Received: 4 December 2020 | Revised: 21 May 2021

DOI: 10.1111/meca.12362

METROECONOMICA
INTERNATIONAL REVIEW OF ECONOMICS

WILEY

ORIGINAL ARTICLE

Instability, political regimes and economic growth. A theoretical framework

Fernando Tohmé[1] | **M. Ángeles Caraballo**[2,3] | **Carlos Dabús**[1]

[1]CONICET, Universidad Nacional del Sur, Bahía Blanca, Argentina

[2]Departamento de Economía e Historia Económica, Universidad de Sevilla, Seville, Spain

[3]IUSEN, Universidad de Sevilla, Seville, Spain

Correspondence
M. Ángeles Caraballo, Departamento de Economía e Historia Económica, Universidad de Sevilla, Avenida Ramón y Cajal, Seville 41018, Spain.
Email: mcaraba@us.es

Abstract

This paper models the influence of political instability on long-term economic growth. We consider three political systems associated to real-world political systems of increasing participation in policy-making. For each system, society chooses an agent that remains in power unless instability, represented as a shortening of the period in office, sets in, which in turn leads to a shortening of the temporal horizon. The agent in charge reevaluates the optimal consumption program, by increasing the rate of time preference and the consumption. With a positively skewed income distribution, the relationship between participation and growth presents different shapes, depending on the probability of the agent in office being ousted. Our results lend theoretical support to the various findings in the empirical literature on the effects of political systems on economic growth.

KEYWORDS

dictatorship, economic growth, instability, majority rule system, political regimes, proportional representation system

1 | INTRODUCTION

One of the most challenging issues in macroeconomics arises by simply inspecting income data across countries. In 2010, there were five countries with an income per capita below 200 US dollars, while eight countries exceeded 30,000 US dollars. In growth rates, for the period 1960–2010, eight countries showed a negative rate while four countries grew at more than 1,000%. In an effort to explain these differences, neoclassical growth models were focused on their correlates, that is, on the accumulation of physical and human capital as well as on the production technology. Nevertheless, as Acemoglu (2009, p. 109) points out, 'why is that some societies do not improve their technologies, invest in physical capital, and accumulate human capital as much as others? (…) There must be other deeper reasons that we will refer to as 'fundamental causes' of economic growth'.

From among the possible fundamental causes, this paper stresses the role of political instability in explaining long-term economic growth: an issue that has garnered great attention among researchers. Certain paradigmatic cases have been widely studied. A striking example is the case of Botswana, one of the countries that grew more than 1000% in the period 1960–2010. Botswana, a democratically stable country since its independence, elicits comparisons with countries with negative growth rates, such as Niger, which, in the same period, suffered three coups, several frustrated coups and a civil war. Moreover, it is obvious that the richest countries in the world, such as the Nordic countries, USA and Canada have enjoyed a long tradition of political stability.

There exists a substantial body of theoretical and empirical literature explaining this negative association between socio-political instability (SPI) and economic growth—for surveys on this topic, see Alesina and Perotti (1994) and Galor (2009). On the one hand, theoretical analyses argue that unstable executives harm long-term growth because not only do they tend to be more corrupt, but they also suffer from myopia in fiscal policy decisions, which leads to heavier borrowing (Chen & Feng, 1996; Devereux & Wen, 1998; Hopenhayn & Muniagurria, 1996; Riedl, 1999).

The empirical literature widely supports the hypothesis that SPI adversely affects economic growth. In cross-country analyses, Barro (1991) showed that the frequency of coups d'état and the number of political assassinations harm economic growth, while Alesina et al. (1996) found that average per capita growth is lowest in years with coups d'état, slightly higher in years with government change and highest in periods without any such changes. More recently, evidence can be found for a variety of time periods and geographical areas, such as Campos and Karanasos (2008) for Argentina, Gurgul and Lach (2013) for CEE countries in transition, and Okafor (2017) for the countries in the Economic Community of West African States. Moreover, a strand of the empirical literature focuses on the channels through which SPI affects economic growth such as physical and human capital accumulation (Haque et al., 2007), financial development (Roe & Siegel, 2011) and productivity (Aisen & Veiga, 2013). Furthermore, several authors claim that the results depend heavily on the measure of SPI—see Butkiewicz and Yanikkaya (2005) and Jong-A-Pin (2009), among others.

Nevertheless, the impact of the type of political regime remains unclear. For instance, Ghardallou and Sridi (2020) review the theoretical arguments favouring both positive and negative effects of democracy on growth. In fact, while researchers like Bates et al. (2012), Madsen et al. (2015) and Acemoglu et al. (2019) find that democracy does cause growth, other authors suggest that the effects of democracy on growth are uncertain, especially in the case of developing countries—see Rodrik and Wacziarg (2005), Doucouliagos and Ulubasoglu (2008), Murtin and Wacziarg (2014), and references therein. Furthermore, certain types of authoritarian regimes may exert positive effects on growth: Singapore, one of the richest countries in the world today, has experienced growth above 1000% since the 1960s, and Qatar was the country with the highest income per capita in 2017, according to the World Data Bank. However, no general agreement has been reached on the effects of autocracies on economic growth. Hence, for instance, Wright (2008) establishes conditions under which authoritarian regimes promote growth, while Easterly and Pennings (2017) conclude that the contributions of the vast majority of political leaders to growth are irrelevant and Rizio and Skaly (2020) find that, as a rule, autocratic leaders are bad for growth, and only in exceptional cases is their impact positive.

Most of the empirical literature uses a dichotomous dummy to classify political regimes[1] (0 for authoritarian regimes and 1 for democratic regimes) and ignores the distinctions between the various types of democracy and the nuances of regimes deemed as authoritarian. This binary classification is certainly arbitrary and omits several relevant differences among political regimes. For instance, this simplified classification is unable to shed light on those cases in which countries oscillate between elected and autocratic governments. Haggard and Kaufman (2016) analysed 78 transitions to democracy and 25 reversions from democracy to authoritarian regimes since 1980. According to these authors, the reversions from higher social participation to dictatorships can take place in developing countries with high inequality and ensuing social conflict over income distribution. In turn, they argue that authoritarian elites are less likely to resign power in those cases where income and assets are highly concentrated. Similarly, in a long-term study for a wide sample of cases, Albertus and Menaldo (2018) state that democratic institutions are frequently designed by a leaving authoritarian regime to protect incumbent elites, thereby granting them an unfair advantage subsequent to democratisation, and introduce what Acemoglu and Robinson (2008) define as *captured democracies*. This seems to be the case of Chile, where the constitution was drafted during Pinochet's dictatorship. The 2019 and 2020 massive manifestations expressed discontent over that state of affairs and forced the democratic government to allow the reformation of the constitution.

The introduction of finer distinctions between autocracy and democracy yields a possible non-linear relationship between economic growth and the degree of democracy. Libman (2012) surveys the literature, and distinguishes two main types of non-linearities. One type of non-linearity indicates that the degree of democracy of a country influences its growth rate in the form of an inverted U shape: the growth rate is higher for the middle levels of democracy and lower for both the least and the most democratic countries (Barro, 1996a, 1997). A possible explanation for this relation is that dictatorships obstruct the normal functioning of market forces, while more freedom promotes entrepreneurship and investment. In turn, a full democracy, where all political positions influence policy making, appears to promote a higher degree of distribution and therefore lowers the accumulation of capital and the rate of growth. Plumper and Martin (2003) provided theoretical support for this result by arguing that the link between democracy and economic growth is the amount and quality of investment in the provision of public goods.

There are also arguments in favour of a U-shaped relationship, meaning that pure autocracies and pure democracies can exhibit better economic performance than intermediate political systems. For instance, Acemoglu and Robinson (2006) show that a government is unlikely to hinder growth at the extreme ends of the political system, either because of the high degree of political competition in a pure democracy or due to the absence of external threats in a pure dictatorship. However, in the intermediate regimes, the fear of the loss of power that reforms can cause leads the government to postpone such reforms, thereby harming economic growth. Gates et al. (2006) argue that intermediate regimes are more unstable,

[1]Among the indices of democracy used frequently in the literature, are those of Papaioannou and Siourounis (2008), Cheibub et al. (2010), and Boix et al. (2012).

which leads to greater uncertainty and to worse economic results than those achieved by the extreme regimes.

Alternatively, Ma and Ouyang (2016) highlight the role of a country's political history, and find an asymmetrical pattern for the effects of democracy on growth in which only those countries that surpass a threshold of experience in democracy can exhibit a positive effect of the political system on economic growth.

These mixed findings on the effects of democracy on economic growth require further theoretical support. This paper contributes to this task by building a unified framework in which the effects of democracy on growth may differ depending on SPI and the type of political regime. In order to achieve this goal, a simplified model of growth and instability is presented, where three political regimes are distinguished: a dictatorship and two types of democracy depending on the electoral systems, which we call a *proportional representation system* and a *majority rule system*.[2] It is assumed that there is an increasing degree of participation in political power running from the dictatorship to the proportional system. When both the degree of participation and SPI are taken into account, the relationship between the political systems and economic growth can adopt a variety of shapes, as shown by the empirical literature.

In the absence of instability, each system has an associated idiosyncratic steady-state income that is determined by the preferences of the agent selected to run the executive. Instability leads to the shortening of the length of the rule by the chosen agent. The response to SPI is conditioned by the position of the chosen agent in each system in the distribution of income and preferences among the agents, and also by the size of the opposition. Two kinds of dictatorship can therefore be distinguished as can two types of proportional representation systems. Each system has an inherent level of associated instability. The response of each system to the possibility of being overthrown is an increase in consumption. This response is different for each system, but for all of them a higher early consumption leads to a lower steady state growth. Depending on the size of the opposition, different configurations can be found, each of which corresponds to a shape of the relationship between growth and participation found in the literature.

In Section 2, the Ramsey model of optimal economic growth is formulated by considering that agents are homogeneous, except for their rate of time preference and their income level. We postulate that the ordering of agents in terms of their initial income levels remains the same in the future, as does the existence of an inverse relationship between relative income and time-preference distributions, that is, the poorer the agent, the higher the rate of impatience. This feature of the model implies that the steady state depends positively on the level of income: when income is higher, the agent is more patient and thus capital accumulation is higher.

Section 3 introduces the political systems. Three highly stylised political systems are contemplated in order to simplify the analysis. These three systems can be roughly identified as proportional representation, majority rule and dictatorship. Our approach consists of comparing the *steady states* associated to these systems. The reasons for the preeminence of a given political system in a society are not incorporated in our analysis, nor is the way in which coalitions against the incumbents are formed. In particular, only the long-term states of the economies are taken into account, and not the possible transitions among regimes, only taking into account , although we do analyse the effect on consumption and capital accumulation of a possible transition in

[2]The classification of political systems by Persson and Tabellini (2000) is loosely followed herein.

Appendix C. However, the study of the transitions among political regimes is indeed extremely relevant. In this respect, for instance, Acemoglu and Robinson (2000, 2001) have modelled the political transitions from non-democracies to democracies, and have specified the causes that can induce autocratic powers to democratise and the conditions that explains why it is more difficult for democracies to consolidate in certain societies. They refine this discussion by also modelling the conditions under which an autocrat may move towards a pattern of captured democracy (Acemoglu & Robinson, 2008).

In our model, each system selects a different ruler, defined as a particular element of the income distribution: the median as the representative of the majority rule, the mean as the chosen agent of the proportional system and either the maximal or minimal element in the case of dictatorship. Each chosen ruler applies a mechanism of taxes and subsidies to induce a steady state corresponding to their own rate of time preference. We consider that, in the most common situation, the richest people constitute a small percentage of the total. Therefore, the income distribution has positive skewness. Here we adopt the point of view of Lambert (1993): 'The inequality in a typical income distribution is evident from an examination of the three measures of central tendency: mean, median and mode. These are typically configured as follows: mode < median < mean. Thus, evidence suggests that the most common income level is less than halfway up the distribution, and the income halfway up the distribution is itself below average. This points to the presence of positive (or right) skew in the distribution -a drawn-out upper tail of high incomes in the frequency density function'. In this case, in which no instability is present, we show that the relationship between participation in decision-making and the steady state adopts an U shape.

In Section 4, instability is introduced as a shortening of the term in office of the executive. We subsequently show how the ruler increases the consumption program when the prospect of staying in power fades away. The level of adjustment depends on the regime as well as on the size of its opposing coalition: it is higher for dictatorships with large oppositions and lower for the proportional representation system with a fringe opposition, and for the majority system. After having been ousted from power, the ruler stops belonging to the economy.[3] Hence, the perspective of a shorter time horizon causes an increase of the ruler's time preference, and so of the consumption during the period the ruler is in charge of the economy. Patient rulers adjust proportionally more than impatient rulers, because the latter are closer to their desired consumption than the former. In turn, in the most common case of a positively skewed income distribution, the prospect of a shorter time in charge changes the shape of the 'political participation/growth' relationship. In turn, the size of the opposition to the agent in office has an additional impact in this relation. The different cases that may arise lend support to the empirical evidence of eclectic effects of democracy on growth as stated by Ghardallou and Sridi (2020). Finally, in Section 5, conclusions are drawn.

2 | THE ECONOMY

In this section, we develop the economic model, which constitutes the basic framework for the discussion on how instability affects economic growth. The economy is closed and produces a single homogeneous good which can be allocated to consumption and saving. Each agent has a

[3]This assumption intends to capture the common results of SPI: assassination, exile, etc., of the former executive. In any case the ousted leader ceases to participate in the economy.

strictly concave production function that exhibits positive and diminishing marginal productivity, and verifies the Inada conditions. Therefore:

$$y_i = f_i(k_i) \tag{1}$$

where y_i and k_i are agent i's income and capital, respectively. Savings are entirely invested, that is, they are converted into physical capital, which does not depreciate. By denoting c_i and $\dot{k}_i$ as i's consumption and investment, respectively:

$$f_i(k_i) = c_i + \dot{k}_i \tag{2}$$

where a simpler version than the usual one is adopted in which some income must be devoted to renew obsolete capital. For our purposes, it is sufficient to assume that capital does not degrade over time.

On the other hand, agents live forever and are homogeneous in that they valuate instantaneous consumption in the same way. Hence, for each agent i:

$$u_i(c_i(t)) = u(c_i(t))$$

The utility function is increasing and concave, and intertemporal substitution elasticity ($\frac{1}{\sigma} = -\frac{u''(c)c}{u'(c)}$) is constant. The difference between agents resides in their rates of time preference. Each agent i has an idiosyncratic rate of time preference, ρ_i. Therefore, the optimal consumption plan for i optimises the sum of discounted instantaneous utilities

$$\int_0^\infty u(c_i(t))e^{-\rho_i t}dt$$

subject to

$$\dot{k}_i = f_i(k_i) - c_i \tag{2'}$$

$$k_i(0) = k_i^0 \tag{3}$$

$$lim_{t\to\infty} k_i(t) \geq 0 \tag{4}$$

where (2′) indicates how i's capital evolves, while (3) means that capital has an initial value k_i^0, and (4) states that the amount of capital can never become negative.

This dynamic optimisation problem, known as the *Ramsey problem*, always has solutions when the functional forms are the same as in our model. The solutions consist of temporal paths for k_i, c_i and y_i, each converging to a steady-state value, k_i^*, c_i^*, and y_i^*, respectively.[4] In economic terms, agent i has to choose a feasible consumption path in such a way that the discounted sum of utilities is maximal. This implies that the agent's plan has to specify, for each instant, their levels of consumption and investment.

Each rate of time preference corresponds to a different steady-state growth. In fact, each consumption plan is complemented by a capital accumulation plan, which converges to a steady-state value. According to the rate of time preference, the equilibrium will be different:

[4]See Appendix C for further discussion of the dynamic process of convergence to the steady state.

Proposition 1 *To each ρ there corresponds a unique vector of steady-state values $\langle k^*_\rho, c^*_\rho, y^*_\rho \rangle$ as solutions for the Ramsey problem. Moreover, for two values ρ_1, ρ_2, with $\rho_1 > \rho_2$, $\langle k^*_{\rho_1}, c^*_{\rho_1}, y^*_{\rho_1} \rangle <_{\mathbb{R}^3} \langle k^*_{\rho_2}, c^*_{\rho_2}, y^*_{\rho_2} \rangle$ (where $<_{\mathbb{R}^3}$ is the order relation in $\mathbb{R}^3$).*

This result implies that a higher feasible degree of impatience, which forces higher initial consumption, leads to lower investment and steady-state growth. In turn, there exists an inverse relation between the distributions of time preference rates and of relative income. This is associated to a line of research that studies variations in time in the preference parameters, and particularly of the rate of time preference. Several authors assume that ρ_i is a function of the agent's level of income, $y_i(t)$ (e.g. Barro & Sala-i-Martin, 1995; Mantel, 1967; Uzawa, 1968). It is therefore assumed herein that the rate of time preference is a decreasing function of income: a higher income implies a lower rate of time preference (Blanchard & Fisher, 1993; Mantel, 1997). Intuition tell us that to sacrifice consumption in order to accumulate for the future involves a greater privation for lower levels of income (see Fischer, 1930). We work here with a *qualitative* version of this intuition: lower-income agents are more impatient, and hence they have a higher rate of time preference and are led to lower steady-state values. The difference with previous approaches is that they treat the rate of time preference as a function of absolute levels of consumption, while we consider it only in terms of relative income levels. This means that the main element is the *ordering* induced by the income distribution. Therefore, even if the consumption levels of all agents increase, the poorest will still be more impatient while the richer will have the lowest time preferences.

We order the set of agents (the labour force), $\mathscr{L} = [0, L] \subset \mathbb{R}^1$, in terms of their relative incomes at time $t = 0$. Since the rates of time preference are assumed to be constant and the accumulation paths are monotone, this order remains unchanged from then on. More precisely, for any moment t we define $i \prec_t j$ if and only if $y_i(t) < y_j(t)$. The ordering therefore remains unchanged if $i \prec_0 j$ implies that $i \prec_t j$ for $t > 0$. Consequently, we attain[5]:

Proposition 2 $\prec_t$ *is a continuous* weak order.

Moreover, $\prec_t$ has an associated statistic, the proportion of agents according to their position in the ordering, $\pi(i) = \frac{\mu(\{j : y_j(t) = y_i(t)\})}{\mu(\mathscr{L})}$, where μ is the standard Lebesgue measure in $\mathbb{R}^1$. We assume that what matters for the agents, in terms of their rates of time preference, is precisely their relative position according to $\prec_t$, that is, the ordering corresponding to the relative income distribution. As stated earlier, the reason for this is that if an agent consumes less than most of the others, then the agent will tend to become less patient, whereas if an agent is well-off, then the agent will be willing to postpone current consumption to accumulate more.

Formally, we endow $\mathscr{L}$ with the ordering $\prec$ ($\prec \equiv \prec_t$ for all t). Then we have:

Lemma 1 *The probability π associated to $\prec$ is time invariant.*

By denoting $\langle \mathscr{L}, \prec, \pi \rangle$ as the *relative income distribution*, we can define a continuous bijection;

$$\rho : \langle \mathscr{L}, \prec \rangle \to \langle \mathbb{R}^+, < \rangle$$

[5]See the proofs of the main claims in the Appendix A.

such that for a pair of agents, i and j, $i \prec j$, $\rho(j) < \rho(i)$. This function assigns a non-negative real number (the rate of time preference) to each agent, thereby inverting the order induced by the time-invariant relative income distribution. Hence, the *ordering* of agents according to their time preferences induces a concomitant (inverse) ordering on steady states. This implies that the more common case of a positively skewed income distribution is associated with a negative skewed distribution of the rate of time preferences, and vice versa. The following result follows immediately from Proposition 1:

Corollary 1 *Each agent* i *has a steady-state* per capita *income* y_i^*. *Moreover, if* $i \prec j$ *then* $y_i^* < y_j^*$.

This result establishes a direct connection between the distribution of income (through the the rates of time preference) and steady states. However, at this stage, there exists a multiplicity of alternative steady states, and the rules of this stylised economy fail to indicate which one to choose. This problem can be addressed by introducing an additional element into the framework: the institutional structure. This means enlarging the picture by considering the political system that determines which level of steady state should be pursued.

3 | POLITICAL SYSTEMS AND STEADY STATES

In this section, we introduce the political systems into the economy. In our framework, it means that formal methods will be added to select an agent as the 'ruler' of society. Since this agent summarises the preferences of society, via its political system, the agent becomes the ruler, whose preferences determine the path towards a steady state for the entire economy.[6]

At this stage, it is assumed that there is no instability. This means that the agent in charge will be selected at time $t = 0$ and the plan will be implemented during the incumbency period of this ruler.

A *Political System* is understood here as a rule that selects an agent $a \in \mathscr{L}$ from $\langle \mathscr{L}, \prec, \pi \rangle$, called the *executive* or *ruler*. The idea is that this individual is chosen at $t = 0$ and remains in charge of the economy for all future periods or until she/he is overthrown. Once selected, this agent implements an economic policy such that her/his time preference becomes the *aggregate* rate for the entire economy.

Formally, this means that the executive (with discount rate $\overline{\rho}$) solves the Ramsey problem for the per-capita amounts of consumption and capital accumulation. This leads to a steady-state value $\overline{k}$ such that $f'(\overline{k}) = \overline{\rho}$, where $f(\cdot)$ is the aggregate production function of the economy. One way in which this could be achieved is by means of proportional income taxes (or subsidies), specific to the discount rate of each agent in the economy. In other words, agent i, with discount rate ρ_i will have $(1 - \tau_i)y_i(t)$ available at each instant t. In fact, τ_i may be positive or negative, indicating either a tax or a subsidy. Agent i's solution to the Ramsey problem can therefore be summarised by the following differential equations:

$$\frac{\dot{c}_i}{c_i} = \sigma\ ((1-\tau_i)f_i'(k_i) - \rho_i)$$

[6]The use of the term *ruler* should be understood as meaning 'the agent in the executive office'. Only in the case of dictatorship is this ruling autocratic. In the other cases, we assume that the ruler executes policies mandated either by the majority or by the proportional coalition. In turn, notice that this does not mean that there might exist opposition, constituted by discontents and unrepresented agents.

and

$$\dot{k}_i = (1-\tau_i)\, f_i(k_i) - c_i.$$

Therefore, the steady-state value of capital for i, $\overline{k}_i$, verifies that $f_i'(\overline{k}_i) = \frac{\rho_i}{1-\tau_i}$. Given the goals of the ruler, a condition that the rates $\{\tau_i\}_{i\in\mathscr{L}}$ should verify is that the average after-tax steady-state values should be equal to the ruler's desired steady state value (corresponding to the ruler's discount rate $\overline{\rho}$):

$$\int_{\mathscr{L}} f_i'^{-1}\left(\frac{\rho_i}{1-\tau_i}\right) d\pi(i) = f'^{-1}(\overline{\rho}) \tag{5}$$

with the proviso that the tax-adjusted rates of time preference $(\frac{\rho_i}{1-\tau_i})$ preserve the order given by $\prec$.

An additional condition that ensures the sustainability of this policy is that, once in steady state, the subsidies should be less than or equal to the amounts levied in taxes[7]:

$$\int_{\mathscr{L}} \tau_i\, \overline{k}_i\, d\pi(i) \geq 0. \tag{6}$$

The first condition can be seen as given by a Fredholm integral equation of the first kind. General methods of solution for these equations are based on the properties of their *kernels*, which are given by the specific details of the distribution π (Jerri, 1985). The second condition is slightly less difficult to fulfil, but notice that, since it involves events in the very long term, it must be based on the assumption that the executive will be in charge forever. The example in Appendix B may illustrate the difference between these two conditions. It shows that the political system matters for the overall performance of the economy. The example also shows that, depending on the distribution of preferences, a regime that tries to apply a higher rate of capital accumulation may lead to a more unequal distribution. To analyse this in more detail, notice that for a non-finite $\mathscr{L}$ there might exist an infinite number of political systems. For the sake of simplicity, we select only three alternatives, which seem to capture several aspects of real-world political systems:

- *Majority rule*: The executive is chosen as $a_{\text{maj}} = Median(\langle \mathscr{L}, \prec, \pi \rangle)$, the median in the distribution. It is worth noting that, since the distribution π remains invariant over time, once chosen, a_{maj} remains as the median agent for ever.
- *Proportional representation*: The mean agent is chosen. In principle, it could be represented as the $Mean(\langle \mathscr{L}, \prec, \pi \rangle)$, but since this expression is prone to time inconsistencies, a time-invariant notion of 'average' must be sought. Our approach requires several steps. In the first step, let us note that $\{\rho_i : i \in \mathscr{L}\}$ can be embedded in a bounded interval $\Psi = [\rho_{min}, \rho^{max}]$, the interval of feasible time preference rates. Subsequently, we have to note that there exists a natural isomorphism ϕ between $\mathscr{L}/\sim$ (the set of equivalence classes of $\langle \mathscr{L}, \prec \rangle$) and Ψ: for each equivalence class $\bar{i}$ there is one and only one $\rho \in \Psi$ such that $\rho = \rho_i$ for each $i \in \bar{i}$. Since $\prec$ is a continuous weak order, $\mathscr{L}/\sim$ is isomorphic to a closed interval of $\mathscr{L}$. Therefore, $\phi(\bar{i}) = \rho$ establishes an order-preserving continuous transformation. In particular, π remains invariant

[7]This should suffice, since the economy will stay in steady state forever, to compensate for eventual disequilibria in the transient phase.

under ϕ. Then, with a slightly abuse of the language, the average for $\langle \mathscr{L}, \prec, \pi \rangle$ can be defined as $a_{\text{prop}} = \int_{\mathscr{L}/\sim} \pi(\phi^{-1}(\rho))\phi^{-1}(\rho)d\phi^{-1}(\rho)$.
- *Dictatorship*: if $a_{\text{maj}} \prec a_{\text{prop}}$ then $a_{\text{dict}} \in Maximal(\langle \mathscr{L}, \prec, \pi \rangle)$. Otherwise, $a_{\text{dict}} \in Minimal(\langle \mathscr{L}, \prec, \pi \rangle)$. The first case arises when most agents are less patient than the average agent, and hence the dictator is the most patient agent. Conversely, in the second case, the dictator is the least patient agent, in contrast to majority rule, which is patient.

The motivation for each system can be found in real-world political regimes. The majority rule, for instance, is a democratic state of affairs that may arise by pairwise voting among all the alternatives that are optimal for a certain agent. This procedure yields an overall winner, known in the literature of social choice theory as the *median voter* (Black, 1948).[8]

Proportional representation, admittedly the most contrived among the three systems, is such that all the agents participate (in principle) in decision-making, in proportion to how frequent their preferences are among the entire $\mathscr{L}$. We represent this as the average of the 'votes' cast at $t = 0$. In our framework, it follows immediately that the social steady state arises by weighting the individual steady states of all the agents in the economy. This can be interpreted as the balance of forces of proportional electoral systems themselves leading to the election of coalitional executives.[9] In one-dimensional elections, as that implied here, it reduces to the selection of the weighted average of the 'candidates'. In an utilitarian approach, this system can be seen as implementing a *bargaining solution*.

Finally, our notion of dictatorship can be interpreted as representing a type of government where the preferences of a single individual become the rule for the entire society. Our modelling primitive is that the dictator is the agent most opposed to the majority, that is, a member of an extreme minority. Therefore, the rate of preference selected by a dictatorship will be at the extreme position of the distribution, opposed (with respect to the mean) to the majority rule executive. Although this is an oversimplification, since as a political system, a dictatorship cannot be described by an unambiguous definition,[10] our characterisation seems to capture the empirical fact that dictators tend to carry out 'unpopular' policies, although these could favour economic growth. Nevertheless, dictators need to have a certain level of consensus to avoid being overthrown. This matters regarding the issue of instability, which is in Section 4.

To ensure the soundness of our characterisations, the following should be considered:

Proposition 3 *For a given* $\langle \mathscr{L}, \prec, \pi \rangle$, $a_{\text{dict}}, a_{\text{maj}}$, *and* a_{prop} *are in* $\mathscr{L}$ *and are time-invariant.*

Another issue that becomes relevant in the analysis of these systems is whether they are *manipulable* or not, that is, whether there exists an agent in $\mathscr{L}$ that may found which, by declaring a false position in $\prec$, may force an outcome that is better than that obtained with an honest declaration. In other words, if the agents are allowed to declare their positions in $\langle \mathscr{L}, \prec \rangle$ to a 'social planner' unaware of their true initial outcomes, an alternative distribution π' may be attained. In formal terms,

[8]Beck (1978) analysed the implications of the median voter theorem for growth theory.

[9]For a survey of the distinctions between real-world majoritarian and proportional systems, see Persson and Tabellini (2000). Moreover, Hassler et al. (2003) analyse how these different kinds of democracy affect the redistribution of income in the economy.

[10]In this respect, Barro (1996b) asserts that, with respect to economic growth, there are two types of dictators: those whose own interests promote economic development, and others whose personal goals are detrimental to growth.

consider a coalition $\mathscr{M} \subseteq \mathscr{L}$ such that for $i \in \mathscr{M}$, the agents i-th position in $\prec$ declare an alternative position i'. Then, an alternative distribution $\pi'_{\mathscr{M}}$ obtains. The steady state of the ruler (for a given political system) corresponding to $\langle \mathscr{L}, \prec, \pi'_{\mathscr{M}} \rangle$ is denoted by c' while it is c for $\langle \mathscr{L}, \prec, \pi \rangle$. The agents in $\mathscr{M}$ can therefore successfully manipulate the system by means of $\pi'_{\mathscr{M}}$ if $u_i(c') > u_i(c)$ for each $i \in \mathscr{M}$. If there exists a coalition $\mathscr{M}$ such that the manipulation is successful, then the system is said (group) to be manipulable. By extension, it can be stated whether the executive is manipulable:

Proposition 4 *For a given $\langle \mathscr{L}, \prec, \pi \rangle$, a_{maj} is not manipulable while a_{dict} and a_{prop} may be prone to manipulation.*

This indicates that dictatorship and proportional representation can only succeed if it is assumed that, in these systems, society chooses their ruling agent by honest voting, while the majority rule still works under strategic voting. Interestingly, manipulable systems are prone to instability since dishonest voters may act in order to lead the economy to their actual preferred rates of accumulation. Nevertheless, our analysis is framed in the context of honest voting or, alternatively, assuming that the social designer knows $\langle \mathscr{L}, \prec, \pi \rangle$. Instability can ensue independently of the lack of manipulation.

Let us compare the steady states of the alternative political systems. The differences between these systems are not independent of the distribution of income and time preferences, shared by the all three of them, which in the following is characterised in terms of properties of the time-invariant probability distribution π.

The third moment of π, its skewness, is particularly relevant for our analysis. It indicates where the mass of agents is located with respect its mean and the median. Since it is standard, π is a distribution with positive skewness if the median is to the left of the mean, and with negative skewness otherwise. The case of zero skewness is disregarded, since it is not generic. Therefore, two generic cases must be considered:

- π **has negative skewness**: whereby $a_{\text{dict}} \prec a_{\text{prop}} \prec a_{\text{maj}}$. Therefore, $y^*_{\text{dict}} < y^*_{\text{prop}} < y^*_{\text{maj}}$.
- π **has positive skewness**: whereby $a_{\text{maj}} \prec a_{\text{prop}} \prec a_{\text{dict}}$. Hence, $y^*_{\text{maj}} < y^*_{\text{prop}} < y^*_{\text{dict}}$.

As can be observed, the (less common) first case coincides with Barro's inverted U. In this case, the dictator is the least patient agent in society. This fact pushes the dictatorial steady state to the lowest level of output and consumption. An increase in participation in the choice of the social steady state means changing from the dictator to the median agent, who is above the mean, and is therefore more patient. This ensures a higher steady-state value. Proportional representation induces the participation of minorities in decision-making. The preferences of the agents at the minimal levels again influence the social decision and therefore balance the preferences of the majority, and push the economy towards a lower steady-state path, because it is associated to a higher rate of preference. Therefore, the highest steady-state value is that of the majority rule, while the least and most inclusive regimes exhibit lower values.

On the other hand, in the case of positive skewness, the dictator is the most patient, leading to the highest steady state. The majority rule depresses that value by shifting preferences towards a higher rate of time preference. Proportional representation improves the steady-state values but insufficiently for the recuparation of the levels sought by the most patient agents. Figure 1 depicts the positively skewed income distribution and Figure 2 represents the associated steady state of output.

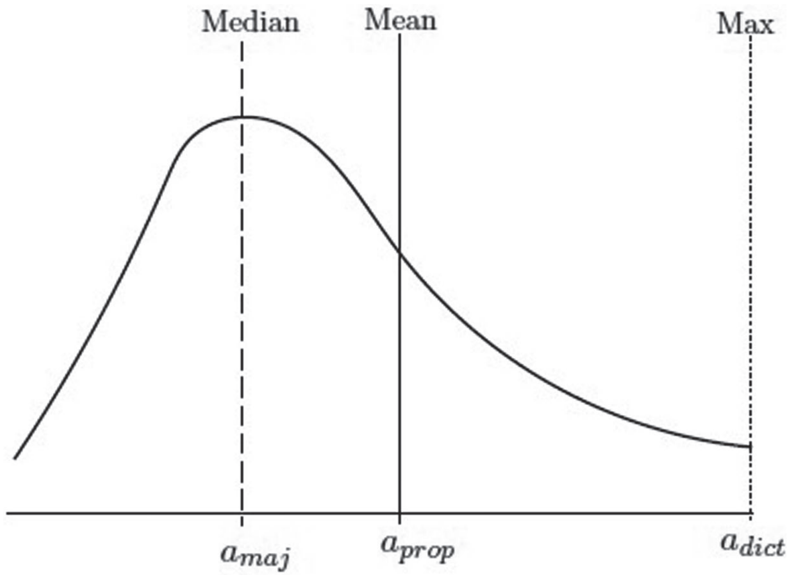

FIGURE 1 A positively skewed distribution and the agents corresponding to each regime

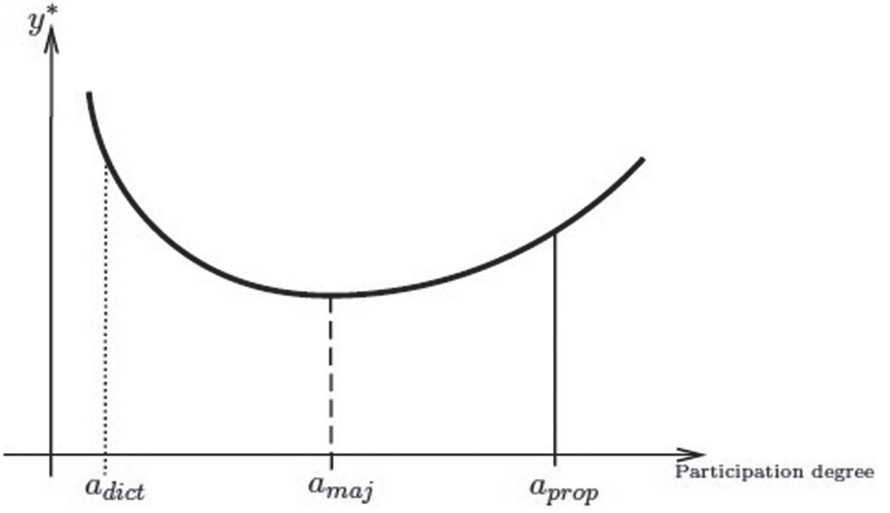

FIGURE 2 Steady states corresponding to a positively skewed income distribution

In short, the more common case of a positive skewed income distribution yields a uniform U shape in the relationship between social decisiveness and steady state. This contrasts sharply with studies that indicate an inverted U shape, such as that by Barro (1997). Nevertheless, it should be borne in mind that, in those contributions, the economies studied exhibit various degrees of social and political instability, while this feature has yet to be introduced. It is now shown that the previous results can be modified if instability is introduced. If executives stay in charge for only a short period, then their plans must be modified, and hence the growth plan for the economy is affected. In fact, in certain cases the inclusion of instability enables an inverted U to be attained for the more common instance of a positive skewed income distribution. This is discussed in the next section.

4 | INSTABILITY

In this section, instability is introduced into the picture, understood as a context of social discontent[11] that leads to a shortening of the permanence of the executive agent in charge. This characterisation strives to capture a very natural insight: in politics, a shorter time in office, and therefore a higher turnover rate, indicates higher instability. Constitutional governments generally experience finite and preordained horizons; political unrest, however, induces at least partial changes in the highest positions of the executive body.

In our stylised presentation, the presence of instability is indicated by a finite horizon, $t < \infty$. A lower t is associated to higher instability. The executive remains uncertain regarding the actual value of the turnover time, t, although the expected value of t, $\bar{t}$, can be estimated.

The optimal growth path for an infinite horizon indicates the planned consumption for all future times. If the horizon is shortened, then the ruler increases the rate of time preference in an effort to compensate for the expected loss of utility after $\bar{t}$. To reduce that loss, the ruler increases the levels of consumption above those that would be set if the utility were being maximised according to the ruler's true time preference. By the end of the period in power, the ruler would have surpassed the amounts specified in their optimal plan up to that point. Hence, the economy tends towards a different steady state, with a higher consumption and lower accumulation in its early stages, and a lower steady state in the long run.

The argument is as follows. The executive faces the decreasing probability of being in charge after time t, $\Omega(t)$, defined for $t \in [0, \infty)$, which approximates the degree of instability. Therefore $\dot{\Omega}(t) < 0$, and for the sake of simplicity, it is assumed that the probability of survival has a constant variation rate $\widehat{\Omega} = \frac{\dot{\Omega}(t)}{\Omega(t)}$ for $t < \bar{t}$, whereas it is 0 afterwards. The main objective of the executive now becomes the optimisation of the lifetime utility while taking this fact into account. Hence, the new goal is to maximise the following function[12]:

$$\int_0^{\infty} \Omega(t) u(c(t)) e^{-\rho t} dt$$

subject to

$$\dot{k} = (1 - s) f(k) - c$$

$$k(0) = k_0$$

$$\lim_{t \to \infty} k(t) \geq 0$$

where the constraints are assumed to be satisfied with probability one. ρ is the discount rate of the ruler, f the aggregate production function, while c and k are the amounts of consumption and capital accumulation that the ruling body may ensure for itself by remaining in office. Notice that, in the definition of $\dot{k}$, only a reduced proportion of the income, $(1 - s)f(k)$, where $s < 1$ is a positive constant, is devoted to consumption and accumulation. The rest, $sf(k)$ is destined to the costs of keeping the opposition at bay.

The Hamiltonian for this problem is:

[11]See, among other characterisations of instability, the presentation in Alesina et al. (1996).

[12]See Yaari (1965) for a discussion on the problem of consumer behavior in the case of uncertain lifetime.

$$H(c,k,\lambda) = \Omega(t)u(c(t))e^{-\rho t} + \lambda((1-s)f(k(t)) - c(t))$$

Its optimisation yields two first-order conditions:

$$\Omega(t)e^{-\rho t}u'(c) = \lambda$$
$$(1-s)f'(k) = -\frac{\dot{\lambda}}{\lambda}$$

By taking logarithmic derivatives and combining both expressions, we attain:

$$-\rho + \frac{u''(c)}{u'(c)}\dot{c} + \widehat{\Omega} = -f'(k)$$

Again, by assuming a constant elasticity of substitution, $\sigma = -\frac{u'(c)}{cu''(c)}$, we find that

$$\frac{\dot{c}}{c} = \sigma\ [(1-s)f'(k) - \rho + \widehat{\Omega}]$$

which, in steady state, yields $(1-s)f'(k) = \rho - \widehat{\Omega}$, for $t < \bar{t}$. Since $\widehat{\Omega} < 0$, this means that the new adjusted time preference, $\rho' = \frac{\rho - \widehat{\Omega}}{1-s}$, is higher than ρ and therefore leads to a lower steady state.

In other words, the ruler a, with the time preference ρ_a and the optimal plan for an infinite horizon, $\{c^*(t)\}_{t=0}^{\infty}$, chooses an alternative time preference $\rho_a' > \rho_a$. The new time preference is such that the corresponding optimal plan, $\{c'(t)\}_{t=0}^{\infty}$, verifies $c'(t) \geq c^*(t)$ for every t, $0 \leq t \leq \bar{t}$.

The rationale for this behaviour is that, although the ruling agent wants to increase consumption while in charge, forcing a collapse of the economy is not an optimal behavior. Since $\bar{t}$ may differ from the actual turnover time, the executive will lack incentives to consume the entire capital stock in a finite period, which explains why the problem that the ruler faces is *not* finite-horizon optimisation. Hence, $k > 0$ for every finite t. It is evident that the ruler has to select an alternative path that leads to non-zero steady states. In turn, since higher consumption benefits the ruler only during the period in charge, there seems to be no advantage in choosing a path where consumption remains higher than in the preferred path after having been overthrown. To show that there exists such a choice we state the following:[13]

Proposition 5 *For each feasible time preference* ρ_a^*, *there exists a unique* ρ_a' *yielding a plan* $\{c'(t)\}_{t=0}^{\infty}$ *such that* $c'(t) \geq c^*(t)$ *for every* t, $0 \leq t \leq \bar{t}$.

Hence, there exists a one-to-one function $\phi_{\bar{t}}: \Psi \to \Psi$, where $\Psi \subseteq \mathbb{R}^+$ is the closed interval of feasible time preferences. This function is such that $\phi_{\bar{t}}(\rho_a) = \rho_a'$. The properties of $\phi_{\bar{t}}$ are summarised in the following:

Theorem 1 *For a given* $\bar{t}$, $\phi_{\bar{t}}$ *is a continuous and differentiable function, such that* $\frac{d\phi_{\bar{t}}}{d\rho} \geq 0$ *and* $\frac{d^2\phi_{\bar{t}}}{d\rho^2} \leq 0$.

Hitherto, the effect of instability on the adjustment towards lower steady-state values has been considered regardless of the political system. However, this disregards the fact that the effects of

[13]See Appendix C for a discussion of the dynamics of the solutions and how a transition may happen after the incumbent is overthrown.

instability may differ from one system to another. In order to compare the consequences of instability on our three systems, let us introduce the idea of an *opposition* for a system with executive a, $\mathcal{M}_a$, as the largest coalition that would like to have the opportunity to force a different outcome in a recall election.

It should be noted that we assume that instability does not depend on the distributive problem. The opposition coalitions are not necessarily made up of those who would like to change their level of consumption, but rather by those who are dissatisfied with the executive. In other words, the executive is chosen for economic reasons, while the reasons for its removal are political and/or social. Otherwise, there would be a violation of the assumption of honest voting made in this model.

A characterisation of the probability of staying in power can be given in terms of the size of $\mathcal{M}$. Since the Ω distributions are assumed to be decreasing with a constant rate of variation, the differences between systems can be represented by a relation of first-order stochastic dominance. Formally: given two political systems, with ruling agents a and a', respectively,

$$\Omega_a \succeq_{f.o.d.} \Omega_{a'} \quad \text{if and only if} \quad |\mathcal{M}_a| \geq |\mathcal{M}_{a'}|$$

where $\succeq_{f.o.d.}$ is the (weak) relation of first-order stochastic dominance and $|X|$ indicates the cardinality of a set X.

The rationale of this characterisation is simple: if more agents would like to have the opportunity to oust an executive, the shorter the period as a ruler will tend to be.

In fact, as mentioned earlier, real-world dictatorships tend to face huge opposition and are not legitimised by any due electoral process. Moreover, as shown by the example in Appendix B, dictatorships worsen the income distribution in the economy, thereby creating extra reasons for resistance among the majority of the population, whose members see their income levels affected by the tax policies implemented by the dictator. As we have already argued and shown in the example, the path towards a higher steady state value may lead to a higher inequality in income and, consequently, to higher instability. As shown by Acemoglu and Robinson (2001), the consolidation of a democracy is more difficult in societies with a highly unequal income distribution. This generates further instability which may lead to an oscillation between democratic and autocratic regimes.

It is worth noting that this claim is supported by empirical evidence, as shown, among others, by Alesina and Perotti (1996), Benabou (1996), Okazaki (2007) and Blanco and Grier (2009). Indeed, SPI is more likely to arise in dictatorial regimes, since they usually carry out economic plans that tend to be more detrimental for lower-income sectors, thereby inducing an increasing social discontent and shortening the time of those incumbents in office. The relationship postulated in our model between income inequality and SPI indicates that dictatorships are more unstable than the other two regimes of proportional representation system and majority-rule system. This is particularly true of the unsuccessful dictatorships in Africa and Latin America. Moreover, similar results have been found by Gurgul and Lach (2013) for 10 countries belonging to the European Union in transition from the Central and Eastern Europe region during the 1990–2009 period.

Nonetheless, staying in power under instability could depend on the success of the economic policies of the autocratic government. In this respect, the difference between failed and successful experiences can be found. In fact, autocratic governments that have promoted economic growth, such as that of Pinochet in Chile and others in several nations in South East Asia and the Persian Gulf, exhibit dictatorships that remained in power for a long time. This may be related to the distinction made by Barro (1996b) and mentioned in footnote 10. On the one hand, there are dictatorships in which personal interests are detrimental to growth, partly induced by greater inequality in income—see Delbianco et al. (2014) and references therein. On the other hand, there could be dictatorships whose own goals promote growth, which may explain the social support received by these dictatorships, and allows us to conjecture that they should be considered variants of majority-rule regimes.

Therefore, the difference in the duration in power of dictatorships may depend on whether their economic performances soften their opposition. Successful experiences may lead to the participation of broader sectors of the population in the benefits of higher economic growth, thereby granting the government greater support from society and therefore a longer duration in power.

In order to formally distinguish between these two cases, we will say that there are dictatorships with a large coalition against, $\mathscr{M}^l_{\text{dict}}$, while there are others in which, in particular, the median voter does not belong to the opposing coalition, $\mathscr{M}^s_{\text{dict}}$. Therefore, the degree of participation in political power is lower when $\mathscr{M}^l_{\text{dict}}$ prevails than when $\mathscr{M}^s_{\text{dict}}$ prevails.

Single-party governments associated to majority-rule regimes, seem to have a consistently better record of stability in comparison with coalitional systems. Bejar et al. (2011) present data on the duration of governments from a sample of 24 OECD and 87 non-OECD (developing country) democracies for the period 1975–2007. They show that coalition governments have shorter durations in office. More precisely, the average length of time that coalition governments survive in office is 23 months, while single-party governments, on average, survive in office for 36 months.

Our proportional representation system can be associated to parliamentary-like electoral systems that often give rise to coalition governments. Even though they are on average less stable than single-party governments, there are cases of remarkable stability[14], with minor opposition on the fringes without the ability of affecting the normal functioning of a government. Let us denote such opposition coalitions $\mathscr{M}^s_{\text{prop}}$. The more frequent kind of proportional representation governments are associated to shifting balances in the preferences of the population, which we may represent with a significant coalition of those that do not make the cut to be in the executive, $\mathscr{M}^l_{\text{prop}}$. This coalition is larger than $\mathscr{M}^s_{\text{prop}}$ mostly because mainstream citizens may become members of $\mathscr{M}^l_{\text{prop}}$ at some point. As for dictatorships, the participation degree in political power is lower with $\mathscr{M}^l_{\text{dict}}$ than with $\mathscr{M}^s_{\text{dict}}$.

We can safely assume that the three following configurations cover all the possibilities:

1. $|\mathscr{M}^l_{\text{dict}}| \geq |\mathscr{M}^s_{\text{dict}}| > |\mathscr{M}^l_{\text{prop}}| > |\mathscr{M}^s_{\text{prop}}| > |\mathscr{M}_{\text{maj}}|$.
2. $|\mathscr{M}^l_{\text{dict}}| \geq |\mathscr{M}^l_{\text{prop}}| > |\mathscr{M}^s_{\text{dict}}| > |\mathscr{M}^s_{\text{prop}}| > |\mathscr{M}_{\text{maj}}|$.
3. $|\mathscr{M}^l_{\text{dict}}| \geq |\mathscr{M}^l_{\text{prop}}| > |\mathscr{M}^s_{\text{prop}}| > |\mathscr{M}^s_{\text{dict}}| > |\mathscr{M}_{\text{maj}}|$.

The induced ordering of the Ω distributions leads immediately to the following result:

Proposition 6 *The finite time horizons induced by instability for the different regimes are as follows. For Case 1 above:*

$$\bar{t}^l_{\text{dict}} < \bar{t}^s_{\text{dict}} < \bar{t}^l_{\text{prop}} < \bar{t}^s_{\text{prop}} < \bar{t}_{\text{maj}}$$

[14] German governments in the last forty years seem to be of this type.

for Case 2 we have:

$$\bar{t}^{l}_{\text{dict}} < \bar{t}^{l}_{\text{prop}} < \bar{t}^{s}_{\text{dict}} < \bar{t}^{s}_{\text{prop}} < \bar{t}_{\text{maj}}$$

while for Case 3:

$$\bar{t}^{l}_{\text{dict}} < \bar{t}^{l}_{\text{prop}} < \bar{t}^{s}_{\text{prop}} < \bar{t}^{s}_{\text{dict}} < \bar{t}_{\text{maj}}$$

A consequence of the previous discussions is that the higher the agent's patience, the higher the adjustment of time preference when instability is introduced. This is due, on the one hand, to the fact that a patient agent at period $\bar{t}$ is far from being in a steady state, and therefore, this agent is more willing to increase consumption than an agent who is close to their own steady state. On the other hand, this is reinforced by the adjustment induced by the aforementioned inherent instability of the political regimes. So, in the most common case of a positively skewed income distribution, the dictator (who is already the most patient agent) with a large opposition has to face the shortest finite horizon $\bar{t}^{l}_{\text{dict}}$. Therefore, this ruler has to increase consumption in such a way that it will lead to a lower steady state, *both* because their ideal steady state is far away in any case, and because their time in charge is shorter than that of the rulers of the other systems. This places their *actual* time preference closer to that of the other chosen agents, and may even cause them to exchange positions.

Formally, when the temporal horizon changes to $\bar{t}' < \bar{t}$, the new adjustment function $\phi_{\bar{t}'}$ exhibits properties analogous to $\phi_{\bar{t}}$. The new time preference, ρ''_a, yields a lower steady state, as shown in the following

Proposition 7 *$\phi_{\bar{t}'}$ is such that the plan corresponding to $\rho''_a = \phi_{\bar{t}'}(\rho_a)$, $\{c''(t)\}_{t=0}^{\infty}$ tends towards a steady-state value $c''^{*} < c'^{*}$, where c'^{*} is the steady state corresponding to $\rho'_a = \phi_{\bar{t}}(\rho_a)$.*

In other words, a shorter horizon yields a lower steady state. In terms of the relationship between participation and growth, this means that the steady states corresponding to each political system are lower when the ruler faces a shorter time in charge. An analogous argument shows that the most affected are again the most patient agents.

In the case of a distribution π with positive skewness, if it is assumed that $y^{l*}_{\text{dict}} = y^{s*}_{\text{dict}}$ and $y^{l*}_{\text{prop}} = y^{s*}_{\text{prop}}$ without instability,[15], then:

Proposition 8 *In Case 1 of the ordering of shortened time horizons in Proposition 6, we have that y^{l}_{dict}, y^{s}_{dict} are reduced the most, followed by y^{l*}_{prop} and y^{s*}_{prop}, and the least affected is y_{maj}. In Case 2, the most affected are y^{l}_{dict}, y^{l}_{prop}, followed by y^{s*}_{dict} and y^{s*}_{prop}, and again the least affected is y_{maj}. Finally, in Case 3, the reductions are, in decreasing order, y^{l}_{dict}, y^{l}_{prop}, then y^{s*}_{prop} and y^{s*}_{dict}, and finally y_{maj}.*

This means that Case 1 may lead to steady-states that increase monotonically with the degree of political participation and least opposition. Case 2, in turn, tends to assign the highest steady states to the majority regime as well as to those proportional and dictatorial regimes with minor opposition. Finally, Case 3 shows again a non-linear relationship, but in this case the highest steady state may correspond to the dictatorship with a minor opposition. Figures

[15]That is, the existence of small or large oppositions does not affect the steady-state of the executives if they do not face the risk of being overthrown.

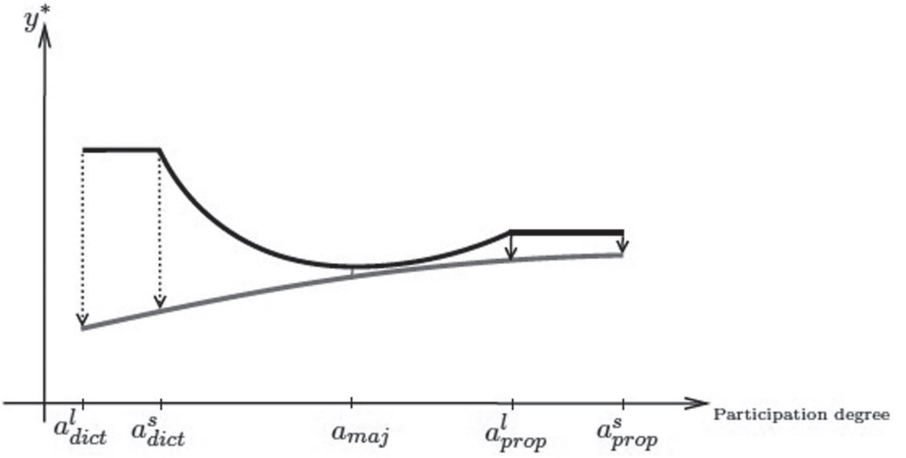

FIGURE 3 Case 1 of the response to instability

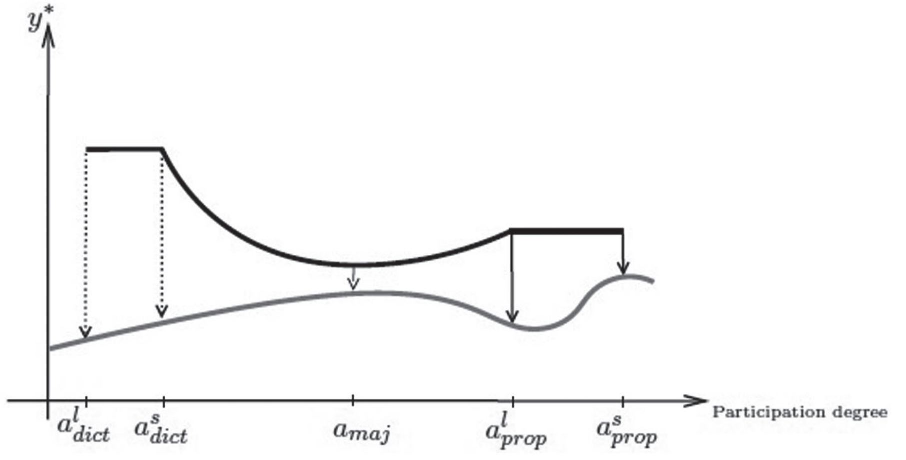

FIGURE 4 Case 2 of the response to instability

3–5 show each case respectively. The dark curve depicts the steady states when SPI is not considered, while the grey curve indicates the change in the steady states when instability is taken into account.

5 | CONCLUSIONS

In this paper, we find a theoretical negative relationship between instability and economic growth. As a first step, our model emphasises that time preferences affect growth: a higher degree of impatience leads to a lower steady state. By introducing political systems, the relative magnitude of steady states depends on the shape of the income distribution. Each system chooses an agent who is at a particular position in the distribution (the mean, the median, and a maximal or a minimal agent). The relative position of the ruling agents is determined by the skewness of the income distribution. This yields a relation between political participation and steady states.

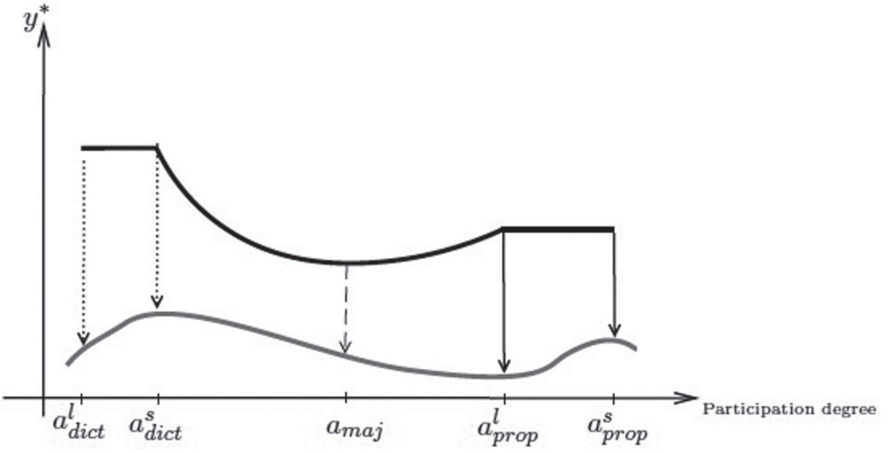

FIGURE 5 Case 3 of the response to instability

Without instability, only the less common case of negative skewness leads to an inverted U shape of the relation between political participation and economic growth.

Instability, represented here by the shortening of the time horizon faced by the executive, changes the steady states. Rulers who face a shorter period in office adjust their consumption paths in order to consume more while in power, but without exhausting the capital stock of the economy. This is done in such a way that the more patient rulers end up adjusting their consumption paths far more than those who are less patient. This arises since a patient agent is, at a given time period, farther away from the steady state than a more impatient individual, who cannot greatly increase consumption without falling onto an infeasible path. Moreover, a very patient ruler has to induce a regressive distribution of income in order to achieve the desired steady-state value, which in turn generates a high degree of instability.

Therefore, once instability is introduced, in the most common case of positive skewness of the income distribution, the dictator, who is the most patient ruler, may face the strongest instability, depending on the size of the coalition of opponents. On the other hand, in our model, the majority-rule executive is the most impatient and also the most stable. If all the ruling agents know they will stay in charge only for a finite period, they will adjust their consumption paths. The expectations of time in power differ from system to system, depending on their inherent amounts of conflict, represented by the size of the regret coalition, that is, the largest group that can manipulate the outcome of the system under strategic voting. The dictator facing a greater coalition in opposition will adjust proportionally more than other rulers, while the majority-rule executive will adjust less. If the time preferences of the rulers differ widely, it is possible that the U shape becomes transformed into a linear relationship or, in certain cases, into a relationship more closely resembling Barro's inverted U shape, as suggested by the empirical evidence mentioned throughout the paper and the theoretical arguments for both positive and negative effects of democracy on growth presented in Ghardallou and Sridi (2020). Instability, in the context of the three political systems presented in this paper, therefore provides an alternative explanation for the empirical evidence on socio-political instability and growth.

ACKNOWLEDGEMENTS

The authors thank the two anonymous referees for their excellent comments which have greatly improved the paper. The authors also acknowledge financial support from the Universidad de

Sevilla (for covering the open access APCs), the Secretaría de Ciencia y Tecnología (Universidad Nacional del Sur, Argentina), project PGI 24/E137 and Junta de Andalucía (Spain), Research Group SEJ258-Research in Applied Economics.

REFERENCES

Acemoglu, D. (2009). *Introduction to modern economic growth*. Princeton University Press.

Acemoglu, D., Naidu, S., Restrepo, P., & Robinson, J. A. (2019). Democracy does cause growth. *Journal of Political Economy*, *127*(1), 47–100. https://doi.org/10.1086/700936

Acemoglu, D., & Robinson, J. A. (2000). Democratization or repression? *European Economic Review*, *44*(4–6), 683–693. https://doi.org/10.1016/S0014-2921(99)00040-9

Acemoglu, D., & Robinson, J. A. (2001). A theory of political transitions. *American Economic Review*, *91*(4), 938–963. https://doi.org/10.1257/aer.91.4.938

Acemoglu, D., & Robinson, J. A. (2006). Economic backwardness in political perspective. *American Political Science Review*, *100*, 115–131. https://doi.org/10.1017/S0003055406062046

Acemoglu, D., & Robinson, J. A. (2008). Persistence of power, elites, and institutions. *American Economic Review*, *98*(1), 267–293. https://doi.org/10.1257/aer.98.1.267

Aisen, A., & Veiga, F. (2013). How does political instability affect economic growth? *European Journal of Political Economy*, *29*, 151–167. https://doi.org/10.1016/j.ejpoleco.2012.11.001

Albertus, M., & Menaldo, V. (2018). *Authoritarianism and the elite origins of democracy*. Cambridge University Press.

Alesina, A., Ozler, S., Roubini, N., & Swagel, P. (1996). Political instability and economic growth. *Journal of Economic Growth*, *1*, 189–212. https://doi.org/10.1007/BF00138862

Alesina, A., & Perotti, R. (1994). The political economy of growth: A critical survey of the recent literature. *The World Bank Economic Review*, *8*, 351–371. https://doi.org/10.1093/wber/8.3.351

Alesina, A., & Perotti, R. (1996). Income distribution, political instability, and investment. *European Economic Review*, *40*, 1203–1228. https://doi.org/10.1016/0014-2921(95)00030-5

Barro, R. (1991). Economic growth in a cross-section of countries. *Quarterly Journal of Economics*, *106*, 407–444. https://doi.org/10.2307/2937943

Barro, R. (1996a). Democracy and growth. *Journal of Economic Growth*, *1*, 1–27. https://doi.org/10.1007/BF00163340

Barro, R. (1996b). *Getting it right*. MIT Press.

Barro, R. (1997). *Determinants of economic erowth: A cross-country empirical study*. MIT Press. https://doi.org/10.3386/w5698

Barro, R., & Sala-i-Martin, X. (1995). *Economic growth*. McGraw-Hill.

Bates, R. H., Fayad, G., & Hoeffler, A. (2012). The state of democracy in Sub-Saharan Africa. *International Area Studies Review*, *15*(4), 323–338. https://doi.org/10.1177/2233865912462373

Beck, N. (1978). Social choice and economic growth. *Public Choice*, *33*, 33–38.

Bejar, S., Mukherjee, B., & Moore, W. H. (2011). Time horizons matter: The hazard rate of coalition governments and the size of government. *Economics of Governance*, *12*, 201–235. https://doi.org/10.1007/s10101-011-0096-0

Benabou, R. (1996). *Inequality and growth. NBER Macroeconomics*, *11*, 11–92. https://doi.org/10.1086/654291

Black, D. (1948). On the rationale of group decision-making. *Journal of Political Economy*, *56*, 23–34. https://doi.org/10.1086/256633

Blanchard, O., & Fischer, S. (1993). *Lectures on macroconomics*. MIT Press.

Blanco, L., & Grier, R. (2009). Long live democracy: The determinants of political instability in Latin America. *Journal of Development Studies*, *45*, 76–95. https://doi.org/10.1080/00220380802264788

Boix, C., Miller, M., & Rosato, S. (2012). A complete data set of political regimes, 1800–2007. *Comparative Political Studies*, *46*(12), 1523–1554. https://doi.org/10.1177/0010414012463905

Butkiewicz, J., & Yanikkaya, H. (2005). Sociopolitical instability and long run economic growth: A cross country empirical investigation. *Journal of Policy Modeling*, *27*, 629–645. https://doi.org/10.1016/j.jpolmod.2005.04.001

Campos, N. F., & Karanasos, M. G. (2008). Growth, volatility and political instability: Non-linear time-series evidence for Argentina, 1896–2000. *Economics Letters*, *10*, 135–137. https://doi.org/10.1016/j.econlet.2007.12.013

Cheibub, J. A., Gandhi, J., & Vreeland, J. R. (2010). Democracy and dictatorship revisited. *Public Choice*, *143*(1–2), 67–101. https://doi.org/10.1007/s11127-009-9491-2

Chen, B., & Feng, Y. (1996). Some political determinants of economic growth: Theory and empirical implications. *European Journal of Political Economy*, *12*, 609–627. https://doi.org/10.1016/S0176-2680(96)00019-5

Delbianco, F., Dabús, C., & Caraballo, M. A. (2014). Income inequality and economic growth: New evidence from Latin America. *Cuadernos de Economía*, *33*, 381–398. https://doi.org/10.15446/cuad.econ.v33n63.45338

Devereux, M., & Wen, J.-F. (1998). Political instability, capital taxation, and growth. *European Economic Review*, *42*, 1635–1651. https://doi.org/10.1016/S0014-2921(97)00100-1

Doucouliagos, H., & Ulubasoglu, M. A. (2008). Democracy and economic growth: A meta-analysis. *American Journal of Political Science*, *52*, 61–83. https://doi.org/10.1111/j.1540-5907.2007.00299.x

Easterly, W., & Pennings, S. (2017). *Shrinking dictators: How much economic growth can we attribute to national leaders?* (DRI Working Paper No. 94).

Fischer, I. (1930). *The Theory of Interest*. Mac Millan.

Galor, O. (2009). *Inequality and economic development: An overview* (Brown University Working Paper No. 2009-3).

Gates, S., Hegre, H., Jones, M. P., & Strand, H. (2006). Institutional inconsistency and political instability: Polity duration, 1800–2000. *American Journal of Political Science*, *50*, 893–908. https://doi.org/10.1111/j.1540-5907.2006.00222.x

Ghardallou, W., & Dorsaf, S. (2020). Democracy and economic growth: A literature review. *Journal of the Knowledge Economy*, *11*, 982–1002. https://doi.org/10.1007/s13132-019-00594-4

Gurgul, H., & Lach, L. (2013). Political instability and economic growth: Evidence from two decades of transition in CEE. *Communist and Post-Communist Studies*, *46*, 189–202. https://doi.org/10.1016/j.postcomstud.2013.03.008

Haggard, S., & Kaufman, R. R. (2016). *Dictators and democrats: Masses, elites, and regime change*. Princeton University Press, https://doi.org/10.1515/9781400882984

Haque, A., Santhirasegaram, S., & Younts, M. (2007). Sociopolitical instability and capital accumulation in developing countries: Cross country pooled data evidence. *Journal of Social Sciences*, *3*, 208–212.

Hassler, J., Storesletten, K., & Zilibotti, F. (2003). Dynamic political choice in macroeconomics. *Journal of the European Economic Association*, *1*, 543–552. https://doi.org/10.1162/154247603322391189

Hopenhayn, H., & Muniagurria, M. (1996). Policy variability and economic growth. *Review of Economic Studies*, *63*, 611–626. https://doi.org/10.2307/2297796

Jerri, A. (1985). *Introduction to integral equations with applications*. Dekker.

Jong-A-Pin, R. (2009). On the measurement of political instability and its impact on economic growth. *European Journal of Political Economy*, *25*, 15–29.

Lambert, P. (1993). *The distribution and redistribution of income: A mathematical treatment*. Manchester University Press.

Libman, A. (2012). Democracy and growth: Is the effect non-linear? *Economic Research Guardian*, *2*, 99–120.

Ma, T. C., & Ouyang, L. (2016). Democracy and growth: A perspective from democratic experience. *Economic Inquiry*, *54*(4), 1790–1804. https://doi.org/10.1111/ecin.12337

Madsen, J. B., Raschky, P. A., & Skali, A. (2015). Does democracy drive income in the world, 1500–2000? *European Economic Review*, *78*, 175–195. https://doi.org/10.1016/j.euroecorev.2015.05.005

Mantel, R. (1967). *Maximization of utility over time with a variable rate of time preference*. Cowles Foundation for Research in Economics, CF-70525 2.

Mantel, R. (1997). Optimal economic growth with recursive preferences: Decreasing rate of time preference. *Anales de la XXXII Reunión Anual de la Asociación Argentina de Economía Política*, *2*, 83–100.

Murtin, F., & Wacziarg, R. (2014). The Democratic transition. *Journal of Economic Growth*, *19*, 141–181. https://doi.org/10.1007/s10887-013-9100-6

Okafor, G. (2017). The impact of political instability on the economic growth of ECOWAS member countries. *Defence and Peace Economics*, *28*(2), 208–229. https://doi.org/10.1080/10242694.2015.1092206

Okazaki, T. (2007). Economic development, income inequality and social stability in prewar Japan: A prefecture-level analysis. *CIRFE Discussion Paper*, *500*.

Papaioannou, E., & Siourounis, G. (2008). Democratisation and growth. *Economic Journal*, *118*(532), 1520–1551. https://doi.org/10.1111/j.1468-0297.2008.02189.x

Persson, T., & Tabellini, G. (2000). *Political economics: Explaining economic policy*. MIT Press.

Plumper, T., & Martin, C. W. (2003). Democracy, government spending, and economic growth: A political-economic explanation of the Barro-effect. *Public Choice*, *117*(1–2), 27–50. https://doi.org/10.1023/A:1026112530744

Riedl, A. (1999). *Social (in)stability, distributive conflicts, and investment in poor and rich economies*. University of Amsterdam Working Paper.

Rizio, S., & Skali, A. (2020). How often do dictators have positive economic effects? Global evidence, 1858–2010. *The Leadership Quaterly*, *31*, 1–18. https://doi.org/10.1016/j.leaqua.2019.06.003

Rodrik, D., & Wacziarg, R. (2005). Do democratic transitions produce bad economic outcomes? *AEA Papers and Proceedings*, *95*, 50–55. https://doi.org/10.1257/000282805774670059

Roe, M., & Siegel, J. (2011). Political instability: Its effects on financial development, its roots in the severity of economic inequality. *Journal of Comparative Economics*, *39*, 279–309. https://doi.org/10.1016/j.jce.2011.02.001

Rothstein, P. (1991). Representative voter theorems. *Public Choice*, *72*, 193–212. https://doi.org/10.1007/BF00123744

Uzawa, H. (1968). Time preference, the consumption function and optimum asset holding. In J. Wolfe (Ed.), *Value, capital and growth: Papers in honor of Sir John Hicks*. University of Edinburgh Press.

Wright, J. (2008). Do authoritarian institutions constrain? How legislatures affect economic growth and investment. *American Journal of Political Science*, *52*, 322–343. https://doi.org/10.1111/j.1540-5907.2008.00315.x

Yaari, M. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, *32*, 137–150. https://doi.org/10.2307/2296058

How to cite this article: Tohmé, F., Caraballo, M. A., & Dabús, C. (2021). Instability, political regimes and economic growth. A theoretical framework. *Metroeconomica*, 00, 1–27. https://doi.org/10.1111/meca.12362

APPENDIX A

Proof of Proposition 1 We have to prove that $\prec_t$ is complete and transitive to show that it is a weak order:

- *completeness*: given two elements i and j, either $i \preceq_t j$ or $j \prec_t i$ since either $y_i(t) \leq y_j(t)$ or $y_j(t) < y_i(t)$ (since $<$ on $\mathcal{R}^1$ is complete).
- *transitivity*: given $i \prec_t j$ and $j \prec_t k$, it follows that $y_i(t) < y_j(t)$ and $y_j(t) < y_k(t)$. Therefore $y_i(t) < y_k(t)$, that is, $i \prec_t k$.

To prove that $\prec_t$ is continuous, we have to show that $Up(i) = \{j : i \prec_t j\}$ is an open set. This is equivalent to showing that $Up(y_i(t)) = \{y_j(t) : y_i < y_j(t)\}$ is open. This is true since $Up(y_i(t))$ is a left-open interval in $\mathcal{R}^1$.□

Proof of Lemma 1 If $y_j(0) = y_i(0)$ then both $i \preceq j$ and $j \preceq i$, that is, $i \sim j$, where $\sim$ is the derived equivalence relation for $\prec$. Since $\prec$ does not change over time, the equivalence classes remain constant and therefore their measures are time invariant. Therefore, for every i, $\mu(\{j : j \sim i\})$ remains constant through time.

Proof of Proposition 2 By definition, a_{dict} and a_{maj} are particular elements in $\mathscr{L}$. On the other hand, since $a_{\text{prop}} = \int_{\mathscr{L}} \pi(\phi^{-1}(\rho))\phi^{-1}(\rho)d\phi^{-1}(\rho)$, it is the mean of $\phi^{-1}(\rho)$. However, ϕ is a continuous bijection and therefore $a_{\text{prop}} = \phi^{-1}(Mean(\Psi))$. $Mean(\Psi) \in \Psi$ since Ψ is a closed interval. Therefore, $a_{\text{prop}} \in \mathscr{L}$.

Finally, since π is time invariant, a_{prop} must also be time invariant. Since a_{maj} is also, by definition, time invariant, a_{dict} must be (since both depend on a_{maj} and a_{prop}) constant through time. □

Proof of Proposition 3 From the strategic version of the median voter theorem with single-peaked preferences (Rothstein, 1991), it immediately follows that a_{maj} is not manipulable. In fact, since the u_i are strictly concave, they are single-peaked.

On the other hand, for a_{dict}, the only possible alternative outcome of the system is the minimal (maximal) agent in $\prec$ if a_{dict} is the maximal (minimal) agent in that ordering. However, in order to induce this reversion, there must be a corresponding reversion in the skewness of the distribution. Hence, without any loss of generality, if a_{dict} is the maximal in $\langle \mathscr{L}, \prec \rangle$, consider $\mathscr{M} = \{i : i \preceq a_{\text{dict}} \textit{ and } u_i(c_{\text{dict}}) < u_i(c_{min})\}$, where c_{dict} and c_{min} are the steady states of a_{dict} and the minimal of $\prec$, respectively. Each $i \in \mathscr{M}$ may declare $i' = Maximal(\prec)$. A distribution $\pi_{\mathscr{M}}$ obtains, with more than half of the agents on $Maximal(\prec)$, since all the agents $i \preceq Median(\prec)$ will belong to $\mathscr{M}$. Therefore, $Mean(\pi_{\mathscr{M}}) \leq Median(\prec)$ and hence, $a^{\mathscr{M}}_{\text{dict}}$, the dictator chosen under $\pi_{\mathscr{M}}$, is $Minimal(\prec)$.

The same is true for a_{prop}: if, say, a position i is to the left of a_{prop}, then the corresponding agents may declare a position more to their left, thereby changing the average towards a lower value, closer to their optimum. □

Proof of Proposition 4 The solution of the Ramsey problem yields a unique monotonic growth path for a given feasible rate of time preference. Two paths, $\{c_1(t)\}_{t=0}^{\infty}$ and $\{c_2(t)\}_{t=0}^{\infty}$, each corresponding to a different time preference, say $\rho_1 > \rho_2$, are such that $c_1(0) > c_2(0)$ while their steady states verify $c_1^* < c_2^*$. Consider two cases: (i) that the initial endowment of capital $k_0 < k_1^*, k_2^*$; and (ii) that $k_1^* \leq k_0 < k_2^*$, where $k_1^* < k_2^*$ are the corresponding steady-state values of capital.

In case (i), the paths will both be monotonically increasing, but with a higher long-term limit for c_2. Thus, for each time t, until $\bar{t}$ the trajectory of c_1 will be below that of c_2. On the other hand, in case (ii), the time path of c_1 is decreasing while that of c_2 is increasing. Thus, for any $t < \bar{t} c_1(t) < c_2(t)$, the optimal solution could not have been c_1, which is a contradiction. Therefore, $c_1(t) > c_2(t)$ for all $t < \bar{t}$. □

Proof of Theorem 1

- *continuity*: Given an $\epsilon \approx 0$ and two rates of time preference $\rho_1, \rho_2 \in \Psi$, such that $|\phi_{\bar{t}}(\rho_1) - \phi_{\bar{t}}(\rho_2)| < \epsilon$, it is clear that the corresponding growth paths are close. In fact, since in steady state $f'(k_i^*) = \phi_{\bar{t}}(\rho_1)$ and $f'(k_{ii}^*) = \phi_{\bar{t}}(\rho_2)$, $|f'(k_i^*) - f'(k_{ii}^*)| < \epsilon$. Therefore, as f' is a continuous function, $k_i^* \approx k_{ii}^*$, and consequently $c_i^* \approx c_{ii}^*$ and $y_i^* \approx y_{ii}^*$. Moreover, $c_i(t) \approx c_{ii}(t)$ for any $t > 0$. Since the corresponding paths for ρ_1 and ρ_2 are such that $c_i(\bar{t}) = c_1(\bar{t})$ and $c_{ii}(\bar{t}) = c_2(\bar{t})$, by transitivity it follows that $c_1(t) \approx c_2(t)$, that is, $\rho_1 \approx \rho_2$. More precisely, there exists a small δ such that $|\rho_1 - \rho_2| < \delta$.
- *differentiability*: Suppose that $\phi_{\bar{t}}$ is not differentiable. Therefore, an $\epsilon > 0$ must exist such that for every $r \in \mathcal{R}$ and for all $\delta > 0$ it must be true that, for any pair $\rho_1, \rho_2 \in int(\Psi)$, $|\rho_1 - \rho_2| < \delta$ and $|\frac{\phi_{\bar{t}}(\rho_1) - \phi_{\bar{t}}(\rho_2)}{\rho_1 - \rho_2} - r| > \epsilon$. In other words, no matter how close ρ_1 is to ρ_2, $\frac{\phi_{\bar{t}}(\rho_1) - \phi_{\bar{t}}(\rho_2)}{\rho_1 - \rho_2}$ is beyond any bound. However, if ρ_1 is close to ρ_2, then in steady state (since $f' = \rho$) $f'(k_1^*), f'(k_2^*)$ are also close. Moreover, by continuity, $f'(k_i^*)$ and $f'(k_{ii}^*)$ are also close.

Finally, $\frac{f'(k_i^*)-f'(k_{ii}^*)}{f'(k_1^*)-f'(k_2^*)}$ is bounded, since f' is differentiable. This is a contradiction and therefore $\phi_{\bar{t}}$ is differentiable.

- *first-order condition*: by definition $\phi_{\bar{t}}(\rho) \geq \rho$. Therefore, $\phi_{\bar{t}}$ is monotonically increasing. Since it is also differentiable, it verifies that $\frac{d\phi_{\bar{t}}}{d\rho} \geq 0$.
- *second-order condition*: first of all, the derivative of a monotonically increasing continuous function is also continuous and differentiable. Moreover, it is can be either a constant, a monotonically increasing or a monotonically decreasing function. We want to show that $\frac{d\phi_{\bar{t}}}{d\rho}$ is monotonically decreasing. Suppose that it is not. This means that

$$\frac{d\phi_{\bar{t}}(\rho_1)}{d\rho} \leq \frac{d\phi_{\bar{t}}(\rho_2)}{d\rho}$$

for $\rho_1 < \rho_2$. That is, for an arbitrarily small $\Delta\rho > 0$,

$$\frac{\phi_{\bar{t}}(\rho_1+\Delta\rho)-\phi_{\bar{t}}(\rho_1)}{\Delta\rho} \leq \frac{\phi_{\bar{t}}(\rho_2+\Delta\rho)-\phi_{\bar{t}}(\rho_2)}{\Delta\rho}$$

which is equivalent to

$$\phi_{\bar{t}}(\rho_1') - \phi_{\bar{t}}(\rho_1) \leq \phi_{\bar{t}}(\rho_2') - \phi_{\bar{t}}(\rho_2)$$

where $\rho_j' = \rho_i + \Delta\rho$ for $j = 1, 2$, and therefore $\rho_j' > \rho_j$. Since $\phi_{\bar{t}}$ is monotonic, we have that $\rho_i' = \phi_{\bar{t}}(\rho_1') > \phi_{\bar{t}}(\rho_1) = \rho_i$ and $\rho_{ii}' = \phi_{\bar{t}}(\rho_2') > \phi_{\bar{t}}(\rho_2) = \rho_{ii}$. In consequence

$$\rho_i' - \rho_i \leq \rho_{ii}' - \rho_{ii}$$

This implies, in steady state,

$$f'(k_i'^*) - f'(k_i^*) \leq f'(k_{ii}'^*) - f'(k_{ii}^*)$$

where $k_i'^* < k_i^*$ and $k_{ii}'^* < k_{ii}^*$. However, f' is a decreasing function. Therefore,

$$f'(k_i'^*) - f'(k_i^*) > f'(k_{ii}'^*) - f'(k_{ii}^*)$$

This is absurd and thus $\frac{d^2\phi_{\bar{t}}}{d\rho^2} \geq 0$. □

Proof of Proposition 5 Since $\mathcal{M}_{\text{maj}}$ is the smallest of the opposing coalitions, Ω_{maj} always first-order stochastically dominates Ω^l_{prop}, Ω^s_{prop}, Ω^l_{dict} and Ω^s_{dict}. On the other hand, since $\mathcal{M}^l_{\text{dict}}$ is the largest coalition, it is first-order dominated by all the other distributions. Accordingly, the shortened time horizons lie in the interval $[\bar{t}^l_{\text{dict}}, \bar{t}_{\text{maj}}]$. The three cases depend on the relative sizes of $\mathcal{M}^l_{\text{prop}}$, $\mathcal{M}^s_{\text{prop}}$ and $\mathcal{M}^s_{\text{dict}}$. □

Proof of Proposition 6 $\{c''(t)\}_{t=0}^{\infty}$ verifies that $c''(t) \geq c^*(t)$ for every $t, 0 \leq t \leq \bar{t}'$, $c''(\bar{t}') = c^*(\bar{t}')$, and $c''(\bar{t}) < c^*(\bar{t}')$ for every $t > \bar{t}'$. Comparing the new path to the path obtained from ρ_a', $\{c'(t)\}_{t=0}^{\infty}$, it follows that $c''(t) \leq c^*(t) \leq c'(t)$ for $\bar{t}' \leq t \leq \bar{t}$, and particularly that $c''(\bar{t}) < c^*(\bar{t}) = c'(\bar{t})$. Since the paths are monotonic, $c''(t) < c'$ for $t > \bar{t}'$. Therefore, the steady states are such that $c''^* < c'^*$. □

Proof of Proposition 7 Immediate from Propositions 6 and 7.□

APPENDIX B

Example Consider an economy in which agents may have three different discount rates: $\rho_1 = 0.9$, $\rho_2 = 0.6$ and $\rho_3 = 0.1$. The proportions of the three types are as follows: $\pi_1 = 0.3$, $\pi_2 = 0.5$, and $\pi_3 = 0.2$. Suppose that each agent has the same production function: $f(k) = \ln(k+1)$. Therefore, the steady states for each one are given, in absence of a political system, by: $f'(\overline{k}_i) = \rho_i$, that is, $\overline{k}_1 = \frac{1}{9}$, $\overline{k}_2 = \frac{2}{3}$ and $\overline{k}_3 = 9$, respectively. If we assume that the initial amounts of capital are $k_1^0 = 0.1$, $k_2^0 = 0.5$ and $k_3^0 = 5$, then this means that the growth paths are monotonically increasing. Now suppose that the most patient agent is selected. According to (5):

$$f'\left(0.3\frac{1-\tau_1}{0.9}+0.5\frac{1-\tau_2}{0.6}+0.2\frac{1-\tau_3}{0.1}-1\right)=0.1$$

Working out this condition we end up with the following equation:

$$41 = -2\tau_1 - 5\tau_2 - 12\tau_3$$

which admits (among others) $\tau_1 = 1$, $\tau_2 = 1$ and $\tau_3 = -4$ as solutions. In this case, the poorest subsidise the richest agents in society. Even when with these rates the ordering of agents according to their discount rates remains constant, the initial amounts of capital and the proportions of agents make this policy unsustainable. In fact, the resources extracted from the poorest are not enough to compensate for the large subsidy granted to the rich. On the other hand, the adjusted steady-state values of k (if they were attainable) are negative for the agents with the highest discount rates, while enormously higher (49 instead of 9) for the most patient.

If, instead, a median agent is selected as the executive (i.e., one with discount rate 0.6), then condition (5) becomes:

$$f'\left(0.3\frac{1-\tau_1}{0.9}+0.5\frac{1-\tau_2}{0.6}+0.2\frac{1-\tau_3}{0.1}-1\right)=0.6$$

or

$$-9 = -2\tau_1 - 5\tau_2 - 12\tau_3$$

which admits a solution $\tau_1 = -0.1$, $\tau_2 = -0.01$, and $\tau_3 = 0.77$, which involves a progressive distribution that is sustainable in time (albeit wasteful of resources).

APPENDIX C

The solution of the optimisation problem of the representative agent yields two differential equations, $\dot{c} = 0$ and $\dot{k} = 0$. In Figure C1, it can be observed how these equations define a dynamical system guiding the economy:

Consider first the case of two agents, with time preference rates ρ_1 and ρ_2, such that $\rho_1 > \rho_2$. This means that 1 is more patient than 2. It can be observed that in the case of 1, the geometrical representation of $\dot{c}_1 = 0$ in the phase diagram, (k, c) is the vertical line consisting of all the points (c_1, k_1^*) where k_1^* is such that $f'(k_1^*) = \rho_1$. In turn, $\dot{k}_1 = 0$ corresponds to the function $c_1 = f(k_1)$. At (k_1^*, c_1^*) (point C in the graphical representation), the two conditions are satisfied, meaning that once reached, there will be no further changes in the values of consumption and capital. In other words, (k_1^*, c_1^*) is the *steady state* corresponding to the preferences of 1.

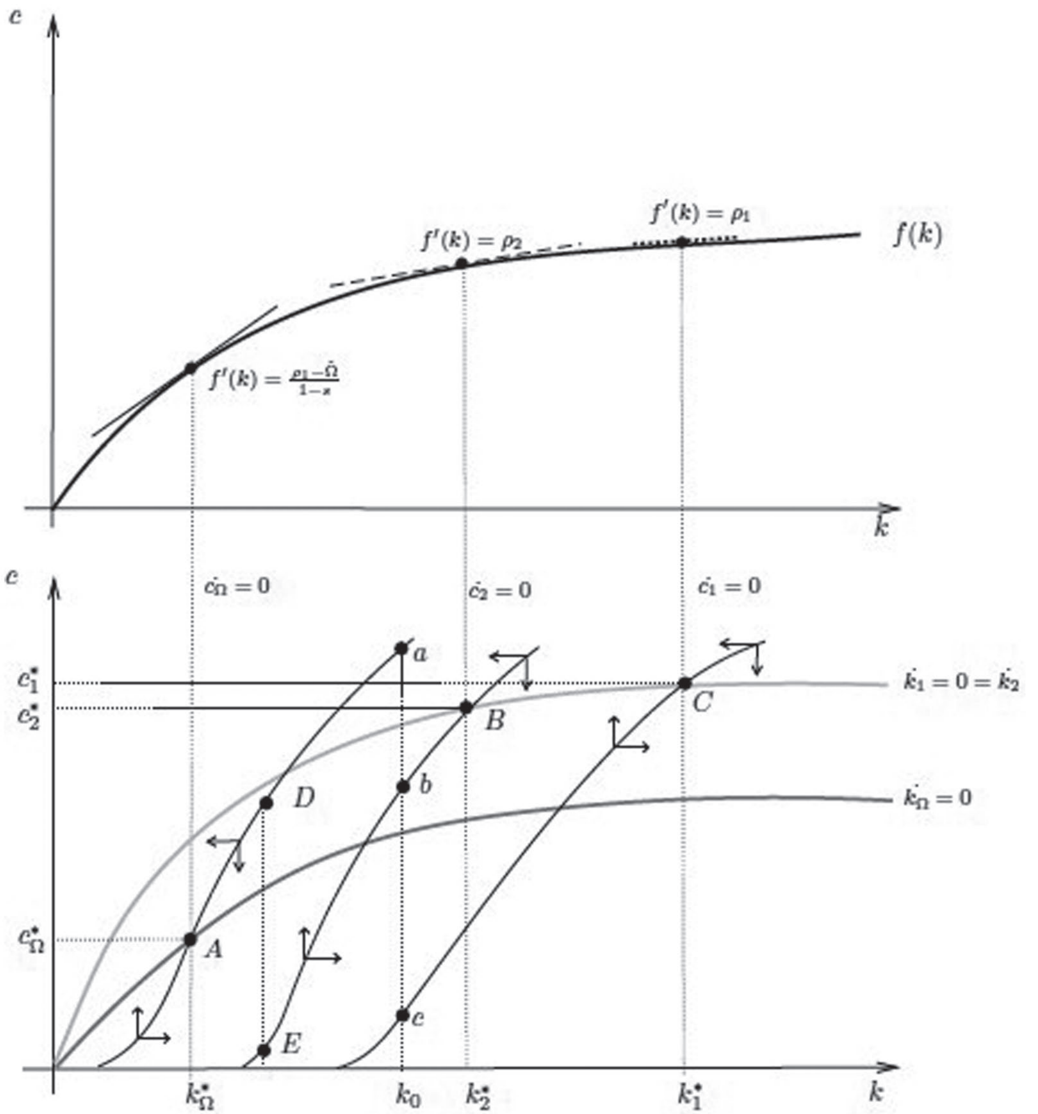

FIGURE C1 The dynamics of the economy with and without instability

It is easy to check that for $k_1 < k_1^*, \dot{c}_1 > 0$, while for $k_1 > k_1^*, \dot{c}_1 < 0$. In turn, for $c_1 < f(k_1), \dot{k}_1 > 0$ and for $c_1 > f(k_1), \dot{k}_1 < 0$. This means that $\dot{c}_1 = 0, \dot{k}_1 = 0$ divide the phase space into four regions, but only in the south-west and north-east regions do the dynamics tend *towards* the steady state, while in the other two regions the dynamics tend *away* from it. This creates the condition for a *saddle-point path*, that is, a curve in the phase space, that passes through the steady state in which if the system starts at one of its points, it will lead towards the steady state. In Figure C1, the point c, corresponding to the initial amount of capital k_0 leads to a monotonical process of increasing both consumption and capital, which leads, in the long run, to the steady state C.

Likewise for 2, it can be graphically observed that, starting at point b, which again corresponds to k_0 on the saddle point path, the system increases both c_2 and k_2 towards the steady state (k_2^*, c_2^*) (B in the phase space). It can be observed that both $c_2^* < c_1^*$ and $k_2^* < k_1^*$ correspond to the higher impatience of 2.

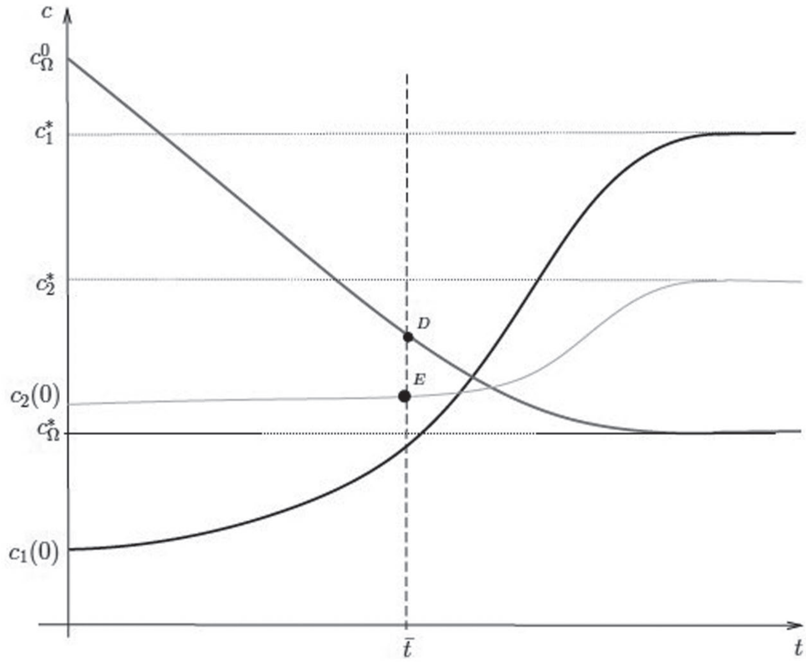

FIGURE C2 Time paths of consumption levels

Now let us assume that 1 is the incumbent, while 2 might be the replacement if 1 is ousted. In this scenario, 1 estimates a time in which this might happen with the ensuing decreasing probability function $\Omega(t)$ of the probability of staying in office. The ruler also diverts resources $(sf(k_1(t)))$ in order to repress opponents. This means that the ruler readjusts the time preference to $\frac{\rho_1 - \hat{\Omega}}{1-s}$. Let c_Ω and k_Ω denote the resulting consumption and capital paths. $\dot{c}_\Omega = 0$ is therefore obtained at the points (k_Ω^*, c_Ω) where k_Ω^* is such that $f'(k_\Omega^*) = \frac{\rho_1 - \hat{\Omega}}{1-s}$, while $\dot{k}_\Omega = 0$ corresponds to the function $c_\Omega = (1-s)f(k_\Omega)$. The new steady state is (k_Ω^*, c_Ω^*) given as A in Figure C1.

It can be observed that the steady state involves values not only lower than those corresponding to the actual preferences of 1, but also lower than those of the possible replacement, 2. Moreover, the saddle-point path starting at point a (with $k_\Omega(0) = k_0$ now leads to a *decreasing* dynamical trajectory towards the steady state. Nevertheless, up to the moment in which 1 leaves office (at point D of that trajectory) the amount of consumption surpasses that in the saddle-point path under the true preferences of 1.

When 1 is overthrown and is replaced by 2, the new incumbent has to adjust the trajectory of the economy. Since the economy is closed and thus it is impossible to borrow, the only possibility is to drastically reduce consumption to put the system in the saddle-point path towards (k_2^*, c_2^*). In Figure C1, this means jumping from D to E. Subsequently, the economy starts to grow again. Figure C2 shows the trajectories of consumption starting from $c_1(0)$, $c_2(0)$, and $c_\Omega(0)$ corresponding to e, b, and a in Figure C1. It can be observed that the consumption values of c_Ω are higher than those of c_2 and c_1 at all periods $t \leq \bar{t}$. At $\bar{t}$, the change of incumbent implies that $c(\bar{t})$ is reduced from $c_\Omega(\bar{t})$ to $c_2(\bar{t})$, that is, from D to E.

Similar analyses can be carried out by assuming different orderings of ρ_1, ρ_2 and $\frac{\rho_1 - \hat{\Omega}}{1-s}$. While there exist many possible configurations of trajectories towards steady states, the main features analysed here remain valid.