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A stable MPC with zone control

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Abstract

Several MPC applications implement a control strategy in which some of the system outputs are controlled within specified ranges or zones, rather than at fixed set points [J.M. Maciejowski, Predictive Control with Constraints, Prentice Hall, New Jersey, 2002]. This means that these outputs will be treated as controlled variables only when the predicted future values lie outside the boundary of their corresponding zones. The zone control is usually implemented by selecting an appropriate weighting matrix for the output error in the control cost function. When an output prediction is inside its zone, the corresponding weight is zeroed, so that the controller ignores this output. When the output prediction lies outside the zone, the error weight is made equal to a specified value and the distance between the output prediction and the boundary of the zone is minimized. The main problem of this approach, as long as stability of the closed loop is concerned, is that each time an output is switched from the status of non-controlled to the status of controlled, or vice versa, a different linear controller is activated. Thus, throughout the continuous operation of the process, the control system keeps switching from one controller to another. Even if a stabilizing control law is developed for each of the control configurations, switching among stable controllers not necessarily produces a stable closed loop system.

Here, a stable MPC is developed for the zone control of open-loop stable systems. Focusing on the practical application of the proposed controller, it is assumed that in the control structure of the process system there is an upper optimization layer that defines optimal targets to the system inputs. The performance of the proposed strategy is illustrated by simulation of a subsystem of an industrial FCC system.

Keywords: Model predictive control; Zone control; Infinite horizon; Stability

1. Introduction

Model predictive control (MPC) is an optimization based strategy that uses a plant model to predict the behavior of the controlled system. At each time step, an open-loop optimization problem is solved, and the first element of the input profile is injected into the plant. This is usually called the receding horizon strategy. At the next time step, the optimization parameters are updated by means of an output feedback, and a new open-loop optimization is performed. Since MPC is formulated as an optimization problem, inequality constraints are naturally incorporated to the resulting control law.

In modern processing plants, MPC controllers are usually implemented as a part of multilevel hierarchy of control functions [2]. At the top of this structure, a plant-wide optimizer determines optimal steady state settings for each process unit of the chemical plant. These settings may be directed to local real-time optimizers at each process unit, which run more frequently than the plant-wide optimizer. As a part of this complex structure, the unit optimizer computes an optimal economic steady state and passes this information to the MPC level. The MPC algorithm must move the plant from one steady state to a more profitable operating point by changing the set points to the PID regulatory level.

In some chemical processes, the aim of the MPC is not to guide all the controlled variables to set points or desired values, but only to maintain them inside appropriate ranges or zones. This is what is called zone control [3].

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Zanin et al. [10], study the case of a FCC system in which only the manipulated inputs need to be guided to specific values, while the controlled outputs merely need to be maintained within specified feasible ranges.

On the other hand, there is a number of research works that treat the problem of how a stable offset-free MPC can be formulated when a supervisory layer produces optimal output set points. Rawlings [8], Pannochia and Rawlings [6], Muske and Badgwell [4] show several ways of incorporating disturbance models to assure that the inputs and states are led to the desired values without offset. They use a reduced model in the MCP algorithm (which works as a dynamic state regulator) and an augmented model in the state estimator. This augmented model contains additional states that represent input, state, and output disturbances. The estimated disturbances are then passed to the target calculation stage, which computes steady state targets (as close as possible to the set point values) for both, states and inputs. To do that, the augmented model must be detectable in order to efficiently estimate the disturbances. In Muske and Badgwell [4] and Pannochia and Rawlings [6], several rank conditions were established to assure the detectability of the augmented model.

In this work, we develop a one-stage nominally stable MPC controller that considers the zone control of the system outputs and incorporates steady state economic targets in the control cost function. Classical stability proofs are extended to the zone control strategy by considering the output set points as additional decision variables of the control problem. Furthermore, sufficient conditions are found for the cost weighting matrices in order to guarantee convergence of the system inputs to the targets.

2. Velocity model

For a system with nu inputs and ny outputs, assuming that the poles related to input u_i and output y_j are non-repeated, a state space model that is suitable to the implementation of an offset-free MPC can be represented in the following form [5]:

$$\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \end{bmatrix} = \begin{bmatrix} I_{ny} & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} + \begin{bmatrix} D^0 \\ D^d \end{bmatrix} \Delta u(k), \tag{1}$$

$$y(k) = \begin{bmatrix} I_{ny} & \Psi \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix}, \tag{2}$$

where

$$x^s = \begin{bmatrix} x_1 & \cdots & x_{ny} \end{bmatrix}^{\mathrm{T}}, \quad x^s \in \mathbb{R}^{ny},$$

$$x^d = \begin{bmatrix} x_{ny+1} & x_{ny+2} & \cdots & x_{ny+nd} \end{bmatrix}^{\mathrm{T}}, \quad x^d \in \mathbb{C}^{nd},$$

$$F \in \mathbb{C}^{nd \times nd}, \quad \Delta u(k) = u(k) - u(k-1),$$

$$\Psi = \begin{bmatrix} \Phi & & 0 \\ & \ddots & \\ 0 & & \Phi \end{bmatrix}, \quad \Psi \in \mathbb{R}^{ny \times nd}, \quad \Phi = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix},$$

$$\Phi \in \mathbb{R}^{nu\ na}.$$

In the state equation defined in (1), the state component x^s corresponds to the integrating poles produced by the incremental form of the model, and x^d corresponds to the system modes. For stable systems, it is easy to show that when the system approaches steady state, component x^d tend to zero. F is a diagonal matrix with components corresponding to the poles of the system. The system has nd stable poles. Matrix D^0 is the static gain of the system. To build up matrix Φ, it is assumed that na is the number of poles associated to any pair (u_i, y_j).

3. Control structure

The control structure considered in this work is represented in Fig. 1. In this structure, at time step k, the real-time economic optimization (RTO) stage, which is based on a rigorous stationary model, computes the optimal target, $u_{des,k}$, for the manipulated input variables. Another strategy, where the method proposed here can be easily adapted, corresponds to the case in which the economic optimization level sets the ranges for some of the outputs. Here, it is assumed that the control stage corresponding to the MPC is dedicated to guide the manipulated inputs to the desired targets defined by the supervisory economic stage, while keeping the outputs within specified zones. In Fig. 1, it is assumed that the PID regulatory level is included in the system level and that the regulatory level is capable of enforcing the set points determined by the MPC level. In general, the target $u_{des,k}$ will vary whenever a disturbance enters the process, or there is a change in the operating objective of the RTO layer. The input targets should satisfy the following constraints:

$$\begin{aligned} & u_{\min} \leqslant u_{des,k} \leqslant u_{\max}, \\ & y_{\min} \leqslant D^0(u_{des,k} - u(k-1)) + \hat{x}^s(k) \leqslant y_{\max}, \end{aligned} \tag{3}$$

where $u_{\min}$ and $u_{\max}$ represent the lower and upper bounds of the input, $y_{\min}$ and $y_{\max}$ represent the lower and upper bounds of the output zones and $\hat{x}^s(k)$ is an estimation of the state components associated with the integrating modes. In this case, since the model adopted here has integral action (given by the incremental form of the input),

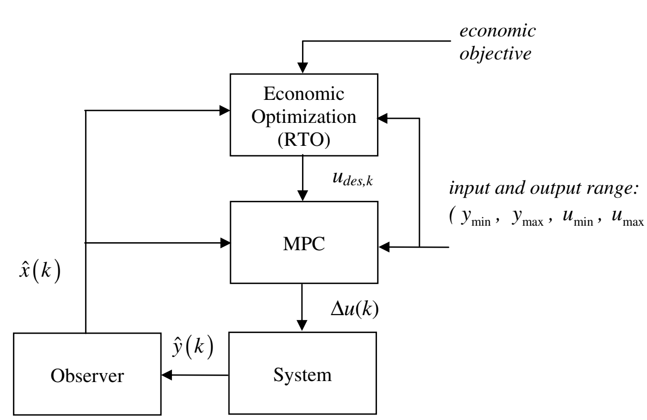

Fig. 1. Control structure.

$\hat{x}^s(k)$ equals the estimated output at steady state. To clarify this point, consider the equation that defines the state observer at time $\bar{k}$ large enough to approach steady state

$$\begin{bmatrix} \hat{x}^s(\bar{k}) \\ \hat{x}^d(\bar{k}) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} \hat{x}^s(\bar{k}) \\ \hat{x}^d(\bar{k}) \end{bmatrix} + \begin{bmatrix} D^0 \\ D^d \end{bmatrix} \Delta u(\bar{k}) + \begin{bmatrix} L_s \\ L_d \end{bmatrix} \left[y(\bar{k}) - [I_{ny} \quad \Psi] \left(\begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} \hat{x}^s(\bar{k}) \\ \hat{x}^d(\bar{k}) \end{bmatrix} + \begin{bmatrix} D^0 \\ D^d \end{bmatrix} \Delta u(\bar{k}) \right) \right],$$

where $\begin{bmatrix} L_s^{\mathrm{T}} & L_d^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ is the observer gain, $\hat{x}^s(\bar{k})$ and $\hat{x}^d(\bar{k})$ are the estimated states at time $\bar{k}$ and $y(\bar{k})$ is the measured steady state output corresponding to the actual plant. Assuming that $\Delta u(\bar{k}) = 0$ and knowing that $\hat{x}^d(\bar{k}) = 0$ at steady state as this state component corresponds to the stable modes of the system, the observer equation for component x^s becomes

$$\hat{x}^s(\bar{k}) = \hat{x}^s(\bar{k}) + L_s[y(\bar{k}) - \hat{x}^s(\bar{k})].$$

The above relation implies that, if $L_s \in \mathbb{R}^{ny \times ny}$ is full rank, then, $\hat{x}^s(\bar{k}) = y(\bar{k})$, and the output predictions will be unbiased with respect to the output measurements.

Therefore, condition (3) implies that, for time $\bar{k}$ large enough, the desired input values should be such that

$$\begin{aligned} y_{\min} &\leqslant D^0 u_{des,\bar{k}} + [y(\bar{k}) - D^0 u(\bar{k})] \leqslant y_{\max}, \\ y_{\min} &\leqslant D^0 u_{des,\bar{k}} + d(\bar{k}) \leqslant y_{\max}, \\ y_{\min} &\leqslant y^c_{\mathrm{des},\bar{k}} \leqslant y_{\max}. \end{aligned} \tag{4}$$

In (4), we can define $y^c_{des,k} = D^0 u_{des,\bar{k}} + d(\bar{k})$ as the desired output corresponding to the input target, and d as the output bias corresponding to the difference between the actual output at steady state and the prediction of the output based on the independent model, that is, the model that does not consider any feedback information to build the output predictions. Note that, since $u(\bar{k}) = \sum_{j=0}^{\bar{k}} \Delta u(j)$, the term $D^0 u(\bar{k})$ represents the output prediction based only on the past inputs.

4. MPC with zone control and input target

The zone control strategy is implemented in applications where the exact values of the controlled outputs are not important, as long as they remain inside a range with specified limits. Maciejowski [3] proposes to reduce the zone control to the standard MPC problem by setting to zero the weight on the error of the predicted outputs and leaving only constraints on the controlled outputs to define the performance objectives. Honeywell's RMPCT product implements a zone control strategy where the output zone is a funnel defined by the output set point and a straight line connecting the last output measurement to the set point. The funnel constraint is introduced in the control problem as a soft constraint, by switching in large penalty weights when the funnel boundaries are crossed or approached [7]. This strategy implies that the number of outputs that are effectively being controlled may change depending on the output predictions. This means that, along the continuous operation of the process, the control structure may change, and switch from one controller to another. Consequently, the usual implementation of the zone control does not guarantee stability even if a stable controller is synthesized for each control configuration.

Recently, González et al. [1] presented an infinite horizon MPC based on the incremental model defined in Eqs. (1) and (2) that takes into account the stationary optimization of the plant operation. The controller was designed specifically for a heat exchanger network with a degree of freedom greater than zero. Stability and offset elimination were assured only when the model and plant gains were coincident. The ideas of González et al. [1] will be extended here to the zone control of systems in which there are economic targets. For this purpose, consider the following cost function:

$$V_k = \sum_{j=0}^{\infty} (y(k+j/k) - y_{sp,k})^{\mathrm{T}} Q_y (y(k+j/k) - y_{sp,k}) + \sum_{j=0}^{\infty} (u(k+j/k) - u_{des,k})^{\mathrm{T}} Q_u (u(k+j/k) - u_{des,k}) + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} R \Delta u(k+j/k), \tag{5}$$

where $\Delta u(k+j/k)$ is the control move computed at time k to be applied at time $k+j$, m is the control or input horizon, Q_y, Q_u, R are positive weighting matrices of appropriate dimensions, $y_{sp,k}$ is the output set point and $u_{des,k}$ is the input target. Assuming that $u_{des,k}$ corresponds to the optimal operating point, then the cost function defined in (5) explicitly incorporates an input deviation penalty that tries to force the system to the optimal operating point. However, since an infinite horizon is used and the model defined in (1) and (2) has integrating modes, terminal constraints must be added to prevent the cost from becoming unbounded. These constraints can be written as follows [1]:

$$x^s(k) + \widetilde{D}^0 \Delta u_k - y_{sp,k} = 0 \quad (x^s(k+m/k) - y_{sp,k} = 0), \tag{6}$$

$$u(k-1) + \widetilde{D}^u \Delta u_k - u_{des,k} = 0 \quad (u(k+m-1/k) - u_{des,k} = 0), \tag{7}$$

where

$$\Delta u_k = \begin{bmatrix} \Delta u(k)^{\mathrm{T}} & \Delta u(k+1)^{\mathrm{T}} & \cdots & \Delta u(k+m-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

$$\widetilde{D}^0 = \Big[\underbrace{D^0 \quad \cdots \quad D^0}_{m} \Big], \quad \widetilde{D}^u = \Big[\underbrace{I_{nu} \quad \cdots \quad I_{nu}}_{m} \Big].$$

The constraints defined in (6) and (7) imply that the output and input errors should be null at the end of the control horizon m. Since the input increments are also generally bounded, the terminal constraints (6) and (7) may result infeasible. This is so because it may not be possible for the system to achieve the targets in m time steps, given that m is frequently small to reduce the computation effort. To enlarge the region where the controller is feasible, an alternative is to incorporate slack variables into the control problem. In this case, the cost defined in (5) is rewritten as follows:

$$V_{k,u} = \sum_{j=0}^{\infty}(y(k+j/k) - y_{sp,k} - \delta_{y,k})^{\mathrm{T}}Q_y(y(k+j/k) - y_{sp,k} - \delta_{y,k}) + \sum_{j=0}^{\infty}(u(k+j/k) - u_{des,k} - \delta_{u,k})^{\mathrm{T}}Q_u(u(k+j/k) - u_{des,k} - \delta_{u,k}) + \sum_{j=0}^{m-1}\Delta u(k+j/k)^{\mathrm{T}}R\Delta u(k+j/k) + \delta_{y,k}^{\mathrm{T}}S_y\delta_{y,k} + \delta_{u,k}^{\mathrm{T}}S_u\delta_{u,k}, \tag{8}$$

where S_y, S_u are positive matrices of appropriate dimensions, and $\delta_{y,k} \in \mathbb{R}^{ny}$, $\delta_{u,k} \in \mathbb{R}^{nu}$ are the slack variables that eliminate a possible infeasibility of the terminal constraints. With the cost defined in (8), the terminal constraints become

$$x^s(k) + \widetilde{D}^0\Delta u_k - y_{sp,k} - \delta_{y,k} = 0,$$
$$u(k-1) + \widetilde{D}^u\Delta u_k - u_{des,k} - \delta_{u,k} = 0.$$

Now, we define the MPC optimization problem that implements the zone control strategy and enforces economic target. This controller results from the solution to the following problem:

Problem P1

$$\min_{\substack{\Delta u_{f,k}, y_{sp,k},\\ \delta_{y,k}, \delta_{u,k}}} V_{k,u}$$

subject to :

$$-\Delta u_{\max} \leqslant \Delta u(k+j/k) \leqslant \Delta u_{\max}, \quad j = 0, 1, \ldots, m-1, \tag{9}$$
$$\Delta u(k+j/k) = 0; \quad j \geqslant m, \tag{10}$$
$$u_{\min} \leqslant u(k-1) + \sum_{i=0}^{j}\Delta u(k+i/k) \leqslant u_{\max}, \quad j = 0, 1, \ldots, m-1, \tag{11}$$
$$y_{\min} \leqslant y_{sp,k} \leqslant y_{\max}, \tag{12}$$
$$x^s(k) + \widetilde{D}^0\Delta u_k - y_{sp,k} - \delta_{y,k} = 0, \tag{13}$$
$$u(k-1) + \widetilde{D}^u\Delta u_k - u_{des,k} - \delta_{u,k} = 0. \tag{14}$$

In problem P1, the output set point $y_{sp,k}$ is an additional vector of decision variables that will be calculated through the solution to problem P1. Note that, in order to minimize the control cost, $y_{sp,k}$ can assume any value inside the output zone. If the output zone is specified such that the upper bound equals the lower bound, then the problem reduces to the conventional set point tracking problem. In general, the output slack, $\delta_{y,k}$, can be made equal to zero if and only if the predicted steady state of the output lies inside the corresponding zone.

Now, as the terminal constraints (13) and (14) are always satisfied, the cost defined in (8) can be written as follows:

$$V_{k,u} = \sum_{j=0}^{m-1}\{(y(k+j/k) - y_{sp,k} - \delta_{y,k})^{\mathrm{T}}Q_y(y(k+j/k) - y_{sp,k} - \delta_{y,k}) + (u(k+j/k) - u_{des,k} - \delta_{u,k})^{\mathrm{T}} \times Q_u(u(k+j/k) - u_{des,k} - \delta_{u,k})\} + \sum_{j=0}^{m-1}\Delta u(k+j/k)^{\mathrm{T}}R\Delta u(k+j/k) + (x^d(k+m/k))^{\mathrm{T}}\overline{Q}x^d(k+m/k) + \delta_{y,k}^{\mathrm{T}}S_y\delta_{y,k} + \delta_{u,k}^{\mathrm{T}}S_u\delta_{u,k},$$

where matrix $\overline{Q}$ is computed as a solution to the following Liapunov equation:

$$\overline{Q} = \Psi^{\mathrm{T}}Q_y\Psi + F^{\mathrm{T}}\overline{Q}F. \tag{15}$$

When the system state is measured, or the state observer is fast enough such that the state estimation converges to the true system state in negligible time, the controller produced by the solution to problem P1 will stabilize the closed loop system as shown in the theorem below.

Theorem. *For a system with stable modes that remains controllable at the equilibrium point corresponding to the desired input targets and output zones, if problem* P1 *is feasible at time k, it will remain feasible at any subsequent time step. Also, if weight S_u is sufficiently large, then the control sequence obtained from the solution to problem* P1 *at successive time steps drives the input to the desired target and the output of the closed loop system asymptotically to a point within its corresponding zone.*

Proof. Let us first prove the recursive feasibility of the proposed controller. Assume that the state is known and no disturbance enters the system; that is $d(k) = 0$. Also, assume that $\Delta u_k^* = \left[\Delta u^*(k/k)^{\mathrm{T}} \quad \cdots \quad \Delta u^*(k+m-1/k)^{\mathrm{T}}\right]^{\mathrm{T}}, y_{sp,k}^*, \delta_{y,k}^*$ and $\delta_{u,k}^*$ correspond to the optimal solution to problem P1 at time k. The cost corresponding to this solution is then

$$V_{k,u}^* = \sum_{j=0}^{\infty}\{(y^*(k+j/k) - y_{sp,k}^* - \delta_{y,k})^{\mathrm{T}}Q_y(y^*(k+j/k) - y_{sp,k}^* - \delta_{y,k}) + (u^*(k+j/k) - u_{des,k} - \delta_{u,k})^{\mathrm{T}} \times Q_u(u^*(k+j/k) - u_{des,k} - \delta_{u,k})\} + \sum_{j=0}^{m-1}\Delta u^*(k+j/k)^{\mathrm{T}}R\Delta u^*(k+j/k) + \delta_{y,k}^{*}{}^{\mathrm{T}}S_y\delta_{y,k}^* + \delta_{u,k}^{*}{}^{\mathrm{T}}S_u\delta_{u,k}^*. \tag{16}$$

Consider now the following set of variables

$$\Delta\tilde{u}_k = \left[\Delta u^*(k+1/k)^{\mathrm{T}} \quad \cdots \quad \Delta u^*(k+m-1/k)^{\mathrm{T}}\, 0\right]^{\mathrm{T}}, \times y_{sp,k}^*, \ \delta_{y,k}^* \text{ and } \delta_{u,k}^*. \tag{17}$$

It is easy to show that the set defined above satisfies constraints (9)–(14), and then, it is a feasible solution to problem P1 at time $k+1$. This proves the recursive feasibility, which means that if problem P1 is feasible at time step k, then, it will remain feasible at all successive time steps $k+1, k+2, \ldots$

To prove the convergence of the proposed controller, we need to prove the convergence of the cost function of problem P1. But, since this function includes the slacks $\delta_{y,k}$ and $\delta_{u,k}$, it is easy to show that the cost can converge to a minimum, while the numerical values of the slacks remain different from zero and, consequently, the inputs do not converge to their targets. Then, the proof of convergence of the proposed controller should also include the proof of the convergence of the slacks to zero.

Let us now start the proof of the convergence of the proposed controller, proving the convergence of the cost defined in (8). At time step $k+1$, the solution defined in (17) corresponds to the cost

$$\begin{aligned}\widetilde{V}_{k+1,u} = \sum_{j=0}^{\infty}\{ & (y^*(k+j+1/k) \\ & - y^*_{sp,k} - \delta^*_{y,k})^{\mathrm{T}} Q_y (y^*(k+j+1/k) - y^*_{sp,k} - \delta^*_{y,k}) \\ & + (u^*(k+j+1/k) - u_{des,k} - \delta^*_{u,k})^{\mathrm{T}} \\ & \times Q_u (u^*(k+j+1/k) - u_{des,k} - \delta^*_{u,k})\} \\ & + \sum_{j=0}^{m-1} \Delta u^*(k+j+1/k)^{\mathrm{T}} R \Delta u^*(k+j+1/k) \\ & + \delta^*_{y,k}{}^{\mathrm{T}} S_y \delta^*_{y,k} + \delta^*_{u,k}{}^{\mathrm{T}} S_u \delta^*_{u,k}. \end{aligned} \tag{18}$$

Observe that, since the input sequence defined in (17) is inherited from the optimal input sequence computed at step k, the predicted state and output trajectories corresponding to this input sequence will be the same as the optimal predicted trajectories at step k. That is, for any $j \geqslant 1$, we have

$$\begin{aligned} x^s(k+j/k+1) &= x^s(k+j/k), \\ x^d(k+j/k+1) &= x^d(k+j/k), \\ y(k+j/k+1) &= y(k+j/k). \end{aligned}$$

Now, subtracting (18) from (16), we have the following equation:

$$\begin{aligned} V^*_{k,u} - \widetilde{V}_{k+1,u} = {} & (y(k) - y^*_{sp,k} - \delta^*_{y,k})^{\mathrm{T}} Q_y (y(k) - y^*_{sp,k} - \delta^*_{y,k}) \\ & + (u^*(k/k) - u_{des,k} - \delta^*_{u,k})^{\mathrm{T}} Q_u (u^*(k/k) \\ & - u_{des,k} - \delta^*_{u,k}) + \Delta u^*(k)^{\mathrm{T}} R \Delta u^*(k). \end{aligned}$$

Then, the optimal solution to problem P1 at time $k+1$ will satisfy

$$\begin{aligned} V^*_{k,u} - V^*_{k+1,u} \geqslant {} & (y(k) - y^*_{sp,k} - \delta^*_{y,k})^{\mathrm{T}} Q_y (y(k) - y^*_{sp,k} - \delta^*_{y,k}) \\ & + (u^*(k/k) - u_{des,k} - \delta^*_{u,k})^{\mathrm{T}} Q_u (u^*(k/k) - u_{des,k} - \delta^*_{u,k}) \\ & + \Delta u^*(k)^{\mathrm{T}} R \Delta u^*(k). \end{aligned} \tag{19}$$

Since the right hand side of (19) is positive definite, the successive costs are strictly decreasing and for a large enough time $\bar{k}$ we have $(V^*_{\bar{k},u} - V^*_{\bar{k}+1,u}) = 0$, which proves the convergence of the cost.

The convergence of $V_{k,\,u}$ means that, at steady state, the following relations should hold:

$$\begin{aligned} & y(\bar{k}) - y^*_{sp,\bar{k}} = \delta^*_{y,\bar{k}}, \\ & u^*(\bar{k}/\bar{k}) - u_{des,\bar{k}} = \delta^*_{u,\bar{k}}, \\ & \Delta u^*(\bar{k}) = 0. \end{aligned}$$

To prove the convergence of the input to the desired target and the output to a point within the control zone, we must show that slacks $\delta_{u,\bar{k}}$ and $\delta_{y,\bar{k}}$ will converge to zero. Note that, since there is not a fixed output set point, the desired input values may be achieved even in the presence of bounded disturbances. In fact, with the proposed approach, $y_{sp,k}$ may follow in some sense the predicted output values. Let us now assume that the system is stabilized at a point where both, $\delta_{u,\bar{k}}$ and $\delta_{y,\bar{k}}$ are different from zero although the system is still controllable, which means that it is possible to stabilize the system with both slacks equal to zero. In addition, we assume that the desired input target is constant at $u_{des,\bar{k}}$. Then, at time $\bar{k}$, the optimal cost will be reduced to

$$V^*_{\bar{k}} = \delta^{\mathrm{T}}_{y,\bar{k}} S_y \delta_{y,\bar{k}} + \delta^{\mathrm{T}}_{u,\bar{k}} S_u \delta_{u,\bar{k}} \tag{20}$$

and from Eqs. (13) and (14), we have the following relations:

$$x^s(\bar{k}) - y_{sp,\bar{k}} = \delta_{y,\bar{k}} \quad \text{and} \quad u(\bar{k}-1) - u_{des,\bar{k}} = \delta_{u,\bar{k}}.$$

Now, we will show that a suitable selection of S_u makes it possible to guide the system to a point in which the slacks are null and the corresponding cost is smaller than the cost defined in (20). Assume, for simplicity, that $m=1$ and the input constraints are not active. Then, at time $\bar{k}$, let us consider the following candidate solution to problem P1:

$$\Delta\bar{u}(\bar{k}/\bar{k}) = u_{des,\bar{k}} - u(\bar{k}-1) = -\delta_{u,\bar{k}} \tag{21}$$

and

$$\bar{y}_{sp,\bar{k}} = x^s(\bar{k}) - D^0 \delta_{u,\bar{k}}. \tag{22}$$

The set point given in (22) is the steady state value of the output corresponding to the input increment given in (21). Note that, this new set point variable is feasible, because it is assumed that the system is controllable at steady state (or condition (3) holds true).

The variables defined in (21) and (22) must satisfy constraints (13) and (14) of problem P1, that is,

$$x^s(\bar{k}) - D^0 \delta_{u,\bar{k}} - \bar{y}_{sp,\bar{k}} - \bar{\delta}_{y,\bar{k}} = 0, \tag{23}$$

$$u(\bar{k}-1) - \delta_{u,\bar{k}} - u_{des,\bar{k}} - \bar{\delta}_{u,\bar{k}} = 0. \tag{24}$$

Note that $\bar{u}(\bar{k}) = u(\bar{k}-1) - \delta_{u,\bar{k}}$, $\bar{y}_{sp,\bar{k}}$, $\bar{\delta}_{y,\bar{k}}$ and $\bar{\delta}_{u,\bar{k}}$ constitute the feasible solution that would result if the input increment defined in (21) was implemented at time $\bar{k}$. Now, combining (21) and (24), we conclude that $\bar{\delta}_{u,\bar{k}} = 0$. This means that the input will reach its target at steady state, that is $\bar{u}(\bar{k}) = u_{des,\bar{k}}$. Now, substituting (22) into (23), results $\bar{\delta}_{y,\bar{k}} = 0$, which means that the predicted steady state output is inside the output zone.

The cost corresponding to this feasible solution is the following:

$$\begin{aligned}\overline{V}_{\bar{k}} &= (y(\bar{k}/\bar{k}) - \bar{y}_{sp,\bar{k}} - \bar{\delta}_{y,\bar{k}})^{\mathrm{T}} Q_y (y(\bar{k}/\bar{k}) - \bar{y}_{sp,\bar{k}} - \bar{\delta}_{y,\bar{k}}) \\ &\quad + \underbrace{(u(\bar{k}/\bar{k}) - u_{des,\bar{k}} - \bar{\delta}_{u,\bar{k}})^{\mathrm{T}} Q_u (u(\bar{k}/\bar{k}) - u_{des,\bar{k}} - \bar{\delta}_{u,\bar{k}})}_{=0} \\ &\quad + (x^d(\bar{k}+1/\bar{k}))^{\mathrm{T}} \overline{Q} x^d(\bar{k}+1/\bar{k}) + \overbrace{\Delta u(\bar{k}/\bar{k})^{\mathrm{T}} R \Delta u(\bar{k}/\bar{k})}^{\delta_{u,\bar{k}}^{\mathrm{T}} R \delta_{u,\bar{k}}} \\ &\quad + \overbrace{\bar{\delta}_{y,\bar{k}}^{\mathrm{T}} S_y \bar{\delta}_{y,\bar{k}}}^{=0} + \overbrace{\bar{\delta}_{u,\bar{k}}^{\mathrm{T}} S_u \bar{\delta}_{u,\bar{k}}}^{=0}. \end{aligned} \tag{25}$$

Now, using the model Eqs. (1) and (2), we have

$$\begin{aligned} x^d(\bar{k}+1/\bar{k}) &= F x^d(\bar{k}) + D^d F N \Delta u(\bar{k}/\bar{k}), \\ x^d(\bar{k}+1/\bar{k}) &= F \underbrace{x^d(\bar{k})}_{=0} - D^d F N \delta_{u,\bar{k}} \\ &= -D^d F N \delta_{u,\bar{k}}, \\ y(\bar{k}/\bar{k}) &= x^s(\bar{k}) + \Psi x^d(\bar{k}), \\ y(\bar{k}/\bar{k}) - \bar{y}_{sp,\bar{k}} - \bar{\delta}_{y,\bar{k}} &= x^s(\bar{k}) + \Psi \overbrace{x^d(\bar{k})}^{=0} - \bar{y}_{sp,\bar{k}} \\ &= D^0 \delta_{u,\bar{k}}. \end{aligned}$$

Consequently, Eq. (25) can be written as follows:

$$\overline{V}_{u,\bar{k}} = \delta_{u,\bar{k}}^{\mathrm{T}} S_u^{\min} \delta_{u,\bar{k}},$$

where

$$S_u^{\min} = D^{0^{\mathrm{T}}} Q D^0 + N^{\mathrm{T}} F^{\mathrm{T}} D^{d^{\mathrm{T}}} \overline{Q} D^d F N + R.$$

Finally, if

$$S_u > S_u^{\min}, \tag{26}$$

then, the cost corresponding to the solution defined in (21) and (22) will be smaller than the cost obtained in (20). This means that if inequality (26) is satisfied, then the closed loop system will converge to a steady state where slacks $\delta_{y,k}$ and $\delta_{u,k}$ will be equal to zero. Thus, as long as the system remains controllable, condition (26) is sufficient to guarantee the convergence of the closed loop with the proposed controller.

Now, we can prove the stability of the proposed zone controller under the same assumptions considered in the proof of the convergence. To simplify the proof, we also assume that $m = 1$, and the optimal solution obtained at step $k-1$ is given by $\Delta u_{k-1}^* = \Delta u^*(k-1/k-1)$, $y_{sp,k-1}^*$, $\delta_{y,k-1}^*$ and $\delta_{u,k-1}^*$. The hypothesis of controllability means that the initial state $x^s(k-1)$ is such that the predicted output at steady state, which in our model representation is equal to $x^s(k/k-1)$, is inside the output zone. If this is the case, then, in the solution to problem P1, the set point of the system output $y_{sp,k}^*$ can be made equal to $x^s(k/k-1)$, which implies that $\delta_{y,k-1}^* = 0$. Thus, a feasible solution to problem P1 at time k is given by

$$\Delta\tilde{u}_k = 0, \quad \tilde{y}_{sp,k} = y_{sp,k-1}^*, \quad \tilde{\delta}_{y,k} = \delta_{y,k-1}^* = 0 \quad \text{and} \quad \tilde{\delta}_{u,k} = \delta_{u,k-1}^*. \tag{27}$$

The cost corresponding to the feasible solution defined in (27) is then

$$\begin{aligned}\widetilde{V}_k &= (y(k/k) - y_{sp,k-1}^* - \delta_{y,k-1}^*)^{\mathrm{T}} Q_y (y(k/k) - y_{sp,k-1}^* \\ &\quad - \delta_{k-1}^*) + (u(k/k) - u_{des,k} - \delta_{u,k-1}^*)^{\mathrm{T}} Q_u (u(k/k) \\ &\quad - u_{des,k} - \delta_{u,k-1}^*) + (x^d(k+1/k))^{\mathrm{T}} \overline{Q} x^d(k+1/k) \\ &\quad + \overbrace{\Delta u(k/k)^{\mathrm{T}} R \Delta u(k/k)}^{=0} + \overbrace{\delta_{y,k-1}^{*\,\mathrm{T}} S_y \delta_{y,k-1}^*}^{=0} + \delta_{u,k-1}^{*\,\mathrm{T}} S_u \delta_{u,k-1}^*. \end{aligned} \tag{28}$$

Now from (14), we have

$$\delta_{u,k-1}^* = u(k-1) - u_{des,k} = u(k/k) - u_{des,k} = C_1 \bar{x},$$

$$\text{where} \quad C_1 = [\,0 \quad 0 \quad I_{nu}\,] \quad \text{and} \quad \bar{x}(k) = \begin{bmatrix} x^s(k) - y_{sp,k} \\ x^d(k) \\ u(k/k) - u_{des,k} \end{bmatrix}.$$

Now, considering the model Eqs. (1) and (2), we have

$$\begin{aligned} y(k/k) - y_{sp,k-1}^* - \delta_{y,k-1}^* &= x^s(k) + \Psi x^d(k) - y_{sp,k-1}^* \\ &= \Psi x^d(k) = C_2 \bar{x}(k), \end{aligned}$$

$$x^d(k+1/k) = F x^d(k) + D^d F N \overbrace{\Delta u(k/k)}^{=0} = C_3 \bar{x}(k),$$

where $C_2 = [\,0 \quad \Psi \quad 0\,]$ and $C_3 = [\,0 \quad F \quad 0\,]$. Thus, the cost defined in (28) can be written as follows:

$$\widetilde{V}_k = \bar{x}(k)^{\mathrm{T}} H_1 \bar{x}(k), \tag{29}$$

where $H_1 = C_2^{\mathrm{T}} Q C_2 + C_3^{\mathrm{T}} \overline{Q} C_3 + C_1^{\mathrm{T}} S_u C_1$.

Since this cost corresponds to a feasible solution to problem P1 at time step k, then, the optimal cost will satisfy

$$V_k^* \leqslant \widetilde{V}_k. \tag{30}$$

Now, from condition (19) we have

$$V_{k+n}^* \leqslant V_k^* \quad \text{for any } n > 1. \tag{31}$$

By a similar procedure as above and based on the optimal solution at time $k+n$, we can find a feasible solution to problem P1 at time $k+n+1$, for any $n > 1$, such that

$$\widetilde{V}_{k+n+1} \leqslant V_{k+n}^* \tag{32}$$

and from the definition of $\widetilde{V}_{k+n+1}$ we have

$$\widetilde{V}_{k+n+1} = \bar{x}(k+n+1)^{\mathrm{T}} H_1 \bar{x}(k+n+1). \tag{33}$$

Therefore, combining inequalities (29)–(33) results

$$\bar{x}(k+n+1)^{\mathrm{T}} H_1 \bar{x}(k+n+1) \leqslant \bar{x}(k)^{\mathrm{T}} H_1 \bar{x}(k) \quad \forall n > 1.$$

As H_1 is positive definite, it follows that

$$\|\bar{x}(k+n+1)\| \leqslant \alpha \|\bar{x}(k)\| \quad \forall n > 1,$$

$$\alpha = \left[\frac{\lambda_{\max}(H_1)}{\lambda_{\min}(H_1)}\right]^{1/2}.$$

If we restrict the state at time k to the set defined by

$$\|\bar{x}(k)\| < \rho,$$

then, the state at time $k+n+1$ will be inside the set defined by

$$\|\bar{x}(k+n+1)\| < \alpha\rho \quad \forall n > 1.$$

This proves the stability of the closed loop system, as $\bar{x}$ will remain inside the ball $\|\bar{x}\| < \alpha\rho$, where α is limited, as long as the closed loop starts from a state inside the ball $\|\bar{x}\| < \rho$. Therefore, as we have already proved convergence of the closed loop, we can now assure that under the assumption of state controllability at the final equilibrium point, the proposed MPC is asymptotically stable. □

Remark 1. Only matrix S_u is involved in the convergence condition defined in (26), because the assumption of controllability implies that condition (3) is satisfied. Therefore, we can assure that the output prediction, $y^c_{des,\bar{k}}$, corresponding to the desired input target is in the output zone. In this case, for any positive matrix S_y, in the solution of problem P1, the slack δ_k will be made equal to zero by making the set point variable equal to the steady state output prediction. Matrix S_y, however, must be large enough to avoid any numerical problem in the solution of problem P1.

Remark 2. It is important to observe that, even if condition (3) cannot be satisfied by the input target, or the input target is such that one or more outputs need to be kept outside their zones, the proposed controller will still be stable. This is a consequence of the decreasing property of the cost function (inequality (19)) and the inclusion of the slack variables in the optimization problem, as the open-loop system is assumed to be stable. In this case, the system will evolve to a point in which the slack variables are as small as possible, but not equal to zero. This is an important feature of the proposed controller, as in a practical application a disturbance may move the system to a point from which it is not possible to reach a steady state that satisfies constraints (3). When this happens, the controller will do its best to compensate the disturbance while maintaining the system under control.

5. Alternative formulation

We may reduce the case in which controllability of the system output cannot be assumed, because it is not possible to satisfy the constraints defined in (3), or it is not possible to force the outputs at steady state to lie within their zones, to the case considered in the previous section where controllability was assumed. The alternative approach is represented in Fig. 2.

This structure includes an intermediary layer, called *target calculation*, which computes modified targets $u_{\text{target},k}$ for the inputs in order to guarantee the controllability of the MPC controller. This means that the system input will converge to $u_{\text{target},k}$ while the output will converge to a point within control zone. This intermediary layer solves the following problem:

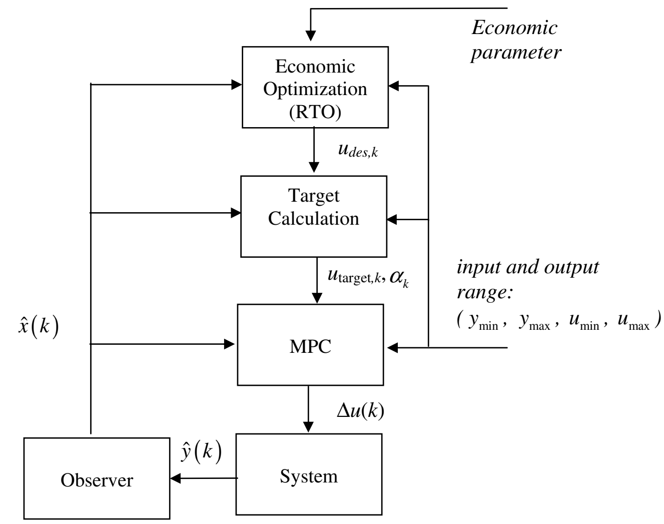

Fig. 2. Extended control structure, considering an intermediary target calculation stage.

Problem P2

$$\min_{u_{\text{target},k},\alpha_k} \quad V_{\text{target}} \triangleq \{(u_{\text{target},k} - u_{des,k})^{\text{T}} \times R_{\text{target}}(u_{\text{target},k} - u_{des,k}) + \alpha_k^{\text{T}} S_{\text{target}} \alpha_k\}$$

subject to :

$$u_{\min} \leqslant u_{\text{target},k} \leqslant u_{\max},$$
$$y_{\min} \leqslant D^0 u_{\text{target},k} + d(k) + \alpha_k \leqslant y_{\max},$$

where $d(k) = \hat{x}^s(k) - D^0 u(k)$ is the output bias based on the comparison between the actual output at steady state ($\hat{x}^s(k)$) and the prediction at steady state based on the independent model. In this problem, R_{target} and S_{target} are weights that penalize the input error and the output zone violation. Since the output zone violation should be tolerated only when there is no input in the range $u_{\min} \leqslant u \leqslant u_{\max}$ that satisfies the output zone, the weights must be such that $R_{\text{target}} \ll S_{\text{target}}$. Note that problem P2 is always feasible while the same problem without the output slack α_k could be infeasible for large disturbances (i.e. if there is no input in the range $u_{\min} \leqslant u \leqslant u_{\max}$ that satisfies the output zone constraint).

A modified problem P1 is then solved within the same time step. The modifications in problem P1 are as follows:

– Replace constraint (12) with

$$y_{\min} \leqslant y_{sp,k} + \alpha_k \leqslant y_{\max}.$$

– Replace $u_{des,k}$ with $u_{\text{target},k}$.

In this new version of problem P1, both $u_{\text{target},k}$ and α_k are assumed to be known.

With this formulation of the control problem, the MPC cost can converge to zero, which means that the permanent offset in terms of the computed target $u_{\text{target},k}$ is eliminated.

Remark 3. Note that the optimization problem P2 is a convex QP problem and consequently has a unique solution. Then, if the desired input value $u_{des,k}$ remains constant, the optimal solution to problem P2 $u_{\text{target},k}$ will also remain constant. In this way, the results about convergence and stability of the MPC layer are not affected by the inclusion of an intermediary target calculation layer.

Remark 4. The inclusion of a target calculation layer increases the complexity of the controller, as well as the number of tuning parameters, while the performance the controller is not significantly affected. So, in practice, the single layer controller should be preferred.

Remark 5. The proposed controller has a larger set of tuning parameters than the conventional MPC. In this set of parameters, the control horizon (m), the output error weight (Q_y), the input error weight (Q_u) and the input increment weight (R) play the same role as in the conventional MPC and should follow the same tuning rules. The weights S_y and S_u on the slack variables should be large enough to guarantee the convergence of the method. The rule defined in (26) should be followed in the selection of S_u, while Odloak [5] provides an expression for the calculation of S_y. However, we observe that the performance of the controller is not significantly affected by the numerical values of S_y and S_u, as long as they are large enough. So, we usually adopt values for these weights, which are several orders of magnitude larger than Q_y and Q_u.

6. Simulation results

The system adopted as an example to test the performance of the controller proposed here is part of the FCC system presented in [9]. This is a typical example of the chemical process industry, in which instead of output set points, the system has output zones. The objective of the controller is to guide the manipulated inputs to the corresponding targets while maintaining the outputs (that are more numerous than the inputs) within specified zones. The system has two inputs and three outputs, and the linear model used by the controller is as follows:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ y_3(s) \end{bmatrix} = \begin{bmatrix} \frac{0.45}{2.98s+1} & \frac{0.20}{1.71s+1} \\ \frac{1.5}{20s+1} & \frac{0.19s-3.81}{17.73s^2+10.83s+1} \\ \frac{1.74}{9.10s+1} & \frac{-6.13}{10.91s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}.$$

In this FCC subsystem, the manipulated input variables are: u_1 the flow rate of air to the catalyst regenerator and u_2 the opening of the regenerated catalyst value. The controlled outputs are: y_1 the riser temperature, y_2 the regenerator dense phase temperature and y_3 the regenerator dilute phase temperature.

The tuning parameters of the controller used in the closed loop simulation are the following: $m = 3$, $T = 1$, $Q_y = \text{diag}(1 \quad 1 \quad 1)$, $Q_u = 0.5\text{diag}(1 \quad 1)$, $R = 0.05\text{diag}(1 \quad 1)$, $S_y = 10^4\text{diag}(1 \quad 1 \quad 1)$ and $S_u = 10^2\text{diag}(1 \quad 1)$.

The input and output constraints, as well as the maximum input increment, are shown in Tables 1 and 2.

The input feasible set, Ω^n, corresponding to this problem can be seen in Fig. 3. This set is computed taking into account the input and output constraints and the model gain as follows:

$$\Omega_n \triangleq \{u : u_{\min} \leqslant u \leqslant u_{\max} \text{ and } y_{\min} - y_{ss} \leqslant D^0 u - D^0 u_{ss} \leqslant y_{\max} - y_{ss}\},$$

where u_{ss} and y_{ss} are the initial stationary values of the input and the output, respectively.

Since the output zones are quite narrow, Ω_n is considerable smaller than the set defined solely by the input constraints.

In the nominal case, conditions (3) require that the desired input targets lie inside the nominal input feasible set. This target was initially settled at $u_{des,k} = [220 \quad 40]$, as can be seen in Fig. 3. The closed loop simulation begins at $u_{ss} = [230.60 \quad 60.26]$ and $y_{ss} = [549.50 \quad 704.27 \quad 690.62]$, which are the values taken from the real FCC system. The control structure represented in Fig. 1 is adopted. The con-

Table 1
Output zones for FCC subsystem

Output	$y_{\min}$	$y_{\max}$
y_1 (°C)	490	570
y_2 (°C)	550	800
y_3 (°C)	550	900

Table 2
Input constraints for the FCC subsystem

Input	$\Delta u_{\max}$	$u_{\min}$	$u_{\max}$
u_1 (ton/h)	25	75	250
u_2 (%)	25	25	101

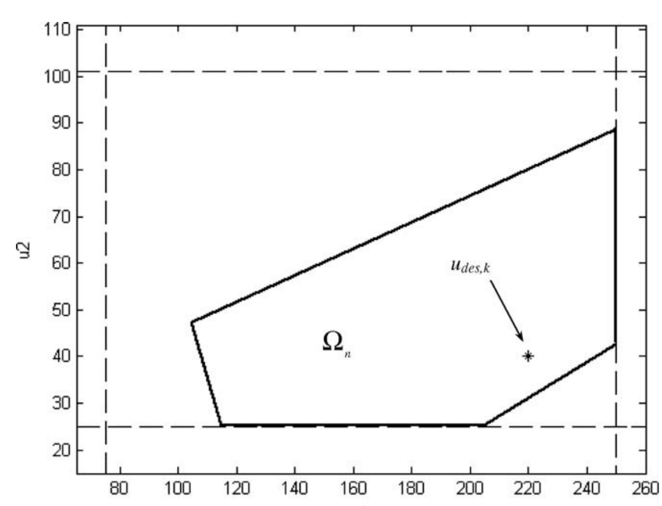

Fig. 3. Input feasible set and input target.

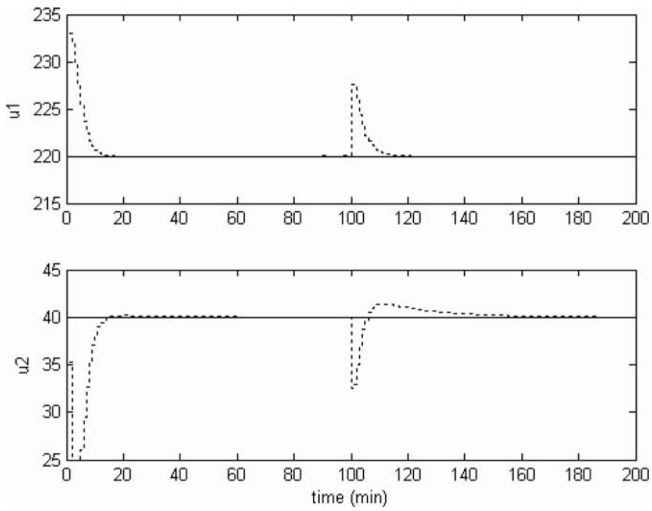

Fig. 4. Inputs of the FCC subsystem.

troller stabilizes the system at a feasible point and then, at time = 100 min, a disturbance is introduced in the state such that the three outputs tend to cross the bounds of the respective zones. The proposed controller guides the system to a new feasible point, in a relatively short time. The inputs (dashed line) and targets (full line), are represented in Fig. 4. In Fig. 5, we can see the outputs (full line), as well as the zones (dashed line) and the set points $y_{sp,k}$ (dotted line) obtained from the solution to problem P1. In addition, Fig. 6 shows the cost function corresponding to different

Table 3
New output zones for the FCC subsystem

Output	$y_{\min}$	$y_{\max}$
y_1 (°C)	510	550
y_2 (°C)	400	500
y_3 (°C)	350	500

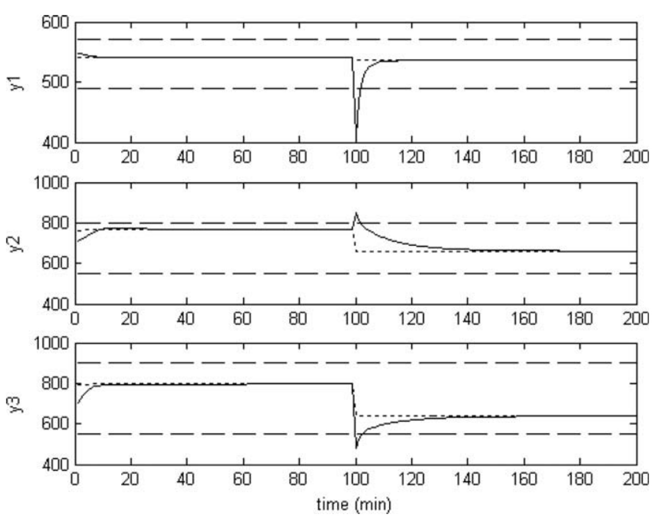

Fig. 5. Controlled outputs (—) and set points (---) of the FCC subsystem.

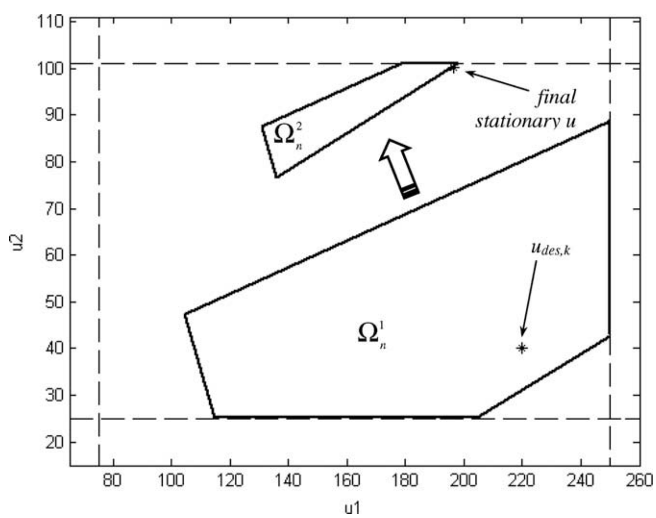

Fig. 7. Input feasible sets for the FCC subsystem.

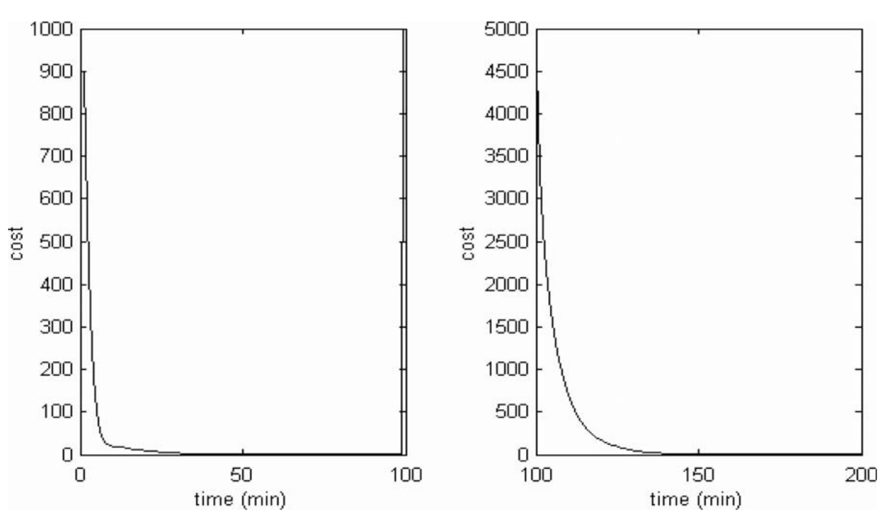

Fig. 6. Cost function of the FCC subsystem.

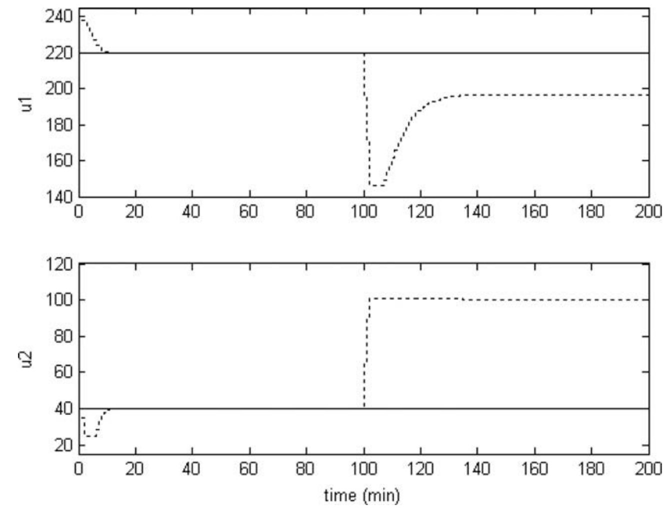

Fig. 8. Manipulated inputs for the FCC subsystem with modified output zones.

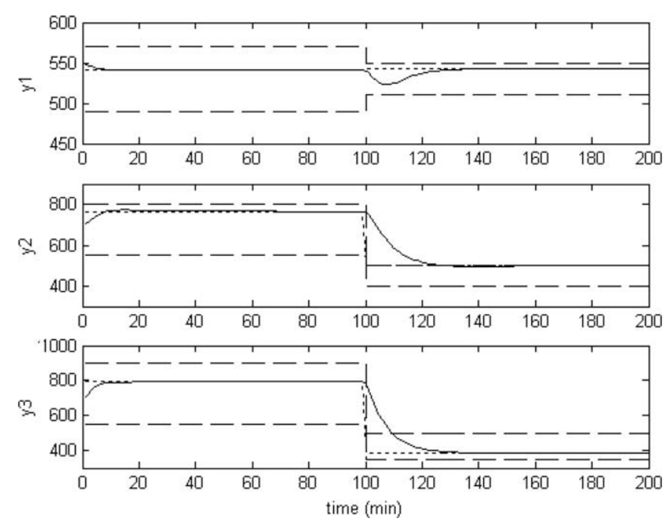

Fig. 9. Controlled outputs and set points for the FCC subsystem with modified zones.

time periods. It can be seen that, as was expected from the stability theorem, the cost is monotonically decreasing.

Next, a sequence of operating changes is simulated. First, the output zones are modified and the new zones are shown in Table 3. As a consequence of the new output zones, the input feasible set changes its dimension and shape significantly as it is shown in Fig. 7. The input target is assumed to remain the same as in the previous case. In Fig. 7, Ω_n^1 corresponds to the initial feasible set and Ω_n^2 represents the new input feasible set. In this simulation, we also adopt the control structure defined in Fig. 1. Since the controller does not include the target calculation stage, and the input target is outside the input feasible set Ω_n^2, it is not possible to guide the system to a point in which the objective cost is reduced to zero at the end of the simulation time. This means that, the controller cannot find an operating point in which the outputs are inside their zones and without offsets in comparison with the input targets. However, if the output weight S_y is kept larger than the input weight S_u, then all

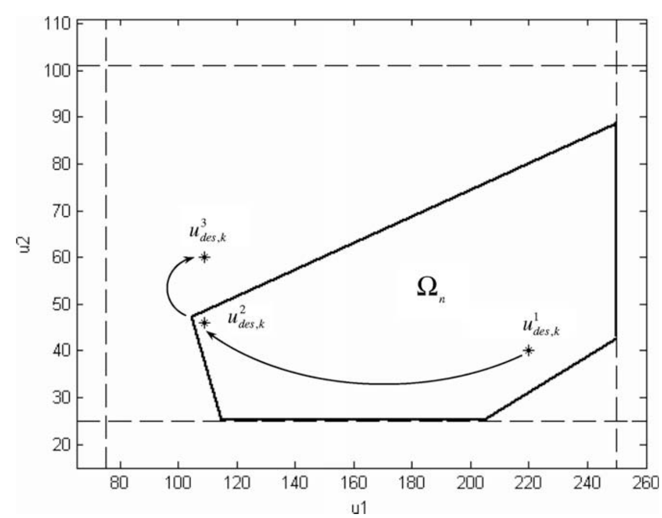

Fig. 11. Input feasible set and input targets.

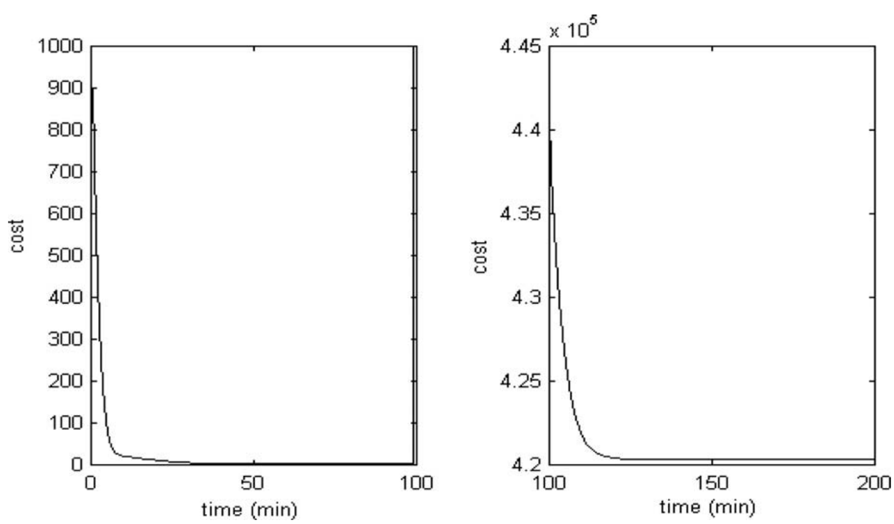

Fig. 10. Cost function for the FCC subsystem with modified zones.

the outputs are guided to their corresponding zones, while the inputs show a steady state offset with respect to the target $u_{des,k}$. The complete behavior of the inputs and outputs of the FCC subsystem, as well as the output set point, can be seen in Figs. 8 and 9, respectively. The final steady state of the input is $u = [196.24 \quad 100.28]$. Fig. 10 shows the control cost along the time.

In the next simulated scenario, we assume that a sequence of changes in the input targets produced by the upper optimization layer enters the controller. The targets are shown in Fig. 11, and their numerical values are: $u^1_{des,k} = [220 \quad 40]$, $u^2_{des,k} = [109 \quad 46]$ and $u^3_{des,k} = [109 \quad 60]$. In Fig. 11, we see that the last input target lies outside the input feasible set. Considering the control structure represented in Fig. 1, which does not include the target calculation stage, the input target is passed directly to the MPC controller. In Figs. 12 and 13, we see that the controller performs adequately and keeps the outputs within their zones while the inputs are led to the desired targets. The only exception corresponds to the case in which the input target is moved to $u^3_{des,k}$ where offset appears in the inputs. However, because the penalization corresponding to the output slacks is larger than those corresponding to the input slacks, the outputs are maintained inside the specified zones. Note that, in the last part of the simulation, output y_3 reaches its lower bound, while inputs u_1 and u_2 deviate from their optimal targets. Fig. 14 shows the control cost corresponding to the different time periods. It can be seen

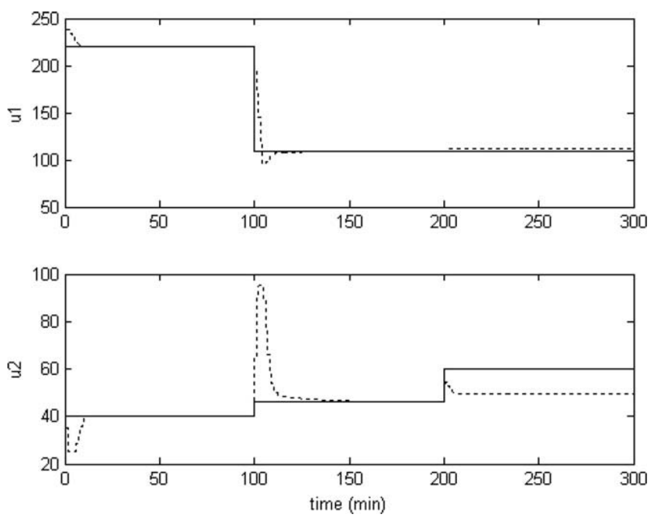

Fig. 12. Manipulated inputs when the input targets are modified.

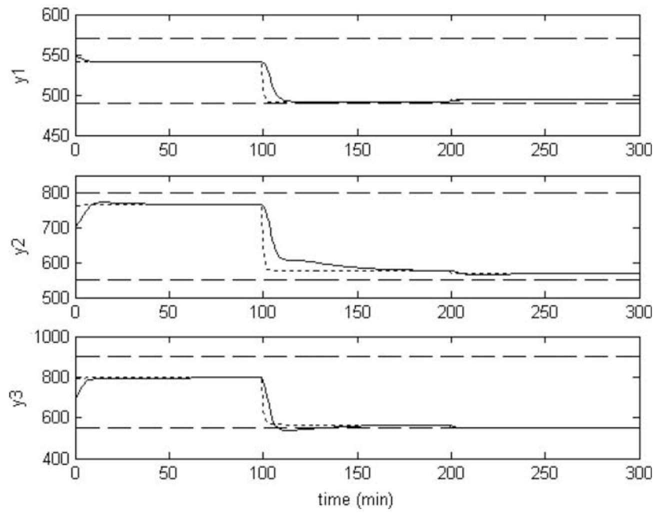

Fig. 13. Controlled outputs when the input targets are modified.

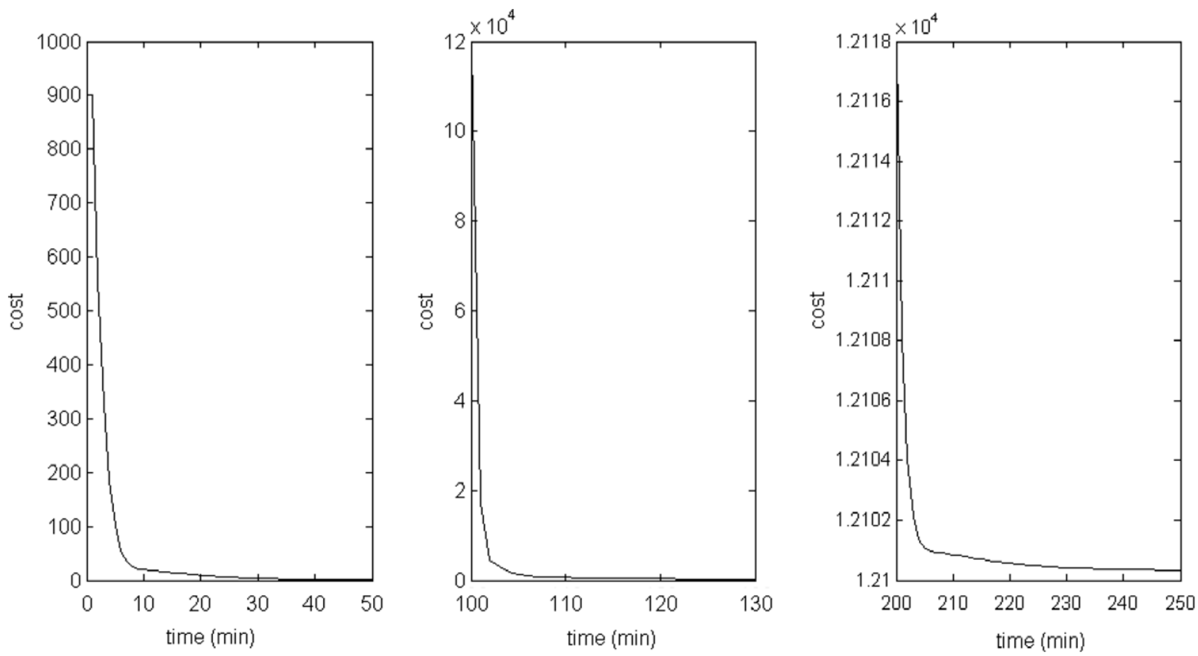

Fig. 14. Cost function for several input target changes.

again that, as established in the convergence theorem, the cost is monotonically decreasing and goes to zero for the two first moves in the input target. In the last one, since the system cannot reach the desired stationary point because of an inconsistent choice of the desired input and the output zones, the cost converges to $\delta_{u,k}^{\mathrm{T}} S_u \delta_{u,k}$, where $\delta_{u,k} = [3.01 \quad -10.58]$.

If the control structure represented in Fig. 2 is adopted in the simulation of this same scenario, the intermediary calculation stage will pass a reachable target to the MPC controller, which will be as close as possible to the point corresponding to the target given by the supervisory optimization level. Fig. 15, shows the location of the calculated input target $u_{\mathrm{target},k}$ corresponding to different weighting matrices:

$$u_{\mathrm{target},k}^1 = [144.78 \quad 58.74] \quad (R_{\mathrm{target}}^1 = \mathrm{diag}(1 \quad 100)),$$
$$u_{\mathrm{target},k}^2 = [112.01 \quad 49.42] \quad (R_{\mathrm{target}}^2 = \mathrm{diag}(1 \quad 1)) \quad \text{and}$$
$$u_{\mathrm{target},k}^3 = [109.03 \quad 48.57] \quad (R_{\mathrm{target}}^3 = \mathrm{diag}(100 \quad 1))$$

Observe that the points corresponding to each of these targets are located on the boundary of the input set, and one of outputs is forced to reach the bound of the zone.

The last scenario simulated here corresponds to the case where a step disturbance, which is unknown by the controller, is introduced in the system output. First, at time = 100 min, the disturbance $d_1(k) = [67 \quad 75 \quad 46]^{\mathrm{T}}$ was injected into the system output and results in the reduction in the input set. Then, once the system stabilized, a second step disturbance ($d_2(k) = [-26.4 \quad 8.3 \quad 33.5]^{\mathrm{T}}$) was introduced at time=200 min. In Fig. 16 Ω_n^1 represents the input set without any disturbance, Ω_n^2 represents the input set after disturbance d_1 was introduced. and set $\Omega_n^3 \triangleq \{u : y_{\min} - y_{ss} \leqslant D^0 u - D^0 u_{ss} \leqslant y_{\max} - y_{ss}\}$ corresponds to the case where disturbance d_2 was introduced in the system. Note that, in this last case, the intersection of Ω_n^3 with the set defined by constraint (11) is an empty set.

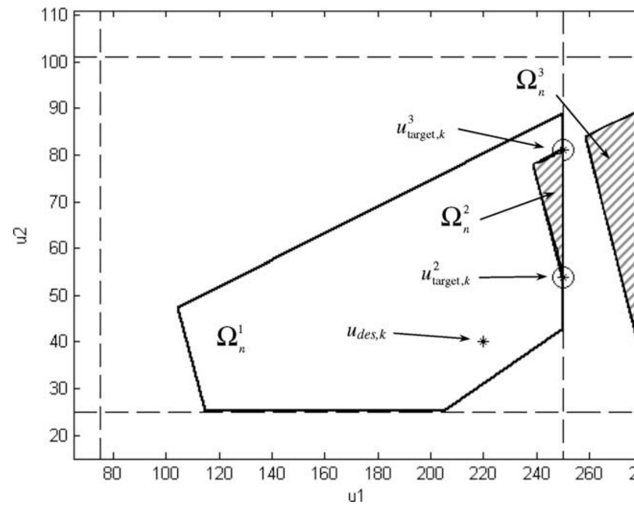

Fig. 16. Input feasible sets produced by output step disturbances.

Fig. 16 shows that for the first disturbance, the target $u_{\mathrm{target},k}^2$ is as close as possible to $u_{des,k}$ and lies on the boundary of the input feasible set, Ω_n^2. On the other hand, when the second disturbance is introduced in system, target $u_{\mathrm{target},k}^3$ is placed as close as possible to Ω_n^3, but inside the set given by $\{u: u_{\min} \leqslant u \leqslant u_{\max}\}$, which is a hard constraint, and no violation is allowed.

Figs. 17 and 18 show the inputs and outputs, respectively. Fig. 18 also shows the calculated output set points. In the first period of time, input u_1 reaches its upper bound while output y_1 is controlled at its lower bound. This means that there is no offset in the outputs or inputs (the targets are consistent). However, in the third period of time, input u_1 remains on its upper bound but the output y_1 lies outside its zone. This is so because the introduction of d_2 produces a null input feasible set, which means that there is no feasible input capable of bringing all the outputs to their zones.

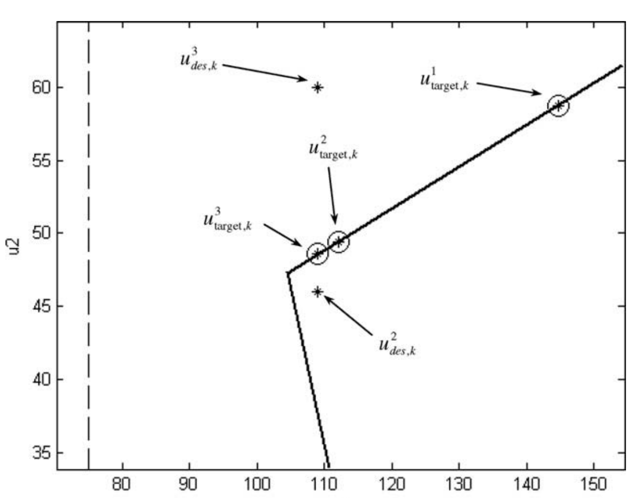

Fig. 15. Location of the intermediary input targets when the target calculation stage is implemented.

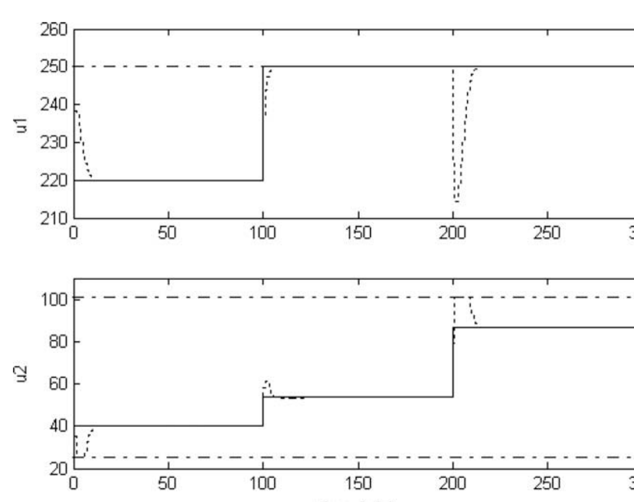

Fig. 17. Manipulated inputs for output step disturbances.

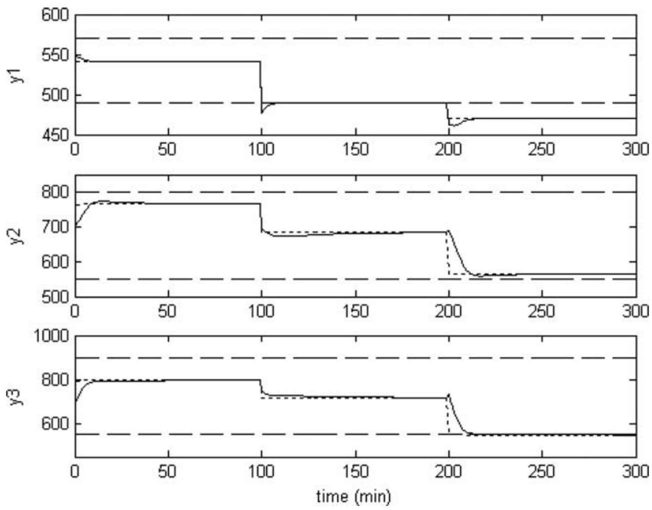

Fig. 18. Controlled outputs for output step disturbances.

7. Conclusion

This work proposes a strategy to implement a MPC controller in which the system outputs are controlled in specified zones and the manipulated inputs have targets associated to the economic objectives of the controlled system. By defining an appropriate cost function, recursive feasibility and stability can be proved for open-loop stable systems. The strategy was shown, by simulation, to have an adequate performance for a 2×3 subsystem of a typical industrial system.

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