperature gradients above which the transport rates increase drastically. The SOC models address the transport properties from a more general approach, compared to those based on turbulence arising from specific plasma instabilities, and provide a better framework for modeling features such as superdiffusion. The studies of space and laboratory plasmas as nonequilibrium systems have been motivated by features such as scaling and critical behavior, and have provided new insights by highlighting the properties that are common with other systems.

Keywords Self-organized criticality · Nonequilibrium systems · Multiscale phenomena · Turbulence · Space plasmas · Tokamaks

1 Introduction

Nonequilibrium phenomena are ubiquitous in many systems, e.g., physical, biological, economic and social, and many approaches are needed to advance the understanding of such systems. In general these phenomena exhibit highly irregular or stochastic features, indicating a high dimensionality of its dynamics. At the same time they clearly show the presence of underlying long range correlations, and the resulting low-dimensionality or coherence. The latter property often results from a fine tuning of the system, as in the case of phase transition in equilibrium systems, and the recognition that this property can develop spontaneously in a system is a key feature of self-organized criticality (SOC) (Bak et al. 1987). The SOC framework and its many applications are reviewed in the companion papers, viz. Aschwanden et al. (2014), McAteer et al. (2015, this issue) and Watkins et al. (2015, this issue). The theoretical foundation of SOC (Jensen 1998; Dhar 2006; Aschwanden 2013b) shares many common features with other approaches, e.g., self-organization, phase transition, critical phenomena, and turbulence. From the dynamics point of view, these approaches target the understanding of complexity in extended dissipative dynamical systems, and have influenced the related research areas significantly. For example, the framework of SOC has stimulated the development of theoretical foundations of nonequilibrium statistical physics. In many systems, in nature and in laboratory, often there is no clear separation of scales (space and/or time) to enable an identification of equilibrium states and consequently their behavior is of nonequilibrium nature. In the absence of a general theory the framework of SOC, with its intuitive and often simplified characteristics, has been used extensively to describe many nonequilibrium phenomena.

Plasmas are inherently nonlinear and their complexity and nonequilibrium behavior are evident in natural as well as laboratory systems (Sharma and Kaw 2005). The collective behavior of plasmas arising from their eigenmodes and their interactions are the underlying causes of complexity in such systems (Anderson 1972, 2011). The complexity of natural plasmas has been studied using many paradigms, e.g. the studies of the Earth's magnetosphere (Sharma 1995; Klimas et al. 1996; Chapman and Watkins 2001; Consolini 2002;

Chang et al. 2003; Sharma and Curtiss 2005; Watkins 2013). Among the different applications of SOC to plasmas the case of solar physics stands out as extensive and there has been many applications in heliophysics and astrophysics (Aschwanden et al. 2014). The complexity in plasmas exhibits many multiscale features, such as turbulence and intermittency, which have been studied using simulations and experiments. The analysis of these features have used SOC and related approaches, e.g., dynamic renormalization techniques (Chang et al. 2004), and topological disorder (Consolini and Chang 2001). In the studies of turbulence in laboratory plasmas, the magnetically confined toroidal devices, viz. tokamaks and stellarators, are most prominent. This is natural as turbulence plays a critical role in the confinement and transport of the plasma in these devices. In particular edge turbulence in tokamaks plays a crucial role in the confinement of the hot plasma and has been a focus of laboratory fusion research. Many studies based on SOC and related approaches have provided a different but complementary approach to the traditional view of turbulence and its role in tokamaks (e.g., Carreras et al. 1996; Dendy and Helander 1997; Sanchez et al. 2003; Jha et al. 2003; Milovanov 2010). Among studies with a unified view of turbulence and intermittency in laboratory and space plasmas, the sandpile model, which is the most widely used model of SOC, has been used for modeling plasmas in different settings (Chapman et al. 1999). Similar studies of turbulent boundary layers in the magnetosphere and fusion devices show scaling features that indicate anomalous behavior, such as superdiffusion (Budaev et al. 2008; Savin et al. 2015).

Earth's magnetosphere, driven by the turbulent solar wind, is an archetypical extended dissipative system in nature, and a major part of this article is devoted to the study of its complex behavior. The time evolution of such systems is essentially determined by the internal dynamics and interaction with its driver, the solar wind. The magnetosphere is thus a driven complex system and is far from equilibrium, especially during geomagnetically active periods (Sharma 2010). In the dynamics of the magnetosphere the substorms are sudden changes in the state of the magnetosphere and is a central theme in solar wind-magnetosphere coupling. While our understanding of substorms has advanced rapidly from the analysis of spacecraft and ground-based measurements, accompanied by theory, modeling and simulations, many key questions remain unanswered. The approach to the understanding of magnetospheric dynamics in terms of the basic plasma processes has limitations due to the inherent complexity arising from the multiscale interactions among its components and the driving by the turbulent solar wind. Many studies characterizing substorms as SOC (Chapman et al. 1998; Uritsky et al. 2001a, 2001b), and nonequilibrium phase transitions (Sitnov et al. 2000; Sharma et al. 2001) have provided a new understanding of their dynamics. These and related studies, reviewed in the following sections, provide a new framework that complements the first-principle approach for the understanding of the complex behavior of the magnetosphere, with the objective to develop a comprehensive picture of the solar wind-magnetosphere coupling. Further, these studies explore the inherent features of the system from extensive data, and are early examples of BIG DATA, data-enabled science and the fourth paradigm (Sharma 2014).

Turbulence plays a critical role in the confinement and transport of the plasma in most laboratory devices, e.g., tokamaks and stellarators. In laboratory systems the tendency for self-organization or relaxation under specified constraints have been recognized as a generic process, e.g., in their evolution to stable or minimum energy states (Hasegawa 1985; Taylor 1986). In tokamaks and stellarators, the nature of radial plasma transport tends to drive the confined plasma to profiles that are close to marginal stability. In these near-marginal regimes that are considered fusion-relevant conditions, many features are observed, such as scaling laws or profile consistency, that are indicative of universality. This phenomenology has motivated the proposal of SOC approaches, starting in the mid- to late-90's, to develop an alternative view that complements the first principles approach (Diamond and Hahm 1995; Carreras et al. 1996; Newman et al. 1996; Dendy and Helander 1997). The adequacy of these approaches is still under debate in the community, although a large amount of numerical (Sect. 4.1.2) and experimental (Sect. 4.1.1) evidence seems to point to a definite role of this type of dynamics in both tokamak and stellarator plasmas. It is worth mentioning that much insight has been gained in this context by using the well-known sandpile SOC paradigm, adapted to the different settings of interest (Newman et al. 1996; Chapman et al. 1999; Sanchez et al. 2001).

This paper is a review of the developments in the study of the nonequilibrium nature of space and laboratory plasmas using SOC and related approaches. The next section presents a brief overview of the approaches that are closely related to nonequilibrium systems in general and to plasmas in particular. Section 3 reviews the understanding derived from the applications to space plasmas, viz. Earth's magnetosphere, solar wind and solar cases, with emphasis on the magnetosphere. Studies of laboratory plasmas are presented in Sect. 4 and the focus is on magnetic confinement devices, in particular the tokamak. The fluctuation analysis and its relationship to SOC related studies are discussed in Sect. 5. The paper concludes with a brief summary of the recent developments.

2 Overview of Nonequilibrium Phenomena

The multiscale nature of nonequilibrium phenomena, often expressed in the form of power law dependence of characteristics features, has stimulated extensive research in many fields. Such behavior follows from the inter-connected concepts such as self-similarity, scale invariance and long-range correlations in the systems under study. These characteristics have been recognized independently in many fields and their universal nature has motivated the search for new ways of understanding them. The development of theoretical methods such as the renormalization group (Fisher 1974; Wilson 1975) and mathematical objects such as fractals have led to new advances in the understanding of such systems in many ways. In these studies modern critical phenomena, scaling, universality and renormalization are generally recognized to be the three main pillars (Stanley 1999). These advances however are confined largely to systems in thermodynamic equilibrium. In nature many systems that exhibit scale invariance, exhibited by the power law dependence of size distributions or the fractal structure, are neither at nor near equilibrium. Rather, they appear to be common to systems far from equilibrium, e.g., earthquakes (Bak and Tang 1989), forestfires (Malamud et al. 1998), and landslides (Hergarten and Neugebauer 1998).

Many approaches to nonequilibrium phenomena, such as nonequilibrium phase transitions, self-organized criticality and catastrophes, are essentially based on the nature of critical phenomena. The critical phenomena in conventional phase transitions arise due to fine tuning of the external parameters driving the system (Stanley 1971; Dixon et al. 1999). However, unlike the laboratory systems which can be studied under controlled conditions, systems in nature usually are not in equilibrium and in general there is an absence of tuning, and the efforts to understand critical phenomena in such situations have led to a range of approaches and applications. A natural approach is to extend the concepts of the underlying equilibrium phenomena to systems that are evolving in time. An example is phase transition and dynamic critical phenomena (Hohenberg and Halperin 1977). However the extension to systems in steady states far from equilibrium is complicated and is tied to the development of nonequilibrium statistical mechanics (Balecu 1997). The SOC concept was motivated by a need for a theory of nonequilibrium critical phenomena and it has led to considerable advances in many fields of study. The original SOC idea was demonstrated using computer models of a sandpile, which is the widely used model for avalanche phenomena. Early studies used the examples of sandpiles to model the dynamics of open and spatially extended systems. In laboratory experiments sandpiles were found to behave in a manner more reminiscent of a first-order phase transition (similar to the ordinary fold catastrophe) than a second order one (Nagel 1992). There are however other avalanche phenomena in the laboratory in which SOC have been observed, e.g., in piles of long grain rice (Jensen 1998). The catastrophe approach (Gilmore 1993) has many features common to these approaches and has the advantage of combining features of the first and second order phase transitions, in nonequilibrium systems. Further, the focus on the nature of the sudden transitions in a self-organizing system has led to other approaches. An example is the highly optimized tolerance, which casts criticality as a way the system optimizes its time evolution, especially in response to external perturbations (Carlson and Doyle 2002). In the studies of nonequilibrium systems the notion of self-organization in the presence of fluctuations has played a key role and is responsible for the emergence of dissipative structures as dynamical states (Nicolis and Prigogine 1977). This approach is formulated using thermodynamic variables such as entropy. The SOC concept was stimulated by the $1/f$ feature in a wide range of physical systems and most claims of SOC behavior are based on such power law spectra. It should be emphasized that fluctuations in nature are much more complicated, due to phenomena such as intermittency and multifractal behavior (Riazantzeva et al. 2015).

The studies of nonequilibrium phenomena have been motivated by the striking similarity in the behavior near the transition (critical point) in systems that are otherwise quite different. Such systems occur in a wide range of conditions in nature and in laboratory, and some examples are turbulent transport in tokamaks (Diamond and Hahm 1995), computational complexity (Monasson et al. 1999), and forest fires (Malamud et al. 1998). Understanding plasma transport in tokamaks is crucial to harnessing their potential as fusion devices and belongs to the broad class of problems of the confinement of matter under nonequilibrium conditions. Many aspects of the transport process have features of SOC (Newman et al. 1996; Carreras et al. 1996; Dendy and Helander 1997). In the studies of computational complexity, the exponential increase in the computing time with the number of variables is the basis of the well known K-SAT problem and the transition has been shown to have characteristics of a second order phase transition (Monasson et al. 1999). Forest fires exhibit power law frequency-area statistics over many orders of magnitude, which is interpreted in terms of SOC (Malamud et al. 1998).

Nonequilibrium phenomena have been studied largely using experimental and observational data of a variety of data from a wide range of systems. In most cases power law dependence of physical variables has been considered as evidence of SOC behavior. However the power spectral densities obtained from observational data show different characteristics in many cases. For example, in the studies of Earth's magnetosphere, the statistics of chorus events seen at the ground and particle injections in the inner magnetosphere (Borovsky 1993) showed departures from the power law scaling expected of avalanching behavior. Also the intensity and occurrence of substorms have a probability distribution that indicate a well-defined mean under strong driving by the solar wind (Sharma et al. 2008). A way to reconcile the global or low-dimensional and the multiscale or scaling behavior of the magnetosphere has been proposed in the context of phase transitions (Sharma et al. 2001; Sitnov et al. 2000). Extensive studies of the SOC-like behavior of the magnetosphere (e.g., Chapman et al. 1998; Uritsky et al. 2001b, 2002; Klimas et al. 2000; Lui et al. 2000; Tam et al. 2000) are reviewed in the next section. Many SOC models of the magnetosphere consider it to be an autonomous system instead of an open system driven by the solar wind. The solar wind variables are the drivers of geomagnetic activity and together with the magnetospheric variables yield a dynamical description that better represents the coupled system. Such an approach used the time delay embedding technique in which the reconstructed vector consists of the AL and solar wind data together (Sitnov et al. 2000). According to this analysis the global behavior of the magnetosphere exhibits signatures of low effective dimension and coherent behavior, and geomagnetic activity such as substorms can be viewed as first order dynamical phase transitions. The multiscale features are largely scale-invariant, consistent with SOC and second order phase transition.

The early explorations of complexity in the magnetosphere addressed the global behavior, based on the use of the long time series data of geomagnetic indices (Vassiliadis et al. 1990). The evidence of large scale coherence in magnetospheric dynamics was first obtained from the time series of the auroral electrojet index AE in the form of low dimensional behavior (Vassiliadis et al. 1990; Sharma et al. 1993). These results are consistent with the phenomenology of the magnetosphere derived from the observational data and theoretical understanding (Siscoe 1991), and numerical simulations using global MHD codes (Lyon 2000). The recognition of the low dimensional dynamics of the magnetosphere has stimulated a new direction in the studies of the solar wind-magnetosphere coupling. Among the outcomes of this research is its capability of forecasting geomagnetic activity, viz. substorms (Vassiliadis et al. 1995) and storms (Valdivia et al. 1996; Sharma 1997), which provided the first forecasts of space weather (Lerner 1995; Sharma 1995; Vassiliadis 2006). Further, the low-dimensional dynamical behavior of the magnetosphere is characterized in terms of Lyapunov exponents computed from the dynamical trajectories in the phase space reconstructed from time series data (Vassiliadis et al. 1991a). However, it should be noted that computing dynamical quantities such as Lyapunov exponents requires good representations of the dynamical trajectories, and the observational data may not yield such features readily.

The multiscale nature of the magnetosphere co-existing with the global behavior has been recognized in many different ways. From the perspective of first-principles of solar wind-magnetosphere coupling, the processes have a wide range, from the electron-scale (Zhou et al. 1996; Jain and Sharma 2009) to the planetary scale. Further the multiscale phenomena are excited by processes such as magnetic reconnection (Sitnov et al. 1998, 2002; Birn et al. 2001). An early recognition of the multiscale behavior in the magnetosphere was, albeit indirectly, in the analogy between the dynamics of the magnetotail dynamics to the turbulence generated by a fluid flow past an obstacle (Rostoker 1984). Subsequently, the power law dependences of the observed time series data, e.g., in the AE index (Tsurutani et al. 1990), have provided quantitative measures of the multiscale behavior. Studies of the auroral electrojet indices using techniques of fluctuation analysis such as the structure function, have shown features that are reconciled with multiscale behavior (Takalo et al. 1993; Setty 2014).

The coexistence of the global coherence with multiscale behavior is consistent with some models that recognize the nonequilibrium nature of magnetospheric transitions. For example, the catastrophe scenario of substorms (Lewis 1991) based on the evolution of the magnetosphere on a cusp manifold embodies the global features with the multiscale behavior. Also, the global features of substorms have been attributed to a global instability of the magnetosphere (Baker et al. 1990, 1999), akin to the catastrophe picture. The idea of criticality in the magnetospheric dynamics (Chang 1992) outlines a framework for a description based on the renormalization group technique (Wilson 1975). Many studies based on first principles also conclude the presence of such behavior. For example, the balance of forces in the magnetotail has shown that the plasma can not be maintained at equilibrium readily and this recognition led Chen and Wolf (1993) to a scenario of substorms that is similar to the boiling of a liquid. Similar conclusions were reached by Sergeev et al. (1996) in a review of the substorm phenomenology and they likened substorm activity to rain. Low dimensional models based on the many physical processes in the magnetosphere (Horton and Doxas 1996) have been motivated, in part, by the critical behavior of the magnetosphere. The common thread in these studies is the presence of both the globally coherent and multiscale features of the magnetosphere and the need to understand these in terms of a comprehensive model that can account for both the large scale features and the coupling across the different scales. The framework of SOC describes a class of systems in which the relevant processes occur on all scales, thus exhibiting multiscale behavior. This complements the features of a low dimensional system, in which the dominant features are global. On the other hand, both the global coherence and multiscale behavior can be described within the framework of phase transitions, in which the first order transitions are global in nature and the second order transitions describe criticality and multiscale behavior. While the theory of phase transitions in equilibrium is well developed, it is not so in the case of nonequilibrium systems. The three approaches, viz. phase transitions, catastrophe and SOC are thus ways to describe nonequilibrium transitions, and complement each other in many ways.

The spatio-temporal dynamics of complex systems have been studied using models such as the complex Ginzburg-Landau equation and coupled map lattices (Cross and Hohenberg 1993). However the observational data cover a wide range of scales and new techniques and approaches are needed to model the inherent features. An approach to quantifying the complexity in extended systems from observational data was developed using the local asymmetry to define a fragmentation parameter (Rosa et al. 1998, 1999). As an example a spatio-temporal distribution of coupled logistic map was used to study their time evolution for different initial conditions. It was found that even when the initial conditions are random, coherent features develop in the system, and these were quantified in terms of the fragmentation parameter, which showed the evolution from a random system to one with coherent patterns (Sharma et al. 1998). Such features of self-organization and development of coherence and cluster formation in coupled maps have been studied in more detail (Jalan and Amritkar 2003).

3 Space Plasmas

Plasmas are pervasive in space and spacecraft missions have provided extensive measurements of the plasma in the heliosphere and enabled detailed studies of its properties and the dynamical behavior. The solar plasma is essentially driven by sub-surface convection and observations, in particular those exhibiting power law dependences, have been modeled using SOC and related approaches. The solar wind is driven by the activity in the solar corona and spacecraft explorations have provided data on a wide range of scales, from those of the short kinetic processes to the longest MHD phenomena. The main approach to the understanding of the properties such as the scaling behavior in the solar wind has been turbulence (Veltri 1999). The applications of SOC to the scaling behavior of the solar wind has complemented the turbulence approach and is reviewed briefly in the following and in more detail in the companion article (Aschwanden et al. 2014). The coupling of the solar wind energy and momentum to the magnetosphere drives many processes with a wide range of scales, from the electron skin depth (~ 5 km) to the system size ($\sim 10^5$ km). The observational data of the magnetospheric phenomena from ground and satellite measurements provide extensive coverage over this range and have enabled detailed studies and are reviewed in this section.

3.1 Solar Phenomena

The application of the SOC concept to solar phenomena is extensive and has been stimulated by the ubiquitous power law dependence, such as the solar flare occurrence rate on the flare size (Lu and Hamilton 1991). The companion article in this volume (Aschwanden et al. 2014) provides a detailed review of SOC models applied to solar phenomena. We summarize here some aspects that are complementary to the theme of space plasmas.

The solar corona is a very dynamic region which is the source of many phenomena such as solar flares, coronal mass ejections, and solar energetic particle (SEP) events. Solar flares are sporadic impulsive bursts of energy releases that are believed to be caused by magnetic reconnection processes in the solar corona. The statistics of any physical parameter measured in flares, such as hard X-ray peak fluxes, fluences, or time durations, exhibit strict power laws (e.g., Crosby et al. 1993) when sampled over a sufficiently large sample, which clearly indicates that solar flares are not governed by a random process with a preferred size scale (which would yield a Gaussian distribution), but rather are triggered by a scale-free mechanism as is typical for nonlinear energy dissipation processes in systems in a state of self-organized criticality. Using the paradigm of sandpile avalanches (Bak et al. 1987), the energy input of a solar flare SOC system is supplied by buoyant magnetic flux tubes that are generated in the solar convection zone (or tachocline) and emerge in active regions on the solar surface, where they become increasingly stressed and twisted by differential rotation, sheared surface flows, and sunspot rotation, until a threshold of nonpotential magnetic energy is exceeded that leads to a loss of equilibrium and triggers an impulsive solar flare episode with dissipation of free magnetic energy (i.e., the difference between non-potential and potential magnetic energy), which is the equivalent to a sandpile avalanche. Thus, the trickling of individual sand grains corresponds to the fragmented generation of magnetic flux tubes in the tachocline, which drift to the surface by buoyancy and magneto-convection. It is no coincidence that the depth of the convective layer in the Sun (about 30 % of the solar radius, i.e., ≈ 200 Mm, coincides with the maximum size of active regions on the Sun), because the largest convection cells spread the magnetic energy coherently over a comparable vertical and horizontal size. Thus, the spatial size L of flares varies from the height of the transition region or photospheric granulation cells (≈ 1–2 Mm) to the maximum size of active regions (≈ 200 Mm), which is about two orders of magnitude. The resulting flare volume $V \approx L^3$ varies over about six orders of magnitudes. The thermal energy of the flare plasma, $E_{th} = 3n_e k_B T_e V$ (with a mean electron density n_e and flare temperature T_e) follows a nonlinear scaling with the volume (see Sect. 3.2.7 in Aschwanden et al. 2014) according to hydrodynamic flare models (Rosner et al. 1978) and predicts a power law distribution $N(E_{th}) \propto E_{th}^{-\alpha_E}$ with a slope of $\alpha_E \approx 1.5$–2.0 (depending on the model), extending over up to 8 orders of magnitude. Similar power law distributions are predicted for other flare parameters (emission measures, thermal energies, nonthermal energies, magnetic energies, etc.) and for flare peak fluxes in almost all wavelengths (from gamma-rays, hard X-rays, soft X-rays, to extreme ultra-violet, white light, and radio wavelengths). Current progress in the application of SOC models to solar flares is mostly driven by modeling the (power law-like) frequency distributions of flare parameters according to physical (hydrodynamic, magnetic reconnection, plasma instability criteria) scaling laws and vice versa. When we ask what is universal about SOC models, and what is specific to solar flare phenomena, we think that the size distributions of spatio-temporal (geometric and temporal) parameters are governed by universal principles, such as the *scale-free probability conjecture* $N(L) \propto L^{-d}$ (Aschwanden 2012), which predicts size distributions of avalanche volumes according to $N(V) \propto V^{-(2-1/d)}$, with d the Euclidean dimension. Also temporal scales (of avalanche durations T) can be described by universal transport processes, $L \propto T^{\beta/2}$, such as random walk ($\beta = 1$), sub-diffusion ($\beta < 1$), hyper-diffusion ($\beta > 1$), or turbulence, which approximate the next-neighbor interactions in sandpiles, earthquakes, solar flares, and cellular automaton models. The size distributions of energies and other physical parameters, and their relationships to spatio-temporal parameters, however, depend on the physical scaling laws (such as $E \propto V^{\gamma}$ in the simplest case), which are specific to the observed phenomenon (tectonic stress for earthquakes, or magnetic field stress for solar flares). Solar flares are one of the most compelling examples of SOC type behavior; their power law distribution covers over eight orders of magnitude (Aschwanden et al. 2014). SEP events have also been found to display power law behavior covering three to four decades when frequency distributions are performed on the data (e.g., Gabriel and Feynman 1996). Furthermore, Robbrecht et al. (2009) presented results that suggest the possibility that the size of coronal mass outflows also follows a power law distribution.

Alternative applications of SOC processes to solar flares have included attempts to express the next-neighbor interactions of cellular automaton models in terms of discretized magneto-hydrodynamic (MHD) equations (Vassiliadis et al. 1998), by including photospheric magneto-convection and MHD turbulence (Georgoulis and Vlahos 1998), by including observed magnetograms and calculating linear force-free fields (Vlahos and Georgoulis 2004), by including nonlinear force-free fields (Dimitropoulou et al. 2011), or by including footpoint-driven random motion and magnetic braiding (Morales and Charbonneau 2008a, 2008b, 2009). Also magnetic reconnection processes have been applied to derive size distributions of SOC systems (Nishizuka et al. 2009). Flare phenomena are also connected with extreme space weather events, geomagnetic substorms, auroral displays, and solar wind fluctuations. Solar flares and coronal mass ejections are drivers of many phenomena in magnetospheric and heliospheric plasmas.

3.2 Solar Wind

The solar wind plasma is composed of electrons, protons, alpha particles, heavy ions, with typical energies of 1–10 keV, which escape the Sun's gravity field because of their high kinetic (supra-thermal) energy and the high temperature of the solar corona. The solar wind has two different regimes, depending on its origin, namely a fast solar wind with a speed of $v \lesssim 800\ \mathrm{km\,s^{-1}}$ originating from open-field regions in coronal holes, and a slow solar wind with a speed of $v \lesssim 400\ \mathrm{km\,s^{-1}}$ originating from low latitudes in the surroundings of coronal streamers. The dynamics of the solar wind was originally explained by Parker (1958) as a supersonic outflow that can be derived from a steady-state solution of the hydrodynamic momentum equation. Later refinements take the super-radial expansion of the coronal magnetic field, the average macro-scale and fluctuating meso-scale electromagnetic field in interplanetary space, and the manifold micro-scale kinetic processes (such as Coulomb collisions and collective wave-particle interactions) into account. The properties of the solar wind that can be measured from the solar corona throughout the heliosphere are plasma flow speeds, densities, temperatures, magnetic fields, wave spectra, and particle composition, which all exhibit complex spatio-temporal fluctuations. Most of the observations of the solar wind were made in-situ (with the Mariner, Pioneer, Helios, ISEE-3, IMP, Voyager, ACE, WIND, Cluster, Ulysses, or STEREO spacecraft), complemented by remote-sensing imaging (with STEREO) and radio scintillation measurements.

The dynamics of the solar wind is often characterized by the MHD turbulent cascade model. The solar wind power spectrum exhibits fully developed turbulence of the Kolmogorov type, $P(\nu) \propto \nu^{-5/3}$, in interplanetary space and near Earth (Fig. 15 in Aschwanden et al. 2014), while the input spectrum in the lower corona is of the $1/f$-noise type, $P(\nu) \propto \nu^{-1}$ (Matthaeus and Goldstein 1986; Nicol et al. 2009). The MHD turbulent cascade starts at the largest scales fed by MHD waves with a $1/f$-noise spectrum in the lower corona, while turbulent interactions produce a cascade of energy through vortices and eddies to progressively smaller sizes with a spectrum of $\nu^{-5/3}$, and final energy dissipation at the smallest scales by heating of electrons, with a spectrum of $\nu^{-11/3}$ (Meyrand and Galtier 2010). The analysis of MHD turbulence in solar wind data includes determining power spectra and structure functions, waiting time distributions of solar-wind bursts, identifying the phenomenology or MHD turbulence (Kolmogorov 1941; Kraichnan 1974), characterizing self-similarity and intermittency, and identifying the most intermittent structures, such as shock waves, small random events, current cores, and 1D current sheets (e.g., Horbury and Balogh 1997; Veltri 1999). With the Alfven wave as the dominant mode the solar wind is essentially considered a magneto-fluid, with the kinetic Alfven wave leading to plasma heating at the shorter scales (Sahraoui et al. 2010). However there are periods with strong Alfven turbulence in which the turbulence and particles seem to be in quasi-equilibrium even at intermediate scales, thus requiring kinetic modeling.

Recent interpretations of the dynamics of the solar wind include self-organization and SOC systems. The strongest argument for a SOC interpretation is the fact that power law size distributions were found for energy fluctuations ($E_B \propto B^2$), durations (T), and waiting times (Δt) in the solar wind (Freeman et al. 2000a; Moloney and Davidsen 2011). Although the power law shape of the waiting time distribution of solar wind bursts is not exponential, hence inconsistent with the original BTW model (Boffetta et al. 1999; Freeman et al. 2000a), it can be reproduced with a non-stationary Poisson process (Fig. 6 in Aschwanden et al. 2014; Wheatland et al. 1998; Aschwanden and McTiernan 2010). This feature is consistent with MHD simulations (Watkins et al. 2001; Greco et al. 2009a, 2009b).

The spatio-temporal structures of systems in SOC exhibit fractal features. The fractal nature of magnetic energy density fluctuations in the solar wind has been verified observationally (Hnat et al. 2007; Rypdal and Rypdal 2010, 2011). Moreover, solar wind turbulence is found to be multifractal, requiring a generalized model with multiple scaling parameters to analyze intermittent turbulence (Maczek and Szczepaniak 2008; Macek and Wawrzasek 2009; Macek 2010), although a single generalized scaling function is sometimes sufficient too (Chapman and Nicol 2009; Rypdal and Rypdal 2011). However, the fractal geometry of solar wind bursts deviate from self-similarity, since the ratio of kinetic (E_k) to magnetic energy ($E_B \propto B^2$) is frequency-dependent, with a magnetic energy spectrum of $\propto E_B^{-5/3}$ and a kinetic energy spectrum of $\propto E_k^{-3/2}$ (Podesta et al. 2006a, 2006b, 2007). It was suggested that the interplanetary magnetic field (IMF) is clustered (self-organized) by low-frequency magnetosonic waves, leading to a fractal structure with a Hausdorff dimension $D = 4/3$ and a power law power spectral density of the fluctuations $P(f) \propto \nu^{-5/3}$ (Milovanov and Zelenyi 1999). Also the clustering phenomena in the IMF were shown to have a feedback effect on underlying magnetosonic turbulence, generating a distinct branch of the excitations termed magnetosonic fractons (Zelenyi and Milovanov 2004). It should be noted that the fluctuation spectra of the solar wind show a break, yielding two characteristic slopes and are accompanied by features of intermittency (Riazantzeva et al. 2015).

3.3 Earth's Magnetosphere

The magnetospheric substorms are the most prominent manifestations of the storage and release of the solar wind energy and momentum in the magnetosphere. They have typical time scales of hours, distinct phases (growth, expansion and recovery), and prominent signatures, viz. aurora brightening, sudden changes in the auroral electrojet indices, turbulence, current disruption, plasmoid formation and release in the magnetotail. The complexity of the magnetosphere during substorms is now well recognized and arises from plasma processes which can be viewed from a dynamical systems perspective. Due to the inherent nonlinearity of plasma processes most linear processes such as instabilities develop into nonlinear states and the interaction among them leads to the cross-scale coupling and multiscale processes. The electrodynamic nature of the interaction between the different parts of the magnetosphere in the presence of the anchor dipole magnetic field leads to a global coherence in its dynamics. At the same time, the magnetosphere is an open system driven by the turbulent solar wind and consequently the dominant processes are essentially nonequilibrium in nature. These and other features of the magnetosphere indicate the limitations in the efforts to understand the complexity of magnetospheric substorms in terms of basic plasma processes alone. In spite of the distributed nature of the physical processes and their apparent irregular behavior, there is a remarkable coherence in the magnetospheric response during substorms and the entire magnetosphere behaves as a global dynamical system (Baker et al. 1990; Siscoe 1991) consistent with the low dimensionality obtained from time series data (Vassiliadis et al. 1990). This has led to a number of studies based on the low-dimensional nonlinear dissipative behavior of the magnetosphere, which exhibits complex and irregular behavior due to effects like dynamical chaos (Vassiliadis et al. 1990, 1991a, 1991b; Sharma 1995).

There are however many essential features of the magnetosphere beyond the low-dimensional behavior. The power spectrum of the AE index was studied using 5-min averaged data from 1967 to 1980 (Tsurutani et al. 1990). The power spectrum was found to have a break at about 5 hrs, and at frequencies less than this value the spectrum was close to $1/f$ and at higher frequencies the spectral index was -2.2. The same index for the solar wind, represented by the product VBs of the flow speed V in the sun-earth direction and the southward component Bs of the magnetic field component Bz, was found to be -1.42. The power spectral nature of the magnetospheric response was studied using the structure function, which characterizes the fractal nature, and the break in the spectrum was interpreted in terms of bicolored noise (Takalo et al. 1993). These results have shown that the fractal nature, and hence the self-similarity and scale invariance, are indications that the dynamics deviate from that of a simple low dimensional system. The multiscale and intermittent behavior of the magnetosphere has been investigated using many approaches and data sets. For example, the multifractal approach (Halsey et al. 1986) provides a characterization of the time series data of magnetospheric variables (Consolini et al. 1996; Consolini 1997). This result, based on the probability scale distribution computed from the AE index data, further emphasized the presence of the multiscale behavior in the magnetospheric dynamics. This has motivated a view of the magnetosphere as a complex system and SOC has been used as a new way to study its complexity and multiscale nature. The multifractal behavior also provides a way to distinguish the characteristics of the magnetosphere and the solar wind with the degree of multifractality computed from the data (Thomas et al. 2001; Setty 2014).

While many studies have been based on the auroral indices derived from the data from distributed ground magnetometer stations, many other studies have used the spacecraft data, notably from the fleet of the International Solar-Terrestrial Physics (ISTP) spacecraft. Studies of the magnetic field fluctuations during the disruption of the magnetotail current have shown power spectrum dependence (Ohtani et al. 1995, 1998). The plasma flows in the inner plasma sheet measured by Geotail and Wind spacecraft have been used to study the nature of the intermittency in the magnetosphere (Angelopoulos et al. 1999). The probability density of the magnitudes of the bursty bulk flows show power law dependence in time and their distribution is non-Gaussian, features consistent with SOC. A superposed epoch analysis of the plasma flows in the tail during substorm related current disruptions show power law dependence and multifractality, and this is consistent with SOC (Lu 1995). The UVI (UltraViolet Imager) images from the Polar spacecraft was used to study the nature of the dynamics during global auroral energy deposition events (Lui et al. 2000). In this study using more than 9000 frames of auroral images the internal scales of the magnetosphere were found to have the same power law in both quiet and active periods. The global energy dissipation during geomagnetically active periods however had a different scale. These features were interpreted as consistent with an avalanching system that exhibits criticality.

The power law nature of magnetospheric variables have two main origins, viz. the internal magnetospheric dynamics and the turbulent solar wind, and their inter-relationship has been studied using correlated datasets. Studies of the solar wind induced electric field VBs and the energy input into the magnetosphere have been found to have power law dependences (Freeman et al. 2000a). The analysis of the probability density functions of the solar wind variables also show power law dependences. The power law form of the inter-burst intervals in the solar wind was found to be distinct from that of ideal SOC but not from SOC-like sandpile models. This result has a wider implication on the signatures of SOC. For example, solar flares have been considered to be in SOC based on the power spectral form (Lu and Hamilton 1991; Einaudi and Velli 1999). This brings to focus the indistinguishability from turbulence (Boffetta et al. 1999). These results are in apparent conflict and their resolution will lead to a better understanding of SOC, related models and their distinction from turbulence, all of which have power spectral dependences (Aschwanden et al. 2014). When identifying a system to be in SOC it is of crucial importance "to demonstrate a property unique to the process of self-organization to criticality rather than simply observing the avalanche phenomena that SOC was designed to account for" (Freeman et al. 2000b).

3.3.1 SOC Models of Magnetospheric Substorms

The data of magnetospheric phenomena from ground and spacecraft measurement show power law dependence in various forms (Aschwanden et al. 2014). These include spacecraft images of auroras (Lui et al. 1988; Uritsky et al. 2002), all-sky camera images (Kozelov et al. 2004), magnetic field measurements in the magnetotail (Angelopoulos et al. 1996, 1999; Hoshino et al. 2005), ionospheric velocity fluctuations (Bristow 2008) and microsatellites data (Crosby et al. 2005). The early applications of SOC to Earth's magnetosphere largely used the sandpile model of avalanches, which is a simple way to visualize the dynamical behavior of systems made up of large number of interacting parts. A sandpile builds up by addition of sand grains and as it gradually gets steeper avalanches begin to occur, becoming bigger as the pile grows. The sandpile eventually reaches a critical state and the system regulates itself by balancing the accumulated amount of sand by that carried away by avalanches. The avalanching behavior exhibited by model sandpiles have motivated many models of magnetospheric dynamics which are essentially cellular automaton models with the simplified rules chosen from considerations of the known properties of the system. They can be put into the categories of sandpile models (Chapman et al. 1998; Uritsky et al. 2001a, 2001b) and coupled-map lattice models (Takalo et al. 1999a, 1999b). Another category of models have sought a closer connection to magnetospheric physics, e.g., Liu et al. (2006) developed a 1-D cellular automaton based on the notion that energy is slowly added to the central

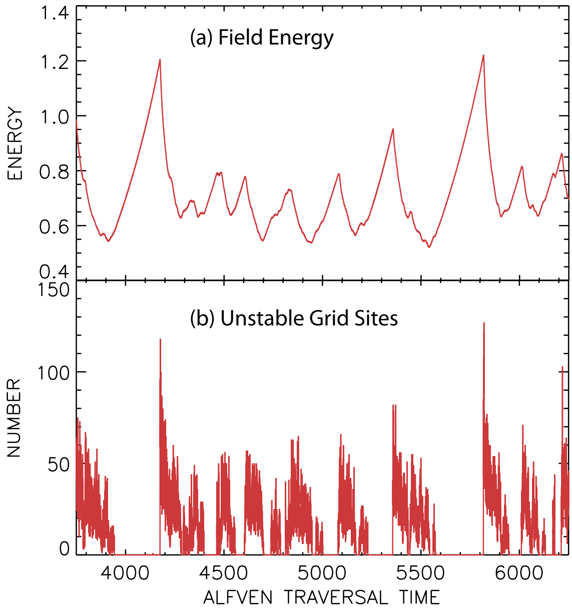

Fig. 1 Illustrating the loading-unloading cycle in the 2-D driven current-sheet model. (**a**) Total magnetic field energy and (**b**) number of grid sites at which current-driven instability is excited. Field energy grows during quiet intervals and decays rapidly during avalanching intervals. Taken from Klimas et al. (2004)

plasma sheet. In the event of an instability, a part of the energy of the unstable node was redistributed to the rest of the lattice. This scheme produced avalanches obeying a scale-free distribution that co-existed with quasi-periodic intermittencies. This model was generalized into 2-D by Vallières-Nollet et al. (2010). Finally Liu et al. (2011) presented a model using the z-component of the magnetic field as the basic dynamical element of a cellular automaton. This numerical model produced spatially and temporally intermittent, avalanche-like release of magnetic energy, with frequency distributions of avalanche size parameters in the form of power laws.

The results of the spatiotemporal UVI emissions analyses (Uritsky et al. 2002, 2003, 2006, 2008; Kozelov et al. 2004) have shown that bright emissions in the night-side aurora evolve as avalanches in cellular automata models of SOC. Energy, size, and duration probability distributions, growth and survival dynamic distributions, relationships between energy, size, and duration, fractal properties such as roughness, growth, and boundary dimensions, and more have been shown to exhibit power law forms over significant ranges. Moreover, the defining power law exponents have been shown to be interrelated, satisfying important scaling relations that should be satisfied for systems in SOC.

To link these observations with plasma dynamics in the magnetotail, Klimas et al. (2004, 2005) have developed a 2-dimensional numerical driven current-sheet model that incorporates an idealized representation of a current-driven instability into a resistive MHD system. The instability is excited when and where the MHD current density exceeds a critical threshold and its excitation leads to the growth and saturation of an anomalous resistivity-producing wave-field; the resistivity is fed back into the resistive MHD system through a simple hysteretic equation adapted from Lu (1995). It has been found that this hysteresis leads directly to a global loading-unloading cycle in the model (Klimas et al. 2005) and to scale-free avalanching in the transport of electromagnetic (primarily magnetic) energy through the model (Fig. 1). The avalanche statistics displayed by the model are similar to those of the auroral emission regions (Figs. 2 and 3); the ranges of scales and power law indices in the auroral distributions are well represented. The manner in which the statistics

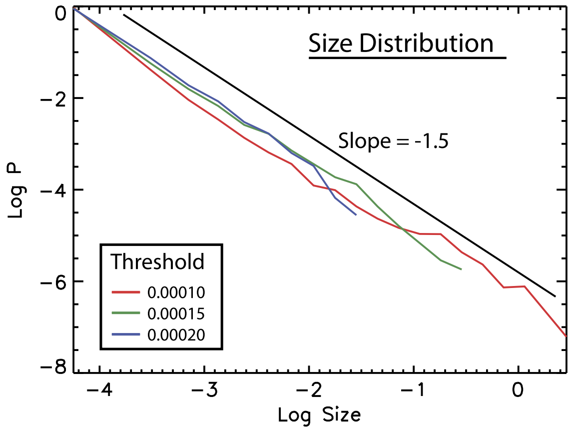

Fig. 2 Size probability distribution of high-transport (HT) regions in the evolving Poynting flux magnitude. The HT regions have been constructed using three threshold values as indicated. Taken from Klimas et al. (2004)

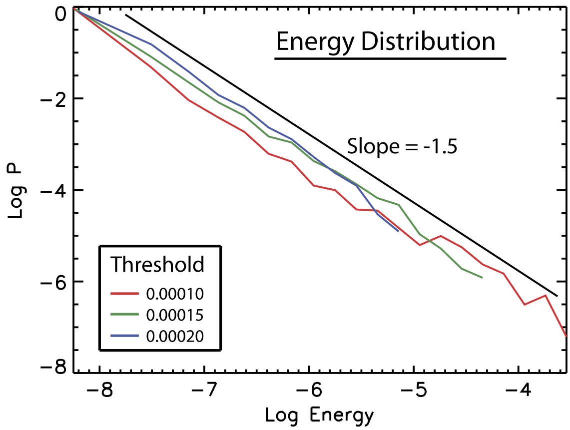

Fig. 3 Energy probability distribution of high-transport (HT) regions in the evolving Poynting flux magnitude. The HT regions have been constructed using three threshold values as indicated. Taken from Klimas et al. (2004)

of the magnetotail dynamics are mapped into those of the auroral emission regions is unexplored at present but the similarity of the 2-D current-sheet model statistics and those of the auroral dynamics revealed in these studies suggests that the mapping may produce only small modifications.

The idea of SOC has been extended to include the forcing in the system and such a system may then exhibit forced SOC (FSOC) and/or SOC. An application of the renormalization group approach to FSOC has given the power law index for the avalanches in the sandpile model of the magnetosphere (Tam et al. 2000). The renormalization group technique, developed in the study of critical phenomena (Fisher 1974; Wilson 1975), is based on scale invariance and is implemented through recognition of similar patterns at different scales by continued coarse-graining and rescaling of the system. The main objective of this technique is to obtain the critical exponents characterizing the critical state, and thus holds promise of yielding deeper insight into the criticality in the solar wind-magnetospheric coupling.

The recognition of the role of criticality in magnetospheric dynamics has motivated the development of models based on simplified physics considerations. While these models are not fully self-consistent they provide a means to study the relative roles of different physical processes known to be important in the magnetosphere. The WINDMI model (Horton et al. 1999) is based on the physical processes relevant to the coupled solar wind-magnetosphere-ionosphere system and describes many global and multiscale features. This low dimensional model naturally describes the global dynamics and the multiscale features arise due to the sequence of inverse bifurcations. A model of localized reconnection in the magnetotail based on resistive MHD considerations (Klimas et al. 2000) yields a power law spectrum and different features of internal and global dynamics. The model consists of simple equations of the type of a forced nonlinear diffusion equation, and the characteristics of the system under different forcing conditions are studied. The strength and nature of the forcing naturally plays an important role and the model yields avalanche phenomena, which is consistent with SOC.

Percolation theory is intimately connected with the fractal structure and multiscale behavior and substorms have been modeled in terms of a percolating network of cross-tail currents (Milovanov et al. 2001a, 2001b; Arzner et al. 2002). In this scenario the network has different levels of fractal measure and at a critical level the structural stability breaks down and the onset corresponds to a topological phase transition. In a related approach, the plasma turbulence in the distant magnetotail has been found to have self-organizing properties and these have been described in terms of fractal geometric properties (Zelenyi et al. 1998). The power spectrum of the magnetotail dynamical response to a white noise-like forcing is obtained using the fluctuation-dissipation theorem and universality of ac conduction at the percolation point (Milovanov and Rasmussen 2001). The result of this calculation gives a power spectral density distribution that is a power law: $S(\omega) \propto \omega^{-4/3}$ (Milovanov 2013), where ω is the frequency and is smaller than the characteristic turn-over frequency imposed by convection (~ 0.01 Hz). By analogy with dielectric-relaxation phenomena it has been shown that the spectrum $S(\omega)$ represents the power spectral density of the magnetic field fluctuations and more precisely of their normal components. A lattice toy-model for the magnetotail SOC current sheet is obtained as a transport model for electric charges of different kinds (conditionally described as free charges, polarization charges, and holes) on a self-adjusting percolation cluster (Milovanov 2010, 2011). The model, dubbed dynamic polarization random walk (DPRW) model, belongs to the universality class of the Bak-Tang-Wiesenfeld sandpile and relies on the established mathematical formalism of random walks in a fractal geometry (e.g., Havlin and ben Avraham 2002). It is found using the DPRW formalism that the relaxation of a supercritical state to SOC is non-exponential (non-Debye) in that it involves multiple relaxation events with a broad distribution of relaxation times (Milovanov 2009, 2013). Indeed the relaxation dynamics corresponded to a Mittag-Leffler relaxation pattern and fractional relaxation equation with long-time memory (see review: Metzler and Klafter 2000). These studies have led to the conclusion that the dynamical response of the geomagnetic tail as a complex system is non-Markovian by nature and involves long-time correlation effects consistent with the implication of SOC.

It is understood that the time varying magnetic perturbation generates a time and spatially varying electric field because of Faraday's law. An important feature which arises in this induction process is gradual heating and energization of the plasma, leading to the occurrence of slowly decaying, high-energy non-thermal wings in the particle energy distribution function (Milovanov and Zelenyi 2001, 2002; Zelenyi and Milovanov 2004). The phenomenon was demonstrated in direct numerical simulation of multiscale electromagnetic turbulence in Zelenyi et al. (2008). Often in magnetospheric plasma research those distributions with wings are modeled by non-thermal the so-called "kappa" distributions (e.g., Christon et al. 1989; Ma and Summers 1998) which interpolate between the initial low-energy exponential forms and the asymptotic inverse power law behavior. The significance of the "kappa" distributions lies in the fact (Milovanov and Zelenyi 2000) that they appear as canonical distributions in the non-extensive thermodynamics due to Tsallis (1988). A defining feature of the Tsallis entropy is its *non-additivity*; implying that there is a deficit or surplus of entropy when two subsystems are merged. Then the entropy deficit case accounts for the macroscopic ordering phenomena beyond the range of applicability of the conventional statistical mechanical paradigm of the Gibbs type, enabling self-organization (Milovanov and Zelenyi 2000).

The SOC models of the multiscale behavior of the magnetosphere are inspired by the observational features such as the power law spectra of auroral electrojet indices and are based on simplified physics considerations. In this sense they are essentially like sandpile models, and like most such cellular automaton models (Jensen 1998) characterize the system in terms of the local slope, which is updated using a chosen rule, which to a large extent may be arbitrary. The distribution of the avalanches in such models and its comparison with the data of the physical systems usually yields a measure of the SOC model. The first such model (Chapman et al. 1998) showed two types of avalanches, corresponding to internal reorganization, with power distribution and SOC behavior, and to systemwide discharges which do not exhibit SOC. Subsequently, SOC as the framework for combining the globally coherent and multiscale features of the magnetosphere has been tested using different considerations (Watkins et al. 1999, 2001). Among the features considered are the power spectra and breaks in them, avalanche and lifetime distributions, and intermittency. These features are evident in the SOC models as well as in the fractional Brownian motion.

While the SOC models of substorms have given many interesting results, it should be noted that real sandpiles may behave in a manner more reminiscent of a first-order transition similar to the fold catastrophe than a second order one (Nagel 1992). As noted earlier, in the case of substorms similar deviations from the simplest SOC picture are evident in the form of the statistics of chorus events seen in the ground-based data and spacecraft data of particle injections in the near-Earth magnetosphere (Borovsky 1993; Pritchard et al. 1996; Smith and Horton 1998), which showed that the intensity and occurrence rate of substorms have a probability distribution with a well defined mean. Independent studies of SOC models have shown that the critical points in some of them are not attractive. Also typical SOC models imply some specific tuning of either the state (Gil and Sornette 1996) or control (Vespignani and Zapperi 1998) parameters. The substorms seem similar to the case of sandpile experiments in which only relatively small avalanches (from 3 to 80 grains) (Jensen 1998) demonstrate power law behavior, while larger avalanches to the contrary demonstrate the distinctive features of the first-order transitions including the hysteresis phenomenon.

A SOC model consistent with the sandpile experiments has been proposed within the framework of Landau-Gizburg theory of SOC (Gil and Sornette 1996). Similar advanced SOC models have taken into account the global coherence of the dynamics (Chapman et al. 1998). More recently, a general theoretical scheme for SOC has been formulated, using the idea of discrete Anderson nonlinear Schrödinger equation (DANSE) with randomness and self-adjusting nonlinearity (Milovanov 2013; Milovanov and Iomin 2014). The model is characterized by a non-perturbative feedback between the probability of site occupancy on a lattice and the strength of nonlinear interaction, generating SOC dynamics in Hilbert space. Also the DANSE equation has been applied to predict delocalization of dynamical chaos by nonlinear interaction and a critical strength of the nonlinearity parameter permitting unlimited spreading of the wave function in random lattices (Milovanov and Iomin 2012, 2014).

3.3.2 Phase Transition-Like Behavior of Substorms

The characteristic feature of magnetospheric substorms is the sudden change in the state of the magnetosphere, thus highlighting its similarity to phase transition. The correlated data of the coupled solar wind-magnetosphere compiled by Bargatze et al. (1985) has been used to study the phase transition-like behavior of substorms (Sitnov et al. 2000; Sharma et al. 2001). The dataset consists of 34 intervals of correlated solar wind input and the magnetospheric response as an output. The solar wind input is the induced electric field VBs, where Bs is the southward component of the interplanetary magnetic field (IMF) and V is the component of the solar wind velocity along the Earth-Sun direction. The magnetospheric response to the solar wind is represented by the auroral electrojet index AL.

The reconstruction of magnetospheric dynamics from the time series data using time delay techniques (Abarbanel et al. 1993) have provided the key features such as effective low-dimensionality (Vassiliadis et al. 1990, 1991a). Among the techniques for the reconstruction of dynamics, the singular spectrum analysis (SSA) (Broomhead and King 1986) provides an orthonormal vector space whose metric is well defined. This technique was used to reconstruct the autonomous dynamics of the magnetosphere from the auroral electrojet indices (Sharma et al. 1993). The singular spectrum analysis is based on the singular value decomposition and uses the properties of the trajectory matrix constructed from the time series data by time delay embedding. The VBs-AL data can be used to construct the time delay vectors and consequently the trajectory matrix. Since the averaging time for this dataset is 2.5 min the value of the embedding dimension m of the reconstructed space was chosen to be 32 to provide a time window of 80 min, appropriate for substorms. The singular value decomposition of this matrix provides orthogonal eigenfunctions corresponding to different eigenvalues. The original idea of the autonomous version of SSA was that there should be a noise floor and the number of eigenvalues with magnitudes greater than the noise floor is an estimate of the effective dimension of the system. In many real systems the SSA spectrum has a clear power law form, with no clear noise floor. In such cases, the limited number of SSA projections may serve as a good approximation of the system, as a mean-field or Landau approximation often used in phase transition theory as a zeroeth-level approximation (Sitnov et al. 2000). This approximation may then eliminate those features connected with multiscale or high dimensional behavior corresponding to either SOC or second order phase transitions.

In the reconstruction of dynamics using SSA the time-delay embedding is used to create a m-dimensional space from the time series data and a singular value decomposition of the resulting trajectory matrix yields a orthonormal space characterized by the singular eigenvectors and corresponding eigenvalues. A projection of the trajectory matrix on this space then yields the principal components that embodies the dynamical features inherent in the data. Although there are m principal components corresponding to the time-delay embedding space of the same dimension, in most cases only a small number of leading principal components are needed to model the dynamical behavior. The first principal component P1 is the measure of the solar wind input averaged over the interval of about 80 min while the second principal component P2 is of the similarly averaged AL index. The third component P3 reflects the changes in the input with time. Figure 4 shows the global dynamics of the magnetosphere given by the leading components (P1, P2, P3) derived from the first 15 intervals of Bargatze et al. (1985) dataset. The set of points representing the trajectory of the magnetosphere in this 3D space lie on a 2D surface (Sitnov et al. 2000; Ukhorskiy 2003). Also shown in Fig. 4 are the arrows showing the circulation flows, which represent the evolution of the system. The growth phase of substorms is reflected in Fig. 4 by the right hand side of the surface (marked red), while the recovery phase corresponds to the left hand side (green). The substorm onset is located close to the middle of the figure where the transition occurs.

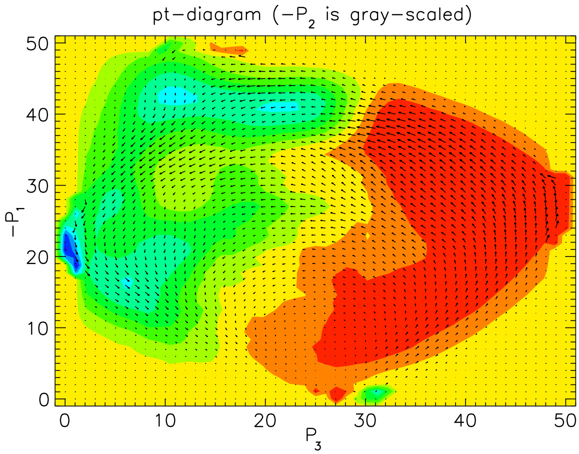

Fig. 4 The global features of magnetospheric dynamics obtained from the correlated solar wind-magnetosphere data. The two-level structure consists of the higher (*red*) and lower (*green*) levels, and the *arrows* show the flow vectors. The sudden changes in the state show the phase transition-like behavior of the magnetosphere (Sitnov et al. 2000)

The reconstructed surface shown in Fig. 4 resembles the so-called temperature-pressure-density (TPD) diagram typical for the equilibrium first order phase transitions. The dynamical or nonequilibrium transitions exhibit a hysteresis phenomenon in which different values of output parameter (AL index) may correspond to the same set of input parameters (VBs). Such features are important characteristics of the critical behavior or of the second order phase transition. However in the surface shown in Fig. 4 such episodes in the original data set have been excluded in order to bring out the features of first order phase transition clearly. The TPD diagram and the corresponding circulation flows are very similar to the model of a substorm as a cusp catastrophe proposed earlier by Lewis (1991).

Geomagnetic indices are widely used in magnetospheric studies, including nonlinear dynamical studies and prediction. They have been derived continuously from the data from arrays of ground-based magnetometers following well-known procedures. However the indices are known to have many limitations due to the manipulations of the original data by averaging or taking the envelope of many different records. These data handling techniques may introduce artificial features and cause the loss of dynamical information. They also complicate the task of relating the dynamical models based on these indices, the models based on the physical parameters and the studies of the fundamental processes. Another limitation of conventional indices is that they represent only the ground-based measurements of the basic processes, which take place deep in the magnetosphere, and thus are remote sensing data. With the availability of multi-spacecraft data of the magnetosphere, the indices can be substituted by more physical parameters directly related to basic mechanisms of geomagnetic activity. A multi-spacecraft database consisting of the solar wind parameters taken from Wind spacecraft and the magnetic field from Geotail and Interball has been compiled to enable studies using in-situ data (Sergeev et al. 2000). These spacecraft spend limited periods in the lobes and this poses the main limitation on the data base, which consists of seven intervals of nearly continuous measurements each around two or three days in length. To make the data less dependent of the spacecraft two procedures have been adopted. First, based on the results of Petrukovich et al. (1999) the effective lobe magnetic field was calculated using a simple pressure balance formula, which seems to be a good estimate of the lobe field even if the spacecraft is located in the plasma sheet. This procedure was used mainly for Geotail data. Second, the measurements have been reduced to a location along the tail axis using the statistical formula of Fairfield and Jones (1996). The seven datasets are from the period December 1995 to December 1996, and show clearly the global coherence in the

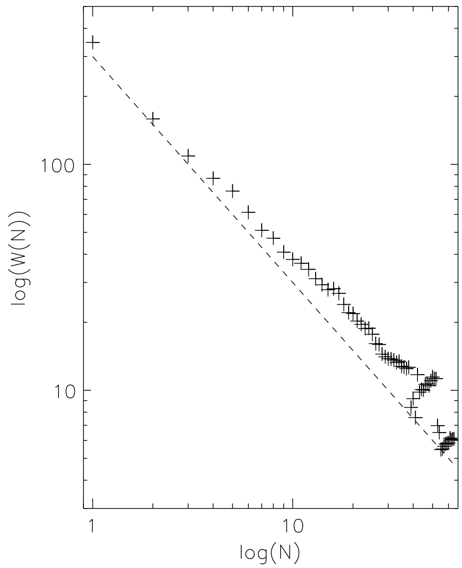

Fig. 5 Singular spectrum for the first 15 intervals of Bargatze et al. (1985) dataset (Sitnov et al. 2000)

magnetosphere. The phase transition manifold was reconstructed from the spacecraft data using the techniques of time delay embedding and singular spectrum analysis. The result shows significant differences from Fig. 4 reconstructed from VBs-AL data, due mainly to the differences in AL and lobe magnetic field. While AL undergoes a sudden drop during a substorm expansion phase and a slow increase during the recovery phase, the lobe field changes were found to be gradual in both the phases (Sharma et al. 2000).

The multiscale behavior of the magnetosphere is evident from the singular spectra of the ground-based data. The eigenvalue spectrum for the first third of the Bargatze et al. (1985) dataset is shown in Fig. 5. The power law nature of the eigenvalues is evident for the significant part of the distribution. A similar spectrum has been obtained for the multi-spacecraft data described above (Sitnov et al. 2000) and shows clear power law dependence with an exponent close to unity, in agreement with the traditional interpretation of critical behavior (Jensen 1998). These features have been studied using the probability distribution functions (pdf) of magnetospheric response to different levels of solar wind driving. The pdfs are readily computed using the AL index as the magnetospheric response to the solar wind electric field VBs, and for the Bargatze et al. (1985) dataset they exhibit different features for different strengths of the solar wind driving (Ukhorskiy et al. 2004a, 2004b). Further analysis using larger datasets show similar features, are shown in Fig. 6. The magnetospheric response is more peaked during strong driving by the solar wind, and there is a clear absence of a simple power law dependence at all levels of driving. The absence of scale-free behavior with a single exponent over the whole range of data and multiple scaling indices are needed to describe the entire spectrum (Hnat et al. 2007; Zaslavsky et al. 2007, 2008).

The theory of phase transitions is a natural framework for the solar wind-magnetosphere coupling mainly because of its input-output nature. The phase transition picture provides a number of relationships between the control (input) and order (output) parameters in terms of critical exponents. These relationships provide a new way of understanding the solar wind-magnetosphere coupling. However, there are many open questions, due mainly to the nonequilibrium nature. Contrary to equilibrium systems, where the critical exponent is determined by the form of the coexistence curve (Stanley 1971), the nonequilibrium system

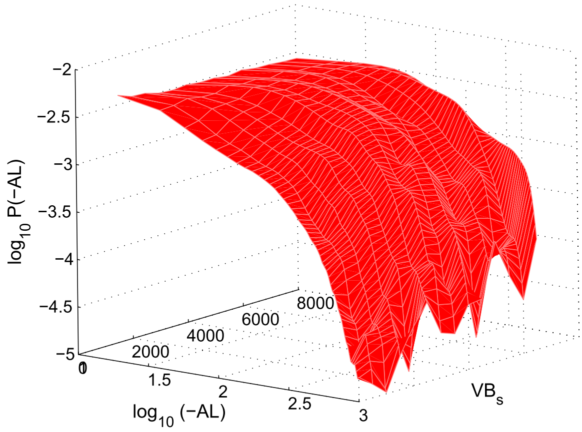

Fig. 6 Dependence of magnetospheric activity on solar wind driving. The probability distribution function of AL index is shown for different values of the solar wind induced electric field VBs. There is no clear scaling with a single exponent for all levels and the deviation from scaling behavior is larger for stronger driving by the solar wind (Sharma et al. 2008)

may often reach the so called spinodal curve corresponding, for instance, to overheated water or overcooled steam (e.g., Gunton 1984). Moreover, due to the finite rate of variation of the control parameters of the system it passes the spinodal curve at finite rate, so that even this metastable state can not be used effectively for determining an analogue of a critical exponent. In an attempt to address such problems a exponent for the solar wind-magnetosphere system was computed from the VBs-AL data (Sitnov et al. 2001).

An important aspect of the reconstruction of phase or state space from time series data of the magnetosphere is its predictive capability (Sharma 1995; Vassiliadis 2006). The first space weather forecasts were made with this technique using the time series data of the geomagnetic indices (Sharma 1995; Vassiliadis et al. 1995; Valdivia et al. 1996), and provided new advances in the understanding of the global and multiscale behavior (Ukhorskiy et al. 2002, 2004a, 2004b; Ukhorskiy 2003; Chen and Sharma 2006; Chen 2007). The nonlinear dynamical modeling and forecasting of space weather using data from a distributed network of magnetometers led to forecasts of the local space weather (Valdivia et al. 1998, 1999; Chen et al. 2008).

3.3.3 Mixed Models: The SOC-Coherent Model

Magnetospheric substorms bear signatures that enable their association with phase transition-like phenomena in the presence of a competing nonlocal ordering (Milovanov and Rasmussen 2005) and that involves other than SOC, a coherent component (Sharma 1995; Chang 1999a, 1999b) which evolves predictably through a sequence of clearly recognizable phases (Baker et al. 1999). The main idea here is that some systems may spontaneously turn into a coherent state before they become SOC, since their dynamical evolution by itself drives these systems into a competition between the SOC and coherent properties as a consequence of subordination between the order parameters (Milovanov 2013). The model, dubbed SOC-coherent model of substorms, is cast in the general theoretical scheme of fractional Ginzburg-Landau equation (Milovanov and Rasmussen 2005), where by “fractional” we mean integro-differential generalizations (Samko et al. 1993; Podlubny 1999) of the gradient term involving algebraically decaying kernels. The fractional Ginzburg-Landau equation is obtained by assuming that the order parameter due to SOC acts as input control parameter for the competing ordering, so that the coherent component is *subordinate* to SOC (Milovanov 2013). Other than substorms, this general approach may include phenomena like the L-H transition in magnetic confinement devices (Freidberg 2007), and the tokamaks as particular case, where the L-phase (low confinement) is associated with SOC, and the H-phase (high confinement) is associated with spontaneously occurring coherent state.

4 Plasmas of Fusion Interest and Laboratory Plasmas

An introduction to the main magnetic configurations currently under investigation as potential fusion reactors can be found in, for instance, Freidberg's book (2007): the tokamaks and their closest relatives the stellarators; the levitated dipole; the field reversed configuration; the reversed field pinch; and finally the spheromak. The tokamak is an axisymmetric configuration with a large toroidal magnetic field and a significant toroidal current. Tokamaks have achieved stable operation at near reactor relevant pressures, temperatures and confinement times and in terms of physical performance have met nearly all requirements for the future power plants. ITER and the future demonstration power plant DEMO will use this configuration to confine and control the hot thermonuclear plasma. It has been discussed that tokamak plasmas in fusion-relevant conditions will squeeze the plasma profiles to their marginal stability, where a SOC-like phenomenology might be expected. Although similar behaviors might (and actually will) be possible in the stellarator configurations as well, it is the outstanding significance of the tokamaks in the contemporary fusion research that is the prime reason why the tokamak confinement has been more often tested for SOC. Even so, the prospective SOC behaviors have been studied for a variety of controlled plasma systems under certain conditions (whether fusion relevant or not), e.g., in simple discharge plasmas (Nurujjaman and Sekar Iyengar 2007); complex (dusty) plasmas (Ratynskaia et al. 2006); and in other laboratory experiments (Pruessner 2013 for a brief review).

4.1 Motivating SOC for Magnetically Confined Fusion Plasma

Understanding transport of particles and heat in tokamaks and stellarators is crucial to their role as prospective fusion reactors. The topic has received considerable attention and study since the beginning of the fusion venture in the late 50s. Many reviews on the general progresses in the field are available (e.g., Wootton et al. 1988; Carreras 1997; Politzer et al. 2002; Huysmans 2005; Diamond et al. 2005; Zonca et al. 2006, 2015; Doyle et al. 2007; Heidbrink 2008; D'Ippolito et al. 2011).

An important feature of a toroidally confined plasma is the existence of critical density and temperature gradients above which the transport rate increases drastically, thus eroding the profiles until they return within their marginally stable shapes, and with the radial transport being again reduced after this has occurred. Nonlinearly, the transport is controlled by the dissipation or loss processes at the plasma edge absorbing the excess free energy from the system. When the conditions allow the profiles to wander in the vicinity of these critical thresholds, the radial transport may exhibit the phenomena of profile stiffness or resilience (e.g., Wesson 2004), and will typically occur in the form of avalanches. It is at this point, where the superdiffusive transport (and the often associated self-similarity features in space and time) will come into play. This phenomenology has motivated the development of theoretical approaches involving the concept of SOC (e.g., Diamond and Hahm 1995; Carreras et al. 1996; Newman et al. 1996; Dendy and Helander 1997; Sanchez et al. 2001; Milovanov 2010; Woodard et al. 2007). The main issue here is that tiny local perturbations to the system trigger a sequence of avalanche like events that dynamically drag the profiles close to marginality. It should be noted, however, that the finite inertia and other finite-size effects lead to the conclusion that the quiescent local slope does not really sit at the marginality itself, but just below it on average. This fact differentiates SOC from other marginal stability models, thus endowing the dynamics with long-time correlations. It is believed that the avalanches are indeed responsible for many of the superdiffusive features observed, and self-organization is capable to equip the transport processes with self-similarity in space and time. By their construction, SOC models are usually less sensitive to the underlying parameters, such as the specific instabilities driving the turbulence, and have the potential to predict some of the statistical and the correlation properties of the associated dissipation events. These questions of stiffness and criticality are of interest for next-step fusion devices such as ITER, where the marginally stable state is expected to occur with relative ease due to the large temperatures involved.

The existence of a dynamic steady state, wandering around its marginal stability, in which the excess particles and free energy are naturally dissipated to boundaries, provides a good candidate for regimes normally foreseen in the low confinement mode (L-mode) of the tokamak operation. But it might also be important in the high confinement modes (H-modes), at least, well inside the edge transport barrier. In this regard, SOC is expected to be an essential key ingredient at describing and controlling the radial superdiffusive transport and the edge-core coupling phenomena. Another important aspect of SOC models is that they are well suited to accommodate the characteristic anomalous scaling of the energy confinement time with the system size, warning against noticeably poorer confinement when compared to a diffusive (collisional or Gaussian quasi-linear) scaling (e.g., Carreras et al. 2001; van Milligen et al. 2004; del Castillo-Negrete 2006; Mier et al. 2008). All in all, they represent a "practical" and valuable approach to construct effective transport models with predictive capabilities under certain conditions.

It has been discussed, based on the existing tokamak phenomenology, that instabilities of low confinement mode plasma generate noises of the Lévy type (as opposed to Gaussian white noise processes: Gnedenko and Kolmogorov 1954) and the associated bursting transport events above a certain threshold value of the average temperature gradient (Milovanov and Rasmussen 2014; references therein). Below that level the transport is slow in that it obeys the conventional transport paradigm of the Gaussian type. This hypothesis has been tacitly assumed in several heuristic so called "fractional" transport models of tokamak plasma turbulence (van Milligen et al. 2004; del Castillo-Negrete 2006; del Castillo-Negrete et al. 2004; Mier et al. 2008; Sect. 4.1.3 below). The existence of critical density and temperature drop-offs in the tokamak L-mode finds support in perturbative experiments with plasma edge cooling and heating power modulation (Mantica and Ryter 2006) and very specifically in some asymmetry between the propagation of perturbations due to heat modulation and cold pulses (del Castillo-Negrete et al. 2008; Naulin et al. 2009; Milovanov and Rasmussen 2014). These kinds of experiments in which both types of perturbations, cold pulses and power modulation, are present for the *same* plasma, have been carried out in JET (Mantica and Ryter 2006). There are several open questions, however, regarding the *in situ* generation of noises of the Lévy type by the intrinsic dynamics of the plasma as implied by the underlying equations of the motion (Fogedby 1994; Jespersen et al. 1999). Several model approaches addressing this fundamental problem have been explored in specialized settings (e.g., Sanchez et al. 2006; Mier et al. 2008), most recently using the ideas of stimulated vortex formation and convective amplification of the turbulence (Milovanov and Rasmussen 2014, 2015). We expect these novel ideas to trigger further research in the coming years.

Other topics currently under investigation involve the transport of momentum and its impact on SOC (Ida and Rice 2014); also in association with the nonlinear dynamics of zonal flows (Diamond et al. 2005); the phenomena of turbulence spreading (Mattor and Diamond 1994; Gurcan et al. 2005; Naulin et al. 2005) and spontaneous rotation reversal (Rice et al. 2012); transport by coherent structures (Naulin et al. 1999), and by blob-filaments as particular case (D'Ippolito et al. 2011); the study of avalanching transport driven by the energetic particles in burning plasma (Chen and Zonca 2007; Zonca et al. 2005, 2006 and 2015).

4.1.1 Experimental Evidence for SOC

There have been a number of studies aiming to prove or disprove the implication of SOC in magnetically confined toroidal plasma. Historically, those studies have looked for avalanches as well as for the direct evidence for memory and self-similarity through time series. Part of the evidence comes from the observation of long-time correlations in plasma edge turbulence (e.g., Carreras et al. 1998, 1999; Budaev et al. 2008). It was argued that the power spectra of the associated noise processes decayed as an inverse power law $S(f) \sim 1/f^\beta$, with $0 < \beta \leq 1$, and that the observed behaviors were universal among the different confinement devices (tokamaks and stellarators). Most of these measurements were made in the edge regions of the various fusion devices, which are accessible to probes and where signals are large. The analysis of the local turbulent fluctuations and fluxes involved the estimates of the Hurst exponent H (a measure of long-term memory in time series[1]) (Carreras et al. 1998; Wen-Hao et al. 2001; Wang et al. 2004; Xu et al. 2005; Carralero et al. 2011; dos Santos Lima et al. 2012), the search for power law regions of decay in the respective power density spectra (Pedrosa et al. 1999; Rhodes et al. 1999; Carreras et al. 1999; Wen-Hao et al. 2001; Wang et al. 2004; dos Santos Lima et al. 2012) and in the waiting-time statistics (Sanchez et al. 2002, 2003; Xu et al. 2005; Carralero et al. 2011), using both rescaling (Carreras et al. 1999; Yuan et al. 2003; Jha et al. 2003; Wang et al. 2004) and multifractal analysis (Carreras et al. 2000; Budaev et al. 2008). Results from those investigations revealed the presence of a meso-range of scales, roughly located between $(10^{-2}\text{–}10^{1})$ ms, where the existence time self-similarity and memory could be well established, consistently with the idea that the edge plasma operates as part of a SOC system, at least in L-mode or Ohmic regimes.

The edge region of confinement devices, e.g., tokamaks and stellarators, share many features with boundary layers in space, e.g., the boundary of the magnetosphere. For example, the transport barrier in the edge region of Asdex tokamak has close similarity with the measurements at the magnetospheric boundary made by the Cluster spacecraft. These regions are turbulent boundary layers and they exhibit many features distinct from those expected from SOC (Budaev et al. 2011). The plasma turbulence in such boundary layers exhibit features such as intermittency and multifractality, which are features that are not compatible with the SOC model that has a power law dependence with a single exponent. It should be noted that the research in turbulent transport in tokamaks is extensive and uses many approaches and models (Benkadda 2008). The understanding of transport, and possibly its control, is critical to the performance of laboratory confinement devices and some of these issues are addressed with the SOC approach.

Concerning transport in the bulk plasma and SOC, the evidence is more scarce due to the problematic accessibility to the region. There has been some evidence of avalanche like transport in the core of the DIII-D tokamak (Politzer 2000; Politzer et al. 2002). These

[1] The Hurst exponent H is related to the spectral index β via the Berry formula $\beta = 2H + 1$. $H = 1/2$ corresponds to white noise, with $\beta = 0$; $H < 0$ to fractional Brownian noise; $H = 0$ to flicker noise; $0 < H < 1$ to fractional Brownian motion.

observations were primarily with the electron temperature fluctuations. Avalanches were identified by their large spatial scale, up to the system size, by the power law behavior of their frequency spectrum and their radial autocorrelation function, and by their radial propagation. Power transport estimates have shown that about half of the heat flux is carried by the avalanche like events under conditions with no magnetohydrodynamic activity. Also Politzer et al. (2002) note that the reported observations are qualitatively similar to results of modeling calculations based on drift-wave turbulence and that SOC might not be the only possible explanation of the avalanching behavior, since several transport mechanisms were simultaneously operative in the plasma. Other authors have analyzed time traces of temperature and density measured at the plasma core by means of different diagnostics (for instance, electron cyclotron emission measurements or reflectometry). Those studies have revealed similar features to those already found at the edge of other devices, such as the characteristic, $1/f$-like bands in the noise power spectra (Rhodes et al. 1999; Politzer et al. 2002; Gilmore et al. 2002; Yu et al. 2003).

Despite the aforementioned progress, effort is still needed to understand the implication of SOC in the case of *core* plasma fluctuations and associated transport. One reason for this is the possible presence of competing transport channels, such as for instance heat pinch phenomena (and other forms of convective transport). Heat avalanches are intermittent bursty transport events with long spatio-temporal correlations and a $1/f$ power spectral density distribution. This type of transport has received considerable attention and study (e.g., Garbet et al. 2004; Naulin et al. 2005; Tokunaga et al. 2012). Mathematically, heat avalanches enter the transport models via the non-diagonal elements of the corresponding transport matrix. It was argued that the heat avalanches in fusion plasmas showed similarity to SOC phenomena and propagated as a consequence of a local temperature gradient, which exceeded the local threshold (Tokunaga et al. 2012).

Transport by avalanches has received further attention in the context of *inward* heat convection in off-axis modulated electron cyclotron heating experiments (e.g., Mantica et al. 2005). The observed phenomenology has been discussed using the drift wave quasilinear model and the electromagnetic global 3D nonlinear model. In perturbative experiments of Mantica and Ryter (2006), inward heat transport has been associated with a rather counter-intuitive behavior of cold edge pulses that rapid local cooling at the edge of magnetically confined plasma leading to an increase in temperature in the plasma core by reversal of the sign of the perturbation. Cold pulse reversal is also related to and occurs under similar conditions as the mysterious effect of spontaneous rotation reversal, where the core rotation velocity of plasma changes sign spontaneously and in the absence of local sources (Rice et al. 2012).

In conclusion, one sees that the integral picture is rather complicated, it involves an interplay between SOC and the heat-pinch phenomena, as well as the multiple threshold conditions (owing to the competing transport channels involved).

Another important ingredient worth noting is the actual *three-dimensionality* of the plasma, which is crucial to several key processes in it, for instance, the excitation of Alfvén instabilities by energetic particles in tokamaks (Sect. 4.3 below), which cannot as a matter of fact be imported from the corresponding slab or cylindrical geometry, and where toroidicity is quite important (e.g., Heidbrink 2008; Zonca et al. 2006, 2007).

4.1.2 Numerical Simulations and SOC

SOC ideas have been tested in the direct numerical simulations of magnetically confined plasmas near marginality. Both fluid and gyro-kinetic (GK) codes have been explored. The latter codes are designed to incorporate plasma behaviors on perpendicular spatial scales comparable to the gyroradius and frequencies much lower than the particle cyclotron frequencies. The achievements so far can be summarized as follows.

In fluid codes, the typical signatures of avalanching behavior and SOC have been found under the condition that the profiles and the fluctuations are dynamically advanced on essentially an equal footing (Carreras et al. 1996; Garbet and Waltz 1998; Sarazin and Ghendrih 1998; Mier et al. 2006). To include the intrinsic self-organization of the plasma, a new forcing scheme has been used, in which sources and sinks are established first, allowing the plasma profiles to dynamically adapt to them. (This is in contrast to a more standard setup which fixes the profiles and only evolves the fluctuations.) Moreover the setup implies that the average outflux compensates the net input drive (at the steady state), hence its name: flux-driven simulations.

Several lessons have been learned out of these flux-driven simulations: First, that a co-evolution of the profiles and the fluctuations on the *same* time scale is indeed a milestone in the development of the SOC state (Carreras et al. 1996; Garbet and Waltz 1998; Sarazin and Ghendrih 1998; Garcia and Carreras 2005). Second, that the presence of radially-sheared zonal flows may decorrelate the avalanches and by doing so may short-circuit the SOC dynamics (Beyer et al. 2000). Last but not least, that an interaction with the competing transport channels, such as for instance the neoclassical diffusion, may deteriorate the mechanisms by which SOC is established (Mier et al., 2006, 2008).[2] These findings confirmed some of the previously made predictions using the sandpiles (Newman et al. 1996; Chapman et al. 1999; Sanchez et al. 2001) and lately extended to gyro-fluid simulations in the presence of the edge transport barriers (Tokunaga et al. 2012). Finally, the phenomena of turbulence spreading (the ability of turbulence to transport itself) have been discussed (e.g., Mattor and Diamond 1994) and their implication in SOC dynamics has been also addressed (Gurcan et al. 2005; Naulin et al. 2005).

The flux-driven setup has recently been applied to the state-of-the-art fusion plasma turbulent simulations: those based on gyro-kinetics (e.g., Idomura et al. 2008, 2009; Ku et al. 2009; Dif-Pradalier et al. 2010; Sarazin et al. 2010, 2011; Nakata and Idomura 2013). It has been confirmed using the GK codes that the transport is indeed endowed with avalanches, in contrast to what has been observed in a fixed-gradient setup. Other properties typical of SOC have been demonstrated: in particular, time self-similarity and the memory properties over a range of meso-scales (beyond the local turbulent scales). Even so, a more systematic characterization of these properties (although extensively studied in fluid simulations) is still lacking in their GK upgrades.

Looking forward to the challenges that lies ahead, several GK specific issues need to be addressed. For instance, some of the flux-driven GK codes have shown that radial transport ceases to be scale-free in stationary enhanced core confinement regimes (in advanced tokamak operation scenario, likely characterized by the existence of core plasma barriers) because of the development of permanent, radially-sheared, poloidal flow structures (known as $E \times B$ staircases), with their own characteristic length (Dif-Pradalier et al. 2010, 2015). The implication is that the spacing of staircase defines the outer scale of the actual avalanche distribution. These staircases seem, however, to be absent in the simulations done with other

[2] Indeed neoclassical transport is classical transport including the effects of toroidal geometry. The transport is still driven by Coulomb collisions—no anomalous transport due to plasma microscopic instabilities is included. The collisional character of the transport implies that there is a characteristic scale posed by the dynamics. So this scale will be always there in the thermodynamic (large-system) limit at odds with the multi-scale features characterizing SOC. The net result is that SOC is to be expected in regimes where Coulomb collisions lose their importance, which is somehow the case of rarefied thermonuclear plasmas.

flux-driven GK codes (Idomura et al. 2008, 2009; Nakata and Idomura 2013). Given the importance of the staircase for energy confinement in a reactor, the comprehension of the above mismatch (and the associated experimental evidence) is kind of urgent. Theoretically, the staircase represents a synergetic cooperation between the transport by avalanches at the meso-scales and the spontaneously occurring zonal flows. The latter constitute a barrier to the avalanche propagation (Beyer et al. 2000). Mathematically, the staircase constitutes a potential well in the corresponding Fokker-Planck equation, leading to an intricate boundary value problem (Milovanov and Rasmussen 2014). There has been some reports claiming the observation of such staircases in plasma discharges in the Tore Supra tokamak using high-resolution, fast-sweeping X-point reflectometry (Dif-Pradalier et al. 2015). It is worth mentioning here that the staircase ideas are fairly general; they have long been advocated in fluid dynamical settings using the concept of potential vorticity and Ertel's theorem (Dritschel and McIntyre 2008) and more recently the turbulent equipartition ideas (Madsen et al. 2015).

To conclude this section, we should stress that the plasma simulations discussed above have not as a matter of fact been specially designed to reproduce the wanted SOC regimes, except that they left the profiles to freely evolve (just given the sources and the sinks as the built-in conditions), and the plasmas to evolve on their own. In this connection, the notion of SOC has merely provided a statistical framework to interpret the results and the outcome of the simulations, but not really the physics flavor of the models behind. We advise the reader against the direct quantitative comparisons with the dynamics of sandpiles in view of the competing channels involved (heat pinches, toroidicity, edge transport barriers, etc.).

4.1.3 Back to Fractional Transport Models

As SOC disregards a characteristic scale for the transport (other than system-size), regular advection-diffusion equations fail to capture the dynamics. In this respect, modified transport equations with thresholded coefficients have been discussed, showing a certain degree of success (Garcia et al. 2002; Tangri et al. 2003; Mantica and Ryter 2006). More attention has been paid to "fractional" transport models, dating back to continuous-time random walks (Montroll and Weiss 1965). Fractional kinetic equations dealing with generalized derivatives in space and time (Podlubny 1999) incorporate in a natural, unified way the key features of non-Gaussianity and long-range dependence that often break down the restrictive assumptions of locality and lack of correlations underlying the conventional statistical mechanical paradigm of the Brownian type (Metzler and Klafter 2000 for review).

Specialized numerical schemes have been devised in the attempt to assess the fractional orders of diffeo-integration in space and time, best suited to describe the actual plasma data (generated numerically or experimentally) (Carreras et al. 2001; del Castillo-Negrete et al. 2004; del Castillo-Negrete 2006; van Milligen et al. 2004, 2005; Sanchez et al. 2005, 2006; Mier et al. 2008). An example of near-marginal (dissipative trapped electron mode) turbulence in a periodic cylinder is shown in Fig. 7. It is found starting from a smooth radial density profile that a dynamical turbulent state occurs near marginality via self-organization of the plasma. In those fluctuation-dominated regimes, the radial velocities of test particles show signatures that enable to associate them with the statistics of the Lévy type. This behavior suggests that the respective transport models could be formulated in terms of fractional diffusion equations (with both fractional time and phase space derivatives). Note that the SOC characteristics weaken as the external diffusive channel strength is increased from zero. This is manifest in the corresponding decrease in the Hurst exponent H and also in some steepening of the tail of the Lagrangian velocity fluctuation function (Mier et al. 2008).

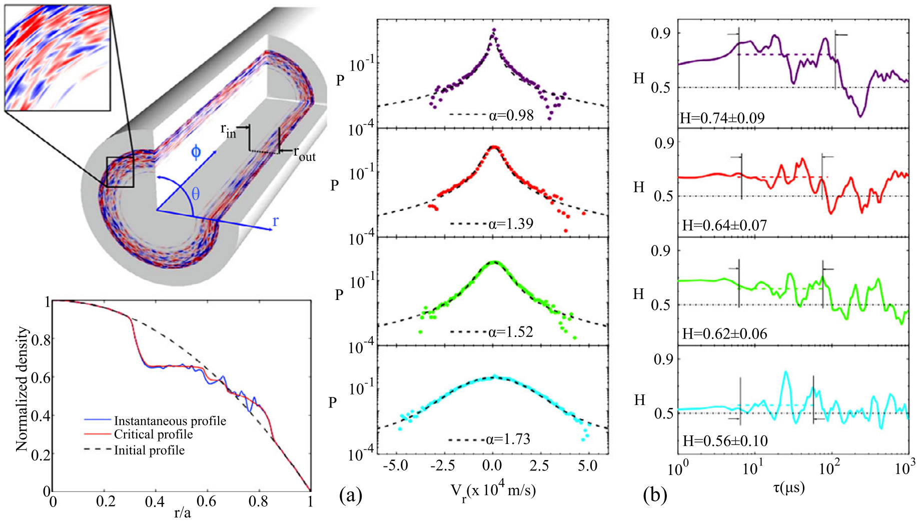

Fig. 7 Simulation of near-marginal (dissipative trapped electron mode) turbulence in a periodic cylinder (*left panel*, *top*). The smooth initial radial density profile (*left panel*, *bottom*) evolves to profiles with clear Levy characteristics of the radial velocity of test particles advanced by the turbulence while in the near-marginal regime (panel (**a**)), as well as the strong correlation of the radial velocity, through the Hurst exponent H (panel (**b**)), as the competing diffusivity increases from zero. This behavior suggests that effective transport models in this situation should be formulated in terms of fractional transport equations (Mier et al. 2008)

A concern raised over these models, however, is that they had merely a phenomenological backing in tokamak plasma turbulence and that their implication in magnetically confined plasma as finite system has only tangentially been addressed. (This not to disregard the fact that the use of a more common, diffusion-advection approach has in some cases been motivated by the correspondingly suitable evidence as well.) Also the use of space fractional operators in finite systems such as laboratory plasmas introduces the (intrinsically controversial) mathematical problem of confined Lévy flight (e.g., Chechkin et al. 2003), in which the nonlocal features due to the flights are limited to the presence of an absorbing boundary at the plasma edge.

Further concerns come from a conflict (Milovanov and Rasmussen 2014) between the nonlocality of the Lévy motion on the one hand and the local (next-neighborlike) character of sandpile interactions on the other hand. Indeed in sandpiles and their variants one is interested in how long-time correlated dynamics will develop via *local* communications between the many degrees of freedom leading to complex patterns (e.g., Zhang 1989; Sornette 1992). Then the local lattice rules assumed in these SOC theories will be incompatible with a nonlocal generalization of the real-space derivative, so that the correlations that are long-ranged enter through fractional differentiation over the time (Milovanov 2009, 2011), rather than the space, variable, thus preserving the local structure of the Laplacian.

A solution to this apparent problem may be proposed in different flavors. Indeed focusing on the "practical" aspects of fractional transport models, one may envisage these as long-time, long-scale "effective" linear limits of the actual non-linear dynamics of fusion plasma as a complex system. The implication is that the individual avalanches—which in practice require a certain amount of time to reach a certain distance in space—lose their identity in the limit $t \to \infty$. Asymptotically, their integrated effect on transport will be represented by "nonlocal" operators, whose exponents can therefore be estimated from the underlying

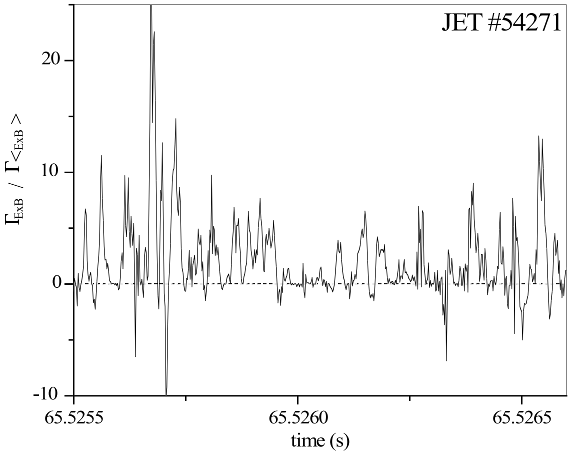

Fig. 8 An outstanding burst of the $E \times B$ flux observed in the JET tokamak (shot 54271). Reported by B. Gonçalves, 11-th European Fusion Physics Workshop, Heraklion, Crete, 8–11 December 2003

microscopic dynamics using the numerical fitting schemes (del Castillo-Negrete et al. 2004; van Milligen et al. 2004; Sanchez et al. 2005, 2006).

Furthermore, going with an "axiomatic" justification of nonlocal behavior, one may exploit the idea of *convective amplification* of two-dimensional fluid turbulence in the presence of an absorbing boundary generating SOC (Milovanov and Rasmussen 2014, 2015). This idea finds its mathematical significance in the fact (Montroll and Shlesinger 1982) that amplification (and amplification of amplification) of an initial distribution of the log-normal type generates after many iteration events a new probability distribution, whose nonanalytical part coincides with the Pareto-Lévy distribution in the sense of stable Lévy motion and generalized central limit theorem (Gnedenko and Kolmogorov 1954). The ensuing fractional transport model (Milovanov and Rasmussen 2015) naturally leads to fractional differentiation over the space variable and also exhibits the Sparre Andersen universality (Metzler and Klafter 2004; references therein) where the first passage time density decays as $\sim t^{-3/2}$ after t time steps ($t \to +\infty$). This behavior has been confirmed through numerical simulation in Chechkin et al. (2003). We consider this universality as a defining property of the nonlocal transport. It has been proposed based on the boundary value problem for Lévy flights that nonlocal transport models with the Sparre Andersen universality adhere to a new universality class of SOC, which is therefore different from the Zhang universality class (Milovanov and Rasmussen 2014). Indeed there is no indication that the Sparre Andersen universality (and the associated advanced SOC models using nonlocal transport) falls within the known universality classes of SOC.

More so, it has been discussed (Milovanov and Rasmussen 2014) that the processes of amplification taking place will manifest themselves in the form of algebraic tails on top of the typically log-normal behavior of the probability distribution function of the flux-surface averaged transport. "Algebraic" means that these tails pertain to a category of processes described by the statistics of the Lévy type, thus paving the way to the derivation of fractional transport models by standard methods. Physically, the algebraic tails shall represent outstanding transport events, which we associate with large intermittent bursts of transport (see Fig. 8).

All in all, the use of fractional transport models to describe SOC has received much comment, from very positive to polemical, and for sure will constitute one exciting problem for future investigations.

4.2 SOC-Turbulence Coupling: A Proxy to Big Events and Disasters?

In a recent investigation of bursty transport in tokamaks, Milovanov and Rasmussen (2014) introduced the theoretical concept of *SOC-turbulence coupling*. The main issue here is that some systems do not develop a pure critical state associable with SOC, since their dynamical evolution involves as a competing key factor an inverse cascade of the energy in reciprocal space.[3] Then relaxation of slowly increasing stresses will give rise to intermittent bursts of transport in the real space (other than the typical avalanching activity representing the fluctuations of the critical state) and outstanding transport events beyond the range of applicability of the "conventional" SOC. The model is a proxy to understand the catastrophic events in complex systems (severe magnetospheric storms, climate disruptions, huge solar flares and outstanding coronal mass ejections).

4.2.1 Time Scale Separation and Nonlinearity Conditions

In the proposed combined SOC-turbulence scenario (Milovanov and Rasmussen 2014, 2015), a guiding role is attributed to the usual picture of eddies and eddy-induced transport associated with plasma instabilities. A paradigmatic framework to understand and explain the phenomenon from first principles is drift-wave turbulence modeled by the well-known Hasegawa-Wakatani equations, as direct numerical investigations show (Naulin et al. 1999, 2004; Basu et al. 2003). Nonlinearly, in a magnetic confinement geometry, the eddy-induced transport reduces the slope of the average profile where the vorticity is maximal, and, at the same time, steepens it in its nearby vicinity, thus increasing the instability in the next radial location. This displacement of the instability is an avalanche in that the step in the gradient moves radially outward, creating an unstable propagating front. The process is analogous to the relaxation in a sandpile or the collapse of dominos. However, because drift-wave turbulence is essentially two-dimensional, there exists an inverse cascade of the energy which is associated with the phenomena of eddy merging and the formation of large-scale coherent structures in the strongly turbulent flow. To this end, the propagation of the unstable front becomes a combined effect due to the next-radial generation of the off-spring eddies and their merging with the ever-growing mother eddy. One sees that turbulence will act as to amplify the avalanches by fueling them with more free energy via the inverse cascade (Milovanov and Rasmussen 2014). The process will stop when excess energy and particles are eventually let out through boundaries, thus reducing the slope of the average profile on the system-size scales. It is noted that the energy reservoir for this behavior is only limited to the size of the confinement system.

It was argued that amplification of unstable fronts by drift-wave turbulence occurs in the parameter range of strong nonlinearity and time scale separation in that the Rhines time in the flow $\tau_{\mathrm{Rh}} \propto 1/\sqrt{u_{E\times B}}$ (here $u_{E\times B}$ is the $E \times B$ drift) must be small compared with the instability growth time. The latter time can be assessed as inverse linear instability growth rate, γ_{L}, which, in its turn, is proportional with nonadiabaticity of the fluctuations (Naulin et al. 1999). The nonadiabaticity parameter $\delta \propto \nu_{ei}$ characterizes the deviation between the potential and the density fluctuations in the Hasegawa-Wakatani model and incorporates via the electron-ion collisional frequency ν_{ei} the parallel (along the magnetic field lines) resistivity. In a basic theory of drift waves it is shown that this deviation leads to an instability

[3]The inverse spectral energy transfer is inherent to fluid-turbulence systems in two dimensions (2D), including the ubiquitous drift-wave turbulence. In 2D turbulence energy cascades from small to large scales because the phenomenon of vortex stretching is forbidden.

with a maximum linear growth rate $\gamma_L \approx \delta/8$. It is therefore convenient to think of $\delta \propto \gamma_L$ as of driving rate for the turbulence. Putting all the various pieces together, we have

$$\tau_{\mathrm{Rh}} \propto 1/\sqrt{u_{E\times B}} \ll \gamma_L^{-1}. \tag{1}$$

The time scale separation in Eq. (1) is favored by near-adiabatic fluctuations (small δ; small turbulence driving rate) on the one hand and by fast $E \times B$ drifts (strong nonlinearity; strong perturbation electric fields) on the other hand. In the latter case the restrictive assumptions of smallness of fluctuations and of vicinity to marginal stability are invalidated.

Furthermore, since the instability drive of the turbulence is controlled by electron-ion collisions in the parallel electron motion (Naulin et al. 1999), the condition for time scale separation in Eq. (1) implies that the particle cross-field transportation time is short compared with the characteristic resistive time. Thus, the avalanches tend to dissipate their content mainly upon reaching the boundaries and merely redistribute the particles and free energy across the system otherwise. These conditions of slow driving and time scale separation combined with the stabilizing role of boundary dissipation constitute a typical set-up for dynamical systems exhibiting SOC (e.g., Aschwanden 2013a). At this point, the propagation of unstable fronts due to the processes of eddy merging and interactions acquires the typical signatures of the avalanching dynamics of the SOC type.

In fluid (and fluid-like, such as the drift-wave) turbulence, one usually deals with a space-filling distribution of vortexes with only limitations posed by the energy injection and dissipation scales. This is usually thought to arise from the fact that a system of fluid vortexes develops its inertial range, characterized by a power law energy density spectrum. If, however, the original process of vortex formation occurs via *stimulated birth* of one or more vortexes due to a nearby recently formed vortex, and if this process occurs near its marginal instability threshold, then a hierarchical structure of vortexes will occur under the condition of time scale separation in Eq. (1) (Milovanov and Rasmussen 2015).[4] The distribution of vortexes obtained by this process produces long-range space correlations (and the associated power law reduced correlation function) and can be modeled as a variant of the Galton-Watson chain process near extinction (see Fig. 9). When seen from the perspective of time scale separation, the condition for strong nonlinearity favors the idea that drift-wave turbulence can acquire features that are otherwise associable with SOC. Indeed the patterning adheres to a situation in which SOC-like and turbulence-like phenomena appear on essentially an equal footing and are not separable statistically. This hybrid SOC-turbulence model (Milovanov and Rasmussen 2014, 2015) naturally leads to a kinetic equation with Lévy fractional derivatives in agreement with the phenomenological statistical model in (del Castillo-Negrete et al. 2008). Moreover it predicts the exact values of the Lévy fractional exponents beyond the usual numerical data-fitting schemes and consistently with the asymptotic character of fractional transport equations (Milovanov and Iomin 2014, 2015).

The coexistence of SOC-like and turbulence-like phenomena in strongly coupled particle systems has been also observed experimentally in two-dimensional complex (dusty) plasma. Indeed optical tracking of particle motion in a complex plasma monolayer revealed high grain mobility and large scale vortex flows coexistent with partial preservation of the global hexagonal lattice structure (Ratynskaia et al. 2006). It was argued that the transport of particles was superdiffusive and ascribed to Lévy statistics on short time scales and to

[4] The idea that activity in one region can stimulate activity in another region, particularly in a nonlinear context, is in fact very general and as such must occur in many applications. Here we mention the processes of stimulated galaxy formation discussed by Schulman and Seiden (1986), who used this to model the hierarchical structure in the distribution of galaxies with power law correlations.

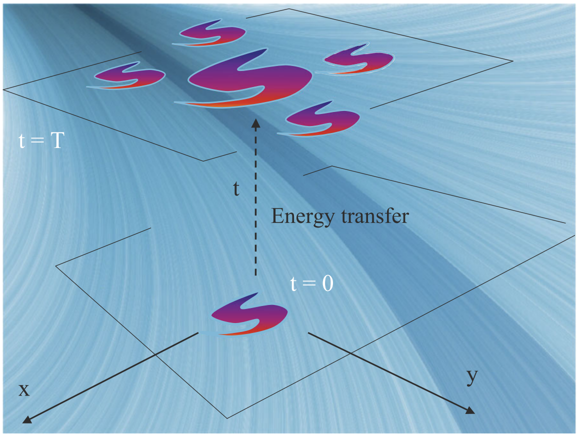

Fig. 9 Stimulated vortex formation as a Galton-Watson chain process in $2+1$ embedding dimensions, with time, t interpreted as the preferred dimension. An artist's view, using a continuum background to accentuate the spatio-temporal character of the dynamics behind the phenomena of SOC. The "arrow of time" looks in the direction of energy transfer. Adapted from Milovanov and Rasmussen (2014)

memory effects on the longer scales influenced by cooperative motion. At these longer time scales, the transport was governed by vortex flows covering a wide spectrum of temporal and spatial scales, offering first direct observation of viscoelastic vortical fluid motions in granular systems adhering to SOC.

4.2.2 *The Dragon-Kings of the Statistics*

It has been discussed that the back-reaction of the inverse cascade of the energy on SOC, mediated by drift-waves, poses significant threat to the projected thermonuclear fusion reactors and future power plants (Aschwanden et al. 2014; Milovanov and Rasmussen 2014) and that some catastrophic events may have clear precursors in the statistical datasets. We would like to know what are the most suitable precursors to forecast such events.[5] The topic is of interest from a fundamental scientific perspective and for the practical realization of diagnostic tools to monitor, forecast, and prevent large-amplitude bursts of transport in fusion power reactors. In this regard, space science might offer a priceless feedback to contemporary fusion research via a focused study of anomalously large events in the geospace plasma processes and solar system plasma (there is an outstanding expertise in the study of precursors to geomagnetic storms via multi-point measurements of the evolving magnetic activity). Other than fusion, the phenomena of SOC-turbulence coupling might be proposed for geophysical flows (using as appropriate the notion of Rhines time), where they shall be responsible for outstanding perturbations beyond the expected weather and climate patterns.

It is understood that forecasting the big events is not at all trivial, partly due to their respective rareness and hence lack of the statistics and partly due to the problem of detection of the *dragon-kings* of the statistics (Sornette and Ouillon 2012). The latter are catastrophic events of anomalously large size, lying off the extrapolation metrics in the corresponding "simpler" systems, and their correct identification and detection through the datasets remains a very delicate and difficult field, if only due to the scarcity of data as well as the

[5]Because of amplification, we expect a steeper drop-off in the energy spectrum of the coupled SOC-turbulence system as compared to the inertial range of the fluid (drift-wave) turbulence. In this connection, we should stress that the avalanching transport is triggered by the explicit radial dependence in the profiles, and, when account is taken for the inverse cascade of the energy, by boundary feedbacks, so that the assumptions of constant energy transfer and of infiniteness of the system, resulting in the fluid-like $-5/3$ behavior, do not really apply here.

extraordinary important implications with respect to hazard assessment, risk control and predictability. The problem of the dragon-kings has the potential to become an extrapolation problem from present-day small- and medium-size tokamaks to the fusion power reactors involving unexplored new physics. Another emerging problem here is coexistence of turbulence and energetic particle physics leading to feedback dynamics in a reactor (Zonca et al. 2015). In many ways the phenomenon of the dragon-king echoes the Anderson's "More and different" (2011) and brings up, rephrasing de Gennes: *The idea ("to put the sun into a box") is pretty. The problem is that the sun ought not to remain in a box.*

4.2.3 The Phenomenology of Big Events

Big events falling off the usual transport metrics in magnetically confined plasma have been reported in tokamak phenomenology (e.g., Xu et al. 2010; Fig. 8). The typical examples of this behavior include edge localized modes (ELMs) (e.g., Huysmans 2005) and blob-filaments, which are magnetic-field-aligned coherent structures that are considerably denser than the surrounding background plasma and are highly localized in the directions perpendicular to the equilibrium magnetic field lines (D'Ippolito et al. 2011). In experiments and simulations, intermittent filaments are often formed near the boundary between open and closed field lines, and seem to arise in theory from saturation process for the dominant edge instabilities and turbulence. Blob transport is of interest from a fundamental scientific perspective, since it is a general phenomenon occurring in nearly all plasmas. Further examples include self-generation of nonlocal transport patterns through complexity coupling between the edge and the core plasma fluctuations and the associated phenomena of turbulence convective expansion and amplification (Milovanov and Rasmussen 2015). There is a growing appreciation of the importance of the intermittent transport mediated by localized blob-like structures propagating far out in the scrape-off layer (SOL) region and proving a mean density and pressure profiles in the SOL that are rather broad (D'Ippolito et al. 2011). This is particularly the case for high densities and when the divertors get detached: a scenario foreseen for ITER.

4.2.4 Failure of Local Transport Models

The comprehension of fusion plasma as complex system involving SOC (e.g., Zonca et al. 2015 for brief review) implies the failure of local transport models, paving the way to nonlocal generalizations of the familiar Fickian diffusion (e.g., Metzler and Klafter 2000, 2004; Sokolov et al. 2002). There is a growing understanding that multiple interactions between dynamical degrees of freedom act to violate the assumptions of locality and the lack of correlations behind Fick's second law[6] and the conventional transport paradigm of the diffusion type, giving rise to large levels of transport across the magnetic field lines, known as anomalous transport, and to intermittent bursts of particle and energy flux beyond the range of predictability of the conventional statistical mechanical paradigm of the Gaussian type.

The fast propagation of a rapid cooling pulse from the edge of magnetically confined plasmas (Mantica and Ryter 2006) is typical for the failure of classical diffusive transport models. The observed phenomenology has been discussed based on the Lévy fractional Fokker-Planck equation in (del Castillo-Negrete et al. 2008; Milovanov and Rasmussen 2014) and using the concept of turbulence spreading in Naulin et al. (2005).

[6] The Fick paradigm states that the internal fluxes are described by a set of local transport coefficients—diffusivities or conductivities—related to the local thermodynamic forces which induce the fluxes through Fick's law. Models based on these assumptions are referred to as local models.

That nonlocal features can occur naturally via self-organization could also be demonstrated based on self-tuning phase transitions, applying the idea of competing order parameter and the formalism of fractional Ginzburg-Landau equation (Milovanov 2013; Milovanov and Rasmussen 2005). This may indeed prove to be a very promising (and quite novel using the notion of convective amplification of turbulence) angle of attack to characterize the origin of nonlocal transport in magnetically confined plasma systems.

4.3 Transport of Energetic Particles and SOC

In magnetically confined plasmas, superthermal energetic particles (EP) often drive shear Alfvén waves unstable via resonances with their orbital motion. These Alfvénic instabilities constitute a fascinating nonlinear system where fluid and kinetic nonlinearities can appear on an equal footing (Heidbrink 2008; Zonca et al. 2015). Avalanching behavior occurs when transport of EPs from one position steepens the spatial gradient at a new position, destabilizing a mode there. A propagating wavefront of this type has been observed in simulation (Zonca et al. 2005, 2006). It has been discussed that the unstable fronts are convectively amplified via self-consistent profile steepening before the final gradient relaxation phase. This phenomenology is strictly related to the resonant character of the energetic particle modes (EPMs), which tend to be radially localized where the drive is strongest. EPMs are destabilized when the EP pressure is comparable to the pressure of the thermal plasma and constitute a separate wave branch with a distinctive dispersion relation (Chen 1994). A threshold in mode amplitude to initiate an avalanche has been hypothesized and more recently demonstrated as a localization-delocalization transition in nonlinear Schrödinger models with disorder (Milovanov and Iomin 2012, 2014). Also a sharp transition from weak to strong EP transport in burning plasmas has been predicted using feedback dynamics on the mode drive and complex nonlinear Schrödinger equation (Zonca et al. 2006). Experimentally, a clear example of transport phenomena with the features of threshold dynamics and avalanches has been observed in the National Spherical Torus Experiment during beam injection (Fredrickson et al. 2006). The typical signatures of avalanching dynamics in the proximity to EPM linear instability threshold have been discussed in terms of sandpile physics involving SOC (Berk et al. 1996; Dendy and Helander 1997), and, when account is taken for the phenomena of convective amplification of the avalanches, in terms of global the so-called "fishbone-like" instabilities of SOC (Milovanov 2010, 2011) within the topics of strong radial transport and significant departures from the state of marginal stability (Chen and Zonca 2007; Zonca et al. 2015).

4.4 The Frontier

Generally speaking, the study of SOC phenomena is currently transitioning from an emphasis on scaling and linear-response theories to an emphasis on understanding and predicting the nonlinear dynamics of systems with many coupled degrees of freedom. In many ways these tendencies are manifest in the introduction of nonlinear models of the Ginzburg-Landau (Gil and Sornette 1996; Milovanov and Rasmussen 2005) and nonlinear Schrödinger (DANSE) type (Milovanov 2013; Milovanov and Iomin 2014, 2015). As this transition occurs, investigations—as much experimental as theoretical and numerical—that pinpoint the nonlinear interactions in complex systems will increase in importance. Theoretically, the study of nonlinear systems with many interacting degrees of freedom is in its infancy. Beyond validation of theoretical models, the future of the field lies in the development of first-principle approaches to SOC, involving Hamiltonian approaches (e.g., Milovanov and Iomin 2014). These may exploit "self-organized" features of ordinary phase transitions along the lines of Ginzburg-Landau theories with a competing order parameter (Milovanov and Rasmussen 2005; Milovanov 2013). In the geo-space plasma research, with the observations becoming multi-spacecraft and/or multi-point in scope, theoretical models are likely to confront issues of nonlocality, self-organization, and build-up of correlations in the presence of many co-existing plasma processes (Zelenyi and Milovanov 2004; Savin et al. 2011).

Similarly to geospace exploration, research activities in fusion plasma are now arriving at a crucial juncture that necessitates the understanding of "complexity" in the accessible and relevant operation regimes of burning plasma. Indeed it is becoming clear that the important questions that will be receiving attention in the coming years, particularly with the development of ITER and DEMO scenarios, are addressed toward the comprehension of burning plasma state as being self-organized, thresholded, nonlinear dynamical system with many interacting degrees of freedom (Chen and Zonca 2007, 2015; Zonca et al. 2006, 2015). The ITER project is a major challenge on the way to controlled fusion burn. The specialized issues of complexity, nonlinear interactions, and SOC have found their significance in the recently formulated Fusion Advanced Studies Torus, or FAST, proposal (Pizzuto et al. 2010), promoting an European Union satellite for ITER. By the time this review is being written, the FAST proposal has been advanced by ENEA for EUROfusion to evolve into the Divertor Test Tokamak (DTT) project (http://fsn-fusphy.frascati.enea.it/DTT/index.php), which is currently a hot topic in the European fusion program. An emergent way of thinking here is to recognize FAST (as well as its DTT upgrade) as having a parallel in the geo-space exploration, the ROY mission concept (Savin et al. 2011)—a project in space research for a constellation of small, probe-like satellites,[7] aiming to investigate the dynamic magnetosphere as complex system (see Fig. 10). We extrapolate that the cross-disciplinary effort of bringing these exciting projects to realization will open new avenues in the study of what proves to be one of the greatest theoretical challenges in the modern nonlinear physics, the paradigm of SOC.

5 Fluctuation Analysis

Characterization of fluctuations is a long standing theme in many areas of research and has a wide range of applications. This is critical to the understanding of the system under study, as is clear in the interpretation of power law dependences, e.g., in the cases of turbulence or SOC. The fluctuation analysis using time series data has been extensively used in many studies for sometime, e.g., the well-known Hurst effect in the flood levels of the Nile (Hurst 1951). Most observational data however contain many other features such as trends and intermittency, and it becomes essential to separate these in the data in order to obtain reliable measures of the fluctuations. This becomes critical in the cases when the power law exponents are used for practical applications, such as in the case of bio-medical applications. Further, with increasing need for the understanding and modeling of extreme events and natural hazards (Sharma et al. 2012) there is an increasing recognition of the importance of statistical techniques. The underlying reason for this is the lack of data on extreme events and the need to use all available data of a phenomenon in order to develop reliable models capable of yielding the characteristic quantities, such as predictability (Sharma et al. 2012). With

[7] The Russian word "roy" means a "swarm" in English.

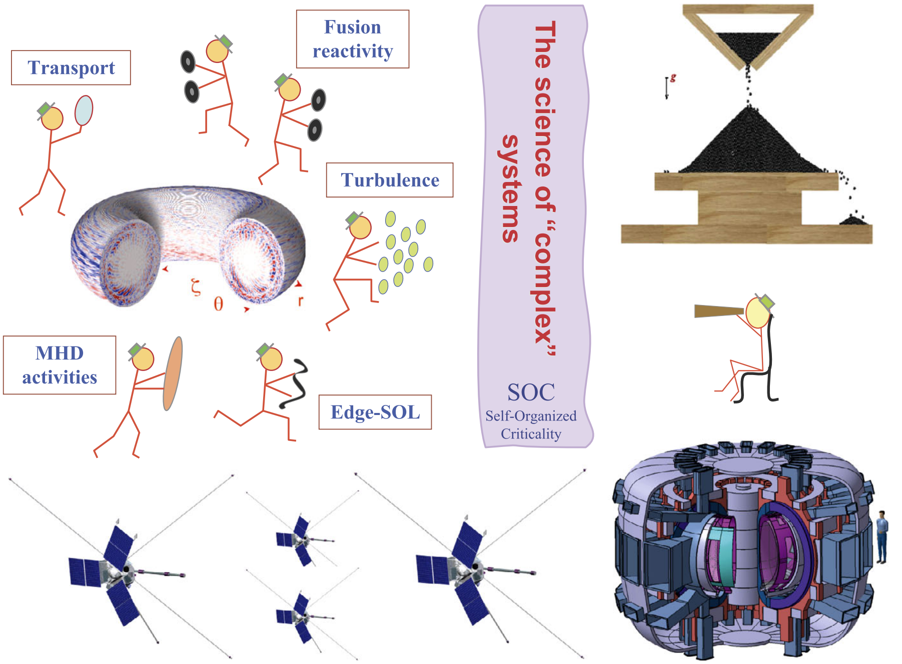

Fig. 10 A cartoon illustrating the many on-going plasma processes in a toroidal magnetic confinement system, viewed through the prism of complexity and SOC. A remake of Fig. 1 from Zelenyi and Milovanov (2004). *In the bottom-right corner* is an artist's view of the FAST tokamak. Courtesy of A. Pizzuto. The four satellites *in the bottom center* represent the ROY project (Savin et al. 2011). Adapted from Milovanov (2013)

the recent phenomenal growth in data (e.g., Big Data) such techniques provide new ways to analyze large datasets to yield the inherent and leading features. Extreme events are features of complex driven systems and because of their nonequilibrium nature the traditional statistical analysis tools are not adequate. Long Range Correlations (LRC) is a key feature of the development of extreme events and is studied using data from diverse physical systems such as temperature records (Bunde et al. 2005), river flows (Mandelbrot and Wallis 1969) heart rate variability (Schäfer et al. 1998) and space weather (Sharma and Veeramani 2011).

Rescaled range analysis (R/S) (Hurst 1951) and fluctuation analysis (FA) are statistical tools developed to estimate the variability of time series through estimation of Hurst exponent H, a statistic which is directly related to the scaling in auto-correlation functions, and, also to the fractal dimension of the time series data. While the scaling exponent, H, is 0.5 for uncorrelated white noise, many natural systems demonstrate values close to 0.7. However, these techniques fail to estimate H in non-stationary data, thus limiting the applicability of the results. Such considerations have led to techniques such as detrended fluctuation analysis (DFA) as a way to obtain improved power law dependences (Peng et al. 1992; Kantelhardt et al. 2001). DFA is widely considered a better technique than R/S or FA considering its capability to detrend the time series data whilst estimating H, making it viable for non-stationary systems. With increased usage of the DFA technique, limitations in its detrending capabilities are now recognized (Bryce and Sprague 2012) and it is evident that there is need for a better alternative detrending schemes for data with atypical trends (e.g., nonlinear trends). In spite of its shortcomings, DFA is widely used for estimation of the Hurst exponent and is an efficient and fast technique.

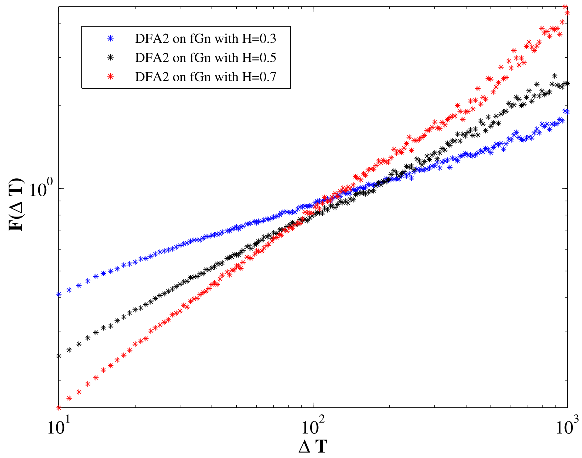

Fig. 11 Application of Detrended Fluctuation Analysis with quadratic detrending (DFA2) to fractional Gaussian noise with Hurst exponents, $H = 0.3, 0.5$ & 0.7 (Setty and Sharma 2015)

In the studies of fluctuations a widely used model is fractional Brownian motion (fBm), a generalization of Brownian motion and a quintessential representative model of the Hurst effect. Since its discovery, there has been increased interest in modeling physical systems as fBm. However, it was quickly realized that imposing a uniform H over the span of the data is in fact a restricting condition as a uniform level of LRC in real life data implies monofractal nature. This is a very strong condition and is not commonly found in real world data. Multifractional Brownian motion (mBm) is a generalization of fBm relaxing this condition (Peltier and Véhel 1995), allowing for variable degrees of self-similarity with non-stationary increments i.e., $H(t)$ over the time span of the data. This also leads to tunable local regularity, a feature of mBm which is of interest in modeling various geophysical systems.

With the increasing use of DFA as a technique to study LRC in time series data, it is now widely recognized that it yields a single Hurst exponent, and thus can not distinguish between multifractal and monofractal systems, e.g., between mBm and fBm, respectively. In fact, most systems exhibit time varying H exponent, but DFA yields a constant H value. Studies have shown the effect of data size used on the Hurst exponent (Lennartz and Bunde 2011), thus requiring caution in the interpretation of the estimated values. This is in direct agreement with our study of the effect of data size on the Hurst exponent estimated by DFA in mBm data, as shown in Fig. 11 (Setty and Sharma 2015).

Although other schemes such as Multi Fractal Detrended Fluctuation Analysis (MF-DFA) were proposed (Kantelhardt et al. 2002) as an alternative, they address the multifractal nature of time series with respect to one fractal dimension at a time and do not provide a solution in estimating the time varying fractal structure of mBm. It is apparent that DFA and other similar techniques were assumed to locally estimate a time averaged Hurst exponent (Carbone et al. 2004; Hu et al. 2001). This assumption helps recreate $H(t)$ by using estimator techniques with sliding windows. It is expected that the success in estimating such a time average depends on local linearity of the Hurst exponents.

Using mBm data generated from linearly varying Hurst exponents, $H(t)$, it was shown that DFA in fact estimates the time average and thus the dependence of the estimated exponent on various data and technique related parameters was tested (Setty and Sharma 2015). The primary motivation for such a study is to establish a bench mark for the performance of DFA technique with respect to its ability to estimate a time averaged Hurst exponents from mBm data, and, identify its limits. This characterizes the performance of DFA on mBm data with linearly varying $H(t)$ and further tests the robustness of estimated time average with

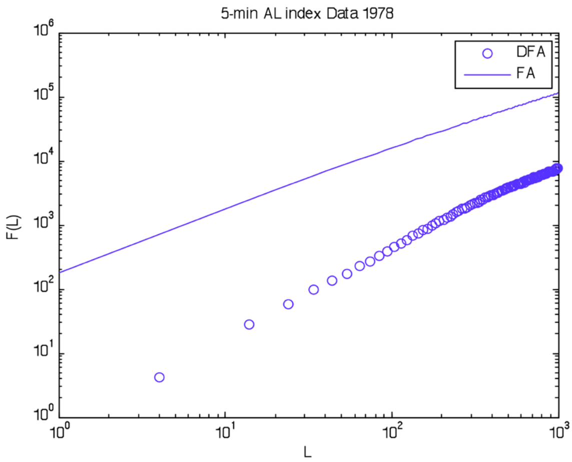

Fig. 12 Detrended fluctuation analysis of 5-min averaged data of AL for 1978. The fluctuation analysis (FA) shows a different scaling exponent for $L \sim 300$ min (Sharma and Veeramani 2011)

respect to various data and technique related parameters. These results serve as a benchmark for using DFA as a sliding window estimator to obtain $H(t)$ in a time series data. Furthermore, given that DFA is one of the primary techniques widely used for fluctuation analysis its applicability to data with time varying fractality is quantified.

Many recent studies have addressed, directly or indirectly, the fluctuation analysis, e.g., the framework of thermodynamics of rare events (Consolini and Kretzschmar 2007), kinetic theory of linear fractional stable motion (Watkins et al. 2009), avalanching with an intermediate driving rate (Chapman and Watkins 2009; Chapman et al. 2001, 2009), and multifractal and fractional Lévy flight models (Zaslavsky et al. 2007, 2008; Rypdal and Rypdal 2010). The early studies of fluctuations in the magnetosphere (Takalo et al. 1993) used the well-known structure function, also referred to as fluctuation analysis (FA). The main shortcoming of FA is its dependence on the computed index to effects such as trends and interrmittency. The 5 min averaged data of AL for the year 1978 was used to study the scaling behavior from FA and DFA (Sharma and Veeramani 2011). As shown in Fig. 12, the scaling exponent obtained with DFA shows a break at ~ 300 min, in agreement with the power sprectrum analysis (Tsurutani et al. 1990) and absent in the FA results. The need for improved computations of the scaling exponents (often the Hurst exponent) has stimulated the development of new techniques. An improved method based on a combination of data-driven modeling approaches provides better estimates of the power law exponents, such as the Hurst exponent (Setty 2014; Sharma and Setty 2015a, 2015b). This technique, referred to as fluctuation analysis after trend elimination (FATE), was demonstrated using data with fractional Gaussian noise ($H = 0.86$) riding on a sinusoidal trend. A notable result is that it yields nearly the same scaling as FA of fGn.

Fluctuations in multiscale phenomena in natural systems often exhibit crossover behavior in the scaling exponents. These exponents represent the nature of correlation in the system and the crossover shows the presence of more than one type of correlation. An accurate characterization of the crossover behavior is thus needed for a better understanding of the inherent correlations in the system, and a multi-step process has been used for analyzing the crossover behavior. The detrended fluctuation analysis is used to remove the trends in the data and the scaling exponents are computed. The crossover point is then computed by a hyperbolic regression technique, with no prior assumptions. The time series data of the magnetic field variations in Earth's magnetosphere is analyzed with these techniques and yields a crossover behavior with a time scale of 4 hrs. A Langevin model of the magnetospheric dynamics yields an excellent fit to the crossover in the scaling exponents and thus provides a model of the non-equilibrium system (Setty 2014; Sharma and Setty 2015a, 2015b).

6 Discussion and Conclusion

The recognition that the dominant features of a physical system are not necessarily specific to the states of the system has stimulated modeling in terms of SOC and similar approaches. The results from these investigations in the laboratory and space plasmas have provided new insights, as reviewed here. This paper complements the companion papers, e.g., on the basic concepts and techniques (McAteer et al. 2015, this issue; Watkins et al. 2015, this issue) and applications to Solar and Astrophysics (Aschwanden et al. 2014).

The SOC concept has stimulated a large number of applications in many areas and although its claims have been somewhat ambitious, it has made important contributions to the study of nonequilibrium phenomena. The current interest in granular matter (de Gennes 1999) was at least partly stimulated by SOC ideas. The transition between fluid and frozen phases of granular matter may in fact have features of phase transition and critical behavior, very much like SOC behavior. While the theory of SOC is not fully developed, a dynamic mean-field theory description in terms of transition probabilities (Vespignani and Zapperi 1998) has been used to determine its relationship to other nonequilibrium phenomena. This approach describes SOC models as nonequilibrium systems which reach criticality by the fine tuning of the control parameters. The original SOC idea is based on the absence of fine tuning and the mean field theory of SOC brings it closer to conventional critical phenomena, rather than the nonequilibrium case.

In a broader sense SOC is viewed as a way in which transitions in nonequilibrium systems take place and this has stimulated extensive research and provided many insights. There are however other approaches that focus on similar phenomena with different levels of success. For example the concept of highly optimized transitions (HOT) is based on similar premises (Carlson and Doyle 2002). Such approaches are of interest in modeling systems in which the convergence to equilibrium states require time scales longer than those of external perturbations. An example outside plasma systems is a cyber network interaction with external action, including adversarial ones. The nonequilibrium nature is a key feature in such cases and modeling techniques based on equilibrium states, e.g., game theory, has limited applicability. Systems perturbed by external perturbations are common and they can be viewed as complex driven systems (Eisler et al. 2005; Sharma 2010, 2014). From the complexity point of view these phenomena have been viewed as self-organized complexity (Turcotte et al. 2002).

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