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Inhomogeneities from quantum collapse scheme without inflation

Gabriel R. Bengochea [a,*], Pedro Cañate [b], Daniel Sudarsky [b]

[a] Instituto de Astronomía y Física del Espacio (IAFE), UBA-CONICET, CC 67, Suc. 28, 1428 Buenos Aires, Argentina
[b] Instituto de Ciencias Nucleares, UNAM, México D.F. 04510, México

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ABSTRACT

In this work, we consider the problem of the emergence of seeds of cosmic structure in the framework of the non-inflationary model proposed by Hollands and Wald. In particular, we consider a modification to that proposal designed to account for breaking the symmetries of the initial quantum state, leading to the generation of the primordial inhomogeneities. This new ingredient is described in terms of a spontaneous reduction of the wave function. We investigate under which conditions one can recover an essentially scale free spectrum of primordial inhomogeneities, and which are the dominant deviations that arise in the model as a consequence of the introduction of the collapse of the quantum state into that scenario.

1. Introduction

Inflation is presently considered as an integral part of our understanding of cosmological evolution. Inflationary models are generally credited with explaining the large scale homogeneity, isotropy and flatness of our universe as well as accounting for the origin of the seeds of cosmic structure. All structures in our universe emerge from a featureless stage described by a background Friedmann–Robertson–Walker (FRW) cosmology with a nearly exponential expansion driven by the potential of a single scalar field, and from its quantum fluctuations characterized by a simple vacuum state.

In inflationary models, the modes relevant to cosmological perturbations are assumed to be born in their ground state at a time when their proper wavelengths are much shorter than the Hubble radius. That is, the state of the quantum field characterizing the seeds of structure is determined by the instantaneous vacuum state corresponding to the static universe that would be obtained by freezing the cosmological evolution in very early epochs (a precise description of such quantum state construction for arbitrary space times can be seen for instance in [1]). The resulting state in the case of a later exponential expansion (which is close to what is given in simple inflationary models) is known as the Bunch–Davies vacuum.

In the Heisenberg picture, the state of the field at later times is, of course, unchanged and the evolution of the field operators is encoded in the evolution of the modes. From such a setting, one obtains a fluctuation spectrum for these modes which corresponds to a scale free spectrum, and thus fits very well with the Harrison–Zel'dovich prediction [2]. Furthermore, that primordial spectrum, upon incorporation of plasma physics effects, taking place before decoupling, leads to a prediction for the Cosmic Microwave Background (CMB) that fits the data extremely well [3,4].

In a recent work [5], Hollands and Wald proposed an alternative model involving a simple fluid, which, under some more or less natural hypothesis, would be equally capable to reproduce the scale free spectrum and thus, in principle, account for the seeds of cosmic structure. Therefore, it may not be necessary to assume that an era of inflation actually occurred and to postulate the existence of a new fundamental scalar such as the inflaton field. The natural hypothesis referred to above concerns the 'birth time' and the initial state of the relevant modes, and will be discussed in more detail below.

In that work, Hollands and Wald also argue that the resolution of the flatness and horizon problems provided by inflation are not truly satisfactory, and that only a much deeper understanding of the conditions determining the initial state could shed light on that matter. We do not wish to engage in this part of the discussion here.

* Corresponding author.
E-mail addresses: gabriel@iafe.uba.ar (G.R. Bengochea), pedro.canate@nucleares.unam.mx (P. Cañate), sudarsky@nucleares.unam.mx (D. Sudarsky).

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The model put forward in [5], starts with the assumption that semiclassical physics applies to phenomena on spatial scales larger than some fundamental length l_0, which, presumably, is of order of the Planck length (l_p) or the grand unification scale. Following this point of view, the authors argue, it would be natural to treat the modes as effectively being born at a time when their proper wavelength is equal to the fundamental scale l_0. Consequently, all the modes would be continuously created over all time. Assuming that the modes are created in their instantaneous ground state (a state that is thus isotropic and homogeneous), the authors obtained a prediction for a scale free spectrum with appropriate amplitude.

The starting point of the analysis corresponds to a universe that is well described by a flat FRW metric, and where there is a large background stress-energy that is linearly perturbed by quantum fluctuations. According to the model, the matter in the early universe can be described, on spatial scales greater than l_0, by a fluid with equation of state $p = w\rho$, where w is a constant. The analysis then continues along a line close to that followed in the standard analysis of inflationary models, by quantizing the perturbations of the coupled Einstein-fluid system, according to Section 10.2 of [6].

Finally, the resulting power spectrum is obtained (as is customary in this kind of analysis), by considering the two point function in the above described state of the quantum field. As noted, this result is such that for modes with wavelength greater than the Hubble radius, at the time of decoupling, one finds a scale free spectrum of density perturbations. The adequate value for the spectrum amplitude is obtained by adjusting the value for l_0, ($l_p/l_0 \sim 10^{-5}$). Furthermore, the authors argue that, within this approach, one can expect a larger ratio of tensor to scalar perturbations than that which are typical for the inflationary models. We plan to study this issue in a future work.

In the present manuscript, we focus on the following issue: it is a fact that in both, the inflationary picture as in the proposal by Hollands and Wald, one must face the difficulties posed when considering a quantum description for the early universe. A form that the problem takes in the present setting is that, according to the two proposals, a completely homogeneous and isotropic stage (evolving according to dynamical equations that cannot break such symmetries), must nevertheless lead, after some time, to a universe containing actual inhomogeneities and anisotropies, presumably characterized by the fluctuation spectrum. This issue has been considered at length in other works, including detailed discussions of the shortcomings of the most popular attempts to address the problem, and we will not repeat such extensive discussions here, except for a brief description intended only as an introduction for the reader who is not familiar with the problem. It is clear that such transition from a symmetric situation to one that is not, cannot be simply the result of quantum unitary evolution, since, as we noted, the dynamics does not break these initial symmetries of the system. As discussed in [7], and despite multiple claims to the contrary (e.g. [8]), there is no satisfactory solution to this problem within the standard physical paradigms. Recently, some books that presents the standard inflationary paradigm have referred to this subject acknowledging to a certain extent the unresolved difficulty (see e.g. [9–12]).

The proposal to handle this shortcoming was considered for the first time in [13]. There, the problem was addressed by introducing a new ingredient into the inflationary account of the origin of the seeds of cosmic structure: the self-induced collapse hypothesis. The basic idea is that an internally induced spontaneous collapse of the wave function of the inflaton field is the mechanism by which inhomogeneities and anisotropies arise at each particular scale. That proposal was inspired on early ones for the resolution of the measurement problem in quantum theory [14–21], which regarded the collapse of the wave function as an actual physical process taking place spontaneously. Also, on the ideas by R. Penrose and L. Diósi [24–29] who assumed that such process should be connected to quantum aspects of gravitation. There are other promising proposals based on Bohemian versions of quantum theory applied to the inflationary field [30], but they will not be considered further in this work.

The simplest way this kind of process can be described is, by assuming that at a certain stage in the cosmic evolution, there was a self-induced jump in the state describing a particular mode of the quantum field, in a manner that is similar to the quantum mechanical reduction of the wave function associated with a measurement. However, the reduction here is assumed to be spontaneous and no external measuring device or observer is called upon as triggering such collapse. A collapse scheme is a recipe to characterize and select the state into which each of the modes of the scalar field jumps at the corresponding time of collapse. The collapse itself is described in a purely phenomenological manner, without reference to any particular mechanism. As reported in, for instance, [13,31,32], the different collapse schemes generally give rise to different characteristic departures from the conventional Harrison–Zel'dovich flat primordial spectrum. There are, of course, more sophisticated theories describing the collapse dynamics, such as those in [14–23], however we will not consider those in the present study, which is meant a first exploration of such ideas in the context of the model proposed by Hollands and Wald.

The main objective of this article is to obtain the effects on the shape of the primordial spectrum, that arise from a particular collapse scheme, into the framework of the proposal of [5]. We will show that with a simple collapse scheme and for a certain range of values of the model parameters, one can effectively recover a scale free spectrum. We will also consider the dominant deviations from the flat spectrum that would arise in this model.

The paper is organized as follows. In Section 2, we develop the necessary formalism of the model, describe the collapse scheme implemented and show the results. Finally, in Section 3, we make our conclusions.

Throughout the work, we will use $c = 1 = \hbar$ and $l_p^2 = \frac{8\pi G}{3}$. Moreover, l_0 is a free parameter and its value is set at the end of our calculations.

2. The original model

2.1. The Einstein-fluid system

Following the original work [5], we start assuming that, on spatial scales grater than l_0, the early universe is dominated by a fluid with pressure p and energy density ρ, which are related by the equation of state $p = w\rho$ where $w \in (0, 1)$ is a constant. We will also assume that the background is well described by a flat FRW metric.

Fixing the gauge (longitudinal) and restricting consideration to the scalar modes, the perturbed metric in this case can be written as

$$ds^2 = a(\eta)^2[-(1+2\Phi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j], \tag{1}$$

where $\Phi = \Phi(\eta, x^i)$ characterizes all deviations from homogeneity and isotropy in the space–time.

The background is completely described by the value of w. In particular, the scale factor is $a(\eta) = B\eta^{\frac{2}{3w+1}}$, where B is fixed so that $a(\eta_{\text{today}}) = 1$, and the background fluid density is $\rho \propto a^{-3(w+1)}$.

The next step in the analysis of this model, consists in the quantization of the perturbations of the coupled Einstein-fluid system. We will adhere to the treatment of the problem in [5], which follows the general approach formulated in [6]. In that work, the authors focus on the variable v, which is a linear combination of the fluid velocity potential φ_v and the metric potential Φ,

$$v \equiv \frac{1}{\sqrt{6}\, l_p}(\varphi_v - 2\, z\, \Phi) \tag{2}$$

where $z \equiv -\frac{a\sqrt{\beta}}{\mathcal{H} c_s}$, with $\beta \equiv \mathcal{H}^2 - \mathcal{H}'$, the conformal Hubble parameter is $\mathcal{H} \equiv \frac{a'}{a}$, and $c_s = \sqrt{w}$ is the speed of sound in the fluid. Here, a prime denotes partial derivative with respect to conformal time η.

The action up to second order in perturbation variables (Eq. (10.62) of [6]) reads

$$\delta^2 S = \frac{1}{2}\int d\eta d^3x \left(v'^2 - c_s^2 (\nabla v)^2 + \frac{z''}{z} v^2 \right) \tag{3}$$

The above action is identical to the standard action of a free scalar field (v) with a time-dependent mass. Moreover, v is analogous to the Mukhanov–Sasaki variable [33], and it is the field that will be treated quantum mechanically.

Then, the Lagrangian density is

$$\delta^2 \mathcal{L} = \frac{1}{2}\left(v'^2 - c_s^2 (\nabla v)^2 + \frac{z''}{z} v^2 \right), \tag{4}$$

the momentum canonical to v is $\pi \equiv \partial \delta^2 \mathcal{L}/\partial v' = v'$, and therefore the Hamiltonian can be written as

$$H(\eta) = \frac{1}{2}\int d\mathbf{x} \left[\pi^2 + c_s^2 (\nabla v)^2 - \frac{z''}{z} v^2 \right]. \tag{5}$$

In the quantization process, the field v and its conjugate momentum π are promoted to operators acting on a Hilbert space $\mathscr{H}$. These satisfy the standard equal time commutation relations

$$[\hat{v}(\eta, \mathbf{x}), \hat{\pi}(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}')$$
$$[\hat{\pi}(\eta, \mathbf{x}), \hat{\pi}(\eta, \mathbf{x}')] = 0 = [\hat{v}(\eta, \mathbf{x}), \hat{v}(\eta, \mathbf{x}')] \tag{6}$$

In the Heisenberg picture, the equations of motion are obtained from

$$i\hat{v}' = [\hat{v}, \hat{H}], \quad i\hat{\pi}' = [\hat{\pi}, \hat{H}] \tag{7}$$

that for the case of $\hat{v}$ it results to be

$$\hat{v}'' + \left(c_s^2 \nabla^2 - \frac{z''}{z} \right) \hat{v} = 0. \tag{8}$$

The standard procedure is to write the general solution to this equation decomposing $\hat{v}$ in terms of the time-independent creation and annihilation operators. For practical reasons, we will work with periodic boundary conditions over a box of size L, where $k_i L = 2\pi n_i$ for $i = 1, 2, 3$. So we write

$$\hat{v}(\eta, \mathbf{x}) = \frac{1}{\sqrt{2}\, L^{3/2}} \sum_{k \neq 0} \hat{v}_{\mathbf{k}}(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} \tag{9}$$

with $\hat{v}_{\mathbf{k}}(\eta) = \hat{a}_{\mathbf{k}} v_k(\eta) + \hat{a}^{\dagger}_{-\mathbf{k}} v_k^*(\eta)$, and the normal modes $v_k(\eta)$ satisfying

$$v_k'' + \left(k^2 c_s^2 - \frac{z''}{z} \right) v_k = 0. \tag{10}$$

The normalization for the modes $v_k(\eta)$ is chosen such that

$$v_k(\eta)\, v_k'^*(\eta) - v_k'(\eta)\, v_k^*(\eta) = 2i, \tag{11}$$

which leads to the standard commutation relations for the creation and annihilation operators,

$$[\hat{a}_{\mathbf{k}}, \hat{a}^{\dagger}_{\mathbf{k}'}] = \delta_{\mathbf{k}\mathbf{k}'}$$
$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = 0 = [\hat{a}^{\dagger}_{\mathbf{k}}, \hat{a}^{\dagger}_{\mathbf{k}'}] \tag{12}$$

and the Fock space can be constructed in the standard way starting with the vacuum state, i.e. the state defined by $\hat{a}_{\mathbf{k}}|0\rangle = 0$ for all $\mathbf{k}$. The choice of the functions $v_k(\eta)$ corresponds to the selection of the vacuum state for the quantum field.

Following [5], we choose as initial state, the vacuum whose mode functions $v_k(\eta)$ satisfy (10) at the birth time η_0^k (i.e. the time of birth of different modes are distinct), and are given by

$$v_k(\eta_0^k) = \frac{1}{\sqrt{kc_s}} e^{-ikc_s\eta_0^k} \quad \text{and} \quad v_k'(\eta_0^k) = -i\sqrt{kc_s} e^{-ikc_s\eta_0^k} \tag{13}$$

That is, the modes are set in the instantaneous vacuum state corresponding at the specific time of birth of each mode.

Using that $\frac{z''}{z} = \frac{a''}{a}$ and defining $A = \frac{2(1-3w)}{(3w+1)^2}$, Eq. (10) can be re-written as

$$v_k'' + \left(k^2 c_s^2 - \frac{A}{\eta^2} \right) v_k = 0 \tag{14}$$

whose general solution is

$$v_k(\eta) = \sqrt{\eta}\Big[C_1 J_n(kc_s\eta) + C_2 Y_n(kc_s\eta) \Big] \tag{15}$$

In the last equation, J_n and Y_n are the Bessel functions of the first and second kind respectively, and $n = \frac{1}{2}\sqrt{1+4A}$. On the other hand, the constants C_1 and C_2 are obtained from the initial data at time η_0^k according to (13). Evaluating these constants we find

$$\begin{aligned} C_1 = {} & \frac{\pi}{4\sqrt{c_s k\eta_0^k}} \Big[(2n + 1 + 2ic_s k\eta_0^k) Y_n(c_s k\eta_0^k) \\ & - 2c_s k\eta_0^k Y_{n+1}(c_s k\eta_0^k) \Big] e^{-ic_s k\eta_0^k} \\ C_2 = {} & -\frac{\pi}{4\sqrt{c_s k\eta_0^k}} \Big[(2n + 1 + 2ic_s k\eta_0^k) J_n(c_s k\eta_0^k) \\ & - 2c_s k\eta_0^k J_{n+1}(c_s k\eta_0^k) \Big] e^{-ic_s k\eta_0^k}. \end{aligned} \tag{16}$$

In order to connect with observations, we must relate our quantum variable, characterizing the perturbation of the coupled Einstein-fluid system through the field v, with the gravitational potential Φ. This is done using the equation (Eq. (12.8) of [6])

$$\nabla^2 \Phi = -\sqrt{\frac{3}{2}} \frac{l_p \beta}{\mathcal{H} c_s^2} \left(\frac{v}{z} \right)' \tag{17}$$

The final hypothesis of the model concerns the time of birth of the modes (as mentioned above). The mode k is born when the scale factor is such that satisfies

$$a_0 = k\, l_0. \tag{18}$$

Then one considers, in the standard manner, the spectrum characterized by the two point function of the field $\hat{v}$ in the vacuum state $|0\rangle$ associated with the above specified mode functions. The point is that, for modes with wavelength greater than the Hubble radius, at the time of decoupling, one finds as in [5] a scale free spectrum of density perturbations:

$$\mathcal{P}_\Phi(k) \sim \frac{l_p^2}{l_0^2} \frac{3w^{1/2}(6w+5)}{4(3w+5)^2} \frac{1}{k^3} \tag{19}$$

However, as was mentioned in the Introduction, it is easy to see that this state $|0\rangle$, is perfectly homogeneous and isotropic (i.e. it is annihilated by the quantum generators of rotations and translations). As the dynamical evolution preserves such symmetries, the state of the system will be symmetric (homogeneous and isotropic) at all times. In fact, there is nothing, given the standard unitary evolution of the quantum theory, that could be invoked to avoid such conclusion. The issue is then, how do we account for a universe with seeds of cosmic structure, not to say an appropriate spectrum for such inhomogeneities, starting from an isotropic and homogeneous state? We will not discuss these conceptual issues in detail here, and we direct the interested reader to see some relevant works in the literature (e.g. [7,13,34]).

In the following sections, we will study the incorporation of the collapse hypothesis, a proposal designed to address the conceptual difficulty mentioned above, into this model. We will consider the modifications on the predicted form of the spectrum that results from the analysis that incorporates such a collapse. As we will see, under certain conditions, one can recover an almost scale free spectrum for scalar perturbations, but generically there would be some characteristic deviations thereof.

2.2. Incorporating the collapse

In the approach taken so far, both the metric and the fluid perturbations are treated at the quantum level. Then, from (17), we must also promote Φ to a quantum operator. Our focus will be the scalar metric perturbation Φ, being this the link with the observations, representing the small anisotropies in the temperature of the CMB. What we need is to find an expression representing the relevant metric perturbation Φ.

We will assume that the classical description is only relevant for those particular states for which the quantity in question is sharply peaked, and that the classical description corresponds to the expectation value of said quantity.

Following previous works (e.g. [31]), where the case has been addressed in detail, we will assume the validity of the identification $\Phi^{\Xi}(x) \equiv \langle\Xi|\hat{\Phi}(x)|\Xi\rangle \equiv \langle\hat{\Phi}(x)\rangle_{\Xi}$, with $|\Xi\rangle$ the corresponding state of the quantum field. That in turn will be either the pre-collapse vacuum state or the post-collapse state of the field $\hat{v}(x)$, characterizing jointly the metric and fluid perturbation, and being sharply peaked in the associated variable $\hat{\Phi}(x)$.

Calculating its expectation value in the vacuum state, in Fourier space, we obtain

$$\langle 0|\hat{\Phi}_{\mathbf{k}}(\eta)|0\rangle = \sqrt{\frac{3}{2}}\frac{l_p\,\beta}{\mathcal{H}c_s^2}\frac{1}{k^2}\left(\frac{\langle 0|\hat{v}_{\mathbf{k}}(\eta)|0\rangle}{z}\right)' \tag{20}$$

Therefore, given that $\langle 0|\hat{v}_{\mathbf{k}}(\eta)|0\rangle = 0$, and without invoking something like a collapse, we will have that $\langle 0|\hat{\Phi}_{\mathbf{k}}(\eta)|0\rangle = 0$ at all times. This explicitly exhibits the above alluded problem, that the symmetries of the initial state do not seem to be compatible with any perturbation of the metric.

Just as in preceding works [7,13], we are lead to introduce something like what we call a *collapse hypothesis*. When the modes are born at time η_0^k, the state $|0\rangle$ is perfectly symmetric. At some collapse time, η_c^k, a transition to a new state $|0\rangle \to |\Xi\rangle$ is produced, which does not have the initial symmetries. And in this new state, we will have that $\langle\Xi|\hat{v}_{\mathbf{k}}(\eta)|\Xi\rangle \neq 0$ for all $\eta \geq \eta_c^k$, and therefore, the collapse process will generate the seeds of cosmic structure.

That process is thought to represent some novel aspect of physics, connecting with properties of quantum gravity, as has been suggested, for instance, by Diósi and Penrose [24–29]. There is a long history of studies about proposals involving something like a collapse of the wave function. Most of them, from the community working on foundations of quantum theory (see e.g. [35]). The manner in which this problematic issues appear in the inflationary cosmological context was considered first in [13].

In this work, we will consider the simple case where only one collapse happens per mode $\mathbf{k}$. Specifically, we will assume that, at η_c^k, a given mode $\mathbf{k}$ undergoes a collapse $|0_{\mathbf{k}}\rangle \to |\Xi_{\mathbf{k}}\rangle$. These collapses will be assumed to take place according to certain specific rules which we will describe in detail and, as we will see, they will depend on a particular *collapse scheme* considered, and will induce a change in the expectation value of $\hat{\Phi}_{\mathbf{k}}(\eta)$. The important point here is that, after the collapse of the mode $\mathbf{k}$, the universe will be no longer homogeneous and isotropic, in regards to that mode.

While the issues related to the transition from a quantum description to a classical one have been the subject of debate (e.g. [7,36]), and the collapse proposal seems better suited to the semi-classical treatments (where the matter fields are quantized and the metric perturbations are not, e.g. [37]), here we will follow the most usual approach in which the composite field v is quantized (see, for instance [31]).

Thus, our equation for the perturbation Φ and for $\eta \geq \eta_c^k$ (in Fourier space) will be:

$$\begin{aligned}\Phi_{\mathbf{k}}^{\Xi}(\eta) &\equiv \langle\hat{\Phi}_{\mathbf{k}}(\eta)\rangle_{\Xi} \equiv \langle\Xi|\hat{\Phi}_{\mathbf{k}}(\eta)|\Xi\rangle \\ &= \sqrt{\frac{3}{2}}\frac{l_p\,\beta}{\mathcal{H}c_s^2}\frac{1}{k^2}\left(\frac{\langle\hat{v}_{\mathbf{k}}(\eta)\rangle_{\Xi}}{z}\right)'\end{aligned} \tag{21}$$

This is the point where we must make contact with the observations.

The small anisotropies observed in the temperature of the CMB radiation, $\delta T(\theta,\phi)/T_0$, can be described in terms the coefficients a_{lm} of the multipolar expansion

$$\begin{aligned}&\frac{\delta T}{T_0}(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi) \\ &\quad a_{lm} = \int \frac{\delta T}{T_0}(\theta,\phi) Y_{lm}^*(\theta,\phi) d\Omega\end{aligned} \tag{22}$$

Here, θ and ϕ are the two-sphere coordinates, $Y_{lm}(\theta,\phi)$ are the spherical harmonics, and $T_0 = 2.725$ K is the CMB average temperature today. At large angular scales (low l) the Sachs–Wolfe effect is the dominant source to the CMB anisotropies. It is well known that for this case,

$$\frac{\delta T}{T_0}(\theta,\phi) = \frac{1}{3}\Phi(\eta_D, R_D) \tag{23}$$

where η_D and R_D are evaluated in the decoupling epoch.

Now, expanding to $\Phi(\eta_D, R_D)$ in Fourier modes and using the spherical Bessel functions of order l, $j_l(k R_D)$, the expression (22) can be written as:

$$a_{lm} = \frac{4\pi i^l}{3L^{3/2}} \sum_{\mathbf{k}} j_l(kR_D)\, Y_{lm}^*(\hat{k}) \Phi_{\mathbf{k}}^{\Xi}(\eta_D) \tag{24}$$

The individual complex quantities a_{lm}, correspond to sums of complex contributions $\Phi_{\mathbf{k}}^{\Xi}(\eta_D)$, each one having a certain randomness, but leading in combination to a characteristic value in just the same way as a two-dimensional random walk made of multiple steps. As in any random walk, the only thing that can be done is to calculate the most likely (ML) value for the total displacement, with the expectation that the observed quantity will be close to that value. Thus, one needs to estimate the most likely value of the quantity

$$|a_{lm}|^2 = \frac{16\pi^2}{9L^2} \sum_{\mathbf{k},\mathbf{k}'} j_l(kR_D)\, j_l(k'R_D) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{k}') \Phi_{\mathbf{k}}^{\Xi}(\eta) \Phi_{\mathbf{k}'}^{\Xi *}(\eta).$$

Following the approach first used in [13], one accomplishes this with the help of an imaginary ensemble of universes (each one corresponding to a possible realization of the collapse). We will identify the most likely value with the observed one, and will estimate its value using the ensemble mean value of the corresponding quantity. That is, $|a_{lm}|^2_{\rm ML} \simeq |a_{lm}|^2_{\rm obs} \simeq \overline{|a_{lm}|^2}$.

Thus, one finds that the quantity that contains the information regarding the theoretical prediction for the primordial spectrum, and that, in an effective way, would correspond to the expression (19), will be obtained once we calculate, for the relevant modes, the quantity $\overline{\Phi^{\Xi}_{\mathbf{k}}(\eta)\Phi^{\Xi *}_{\mathbf{k}'}(\eta)}$, where the over-line indicates the ensemble average. The relevant quantities for the analysis of the seeds of cosmic structure are those characterizing the statistics of the collapse, as we will see next. Finally, a direct connection with the standard results would be obtained if we write

$$\overline{\Phi^{\Xi}_{\mathbf{k}}(\eta)\Phi^{\Xi *}_{\mathbf{k}'}(\eta)} = \mathcal{P}_{\Phi}(k)\, \delta_{\mathbf{k}\mathbf{k}'} \tag{25}$$

Our aim now is to analyze the modifications in the predictions for the primordial scalar fluctuation spectrum that result from incorporating the collapse hypothesis into the model developed in [5].

We will consider weather, and under what circumstances, one can obtain a prediction for a scale free spectrum for those perturbations, for the case of the observationally relevant modes, which have wavelengths greater than the Hubble radius at the time of decoupling (and therefore, within this model, at all previous times).

2.3. Results

In order to proceed to find our results, firstly we decompose the operators $\hat{v}_{\mathbf{k}}(\eta)$ and $\hat{\pi}_{\mathbf{k}}(\eta)$ in their real and imaginary parts,

$$\hat{v}_{\mathbf{k}}(\eta) = \hat{v}^{\rm R}_{\mathbf{k}}(\eta) + i\hat{v}^{\rm I}_{\mathbf{k}}(\eta)$$
$$\hat{\pi}_{\mathbf{k}}(\eta) = \hat{\pi}^{\rm R}_{\mathbf{k}}(\eta) + i\hat{\pi}^{\rm I}_{\mathbf{k}}(\eta)$$

with $\hat{v}^{\rm R,I}_{\mathbf{k}}(\eta) = \frac{1}{\sqrt{2}}\Big(\hat{a}^{\rm R,I}_{\mathbf{k}} v_k(\eta) + \hat{a}^{\rm R,I\dagger}_{\mathbf{k}} v^*_k(\eta)\Big)$ and $\hat{\pi}^{\rm R,I}_{\mathbf{k}}(\eta) = \frac{1}{\sqrt{2}}\Big(\hat{a}^{\rm R,I}_{\mathbf{k}} \times \pi_k(\eta) + \hat{a}^{\rm R,I\dagger}_{\mathbf{k}} \pi^*_k(\eta)\Big)$, where $\hat{a}^{\rm R}_{\mathbf{k}} = \frac{1}{\sqrt{2}}(\hat{a}_{\mathbf{k}} + \hat{a}_{-\mathbf{k}})$ and $\hat{a}^{\rm I}_{\mathbf{k}} = \frac{-i}{\sqrt{2}}(\hat{a}_{\mathbf{k}} - \hat{a}_{-\mathbf{k}})$. Note that according to (3), $\pi_k(\eta) = v'_k(\eta)$. In this manner, $\hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)$ and $\hat{\pi}^{\rm R,I}_{\mathbf{k}}(\eta)$ are Hermitian operators. But now, the commutation relations will be:

$$[\hat{a}^{\rm R}_{\mathbf{k}}, \hat{a}^{\rm R\dagger}_{\mathbf{k}'}] = (\delta_{\mathbf{k},\mathbf{k}'} + \delta_{\mathbf{k},-\mathbf{k}'})$$
$$[\hat{a}^{\rm I}_{\mathbf{k}}, \hat{a}^{\rm I\dagger}_{\mathbf{k}'}] = (\delta_{\mathbf{k},\mathbf{k}'} - \delta_{\mathbf{k},-\mathbf{k}'})$$

with all the other commutators vanishing.

As was mentioned in the Introduction, we assume, in analogy with standard quantum mechanics, that the collapse is somehow analogous to an imprecise measurement (of the Hermitian operators $\hat{v}^{\rm R,I}_{\mathbf{k}}$ and $\hat{\pi}^{\rm R,I}_{\mathbf{k}}$).

The proposal for the collapse scheme, assumes that at certain time η^k_c, which can depend on the mode, the state corresponding to the mode $\mathbf{k}$ undergoes an instantaneous change (or collapse) so that the expectation values of the field operators after such change become:

$$\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta^k_c)\rangle_{\Xi} = x^{\rm R,I}_{\mathbf{k}} \sqrt{\langle(\Delta \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta^k_c))^2\rangle_0} = \frac{x^{\rm R,I}_{\mathbf{k}}}{\sqrt{2}} |v_k(\eta^k_c)| \equiv s_{(k)} \tag{26}$$

$$\langle \hat{\pi}^{\rm R,I}_{\mathbf{k}}(\eta^k_c)\rangle_{\Xi} = 0, \tag{27}$$

where $\langle(\Delta \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta^k_c))^2\rangle_0$ is the quantum uncertainty of the operator $\hat{v}^{\rm R,I}_{\mathbf{k}}$ in the vacuum state $|0\rangle$ at time η^k_c. This is one of the simplest collapse schemes one can consider, where the collapse process changes the expectation value of the field operators but leaves the expectation values of the momenta unchanged. Other schemes are of course possible, such as when the collapse operators affect the later and not the former, or when the collapse process modifies both simultaneously, either preserving, or not, certain correlations present in the quantum state. Previous analysis considering the various kinds of collapse in standard inflationary models indicate that different alternatives lead to similar (but not exactly identical) results. The interested reader can see these differences, in e.g. [32,38]. We will focus our attention here on the simple scheme specified above, that might be viewed as the most straightforward generalization of the collapse into the configuration variable (that is the position operators) and does not modify the expectation value of the conjugate momentum operators, which are used in the context of non-relativistic quantum mechanics [14–21].

The numbers $x^{\rm R,I}_{\mathbf{k}}$ are a collection of independent random quantities (selected from a Gaussian distribution centered at zero with unit-spread) that, as we will see next, will help determine $\Phi^{\Xi}_{\mathbf{k}}(\eta)$ as a kind of random walk. This collapse scheme is a particular case of other more general ones (see, for instance, [31]). As previously mentioned, we will assume that we can estimate the most likely value of the random walk displacement by its ensemble average that is $|a_{lm}|^2_{\rm ML} \simeq \overline{|a_{lm}|^2}$. Therefore, in this case, we can use

$$\overline{x^{\rm R}_{\mathbf{k}} x^{\rm R}_{\mathbf{k}'}} = \delta_{\mathbf{k},\mathbf{k}'} + \delta_{\mathbf{k},-\mathbf{k}'} \tag{28}$$

$$\overline{x^{\rm I}_{\mathbf{k}} x^{\rm I}_{\mathbf{k}'}} = \delta_{\mathbf{k},\mathbf{k}'} - \delta_{\mathbf{k},-\mathbf{k}'}. \tag{29}$$

We must emphasize that our universe corresponds to a single realization of each of these random variables and, therefore, the quantities of interest have a single specific value.

In order to compute $\Phi^{\Xi}_{\mathbf{k}}(\eta)$, we need to know the quantity $\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi}$ for all $\eta \geq \eta^k_c$. In order to obtain this, we make use of Ehrenfest equations,

$$\frac{d}{d\eta}\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} = \langle \hat{\pi}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} \tag{30}$$

$$\frac{d}{d\eta}\langle \hat{\pi}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} = -\Big(c_s^2 k^2 - \frac{A}{\eta^2}\Big)\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} \tag{31}$$

Differentiating (30) with respect to η, and using (31), we can write

$$\frac{d^2}{d\eta^2}\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} = -\Big(c_s^2 k^2 - \frac{A}{\eta^2}\Big)\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} \tag{32}$$

whose general solution is

$$\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} = \alpha^{\rm R,I}_1 f(\eta) + \alpha^{\rm R,I}_2 h(\eta) \tag{33}$$

hence,

$$\langle \hat{\pi}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} = \alpha^{\rm R,I}_1 f'(\eta) + \alpha^{\rm R,I}_2 h'(\eta). \tag{34}$$

Here we have defined:

$$f(\eta) \equiv \sqrt{\eta}\, J_n(c_s k\eta) \tag{35}$$

$$h(\eta) \equiv \sqrt{\eta}\, Y_n(c_s k\eta) \tag{36}$$

Evaluating (33) and (34) at η^k_c, and using our collapse scheme (26)–(27), the constants $\alpha^{\rm R,I}_1$ and $\alpha^{\rm R,I}_2$ (depending on η^k_c) are found to be

$$\alpha^{\rm R,I}_1 = \frac{\pi s_{(k)} h'(\eta^k_c)}{2} \tag{37}$$

$$\alpha^{\rm R,I}_2 = -\frac{\pi s_{(k)} f'(\eta^k_c)}{2} \tag{38}$$

and therefore:

$$\langle \hat{v}^{\rm R,I}_{\mathbf{k}}(\eta)\rangle_{\Xi} = \frac{\pi s_{(k)}}{2}\Big[h'(\eta^k_c) f(\eta) - f'(\eta^k_c) h(\eta)\Big] \tag{39}$$

We restrict our analysis to the cases where the time of birth, the time of collapse for each mode, and the overall evaluation time (the time of interest η, which strictly speaking should be the decoupling time), satisfy that $\eta_0^k < \eta_c^k < \eta$. Furthermore, we recall that we are interested in the observationally relevant modes having $k\eta \ll 1$ corresponding to wavelengths greater than the Hubble radius at the time of decoupling.

Thus, we can use the asymptotic forms for small arguments of Bessel functions, both in the functions $h'(\eta_c^k)$ and $f'(\eta_c^k)$, and for $f(\eta)$ and $h(\eta)$ at the end of the calculation.

In order to proceed, we make some numerical estimates. By taking $w = 0.3$ and $k \sim 10^{-3}$ Mpc^{-1} we find that $\eta_0^k \sim 10^{-50}$ Mpc. Therefore, (39) can be approximated by

$$\langle \hat{v}_{\mathbf{k}}^{\mathrm{R,I}}(\eta)\rangle_\Xi \simeq \frac{\pi s_{(k)}}{2}\left[\frac{\sqrt{2}\Gamma(n)(n-\frac{1}{2})}{\pi\sqrt{kc_s}\eta_c^k}\left(\frac{2}{kc_s\eta_c^k}\right)^{n-\frac{1}{2}} f(\eta) - \frac{\sqrt{kc_s}(n+\frac{1}{2})}{\sqrt{2}\Gamma(n+1)}\left(\frac{kc_s\eta_c^k}{2}\right)^{n-\frac{1}{2}} h(\eta)\right] \quad (40)$$

We have performed separate numerical analysis of this issue and have found that the exact results are essentially indistinguishable from the ones obtained with the above approximations.

Next, we note that for the relevant modes, one can approximate $|v_k(\eta)|$ from (15) by the expression

$$|v_k(\eta)| \simeq \frac{1}{4n\sqrt{c_s k}}\left[(2n-1)\left(\frac{\eta}{\eta_0^k}\right)^{n+\frac{1}{2}} + (1+2n)\left(\frac{\eta_0^k}{\eta}\right)^{n-\frac{1}{2}}\right], \quad (41)$$

which, in combination with the hypothesis of [5], i.e. the modes are born when $a(\eta_0^k) \simeq kl_0$, allow us to write

$$|v_k(\eta)| \simeq \frac{a(\eta)}{4nl_0\sqrt{c_s}k^{3/2}}\left[(2n-1) + (2n+1)\left(\frac{kl_0}{a(\eta)}\right)^{\frac{4n}{2n+1}}\right], \quad (42)$$

where we can see that if $\eta = \eta_0^k$, then $|v_k(\eta_0^k)| = \frac{1}{\sqrt{c_s k}}$. While that for the case of pure radiation, this means $w = \frac{1}{3}$ (or $n = \frac{1}{2}$), $|v_k(\eta)|$ becomes $|v_k(\eta)| = \frac{1}{\sqrt{c_s k}}$ for all η.

Next, using (26), we evaluate the quantum uncertainty of $\hat{v}_{\mathbf{k}}^{\mathrm{R,I}}(\eta_c^k)$ in the state $|0\rangle$. On the other hand, an approximation for $\langle(\Delta\hat{v}_{\mathbf{k}}^{\mathrm{R,I}}(\eta))^2\rangle_0$, evaluated at η_c^k will be

$$\langle(\Delta\hat{v}_{\mathbf{k}}^{\mathrm{R,I}}(\eta_c^k))^2\rangle_0 \simeq \frac{a^2(\eta_c^k)}{32n^2 l_0^2 c_s k^3}\left[(2n-1) + (1+2n)\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}}\right]^2. \quad (43)$$

Now, we note that from the above expression, together with (26), one obtains

$$s_{(k)} \simeq \frac{a(\eta_c^k)}{4n\sqrt{2c_s}l_0 k^{3/2}}\left[2n-1+(1+2n)\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}}\right]x_{\mathbf{k}}^{\mathrm{R,I}}. \quad (44)$$

Given that $z = \gamma\eta^{\frac{2}{3w+1}}$, with $\gamma = \frac{-\sqrt{3w+3}}{\sqrt{2c_s}}\eta_{\mathrm{today}}^{\frac{-2}{3w+1}}$, the expression (21) can now be rewritten as

$$\langle\hat{\Phi}_{\mathbf{k}}^{\mathrm{R,I}}(\eta)\rangle_\Xi = \sqrt{\frac{3}{2}}\frac{l_p\beta}{\gamma\mathcal{H}c_s^2}\frac{1}{k^2}\left(\frac{\langle\hat{v}_{\mathbf{k}}^{\mathrm{R,I}}(\eta)\rangle_\Xi}{\eta^{\frac{2}{3w+1}}}\right)' \quad (45)$$

Then, by using (40) along with (44) into (45), and using the fact that the random variables $x_{\mathbf{k}}^{\mathrm{R,I}}$ satisfy (28)–(29), we can finally write

$$\overline{\Phi_{\mathbf{k}}^\Xi(\eta)\Phi_{\mathbf{k}'}^{\Xi*}(\eta)} = 2\overline{\langle\hat{\Phi}_{\mathbf{k}}^{\mathrm{R,I}}(\eta)\rangle_\Xi\langle\hat{\Phi}_{\mathbf{k}'}^{\mathrm{R,I}}(\eta)\rangle_\Xi} \simeq \frac{3l_p^2\beta^2 a^2(\eta_c^k)}{256n^2\mathcal{H}^2 c_s^5 l_0^2\gamma^2 k^3}\left[2n-1 + (1+2n)\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}}\right]^2\left[\delta_1+\frac{\delta_2}{k^2}\right]^2\delta_{\mathbf{k}\mathbf{k}'} \quad (46)$$

where:

$$\delta_1 \equiv \left(n-\frac{1}{2}\right)\frac{c_s^2\eta}{n(n+1)2^{n+1}}\left(\frac{2}{\eta_c^k}\right)^{n+\frac{1}{2}}$$

$$\delta_2 \equiv \left(n+\frac{1}{2}\right)\frac{2^{n+1}}{\eta^{2n+1}}\left(\frac{\eta_c^k}{2}\right)^{n-\frac{1}{2}} \quad (47)$$

Thus recalling that $a(\eta_c^k) = B(\eta_c^k)^{n+1/2}$ it is easy to see that the right hand side of (46) takes the following form:

$$\overline{\Phi_{\mathbf{k}}^\Xi(\eta)\Phi_{\mathbf{k}'}^{\Xi*}(\eta)} \simeq \frac{3}{2048k^3}\left\{\frac{Bl_p\beta\eta}{(n+1)n^2\sqrt{c_s}\mathcal{H}l_0\gamma}\left[2n-1 + (1+2n)\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}}\right]\left[2n-1 + \frac{4n(2n^2+3n+1)(\eta_c^k)^{2n}}{c_s^2\eta^{2n+2}k^2}\right]\right\}^2\delta_{\mathbf{k}\mathbf{k}'} \quad (48)$$

We note that, when $|2n-1| \gg (1+2n)\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}}$, and $|2n-1| \gg \frac{4n(2n^2+3n+1)(\eta_c^k)^{2n}}{c_s^2k^2\eta^{2n+2}}$, the dominant contribution would lead to the standard scale invariant prediction for the spectrum $\overline{\Phi_{\mathbf{k}}^\Xi(\eta)\Phi_{\mathbf{k}}^{\Xi*}(\eta)} \propto \frac{1}{k^3}$, and its value would be independent of the value of the time of collapse η_c^k of the mode $\mathbf{k}$. However, it is clear that this will not be the case when $n = 1/2$ corresponding to the pure radiation case where $w = 1/3$.

Next, we consider the corrections to the scale invariant component. For this, we must focus on the two above indicated inequalities. When $n \in (0, \frac{2}{3})$, we have $|2n-1| < (1+2n)$. Then, in order to satisfy the first inequality, it is necessary that $\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}} \ll 1$. However, the exponent is such that $0 < \frac{4n}{2n+1} < \frac{3}{2}$. So, although $kl_0 = a(\eta_0^k) < a(\eta_c^k)$ is assured (modes would not collapse before they are born), we need that the collapse to take place sufficiently late so that the two scales differ by, say, at least, one order of magnitude.

In short, we can conclude that the first inequality will be satisfied as long as $0 \ll n < \frac{3}{2}$, and the collapse is such that $a(\eta_0^k)$ and $a(\eta_c^k)$ differ by at least one order of magnitude.

Let us focus now on the second inequality. The first issue we must be concerned with, is the fact that c_s goes to zero when w goes to zero, or equivalently, when n goes to $\frac{3}{2}$. The term $(\eta_c^k/\eta)^{2n}$ is small simply because we are interested on the quantity in (48) at very late times; which clearly must be long after the collapse has taken place for all the relevant modes. This term is further modulated by a $4n(2n^2+3n+1)$ which lies in the range $(0, 60)$. Thus, the second inequality will be satisfied if $0 \ll c_s < 1$ (this means that $0 < n \ll \frac{3}{2}$), together with the conditions that ensure that $c_s k\eta$ is not too small. This, of course, will be most easily satisfied the earlier the collapse for the relevant modes occurs.

Therefore, in the case where the fluid is such that $0 \ll n \ll \frac{3}{2}$ and thus $|2n-1| \gg 0$ (i.e. $0 \ll w \ll 1$ and $|3w-1| \gg 0$),[1] and where the collapse occurs early enough, we find that the expression (48) for the power spectrum is well approximated by

$$\overline{\Phi_{\mathbf{k}}^{\Xi}(\eta)\Phi_{\mathbf{k}'}^{\Xi *}(\eta)} \simeq \frac{3}{2048k^3}\left\{\frac{Bl_p\beta\eta(2n-1)^2}{(n+1)n^2\sqrt{c_s}\mathcal{H}l_0\gamma}\left[1 + \frac{(1+2n)}{(2n-1)}\left(\frac{kl_0}{a(\eta_c^k)}\right)^{\frac{4n}{2n+1}} + \frac{4n(2n^2+3n+1)(\eta_c^k)^{2n}}{c_s^2\eta^{2n+2}(2n-1)k^2}\right]\right\}^2 \delta_{\mathbf{kk}'} \tag{49}$$

which has a leading term corresponding to the standard flat spectrum together with corrections associated with the collapse, and which clearly depends on the collapse times for the relevant modes.

This is very similar to what was found to occur in the standard inflationary model with an instantaneous collapse. We can see this by considering Eq. (88) of Ref. [13]:

$$C(k) = 1 + \frac{2}{(k\eta_c^k)^2}\sin^2(\Delta_k) + \frac{1}{k\eta_c^k}\sin(2\Delta_k), \tag{50}$$

with $\Delta_k = k\eta_D - k\eta_c^k$, where η_D is the conformal time at decoupling, and $C(k)$ is a dimensionless function which encodes all the information about the effects of the details of the collapse scheme on the observational power spectrum. As discussed in that work, it is only if $C(k)$ is independent of k that the standard scale invariant spectrum is recovered. Now, considering only the modes of interest, and taking into account that in the inflationary situation $k\eta_D \approx 0$, then as long as $0 < |k\eta_c^k| \ll 1$ (which would be the case if the collapse of the relevant modes takes place near the end of the inflationary period), the quantity $\Delta_k \to -k\eta_c^k$, and therefore performing a series expansion in (50) one finds that

$$C(k) = 1 + \left[\frac{2(k\eta_c^k)^2}{3} + O(3)\right]. \tag{51}$$

Then, as long as for the relevant modes $|k\eta_c^k| \ll 1$, the dominate terms would lead to a standard scale free spectrum. In that case, just as it happens in the situation considered in this work (see (46)), the dominant part of (51) is independent of the value of the collapse time η_c^k of the mode $\mathbf{k}$.

At this point, it is worth mentioning that once the wave function has collapsed, it keeps evolving according to the Schrödinger equation and thus it will not remain very well localized even if it was so right after the collapse. The general issue of how well localized should be the field and its momentum conjugate at various epochs is one that needs a careful study, which should involve, among other things, the possibility of multiple collapses of each mode. In this regard, we should mention that the analysis contemplating a multiplicity of collapses, within the more traditional inflationary paradigm, has been carried out in [39] indicating that avoiding important deviations from the simple single collapse scheme require some limitation in the number of collapses per mode.

However, one thing which has to be mentioned is that the sharpness in the value of the directly observable quantities (e.g. a_{lm}), is likely to be much bigger than what might be inferred directly from the spread of the individual modes of $v_{\mathbf{k}}$. The reason for this is that, as indicated in Eq. (24), the quantity of interest is the sum of a very large number of terms (one for each wavenumber $\mathbf{k}$), and thus, one should expect something like the standard $1/\sqrt{N}$ decrease in the variance that is characteristic of the statistics of aggregates of large number, N, of uncorrelated contributions. The situation is of course more complicated, among other reasons, due to the different weights with which the various random terms appear in the sum (24). We will leave for future works the detail analysis of this and related issues.

In the case $w \approx \frac{1}{3}$, the model does not lead to the standard scale free spectrum and the details of the predicted shape will depend strongly on the model parameters and the assumptions made regarding the collapse times η_c^k for the observationally relevant modes with $k \in [10^{-4}, 10^{-1}]$ Mpc^{-1}.

From the original proposal [5], it can be inferred that for the case of pure radiation ($w = 1/3$), the scale free spectrum cannot be recovered. We are naturally lead to consider whether the introduction of the collapse hypothesis could modify that conclusion. This possibility arises because, as we saw, the collapse brings an extra set of parameters represented by the collapse times of each of the modes. This can be explored by setting $w = 1/3$ ($n = 1/2$) in Eq. (48). We find that

$$\mathcal{P}_{\Phi}^{w=1/3}(k) \simeq \frac{3l_p^2\beta^2}{2\mathcal{H}^2c_s^5\gamma^2\eta^4k^5}. \tag{52}$$

Once more, we see that the result does not depend on the collapse times, removing the possibility of adjusting the dependence of η_c^k with k, in order to recover the scale free spectrum. Also noteworthy is the fact that the above result is independent of the parameter l_0 characterizing the birth time of the modes.

3. Conclusions

In [5], Hollands and Wald presented a cosmological scheme that leads to similar predictions regarding the spectrum of primordial cosmological fluctuations as those emerging from inflationary models. The proposal is based on a model with a matter content represented by more or less conventional fluids. They studied a quantum treatment of the quantity representing the coupled gravity–matter modes of the perturbations, and showed that, under some simple hypotheses concerning the birth time and the initial state of the relevant modes, it is possible to obtain the same density perturbation spectrum and amplitude, as those emerging from the successful inflationary models.

However, this model (just as the standard inflationary models), has a serious issue regarding the lack of a satisfactory account for the breaking of the symmetry of the initial state. Very briefly, the issue is connected with interpretational difficulties of quantum theory that become extremely exacerbated in the cosmological setting. As has been discussed elsewhere, a satisfactory model should explain the emergence of the seeds of cosmic structure, which clearly correspond to inhomogeneities and anisotropies. Starting from a perfectly symmetric quantum state (isotropic and homogeneous), it cannot be accomplished in standard terms given that the dynamics of the theory preserves such symmetries. The Copenhagen approach to quantum theory cannot explain how this transition would happen in the absence of observers or external measurements.

In previous works, this issue has been extensively discussed, and the arguments were not repeated here, so the interested reader is referred to, for instance, [7,13].

In this manuscript, we showed that, in the framework of the model considered in [5], and with the same hypotheses, one

[1] Our use of the notation '$\ll$' in this discussion is meant to indicate that the inequality is satisfied by a sufficiently large margin, to ensure the dominance of the term in question by a large enough factor, given the values of the other quantities appearing in the corresponding expressions, and the precision of the existing observations.

can obtain under certain circumstances, a prediction for an almost scale free spectrum for scalar perturbations, when a collapse scheme is introduced to allow for the transition from the original state to a state containing actual seeds of cosmic structure. We also showed which would be the dominant corrections that can be expected to arise as a result of such a collapse.

One interesting feature that we found is that, under the appropriate conditions, the amplitude of the dominant term leading to the standard flat spectrum (corresponding to the term proportional to $1/k^3$) does not depend on the collapse time. Thus just as in [5], the value of observed amplitude ($\sim 10^{-10}$) can be obtained simply by taking $l_0 \simeq 5 \times 10^{-55}$ Mpc.

In the original proposal [5], it was found that for the case of pure radiation ($w = 1/3$), the scale free spectrum cannot be recovered. We considered the possibility that the introduction of the collapse hypothesis might lead to a reversal of that conclusion. However, we found that the final result for the spectral shape is again independent of the collapse times. Therefore, such a possibility is excluded, at least for the kind of collapse models explored here.

Furthermore, our result (46) also illustrates the fact that corrections to the scale free spectrum can be expected in association with the introduction of collapse models into the theoretical analysis.

Finally, we should mention that we have focused only on the observational constraints associated with the amplitude and zeroth order form of the power spectrum. We view these as the most robust aspects of the analysis and the ones that are more likely to be roughly independent of the collapse scheme (i.e. the recipe for the selection of expectation values of the field and momentum conjugate in each mode, and the number of collapses per mode). However, other interesting observables do exist, such as the tensor-to-scalar ratio and the scalar spectral index n_S. Regarding the latter, it is worthwhile noting something from Eq. (49). In particular, if the collapse is assumed to occur at some physical fixed scale l_c, i.e. if $a(\eta_c^k) = k\, l_c$ (which is analogous to the assumption for the 'birth time' of each mode), then the first corrective term in (49) is scale invariant, and the second term is proportional to $k^{-(1/2+3w/2)}$ indicating a red spectral index, which is what is favored by current observations. The detailed investigation of this issue is also left for future work. We must acknowledge that the model proposed in [5] leaves various issues unsolved, and it is not completely clear at this point if one can consider it as a truly viable alternative to inflation. However, our main point is that, just as the traditional inflationary paradigm, it requires some mechanism, such as the one explored in this work, to address the conceptual problem that afflict quantum schemes for the emergence of the seeds of cosmic structure, from homogeneous and isotropic initial states.

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