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Infinite horizon MPC with non-minimal state space feedback

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ABSTRACT

In the MPC literature, stability is usually assured under the assumption that the state is measured. Since the closed-loop system may be nonlinear because of the constraints, it is not possible to apply the separation principle to prove global stability for the output feedback case. It is well known that, a nonlinear closed-loop system with the state estimated via an exponentially converging observer combined with a state feedback controller can be unstable even when the controller is stable.

One alternative to overcome the state estimation problem is to adopt a non-minimal state space model, in which the states are represented by measured past inputs and outputs [P.C. Young, M.A. Behzadi, C.L. Wang, A. Chotai, Direct digital and adaptative control by input–output, state variable feedback pole assignment, International Journal of Control 46 (1987) 1867–1881; C. Wang, P.C. Young, Direct digital control by input–output, state variable feedback: theoretical background, International Journal of Control 47 (1988) 97–109]. In this case, no observer is needed since the state variables can be directly measured. However, an important disadvantage of this approach is that the realigned model is not of minimal order, which makes the infinite horizon approach to obtain nominal stability difficult to apply. Here, we propose a method to properly formulate an infinite horizon MPC based on the output-realigned model, which avoids the use of an observer and guarantees the closed loop stability. The simulation results show that, besides providing closed-loop stability for systems with integrating and stable modes, the proposed controller may have a better performance than those MPC controllers that make use of an observer to estimate the current states.

1. Introduction

Most of the initial versions of predictive control rely on a step response model or (equivalently) an impulse response model. These non-parsimonious representations yield a finite number of terms only when the system is open-loop stable. On the other hand, a significant part of the recent research literature on MPC shows contributions based on state-space models. This tendency has been stimulated by the connections found between the standard linear quadratic regulator (LQR) theory and MPC when prediction and control horizon approaches infinity and there are no constraints. In fact, an intensive effort has been based on state-space representations, for instance, to develop the rigorous conditions that guarantee a stable MPC, or to demonstrate recursive feasibility of the sequence of optimal control solutions. Typical developments using state space models to study the stability and feasibility of MPC include Muske and Rawlings [8], Rawlings and Muske [9], and Scokaert and Rawlings [11]. Mayne et al. [7] and Rawlings [10] provide extensive reviews of the theory involved in this formulation of model predictive control.

Some MPC designs based on state space models include a state observer and exploit the separation principle to claim stability. In other cases, the dynamics of the state estimates generated by the state observer is assumed to be much faster than the dynamics of the state feedback control law, and hence, the error signal between the observer and the actual system is assumed to converge to zero at a much faster rate than the system converges to the desired state. However, the presence of constraints in the MPC scheme makes the closed loop non-linear. In other words, the state feedback could be dominated (precisely, when the constraints become active) by nonlinear properties. As a result, the separation principle can no longer be applied to the closed loop system in order to assure stability [15].

One alternative approach to overcome this problem is to include the observer behavior in the stability analysis. Zheng and Morari [15] have developed stabilizing tuning relations for the MPC with output feedback for the case where all the open loop eigenvalues of the system are strictly inside the unit circle. More recently, Mayne et al. [6], propose a robust output feedback MPC that incorporates the error on the state estimation as an additional

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unknown (but bounded) uncertainty that can be accounted for in the design of the controller. These authors define a ‘‘tube” of trajectories based on a controlled invariant set, within which the true state of the system is guaranteed to remain. Goulart and Kerrigan [4] propose a robust output feedback MPC, where the observer dynamics is included in the computation of the domain of attraction of the closed loop system. Ding et al. [2] adopt a dynamic output feedback approach to propose a stable MPC. The method was extended by Ding and Huang [1] to nonlinear systems that can be represented by Hammerstein–Wiener models.

Here, it is proposed a different approach to overcome the output feedback stability problem. The method uses a non-minimal state space model that avoids the use of the state observer [17,14,12]. This type of model has been used by Wang and Young [14] to minimize the closed loop performance deterioration produced by the observer in the presence of input disturbances, and when constraints become activated. This approach produces a simpler control algorithm (and more attractive from the application point of view) in comparison with the one that includes the observer dynamics in the formulation of the control law. However, the non-minimal state space model introduces an additional difficulty to the application of the usual approach to the design of a stable controller, which is based on the adoption of an infinite prediction horizon. This difficulty will be discussed in the next section of this work.

González et al. [3] developed a stable infinite horizon MPC for systems with stable and integrating modes. In their approach, the model is written in such a way that the non-stable modes are separated from the stable modes and, since an infinite horizon cost is used, the non-stable modes are zeroed at the end of the control horizon (this is so, in order to prevent the cost from becoming unbounded). When a non-minimal state space model is used, a direct similarity transformation that produces the separation between the stable and non-stable modes cannot be found, since the resulting transformation matrix is not invertible. As a result, the terminal constraint needed to guarantee stability cannot be written. In González et al. [3] stability is only assured for the case where all the states are measured, which does not usually happen in practice. In this work, this limitation is eliminated through the adoption of a non-minimal state space model in which the state variables correspond to the measured past outputs and inputs. Stable and integrating systems are considered.

The paper is organized as follows: Section 2 describes the non-minimal state space model used for predictions, and the problems that arise when a similarity transformation is needed. This section also shows some model and transformation properties, useful for the formulation of output feedback infinite horizon MPC. Section 3 presents the proposed controller and shows how the infinite cost is bounded for the general case of non-stable systems. Section 4 presents the details of the stable MPC with output feedback for the case of stable and integrating systems. Section 5 shows the simulation analysis of an ethylene oxide reactor system. A comparison between an output feedback controller based on a state observer and the proposed controller is also presented in order to expose the advantages of the new controller. Finally, Section 6 concludes the paper.

2. The non-minimal state space model

The state space model considered here is based on the following difference equation model:

$$y(k) = -\sum_{i=1}^{na} A_i y(k-i) + \sum_{i=1}^{nb} B_i u(k-i), \tag{1}$$

where na is the number of poles of the system, nb is the number of zeros of the system, and the system is assumed to have nu inputs and ny outputs.

Model (1) corresponds to the following state space model in the output realigned form [16,13,5]:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k), \\ y(k) &= Cx(k), \end{aligned} \tag{2}$$

where

$$A = \begin{bmatrix} A_y & A_{\Delta u} \\ 0 & \underline{I} \end{bmatrix}, \quad B = \begin{bmatrix} B_{\Delta u} \\ \bar{I} \end{bmatrix}, \quad C = [\,C_y \quad C_{\Delta u}\,],$$

$$A_y = \begin{bmatrix} I_{ny} - A_1 & A_1 - A_2 & \cdots & A_{na-1} - A_{na} & A_{na} \\ I_{ny} & 0 & \cdots & 0 & 0 \\ 0 & I_{ny} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_{ny} & 0 \end{bmatrix} \in \Re^{(na+1)ny \times (na+1)ny},$$

$$A_{\Delta u} = \begin{bmatrix} B_2 & \cdots & B_{nb-1} & B_{nb} \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{(na+1)ny \times (nb-1)nu},$$

$$\underline{I} = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ I_{nu} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & I_{nu} & 0 \end{bmatrix} \in \Re^{(nb-1)nu \times (nb-1)nu}, \quad B_{\Delta u} = \begin{bmatrix} B_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\bar{I} = \begin{bmatrix} I_{nu} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad C_y = [\,I_{ny} \quad 0 \quad \cdots \quad 0 \quad 0\,] \quad C_y \in \Re^{(na+1)ny},$$

$$C_{\Delta u} = [\,0 \quad \cdots \quad 0 \quad 0\,], \quad C_{\Delta u} \in \Re^{(nb-1)nu}.$$

I_{ny} and I_{nu} are identity matrices of dimension ny and nu, respectively.

Furthermore, the state x is given by

$$x(k) = \begin{bmatrix} x_y(k) \\ x_{\Delta u}(k) \end{bmatrix} \in \Re^{nx},$$

where

$$x_y(k) = \left[\, y(k)^T \quad y(k-1)^T \quad \cdots \quad y(k-na+1)^T \quad y(k-na)^T \,\right]^T \in \Re^{(na+1)ny},$$

$$x_{\Delta u}(k) = \left[\, \Delta u(k-1)^T \quad \Delta u(k-2)^T \quad \cdots \quad \Delta u(k-nb+1)^T \,\right]^T \in \Re^{(nb-1)nu},$$

$$nx = (na+1)ny + (nb-1)nu.$$

The partition of the state is convenient in order to separate the state components related to the system output at past sampling steps, from the state components related to the input at past sampling steps. Also, since the model is written in terms of the input increment (velocity model), model (2) contains the modes of model (1) plus ny integrating modes. Here, we assume that the system represented in (1) has non-repeated stable and integrating modes.

The state matrix A, in model (2), has the following property:

Property 1. *Matrix A is rank deficient. Furthermore*

$$\operatorname{rank}(A^n) = nx - (n)nu, \quad 1 \leqslant n \leqslant nb-1 \quad \text{and}$$
$$\operatorname{rank}(A^n) = nx - (nb-1)nu = (na+1)ny, \quad n \geqslant nb-1.$$

The last equality implies: $A^n = A^{n-nb+1}A^{nb-1}$ for $n \geqslant nb-1$, where $\underline{A} = \begin{bmatrix} A_y & 0 \\ 0 & 0 \end{bmatrix}$.

Proof. It is easy to prove this property if we observe that matrix $\underline{I}$ has nu null rows, $\underline{I}^2$ has $2nu$ null rows, ..., $\underline{I}^{nb-1}$ has $(nb-1)nu$ rows and consequently is a null matrix. □

It is well known that any integrating mode cannot be allowed to proceed over an unbounded time interval without control action. Therefore, in order to implement a MPC based on model (2) with infinite prediction horizon, we need first to find a state transformation that makes explicit the stable and integrating parts of the plant. One straightforward alternative is to adopt the eigenvalue–eigenvector Jordan decomposition:

$$AV_{\mathrm{com}} = V_{\mathrm{com}}A_d, \tag{3}$$

where A_d is a block diagonal matrix (Jordan canonical form) that makes explicit the different dynamic modes of the system, and the columns of V_{com} are the eigenvectors, or generalized eigenvectors, of A.

An unsolved problem related to the transformation defined in (3) is that, since the realigned model defined in (2) is not of minimal order, matrix V_{com} is not invertible. As a result, it is not possible to recover the original states from the transformed states and the main advantage of model (2) (i.e., the avoidance of an observer) is lost. Nevertheless, we can still find states along the prediction horizon where a similarity transformation can be performed (i.e., V_{com} is invertible). To define these states, let us consider the following sequence of input moves:

$$\Delta u(k), \cdots, \Delta u(k+m-1), 0, \ldots \tag{4}$$

Then, taking into account Property 1, and considering the sequence defined in (4), the open-loop state predictions at time instants beyond the control horizon m, and computed at time k, can be written as follows:

$$\begin{aligned} & x(k+m|k) \\ & \quad = \Bigg[x_y(k+m)^{\mathrm{T}} \quad \overbrace{\Delta u(k+m-1)^{\mathrm{T}} \quad \cdots \quad \Delta u(k+m-nb+1)^{\mathrm{T}}}^{x_{\Delta u}(k+m)^{\mathrm{T}}} \Bigg]^{\mathrm{T}} \\ & x(k+m+1|k) = Ax(k+m|k) \\ & \quad = \Bigg[x_y(k+m+1)^{\mathrm{T}} \quad \overbrace{0 \quad \Delta u(k+m-1)^{\mathrm{T}} \quad \cdots \quad \Delta u(k+m-nb+2)^{\mathrm{T}}}^{x_{\Delta u}(k+m+1)^{\mathrm{T}}} \Bigg]^{\mathrm{T}} \\ & \vdots \\ & x(k+m'+j|k) = A^{nb-1+j}x(k+m|k) = \underline{A}^j A^{nb-1}x(k+m|k) \\ & \qquad\qquad = \Bigg[x_y(k+m'+j)^{\mathrm{T}} \quad \overbrace{0 \quad \cdots \quad 0}^{x_{\Delta u}(k+m'+j)^{\mathrm{T}}} \Bigg], \quad j \geqslant 1, \end{aligned}$$

where $m' = m + nb - 1$. This means that beyond time step $k + m'$, the predictions of the last $(nb-1)nu$ state components will be null. In other words, at time steps beyond $k + m'$, the state predictions evolve according to matrix $\underline{A}$, and the input matrix B does not affect the evolution of the state as the input moves are assumed to be null beyond time $k + m'$. In this scenario, we may consider the following transformation:

$$A_y\underline{V} = \underline{V}A_d,$$

where the similarity transformation matrix $\underline{V} \in \Re^{(na+1)ny \times (na+1)ny}$ is now full rank since the modeled system is supposed to have $(na+1)ny$ poles. Matrix A_d is again a block diagonal matrix. Consider now the following augmented matrices:

$$V = \begin{bmatrix} \underline{V} \\ 0 \end{bmatrix}, \quad V \in \Re^{nx \times (na+1)ny}, \quad V_{\mathrm{in}} = \begin{bmatrix} \underline{V}^{-1} & 0 \end{bmatrix}, \quad V \in \Re^{(na+1)ny \times nx}.$$

Then, the following equality holds:

$$\underline{A} = VA_dV_{\mathrm{in}} = [V_{\mathrm{nst}} \quad V_{\mathrm{st}}] \begin{bmatrix} F^{\mathrm{nst}} & 0 \\ 0 & F^{\mathrm{st}} \end{bmatrix} \begin{bmatrix} \widetilde{V}_{\mathrm{nst}} \\ \widetilde{V}_{\mathrm{st}} \end{bmatrix}, \tag{5}$$

where the columns of V_{nst} and V_{st} span the non-stable (integrating) and stable subspaces of the system, respectively. Also, $F^{\mathrm{nst}} \in \Re^{\mathrm{nns} \times \mathrm{nns}}$ is the state block diagonal matrix corresponding to the integrating modes and $F^{\mathrm{st}} \in \Re^{\mathrm{ns} \times \mathrm{ns}}$ is the block diagonal matrix corresponding to the stable modes (nns is the total number of integrating modes and ns is the number of stable modes).

The transformation defined in (5) has the following property related to the state matrix A of the model defined in (2):

Property 2. *$V_{\mathrm{in}}A^n = A_d^{n-nb+1}V_{\mathrm{in}}A^{nb-1}$, for $n \geqslant nb-1$.*

Proof. From (5), we have $V_{\mathrm{in}}\underline{A} = A_dV_{\mathrm{in}}$, which implies $V_{\mathrm{in}}AA^{nb-1} = A_dV_{\mathrm{in}}A^{nb-1}$. Then, from Property 1, $V_{\mathrm{in}}A^{nb} = A_dV_{\mathrm{in}}A^{nb-1}$. Multiplying both sides by A, we have

$$\begin{aligned} V_{\mathrm{in}}A^{nb+1} &= A_dV_{\mathrm{in}}A^{nb}, \\ V_{\mathrm{in}}A^{nb+1} &= A_d(A_dV_{\mathrm{in}})A^{nb-1} = A_d^2V_{\mathrm{in}}A^{nb-1}. \end{aligned}$$

Thus, by induction, we can obtain $V_{\mathrm{in}}A^n = A_d^{n-nb+1}V_{\mathrm{in}}A^{nb-1}$ for $n \geqslant nb-1$. □

Remark 1. For the predicted states where the similarity transformation is possible, the two system representations: $z(k+1) = A_dz(k)$ and $x(k+1) = Ax(k)$ are equivalent. In fact, for any time instant $(k + m' + j)$, we have

$$x(k+m'+j|k) = \underline{A}^jx(k+m'|k)$$

and, using (5), we can write

$$\begin{aligned} x(k+m'+j|k) &= VA_d^jV_{\mathrm{in}}x(k+m'|k), \\ z(k+m'+j|k) &= A_d^jz(k+m'|k), \end{aligned}$$

where $z(k) = V_{\mathrm{in}}x(k)$, $z(k) \in \Re^{nz}$ and $nz = (na+1)ny$.

Now, the integrating and stable states of the transformed model that is equivalent to model (2) can be computed as

$$z(k) = \begin{bmatrix} z^{\mathrm{nst}}(k) \\ z^{\mathrm{st}}(k) \end{bmatrix} = V_{\mathrm{in}}x(k) = \begin{bmatrix} \widetilde{V}_{\mathrm{nst}} \\ \widetilde{V}_{\mathrm{st}} \end{bmatrix} x(k). \tag{6}$$

Furthermore, the system output can be expressed as

$$y(k) = C_d \begin{bmatrix} z^{\mathrm{nst}}(k) \\ z^{\mathrm{st}}(k) \end{bmatrix},$$

where $C_d = CV$, $z^{\mathrm{nst}} \in \Re^{\mathrm{nns}}$, $z^{\mathrm{st}} \in \Re^{\mathrm{ns}}$, nz = nns + ns.

Model $z(k+1) = A_d z(k)$ may have a different structure depending on the nature of the system. In the case of systems with *nun* non-repeated integrating modes (nun poles equal to one), matrix F^{nst} can be written as

$$F^{\mathrm{nst}} = \begin{bmatrix} I_{\mathrm{ny}} & D \\ 0 & I_{\mathrm{nun}} \end{bmatrix} \in \Re^{\mathrm{nns}\times\mathrm{nns}}, \tag{7}$$

where nns = nun + ny, I_{ny} and I_{nun} are identity matrices, and $D \in \Re^{\mathrm{ny}\times\mathrm{nun}}$ with elements $d_{j,k}$ is a particular matrix, where $d_{j,k} = 1$ for $j = k$, and $d_{j,k} = 0$ for $j \neq k$. We may assume, for the sake of simplicity, that nun $\leqslant$ max(ny, nu).

In addition, the state component z^{nst} can be decomposed as $z^{\mathrm{nst}} = \begin{bmatrix} z^i(k) \\ z^{\mathrm{un}}(k) \end{bmatrix}$, and the transformation matrix $\widetilde{V}_{\mathrm{nst}}$, as $\widetilde{V}_{\mathrm{nst}} = \begin{bmatrix} \widetilde{V}_i \\ \widetilde{V}_{\mathrm{un}} \end{bmatrix}$. The state $z^i \in \Re^{\mathrm{ny}}$ corresponds to the integrating states related to the velocity form of the model, and $z^{\mathrm{un}} \in \Re^{\mathrm{nun}}$ corresponds to the integrating states of the system.

Now, for the system structure discussed above, similarity transformation (5) has the following properties:

Property 3.

$$\widetilde{V}_{\mathrm{nst}} A^n = (F^{\mathrm{nst}})^{n-nb+1} \widetilde{V}_{\mathrm{nst}} A^{nb-1} \quad \text{and} \quad \widetilde{V}_{\mathrm{st}} A^n = (F^{\mathrm{st}})^{n-nb+1} \widetilde{V}_{\mathrm{st}} A^{nb-1} \ \text{for}\ n \geqslant nb-1.$$

Proof. The proof is directly obtained from Property 2. □

Property 4.

$$\widetilde{V}_{\mathrm{un}} A^n = \widetilde{V}_{\mathrm{un}} A^{nb-1} \quad \textit{for all } n \geqslant nb-1.$$

Proof. The proof is directly derived from Property 3. □

Property 5. Output set-point transformation *If we define an output set-point* y^{sp}*, then, the set-point of the state of the system represented in* (2) *can be defined as* $x^{\mathrm{sp}} = \begin{bmatrix} y^{\mathrm{sp}^{\mathrm{T}}} & \cdots & y^{\mathrm{sp}^{\mathrm{T}}} & 0 & \cdots & 0 \end{bmatrix}^{\mathrm{T}}$. *Furthermore, we can also define a set-point to the transformed state as* $z^{\mathrm{sp}} = V_{\mathrm{in}} x^{\mathrm{sp}}$*, which satisfies*

$$z^{\mathrm{sp}} = \begin{bmatrix} z^{i,\mathrm{sp}} \\ z^{\mathrm{un,sp}} \\ z^{\mathrm{st,sp}} \end{bmatrix} = \begin{bmatrix} \widetilde{V}_i x^{\mathrm{sp}} \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad z^{i,\mathrm{sp}} = C_d^{i-1} y^{\mathrm{sp}}.$$

Proof. Since x^{sp} corresponds to an equilibrium point, it has to satisfy $(A - I_{nx})x^{\mathrm{sp}} = 0$. Then, the z-set-point is obtained by means of the similarity transformation matrix V_{in}, as $z^{\mathrm{sp}} = V_{\mathrm{in}} x^{\mathrm{sp}}$, satisfies $(A_d - I_{nz})z^{\mathrm{sp}} = 0$.

$$\text{Then, as } (A_d - I_{nz}) = \begin{bmatrix} 0 & D & 0 \\ 0 & 0 & 0 \\ 0 & 0 & F^{\mathrm{st}} - I_{\mathrm{ns}} \end{bmatrix},$$

where D is assumed of full rank, and given that, $F^{\mathrm{st}} - I_{\mathrm{ns}}$ is diagonal full rank matrix, it is easy to show that

$$z^{\mathrm{sp}} = \begin{bmatrix} z^{i,\mathrm{sp}} \\ z^{\mathrm{un,sp}} \\ z^{\mathrm{st,sp}} \end{bmatrix} = \begin{bmatrix} \widetilde{V}_i x^{\mathrm{sp}} \\ 0 \\ 0 \end{bmatrix}.$$

This means that the output set-point for the z-state is concentrated in the first ny components. In addition, we have

$$Cx^{\mathrm{sp}} = y^{\mathrm{sp}} \quad \text{and} \quad Cx^{\mathrm{sp}} = CVz^{\mathrm{sp}} = C_d z^{\mathrm{sp}} = C_d \begin{bmatrix} z^{i,\mathrm{sp}} \\ 0 \\ 0 \end{bmatrix} = C_d^i z^{i,\mathrm{sp}},$$

where $C_d = \begin{bmatrix} C_d^i & C_d^{\mathrm{un}} & C_d^{\mathrm{st}} \end{bmatrix}$, C_d^i is associated with the integrating states related to the velocity form of the model, C_d^{un} is associated with the integrating states of the system, and C_d^{st} is associated with the stable states of the system.

Therefore, $z^{i,\mathrm{sp}} = (C_d^i)^{-1} y^{\mathrm{sp}}$, and the property is proved. □

3. MPC with output feedback

Now, we may formulate an infinite horizon MPC with output feedback based on model (2). The MPC cost is written as

$$V_{1,k} = \sum_{j=0}^{\infty} (Cx(k+j|k) - y^{\mathrm{sp}})^{\mathrm{T}} Q (Cx(k+j|k) - y^{\mathrm{sp}}) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\mathrm{T}} R \Delta u(k+j|k). \tag{8}$$

As usual in the receding horizon strategy, it is assumed that $\Delta u(k+j|k) = 0$ for $j \geqslant m$. The cost defined in (8) will be unbounded unless the integrating states are zeroed at a suitable time step in the prediction horizon. Here, we propose to include in the control problem, the following constraint:

$$z^{\mathrm{nst}}(k+m') - z^{\mathrm{nst,sp}} = 0, \quad m' = m + nb - 1, \tag{9}$$

where $z^{\mathrm{nst,sp}} = \begin{bmatrix} z^{i,\mathrm{sp}} \\ 0 \end{bmatrix}$, and $z^{i,\mathrm{sp}}$ is defined as in Property 5. Constraint (9) can be expressed in terms of the current (and measured) state x, as follows:

$$\widetilde{V}_{\mathrm{nst}} A^{m'} x(k) + \widetilde{V}_{\mathrm{nst}} B_{\mathrm{aug}} \Delta u_k - z^{\mathrm{nst,sp}} = 0, \tag{10}$$

where

$$\widetilde{V}_{\mathrm{nst}} = \begin{bmatrix} \widetilde{V}_i \\ \widetilde{V}_{\mathrm{un}} \end{bmatrix}, \quad B_{\mathrm{aug}} = \begin{bmatrix} A^{m'-1}B & A^{m'-2}B & \cdots & B \end{bmatrix}$$

and

$$\Delta u_k = \begin{bmatrix} \Delta u(k|k)^{\mathrm{T}} & \cdots & \Delta u(k+m-1|k)^{\mathrm{T}} & \underbrace{0 \cdots 0}_{m'-m} \end{bmatrix}^{\mathrm{T}}.$$

Observe that to satisfy constraint (10) we need at least one degree of freedom per integrating state (m×nu $\geqslant$ nns). In addition, we must add to the optimization problem the constraints related to the input and input increment:

$$\Delta u(k+j|k) \in U, \quad j = 1, \ldots, m-1, \tag{11}$$

where

$$U = \left\{ \Delta u(k+j) \left| \begin{array}{l} -\Delta u^{\max} \leqslant \Delta u(k+j) \leqslant \Delta u^{\max} \\ u^{\min} \leqslant u(k-1) + \sum_{i=0}^{j} \Delta u(k+i) \leqslant u^{\max} \end{array} \right. \right\}$$

The cost defined in (8) can also be written as

$$V_{1,k} = \sum_{j=0}^{m'}(Cx(k+j|k) - y^{sp})^T Q(Cx(k+j|k) - y^{sp}) + \sum_{j=m'}^{\infty}(Cx(k+j|k) - y^{sp})^T Q(Cx(k+j|k) - y^{sp}) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^T R \Delta u(k+j|k).$$

Now, taking into account constraint (9), the infinite sum of the above cost can be developed as follows:

$$\begin{aligned}\textstyle\sum_{j=m'}^{\infty}&(Vz(k+j|k) - x^{sp})^T C^T QC(Vz(k+j|k) - x^{sp}) \\ &= \sum_{j=m'}^{\infty}(z(k+j|k) - z^{sp})^T V^T C^T QCV(z(k+j|k) - z^{sp}) \\ &= \sum_{i=0}^{\infty}\left(\begin{bmatrix}(F^{nst})^i(z^{nst}(k+m'|k) - z^{nst,sp}) \\ (F^{st})^i z^{st}(k+m'|k)\end{bmatrix}\right)^T \times V^T C^T QCV\left(\begin{bmatrix}(F^{nst})^i(z^{nst}(k+m'|k) - z^{nst,sp}) \\ (F^{st})^i z^{st}(k+m'|k)\end{bmatrix}\right) \\ &= \sum_{i=0}^{\infty} z^{st}(k+m'|k)^T (F^{st})^{i^T} V_{st}^T C^T QCV_{st}(F^{st})^i z^{st}(k+m'|k),\end{aligned}$$

where x^{sp} and z^{sp} are defined as in Property 5.

This means that the infinite sum of the control cost can be written as follows:

$$\sum_{j=m'}^{\infty}(Cx(k+j|k) - y^{sp})^T Q(Cx(k+j|k) - y^{sp}) = \overbrace{x(k+m'|k)^T \widetilde{V}_{st}^T}^{z^{st}(k+m'|k)^T} P \overbrace{\widetilde{V}_{st} x(k+m'|k)}^{z^{st}(k+m'|k)},$$

where P is computed through the solution to the following Lyapunov equation:

$$P = V_{st}^T C^T QCV_{st} + F^{st^T} P F^{st}.$$

It can be shown that if the optimization problem that minimizes (8) subject to constraints (10) and (11) is feasible at time step k, it will remain feasible at any subsequent time step $k+j$, and the control law resulting from the solution to this optimization problem stabilizes the closed loop system. However, the domain of attraction of this controller is usually too small for practical applications, as the control horizon m is small to reduce the computation effort. This implies that constraints (10) and (11) may become infeasible for disturbances of moderate size in the integrating states. Following González et al. [3], one alternative to enlarge the region where the controller is feasible is to include slack variables into the control problem. Thus, we propose to extend the cost defined in (8) as follows:

$$V_{2,k} = \sum_{j=0}^{\infty}(Cx(k+j|k) - y^{sp} - CV\delta(k,j))^T Q(Cx(k+j|k) - y^{sp} - CV\delta(k,j)) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^T R \Delta u(k+j|k) + \delta_k^{nst^T} S \delta_k^{nst}, \quad (12)$$

where $\delta(k,j) = \begin{bmatrix}\delta^{nst}(k,j) \\ 0\end{bmatrix} = A_d^j \begin{bmatrix}\delta_k^{nst} \\ 0\end{bmatrix} \in \Re^{nz}$, δ_k^{nst} is a vector of slack variables that is introduced in the control problem to enlarge the domain of attraction of the proposed controller and S is a positive matrix of appropriate dimension. As a result of this modification, the terminal constraint defined in (10) becomes

$$z^{nst}(k+m') - z^{nst,sp} - \delta^{nst}(k,m') = 0 \quad (13)$$

that can be expressed in terms of the current (and measured) state x, as follows:

$$\widetilde{V}_{nst} A^{m'} x(k) + \widetilde{V}_{nst} B_{aug} \Delta u_k - z^{nst,sp} + (F^{nst})^{m'} \delta_k^{nst} = 0. \quad (14)$$

Then, cost $V_{2,k}$ can be written as

$$V_{2,k} = \sum_{j=0}^{m'-1}(Cx(k+j|k) - y^{sp} - CV\delta(k,j))^T Q(Cx(k+j|k) - y^{sp} - CV\delta(k,j)) + \sum_{j=m'}^{\infty}(Cx(k+j|k) - y^{sp} - CV\delta(k,j))^T Q(Cx(k+j|k) - y^{sp} - CV\delta(k,j)) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^T R \Delta u(k+j|k) + \delta_k^{nst^T} S \delta_k^{nst}.$$

Now, since at time $(k+m')$ there is a one to one correspondence between states x and z, the infinite sum term of the cost can be written as

$$\begin{aligned}\sum_{j=m'}^{\infty}&(Vz(k+j|k) - \overbrace{Vz^{sp}}^{x^{sp}} + V\delta(k,j))^T C^T QC(Vz(k+j|k) - \overbrace{Vz^{sp}}^{x^{sp}} + V\delta(k,j)) \\ &= \sum_{i=0}^{\infty}\left(\begin{bmatrix}(F^{nst})^i(z^{nst}(k+m'|k) - z^{nst,sp} + \delta^{nst}(k,m')) \\ (F^{st})^i z^{st}(k+m'|k)\end{bmatrix}\right)^T \times V^T C^T QCV\left(\begin{bmatrix}(F^{nst})^i(z^{nst}(k+m'|k) - z^{nst,sp} + \delta^{nst}(k,m')) \\ (F^{st})^i z^{st}(k+m'|k)\end{bmatrix}\right) \\ &= \sum_{i=0}^{\infty} z^{st}(k+m'|k)^T (F^{st})^{i^T} V_{st}^T C^T QCV_{st}(F^{st})^i z^{st}(k+m'|k) \\ &= x(k+m'|k)^T \widetilde{V}_{st}^T P \widetilde{V}_{st} x(k+m'|k),\end{aligned}$$

which represents a bounded quantity.

Making use of the results above, the extended infinite horizon MPC is obtained from the solution to the following optimization problem:

Problem P1

$$\min_{\Delta u_k, \delta_k^{nst}} \quad V_{2,k}$$

subject to (11) and (14).

Although Problem P1 is well posed and we can prove that the cost is always bounded, the control law resulting from the solution to P1 does not necessarily produce an asymptotic converging closed loop system. In order to obtain a stable closed-loop system, we may split Problem P1 into two sub-problems [3], as it is shown in the next section.

Remark 2. Note that in the control cost defined in (12), the infinite sum term has no real physical meaning, as it contains the slack $CV\delta(k,j)$ that is only included to make the cost bounded. However, once the slack $\delta(k,j)$ is zeroed, this infinite sum term penalizes the output error, as usual. This fact justifies the strategy of splitting the control problem in two separate problems: one that assures that the slack δ_k^{nst} converges to zero in finite time, and the other that guarantees that the output error, without the slack, converges to zero asymptotically.

4. A stable MPC with output feedback

To obtain an optimization problem that is equivalent to Problem P1, while producing a controller with guaranteed stability, we first perform the following partition of the vector of slack variables:

$$\delta_k^{nst} = \begin{bmatrix}\delta_k^i \\ \delta_k^{un}\end{bmatrix},$$

where $\delta_k^i \in \Re^{ny}$ is the vector of slack variables corresponding to the integrating states related to the velocity form of the model, and $\delta_k^{un} \in \Re^{nun}$ is the vector of slack variables corresponding to the integrating states of the system. Analogously, the slack penalization matrix can be written as $S^{nst} = \begin{bmatrix} S^i & 0 \\ 0 & S^{un} \end{bmatrix}$. Then, the proposed stable MPC with output feedback results from the solution to the two following optimization problems:

Problem 2a

$$\min_{\Delta u_{a,k},\delta_k^{un}} \quad V_{a,k} = \delta_k^{un^T} S^{un} \delta_k^{un}$$

subject to

$$\Delta u_a(k+j|k) \in U, \quad j = 0,1,\ldots,m-1,$$
$$\widetilde{V}_{un} A^{m'} x(k) + \widetilde{V}_{un} B_{aug} \Delta u_{a,k} - z^{un,sp} + (F^{un})^{m'} \delta_k^{un} = 0 \quad (15)$$

(Notice that, following the definition given in Property 5, $z^{un,sp} = 0$. In addition, notice that Eq. (15) is equivalent to $z^{un}(k+m'|k) + (F^{un})^{m'} \delta_k^{un} = 0$).

Problem 2b

$$\min_{\Delta u_{b,k},\delta_k^{nst}} \quad V_{b,k} = \sum_{j=0}^{m'-1} (Cx(k+j|k) - y^{sp} - CV\delta(k,j))^T Q (Cx(k+j|k) - y^{sp} - CV\delta(k,j)) + x(k+m'|k)^T \widetilde{V}_{st}^T P \widetilde{V}_{st} x(k+m'|k) + \sum_{j=0}^{m-1} \Delta u_b(k+j|k)^T R \Delta u_b(k+j|k) + \delta_k^{iT} S^i \delta_k^i$$

subject to

$$\Delta u_b(k+j|k) \in U, \quad j = 0,1,\ldots,m-1,$$
$$\widetilde{V}_{nst} A^{m'} x(k) + \widetilde{V}_{nst} B_{aug} \Delta u_{b,k} - z^{nst,sp} + (F^{nst})^{m'} \delta_k^{nst} = 0 \quad (16)$$
$$\delta_k^{un} = \delta_k^{un*}. \quad (17)$$

where δ_k^{un*} corresponds to the optimal slack of the integrating states obtained from the solution to Problem 2a. These two optimization problems are solved sequentially at the same time step.

Now, assuming that the system remains controllable at the desired steady-state, the solution to problems 2a and 2b produces a controller that is capable of driving the cost defined in (12) to zero, as shown in the theorems below.

Theorem 1. *For systems with stable and integrating modes, the sequential solution to problems 2a and 2b is always feasible; and the optimal cost is decreasing and converges to stationary point.*

Proof. The proof of this theorem follows almost the same steps as the proof of Theorem 1 in González et al. [3]. Assume that at time step k, the control sequence:

$$\Delta u_{b,k} = \begin{bmatrix} \Delta u_b(k|k)^{*T} & \cdots & \Delta u_b(k+m-1|k)^{*T} & \underbrace{0 \ \cdots \ 0}_{m'-m} \end{bmatrix}^T,$$
$$\delta_k^{nst*} = \begin{bmatrix} \delta_k^{i*} \\ \delta_k^{un*} \end{bmatrix}$$

represent a feasible solution to Problem 2b. This solution satisfies constraint (15) that using Property 4, becomes

$$\widetilde{V}_{un} A^{nb-1} x(k) + \widetilde{V}_{un} A^{nb-1} B \left(\sum_{j=0}^{m-1} \Delta u_b(k+j|k)^{*T} \right) + \delta_k^{un*} = 0. \quad (18)$$

Then, at time step $k+1$

$$\Delta u_{a,k+1} = \begin{bmatrix} \Delta u_b(k+1|k)^{*T} & \cdots & \Delta u_b(k+m-1|k)^{*T} & 0 & \underbrace{0 \ \cdots \ 0}_{m'-m} \end{bmatrix}^T,$$
$$\delta_{k+1}^{un} = \delta_k^{un*} \quad (19)$$

is a feasible solution to Problem 2a, and the corresponding cost is given by $V_{a,k+1} = V_{a,k}^* = \delta_k^{un*^T} S^{un} \delta_k^{un*}$. To prove this assertion, note that the solution defined in (19) satisfies

$$\widetilde{V}_{un} A^{nb-1} x(k+1) + \widetilde{V}_{un} A^{nb-1} B \left(\sum_{j=1}^{m} \Delta u_b(k+j|k)^{*T} \right) + \delta_{k+1}^{un} = 0. \quad (20)$$

Also, for the undisturbed system, we have

$$\widetilde{V}_{un} A^{nb-1} x(k+1) = \widetilde{V}_{un} A^{nb-1} x(k) + \widetilde{V}_{un} A^{nb-1} B \Delta u_b(k|k)^*$$

then, Eq. (20) is exactly the same as (18), which means that the control sequence defined in (19) is feasible and $\delta_{k+1}^{un} = \delta_k^{un*}$.

Now, if the input increment is not constrained, δ_{k+1}^{un} can be made equal to zero by considering the following control sequence:

$$\Delta u_{a,k+1} = \begin{bmatrix} \Delta u_b(k+1|k)^{*T} & \cdots & \Delta u_b(k+m-1|k)^{*T} & \Delta\bar{u} & \underbrace{0 \ \cdots \ 0}_{m'-m} \end{bmatrix}^T,$$

where

$$\Delta\bar{u} = -(\widetilde{V}_{un} A^{nb-1} B)^{-1} \delta_k^{un*}$$

where matrix $\widetilde{V}_{un} A^{nb-1} B$ is assumed to be full rank.

If the input is constrained, then it is easy to show that δ_{k+1}^{un} can be reduced to zero in a number of time steps not larger than $\max_j \frac{|\Delta\bar{u}_j|}{\Delta u_{j,max}}$ where index j designates the components of $\Delta\bar{u}$ and Δu_{max}. This proves that the cost $V_{a,k}$ will converge to zero in a finite number of time steps.

After convergence of $V_{a,k}$ to zero, solving problem 2a becomes equivalent to include in the control problem the following constraint:

$$\widetilde{V}_{un} A^{nb-1} x(k) + \widetilde{V}_{un} A^{nb-1} B \left(\sum_{j=0}^{m-1} \Delta u(k+j|k)^T \right) = 0. \quad (21)$$

Consequently, the solution obtained by solving problems 2a and 2b sequentially becomes equivalent to solving the following problem:

Problem P3

$$\min_{\Delta u_k,\delta_k^{nst}} \quad V_{3,k} = \sum_{j=0}^{m'-1} (Cx(k+j|k) - y^{sp} - CV^i \delta_k^i)^T \times Q (Cx(k+j|k) - y^{sp} - CV^i \delta_k^i) + x(k+m'|k)^T \widetilde{V}_{st}^T P \widetilde{V}_{st} x(k+m'|k) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^T R \Delta u(k+j|k) + \delta_k^{iT} S^i \delta_k^i \quad (22)$$

subject to (16) and (11) and

$$\delta_k^{un} = 0, \quad (23)$$

where $V = [V^i \quad V^{un} \quad V^{st}]$. It is easy to show that the cost defined in (22) is decreasing and converges to a bounded value [3]. Also, it is straightforward to show that if $V_{a,k}$ converges to zero and $V_{3,k}$ converges to a bounded value, then, $V_{2,k}$ will also converge to a bounded value and the theorem is proved. □

Observe that the inclusion of slack δ_k^i in the control problem may allow the term in $V_{3,k}$ related to the error on the output to converge to a stationary value where the slack is not equal to zero. This situation corresponds to the convergence of the closed loop system to a steady state with offset in the output. This may happen when the stable modes are no longer controllable (e.g. the input becomes saturated) or parameter S_i is not properly selected. Now, we show how to select S_i in order to prevent the output offset.

Theorem 2. *For systems with stable and integrating modes that remain controllable at the steady state corresponding to the desired output reference, if weight S^i is sufficiently large, then the control sequence obtained from the solution to problems 2a and 2b at successive time steps drives the output of the closed loop system asymptotically to the reference value and the control cost to zero.*

Proof. Suppose that when $k \to \bar{k}$ (large enough) the state tends to the steady state defined by $x(\bar{k})$. In addition, the solution of the optimisation problem (22) at steady state produces $\Delta u_{\bar{k}} = 0$ which, together with (16) and (23), implies

$$\widetilde{V}_{un}A^{m'}x(\bar{k}) = \widetilde{V}_{un}A^{nb-1}x(\bar{k}) = 0,$$
$$\widetilde{V}_iA^{m'}x(\bar{k}) - z^{i,sp} = -\delta^i_{\bar{k}}.$$

Also, the stable part of the state will tend to zero at this steady state, or $z(\bar{k}+m\prime|\bar{k}) = \widetilde{V}_{st}A^{m\prime}x(\bar{k}) = 0$. Thus, at this steady state, the cost will be given by $V_{3,\bar{k}} = \delta^{iT}_{\bar{k}}S^i\delta^i_{\bar{k}}$. Now, lets us find a control sequence that corresponds to a value of the cost that is smaller than $V_{3,\bar{k}}$. For this purpose, assume that $m = 2$, which is the minimum control horizon to produce an offset free controller, and assume also that none of input constraints is active. Then, let the solution to problem (22) at $\bar{k}$ be given by the sequence

$$\Delta\bar{u}_{\bar{k}} = \begin{bmatrix} \Delta\bar{u}(\bar{k}|\bar{k})^T & \Delta\bar{u}(\bar{k}+1|\bar{k})^T & \underbrace{0 \ \cdots \ 0}_{m'-m=nb-1} \end{bmatrix}^T$$

that has to satisfy the constraints represented in (16), or

$$\overbrace{\widetilde{V}_{un}A^{m'}x(\bar{k})}^{=0} + \widetilde{V}_{un}A^{nb+1}B(\Delta\bar{u}(\bar{k}|\bar{k}) + \Delta\bar{u}(\bar{k}+1|\bar{k})) = 0 \quad (24)$$

and

$$-\overbrace{\delta^i_{\bar{k}}}^{\widetilde{V}_iA^{m'}x(\bar{k})-z^{i,sp}} + \widetilde{V}_iB_{aug}\Delta\bar{u}_{\bar{k}} + \bar{\delta}^i_{\bar{k}} = 0. \quad (25)$$

Now, let us find a control sequence that satisfies (25) and makes $\bar{\delta}^i_{\bar{k}} = 0$. Provided that we have assumed that $\widetilde{V}_{un}A^{nb-1}B$ is full rank, Eq. (24) implies

$$\Delta\bar{u}(\bar{k}|\bar{k}) = -\Delta\bar{u}(\bar{k}+1|\bar{k}).$$

For this case, Eq. (25) becomes

$$\widetilde{V}_i(A^{nb+1} - A^{nb})B\Delta\bar{u}(\bar{k}|\bar{k}) = \delta^i_{\bar{k}}$$

or, taking into account the structure of F^{nst} for integrating systems, the above equation becomes

$$D\widetilde{V}_{un}A^{nb+1}B\Delta\bar{u}(\bar{k}|\bar{k})^T = \delta^i_{\bar{k}}.$$

Consequently, assuming that $D\widetilde{V}_{un}A^{nb+1}B$ is not singular (provided that $\widetilde{V}_{un}A^{nb+1}B$ is assumed to be full rank, if rank(D) = nun, then $D\widetilde{V}_{un}A^{nb+1}B$ is also full rank), a possible control sequence is given by

$$\Delta\bar{u}_{\bar{k}} = M\delta^i_{\bar{k}}, \quad (26)$$

where $M = \begin{bmatrix} (D\widetilde{V}_{un}A^{nb+1}B)^{-1} \\ -(D\widetilde{V}_{un}A^{nb+1}B)^{-1} \\ \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} nb-1 \end{bmatrix}$.

For this control sequence, the value of the cost is given by

$$\overline{V}_{3,\bar{k}} = \delta^{iT}_{\bar{k}}[(A_{ext}M^{-1}_{ss}I_\delta + B_{ext}M)^TC^TQC(A_{ext}M^{-1}_{ss}I_\delta + B_{ext}M) + (\widetilde{V}_{st}B_{aug}M)^TP(\widetilde{V}_{st}B_{aug}M) + M^T\overline{R}M]\delta^i_{\bar{k}},$$

where

$$M_{ss} = \begin{bmatrix} V_{in}A^{m'} \\ I_{\Delta u} \end{bmatrix} \in \Re^{nx\times nx}, \quad I_{\Delta u} = \begin{bmatrix} 0 & I_{(nb-1)nu} \end{bmatrix} \in \Re^{(nb-1)nu\times nx},$$

$$I_\delta = \begin{bmatrix} I_{ny} \\ 0 \end{bmatrix} \in \Re^{nx\times ny}, \quad A_{ext} = \begin{bmatrix} A \\ \vdots \\ A^{nb-1} \end{bmatrix}, \quad B_{ext} = \begin{bmatrix} B & \cdots & 0 \\ \vdots & \ddots & \vdots \\ A^{nb}B & \cdots & B \end{bmatrix}$$

and

$$\overline{R} = \text{diag}([R \quad R \quad 0 \quad \cdots \quad 0])$$

Consequently, $\overline{V}_{3,\bar{k}}$ will be smaller than $V_{3,\bar{k}}$ if

$$S^i > \Big[(A_{ext}M^{-1}_{ss}I_\delta + B_{ext}M)^TC^TQC(A_{ext}M^{-1}_{ss}I_\delta + B_{ext}M) + \Big(\widetilde{V}_{st}B_{aug}M)^TP(\widetilde{V}_{st}B_{aug}M) + M^T\overline{R}M\Big].$$

Analogously, for other values of m, a similar procedure can be used to define a sufficiently large value of S_i, such that the convergence of the output of the closed loop system to the reference is guaranteed. □

5. Simulation results

The system adopted as an example to test the performance of the controller presented here is part of the ethylene oxide reactor system studied by González et al. [3]. This is a typical example of the chemical process industry that exhibits stable and integrating poles. For a sampling period $\Delta T = 1$ min, the simulated system can be represented by the following difference equation model:

$$y(k) = -\begin{bmatrix} -1.8787 & 0 \\ 0 & -1.8964 \end{bmatrix} y(k-1) - \begin{bmatrix} 0.8787 & 0 \\ 0 & 0.8964 \end{bmatrix} y(k-2) + \begin{bmatrix} -0.3800 & -0.5679 \\ -0.2176 & 0.4700 \end{bmatrix} u(k-1) + \begin{bmatrix} 0.3339 & 0.5679 \\ 0.2176 & -0.4213 \end{bmatrix} u(k-2)$$

This system has two stable and two integrating modes. The model defined in (2) introduces two additional integrating modes related to the velocity form of the model. Then, the minimal number of

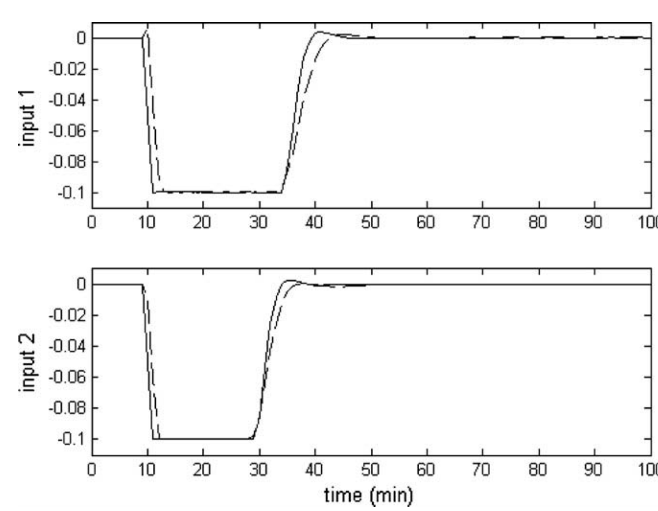

Fig. 1. Inputs for a change in the output set point. Controller I (—) and Controller II (– –).

states of the model of the ethylene oxide reactor is 6. The non-minimal model used here, as described in (2), has 16 states.

In all the cases considered here, the tuning parameters of the controller are the following: $m = 3$, $u_{\max} = [\,0.75 \quad 0.75\,]$, $u_{\min} = [\,-0.75 \quad -0.75\,]$, $\Delta u_{\max} = [\,0.05 \quad 0.05\,]$, $Q = \mathrm{diag}(\,1 \quad 1\,)$, $R = \mathrm{diag}(\,75 \quad 75\,)$, $S^i = \mathrm{diag}(\,1 \quad 1\,) \times 10^2$ and $S^n = \mathrm{diag}(\,1 \quad 1\,) \times 10^4$. Let us designate Controller I the controller defined by the sequential solution to problems 2a and 2b, and Controller II the controller presented in González et al. [3], in its nominal version. Controller II is a two-stage infinite horizon MPC controller that computes the output predictions based on the current estimate of the state and an output prediction oriented state space model (OPOM). The observer used in this structure is a typical Luenberger-like observer, with a gain matrix given by

$$L = \begin{bmatrix} -0.1673 & -0.0450 \\ -0.1293 & -1.1593 \\ 1.7059 & 0.0522 \\ 0.1394 & 2.6457 \\ -2.0850 & -0.0371 \\ 0.0552 & 1.7246 \end{bmatrix}.$$

First, we simulate a change in the output set point. The system starts from the origin and at time step 10 min, the desired output values are changed to $y^{sp} = [\,1 \quad -1\,]^T$. We can see in Figs. 1 and 2 that the system inputs and outputs are almost coincident for the two controllers. This is easy to justify: both controllers use a perfect model, and, since no disturbances enter the system, there is no difference between the estimated and the real states.

Fig. 3 shows the cost functions corresponding to problems 2a and 2b of Controller I. Cost $V_{a,k}$ is null during all the simulation because the system starts from a steady state in which $z^{un}(k+m'/k) = 0$ and, consequently, constraint (15) remains feasible with $\delta_k^{un} = 0$. As established in the Theorems 1 and 2, cost $V_{b,k}$ is asymptotically decreasing since the beginning of the simulation period and converges to zero.

The same controllers were also tested for the regulator case by simulating the closed loop system with an unmeasured disturbance in the system input. This disturbance corresponds to the control move $\Delta u = [\,0.4 \quad -0.4\,]$. The desired output values are kept at $(y_1^{sp}, y_2^{sp}) = [\,0 \quad 0\,]$ during all the simulation time. Figs. 4 and 5 describe the inputs and outputs of the system, respectively. It is clear that with both controllers, the outputs tend to the desired values while the inputs converge to new steady state values that

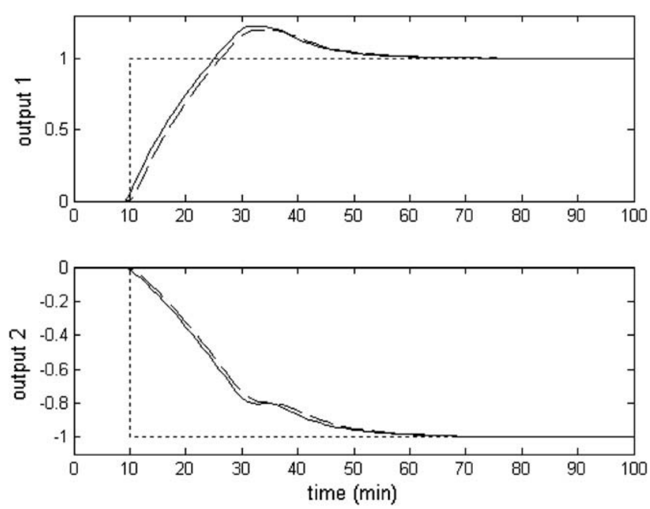

Fig. 2. Outputs for a change in the output set point. Controller I (—) and Controller II (– –).

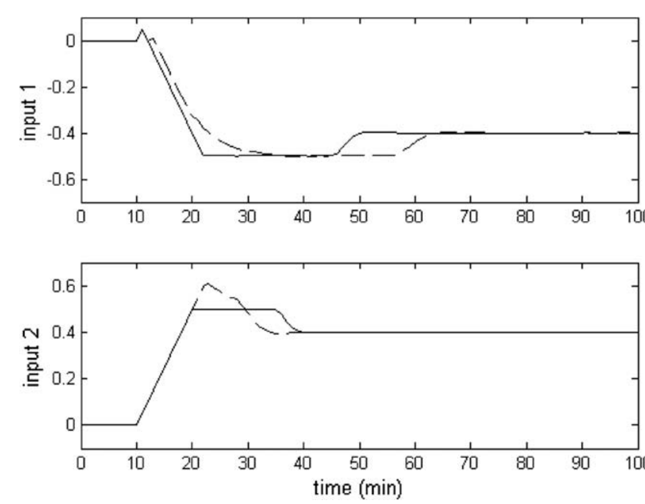

Fig. 4. Inputs for an input disturbance. Controller I (—) and Controller II (– –).

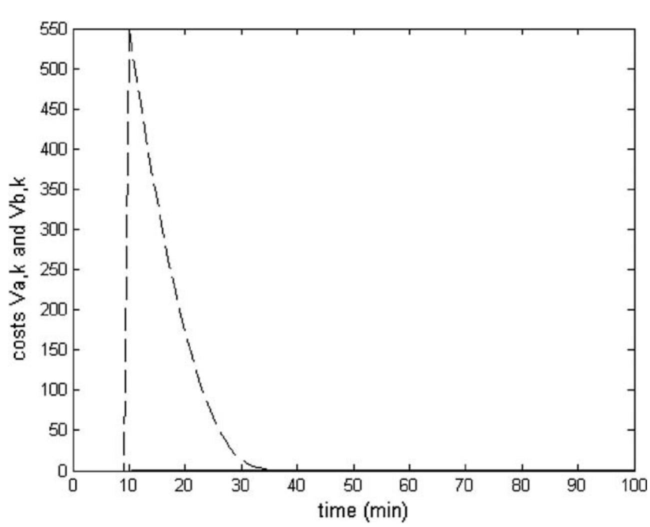

Fig. 3. Costs $V_{a,k}$ (—) and $V_{b,k}$ (– –) of Controller I, for a change in the output set point.

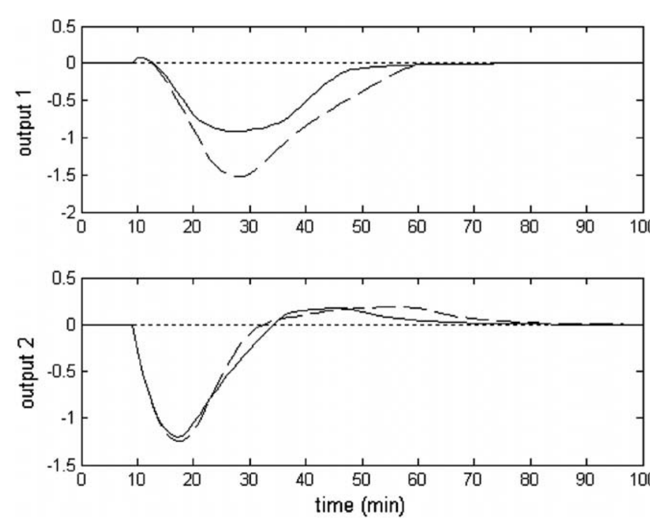

Fig. 5. Outputs for an input disturbance. Controller I (—) and Controller II (– –).

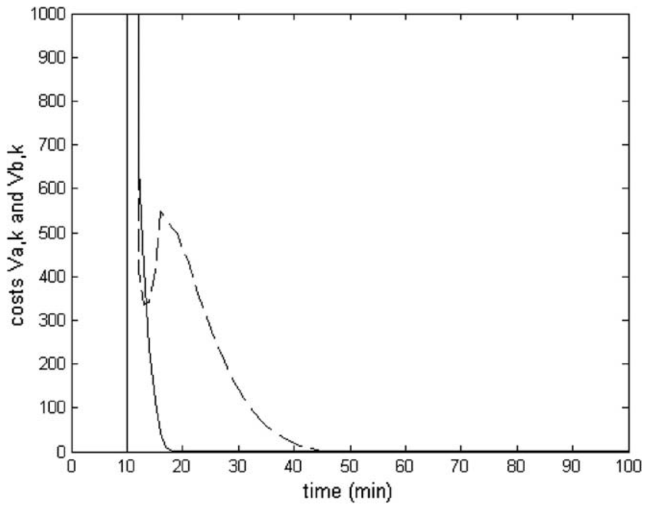

Fig. 6. Costs $V_{a,k}$ (solid line) and $V_{b,k}$ (dashed line) corresponding to problems 2a and 2b of Controller I, for a disturbance in the input.

are not equal to zero. This is so because the inputs need to compensate the effect of the disturbance that is introduced in the input. However, Controller I shows a better performance since the state estimation adds an additional dynamics to the closed loop with Controller II. This effect can be clearly seen in Fig. 4 where the control action applied by Controller II to reject the disturbance is significantly slower than the one applied by Controller I. As a result of the disturbance in the integrating states, Fig. 6 shows that, as established in the Theorem 1, cost $V_{b,k}$ of Controller I is not strictly decreasing until cost $V_{a,k}$ is zeroed (that is, until the slack variable δ_k^{un} is zeroed). In this case, the convergence of cost $V_{a,k}$ to zero can only be reached after eighteen time steps.

6. Conclusions

In this work it was addressed the problem of designing an infinite horizon MPC with output feedback for systems with stable and integrating poles, and constraints in the inputs and input increments. The proposed controller is based on a non-minimal state space model in which the states represent the measured past inputs and outputs and no state observer is needed. No system-model mismatch is considered in the present version of the controller. By means of an appropriate similarity transformation, a terminal constraint is developed and recursive feasibility of the control problem as well as output convergence are obtained. This work presents a contribution towards the practical implementation of the stable MPC with output feedback. The available solutions to this problem that include state observers need additional hypotheses concerning fast observer dynamics and the separation principle to achieve stability; hypotheses that may well not be justified in practical applications. Also, the proposed controller showed an improved performance in comparison with an observer-based MPC, in the simulation study of an industrial reactor system with stable and integrating modes.

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