Flexibility Value of Distributed Generation in Transmission Expansion Planning

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1. Introduction

The efficiency of the classic planning methods for solving realistic problems largely relies on an accurate prediction of the future. Nevertheless, the presence of strategic uncertainties in current electricity markets has made prediction and even forecasting essentially futile. The new paradigm of decision-making involves two major deviations from the conventional planning approach. On one hand, the acceptation the fact the future is almost unpredictable. On the other hand, the application of solid risk management techniques turns to be indispensable.

In this chapter, a decision-making framework that properly handles strategic uncertainties is proposed and numerically illustrated for solving a realistic transmission expansion planning problem.

The key concept proposed in this chapter lies in systematically incorporating flexible options such as large investments postponement and investing in Distributed Generation, in foresight of possible undesired events that strategic uncertainties might unfold. Until now, the consideration of such flexible options has remained largely unexplored. The understanding of the readers is enhanced by means of applying the proposed framework in a numerical mining firm expansion capacity planning problem. The obtained results show that the proposed framework is able to find solutions with noticeably lower involved risks than those resulting from traditional expansion plans.

The remaining of this chapter is organized as follows. Section 2 is devoted to describe the main features of the transmission expansion problem and the opportunities for incorporating flexibility in transmission investments for managing long-term planning risks. The most salient characteristics of the several formulations proposed in the literature for solving the optimization problem are reviewed and discussed along Section 3. The several types of uncertain information that must be handled within the optimization problem are classified and analyzed in Section 4. The proposed framework for solving the stochastic optimization problem considering the value provided to expansion plans by flexible investment projects is presented in Section 5. In Section 6, an illustrative-numerical example based on an actual planning problem illustrates the applicability of the developed flexibility-based planning approach. Concluding remarks of Section 7 close this chapter.

2. The transmission expansion planning

Since the beginning of the power industry, steadily growing demand for electricity and generation commonly located distant from consumption centres have led to the need of planning for adapted transmission networks aiming at transport the electric energy from production sites towards consumption areas in an efficient manner. In the vertically integrated power industry, the responsibility for optimally driving the expansion of transmission networks has typically lied with a centralized planner.

During the last two decades, stimulating competition has been a way to increase the efficiency of utilities as well as to improve the overall performance of the liberalized electricity industry (Rudnick & Zolezzi, 2000; Gómez Expósito). Because of the large economies of scales, a unique transmission company is typically responsible for delivering the power generation to the load points. Under this paradigm, the transmission activity has special significance since it allows competition among market participants. In addition, the transmission infrastructure largely determines the economy and the reliability level that the power system can achieve. For this reasons, planning for efficient transmission expansions is a critical activity. With the aim of solving the transmission expansion planning problem (TEP), a great number of approaches have been devised (Latorre *et al.*, 2003; Lee *et al.*, 2006).

A classic TEP task entails determining *ex-ante* the location, capacity, and timing of transmission expansion projects in order to deliver maximal social welfare over the planning period while maintaining adequate reliability levels (Willis, 1997). Under this traditional perspective, the TEP problem can be mathematically formulated as a large scale, multi-period, non-linear, mixed-integer and constrained optimization problem. In practice, however, such a rigorous formulation is unfeasible to be solved. Planners typically solve the TEP problem under a very simplified framework, e.g. static (one-stage) formulations, where timing of decisions is not a decision variable (Latorre *et al.*, 2003).

2.1 The emerging new TEP problem

The improvement of computing technology with increasingly faster processors along with the option of solving the problem in a distributed computing environment has made possible to handle a bigger number of parameters and variables and even formulate the TEP as a multi-period optimization problem (Youssef, 2001; Braga & Saraiva, 2005). However, jointly with the above mentioned increasing competition brought by the deregulation, relevant aspects such as: the development of new small-scale generation technologies (Distributed Generation, DG), the improvement of power electronic devices (e.g. FACTS), the environmental concerns that makes more difficult to obtain new right-of-way for transmission lines, the lack of regulatory incentives to investing in transmission projects, among others, have increased considerably the dynamic of power markets, the number of variables and parameters to be considered, and the uncertainties involved. Accordingly, the TEP problem is now substantially more complex (Buygi, 2004; Neimane, 2001).

Under this perspective, *ad doc* adjustments of expansion plans or additional contingent investments made in order to mitigate the harmful economic consequences that unexpected events have demonstrated the limited practical efficiency of applying classic TEP models (Añó *et al.,* 2005). In fact, the substantial risks involved in planning decisions emphasize the need of developing practical methodological tools which allow for the assessment and the risk management.

2.2 Nature of transmission investments

Due to some singular characteristics, transmission investments exhibit a distinctive nature with respect to other related investment problems (Kirschen & Strbac, 2004; Dixit & Pindyck, 1994):

Capital intensive: because of the substantial economies of scale, large and infrequent transmission investments are often preferred, involving huge financial commitments.

One-step investments: a substantial fraction of total capital expenditures must be committed before the new transmission equipment can be commissioned.

Long recovering times: transmission lines, transformers, etc. are expected to be paid-off after several years or even decades.

Long-run uncertainties: transmission investments are vulnerable to unanticipated scenarios that can take place in the long-term future. Future demand, fuel costs, and generation investments are uncertain variables at the planning stage.

Low adaptability: transmission projects are typically unable to be adapted to circumstances that considerably differ from the planning conditions. An unadapted transmission system entails considerable loss of social welfare.

Irreversibility: once incurred, transmission investments are considered sunk costs. Indeed, it is very unlikely that transmission equipment can serve other purposes if conditions changes unfavourably. Under these circumstances the transmission equipment could not be sold off without assuming significant losses on its nominal value.

Postponability: In general, opportunities for investing in transmission equipment are not of the type "now or never". Thus, it is valuable to leave the investment option open, i.e. wait for valuable, arriving information until uncertainties are partially resolved. Thus, transmission investment projects can be treated in the same way as a financial call option. The opportunity cost of losing the ability to defer a decision while looking for better information should be properly considered.

Due to the mentioned features, transmission network expansions traditionally respond to the demand growth by infrequently investing in large and efficient projects. Consequently, traditional solutions to the TEP inevitably entail two evident intrinsic weaknesses:

- Because only large projects are economically efficient, planners have a limited number of alternatives and consequently the solutions found provide low levels of adaptability to the demand growth, and
- To drive the expansion, enormous irreversible upfront efforts in capital and time are required.

The huge uncertainties of the problem interact with the irreversible nature of transmission investments for radically increasing the risk present in expansion decisions. Such interaction has been ignored in traditional models at the moment of evaluating expansion strategies. More recently, it has been recognized that conventional decision-making approaches usually leads to the wrong investment decisions (Dixit & Pindyck, 1994). Therefore, the interaction between uncertainties and the nature of transmission investments must be properly accounted for.

2.3 New available flexible options

Although the major negative concerns regarding classic TEP models have been analyzed, in this work potential positive aspects are also considered and exploited. In fact, available *technical and managerial embedded options* exhibit some desirable features such as: modularity, scalability, short lead times, high levels of reversibility, and smaller financial commitments. This option can be incorporated as novel decision choices that a planner has available for reducing the planning risks as well as for improving the quality of the found solutions.

In this sense, planners must rely on an expansion model able to capture all major complexities present in the TEP in order to properly manage the involved huge long-term uncertainties and deal with the problem of dimensionality.

The key underlying assumption of conventional probabilistic models is the passive planner's attitude regarding future unexpected circumstances. In fact, available choices for reacting to the several scenarios which could take place overtime are ignored during the planning process. However, in practice planners have the ability to adapt their investment strategies in response to undesired or unanticipated events.

Hence, planning for contingent scenarios by exploiting technical and managerial options embedded in transmission investment projects is a effective mean for satisfactorily dealing with the current TEP problem

2.4 The flexibility value of Distributed Generation

Distributed Generation is defined as a source of electric energy located very close to the demand (Ackerman *et al.*, 2001; Pepermans *et al.*, 2003). Usually, DG investments are neither more efficient nor more economic than conventional generation or transmission expansions, which still enjoy of significant economies of scale such. Nevertheless, important contributions of DG occur when: energy T&D costs are avoided, demand uses it for peak shaving, losses are reduced, network reliability is increased, or when it lead to investment deferral in T&D systems (Jenkins *et al.*, 2000; Willis & Scott, 2000; Brown *et al.*, 2001; Grijalva & Visnesky, 2005).

DG seems a plausible means of improving the traditional way of driving the expansion of the transmission systems. Delaying investments in T&D systems by investing in DG is one of the major motivations and research topics of this work (Brown *et al.*, 2001; Daly & Morrison, 2001; Vignolo & Zeballos, 2001; Dale, 2002; Vásquez & Olsina, 2007).

The fact of considering DG projects as new decision alternatives within the TEP, involves the incorporation of additional parameters such as investment and production costs of DG technologies, firm power, etc.

Based on the typical short lead times of DG projects and their lower irreversibility, the uncertainty present in DG project investment decisions and investment costs can be neglected. Provided that the DG technologies considered in this work are fuel-fired plants, the availability of the DG could be modelled by assessing only availability factors (Samper & Vargas, 2006).

3. State-of-art of the TEP optimization approaches

The successful development of an efficient and practical expansion model primarily depends on considering the following topics: the planner's objectives, the availability and quality of the information to be handled as well as the depth level at which the planner

decides to face the problem. In this sense, a set of basic elements that the planner must consider and specify before mathematically formulating the problem are summarized in the Table 1.

Topic	Concern	Recommended Value	Symbol
Scales of time		Planning horizon	10 to 15 years	T
		Decision periods	≥ 1 year	p
		Sub-periods resolution	Weekly, monthly, seasonally	$subp$
		Demand duration curve	Peak, valley, mid-load	$P(t)$, $Q(t)$
Decision alternatives	Alternatives that planner has available for driving the expansion	Expansion strategy	S_k, S_f
		Large transmission projects	$D_k(p)$
		Defer transmission projects	$O_k(p)$
		Invest in DG projects	
		Type of alternative	$[0,1,2,3..n]$
		Investment decision timing	p
		Decision alternative location	$\bar{f}(bus)$
Objective function (C_k) components	Efficiency in investments, operative efficiency, reliability and technical feasibility	Investment costs	C_I, C_{IDG}
		Operative costs	C_G, C_{GDG}
		O&M costs	$C_{O\&M}$
		VOLL or EENS costs	C_{LOL}
		Active power losses costs	-
Constraints	Transmission expansion plans performance assessment subject to:	Power balance	$S_G + S_D = S_L$
		Voltage limits	$V_{j\,min}$, $V_{j\,max}$
		Generators capacity limits	$P_{i\,min}$, $P_{i\,max}$
		DG plants capacity limits	$DG_{i\,min}$, $DG_{i\,max}$
		Transmission lines power flow limits	F_l
		Budgetary constraints	-
Input parameters	Certain	Certain	$S(t)$
	Uncertain	Random	$X(t)$
		Truly uncertain	
		Fuzzy	-

Table 1. Basic elements to be defined before devising a TEP methodology

The current TEP problem can be described as the constant planners' dilemma of deciding on a sequential combination of large transmission projects and new available flexible options, which allows the planners to efficiently adapting their decisions to unexpected circumstances that may take place during the planning period.

Under this novel paradigm, TEP is a multi-period decision-making problem which entails determining *ex-ante* the right type, location, capacity, and timing of a set of available decision options in order to deliver a maximal expected social welfare as well as suitably reducing the existing risks over the planning period.

Probabilistic decision theory, i.e. the probabilistic choice paradigm, is well-known and has been extensively applied in several stochastic optimization problems. However, a probabilistic decision formulation within the TEP is an intractable task and its application

has only been feasible when very strong simplifications are adopted by planners (Neimane, 2001). This work proposes a practical framework for treating the TEP. Even though a number of simplifications are still necessary, the main features of the new TEP problem are retained.

The analysis of the state-of-art of the TEP solutions approaches sets as a start point the classic stochastic optimization problem formulation. Under the assumption of inelastic demand behaviour, the optimization problem can be rigorously stated as follows:

$$\underset{S_{opt} \in \overline{S}_f}{opt} \left\{ E[OF|_S] \right\} = \underset{S_{opt} \in \overline{S}_f}{opt} \left\{ \int_0^T \overbrace{\int \ldots \int_\Omega}^{i} OF(C) dF(C) \right\} \tag{1}$$

where, the performance measure of the optimization is the expected present value of the objective function E[*OF(C)*] evaluated over a planning horizon *T*, for a proposed expansion strategy *S*. $\overline{S}_f$ is the set of all feasible states of the network, *F(C)* is the distribution function of the expansion costs function $C(C_1, C_2, C_3, ..., C_i)$. The planning period *T* usually only can take discrete values $t_0, t_1, t_2, t_3, ..., t_p$, and Ω is the domain of existence of *C(X,S)*. The expansion costs function depends on several uncertain input parameters $X(x_1(t), x_2(t), x_3(t), ..., x_n(t))$ which change over the time, as well as depending on the state of the network, which also varies over the time $S(s_1(t), s_2(t), s_3(t), ..., s_d(t))$. It is important to note that the problem is subject to a set of constraints, namely Kirchhoff's laws, upper and lower generation plants capacity limits, transmission lines capacity limits, upper and lower voltage and phase nodes limits, and budgetary constraints, among others, which are represented by means of equality and inequality equations. With these considerations, (1) can be rewritten as follows:

$$\underset{S_{opt} \in \overline{S}_f}{opt} \left\{ E[OF] \right\} = \underset{S_{opt} \in \overline{S}_f}{opt} \left\{ \overbrace{\int \ldots \int_\Psi}^{n.(p+1)} OF[C(X,S)] d\Phi(X) \right\} \tag{2}$$

subject to:

$$P_A(X,S) + P_B(X,S) = P_L(X,S)$$

$$b_{1\min} \leq g_1(X,S) \leq b_{2\max}$$

$$b_{1\min} \leq g_2(X,S) \leq b_{2\max}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$b_{m\min} \leq g_m(X,S) \leq b_{m\max}$$

where $\Phi(X)$ is the $n(p+1)$-dimensional function of probability distributions of input parameters and Ψ is the domain of existence of the input parameters *X*.

Formulating $\Phi(X)$, which incorporates the information about the uncertainties that largely influence the solution, is a complex task as it involves determining probabilities and distribution functions of $n(p+1)$ uncertain parameters. However, the more difficult (and in some cases impossible) task is the formulation of the objective function *OF(C)*. In this sense

the most common simplification considered by TEP models is $OF[C(X,S)] = C(X,S)$ and (2) can be rewritten as:

$$\min_{S_{opt} \in S_f} \left\{ E[C|_S] \right\} = \min_{S_{opt} \in S_f} \left[\overbrace{\int \cdots \int_{\Psi}}^{n.(p+1)} C(X,S) \, d\Phi(X) \right] \tag{3}$$

which implies that the objective function can be entirely described by the expansion costs function. In this case the planning problem is often reduced to the minimization of the expected total expansion costs. Although the complexity of the problem is greatly reduced, such a formulation does not take into account desires of the decision-maker for reducing risks present in the expansion decisions. Eventually, this risk neutral formulation may lead to wrong decisions.

On the other hand, considerable difficulties are related to the computational effort necessary for efficiently assess the multidimensional integral and for proposing the corresponding optimization procedure. The only method for dealing with (3) as strict as possible, given that the $n(p+1)$-dimensional integral must be solved, is applying Monte-Carlo simulation techniques for evaluating the attributes of the objective function.

There are $(n+d)(p+1)$ input parameters in the expansion costs function $C(x_{1,0}, \ldots x_{n,0}, \ldots, x_{n,p}, \ldots s_{1,0}, \ldots s_{d,p})$, from which $d(p+1)$ are decision variables. Assuming as I the number of available decision choices in each possible right-of-way d, the number of possible candidate solutions are $I^{d(p+1)}$. Additionally, by denoting as N the number of simulations that requires the Monte Carlo simulation, the number of simulations to be performed depends on the number of periods of time as $N(p+1)$. It is important to mention that N depends on the degree of confidence that the planner demands on the results. Under these considerations, the number of required computations for rigorously evaluating the multidimensional integral and therefore for finding the global optimum is $N(p+1)I^{d(p+1)}$. Unfortunately performing this task in a real multi-period TEP is not possible since the number of simulations dramatically increases with the result of multiplying the possible links and the time periods $d(p+1)$. Due to this fact, researchers have proposed diverse approaches in order to make the TEP feasible and, in some cases, to incorporate the desires of the decision-maker for reducing the planning risks. According to the reviewed literature such simplifications can be categorized as static, deterministic and non-deterministic formulations of the TEP.

3.1 Static formulation

When the planner demands on further simplifying a deterministic formulation, the intertemporal dependences and the dynamic nature of the TEP problem is not considered. Such a formulation is named static. This is a deterministic formulation that entails finding the optimal state of the network for a future fixed year. Consequently, the input parameters X do not change during the whole solving process. In this case, there are $n+d$ input parameters within the expansion costs function $C(x_1, x_2, \ldots, x_n, \ldots s_1, \ldots s_d)$ from which d are

decision variables. Assuming as I the number of available decision choices in each possible right-of-way d, the number of possible solutions is I^d. For instance, in a small TEP problem with $d = 11$ and five decision choices on each right-of-way $I = 5$, the number of possible combinations is $5^{11} = 4.88 \cdot 10^7$.

3.2 Deterministic formulation

Deterministic models are nowadays widely used in practice for transmission network planning. This type of models assumes that all the input parameters and variables are known with complete certainty and, therefore, there is a unique and known scenario for the evolution of all input parameters. Consequently, there is no need to use probability distribution functions and the complexity of the optimization process is greatly reduced. Thus, deterministic formulation entails finding the optimal state of the network over a planning horizon T, given that the evolution of X along the time is known with certainty.

There are $(n+d)(p+1)$ input parameters inside the expansion costs function $C(x_{1,0},...x_{n,0},...,x_{n,p},...s_{1,0},...s_{d,p})$ from which $d(p+1)$ are decision variables. Assuming as I the number of available decision choices in each possible right-of-way d, the number of possible solutions to be evaluated for finding the global optimum is $I^{d(p+1)}$. For instance, in a small TEP problem with eleven possible new right-of-ways $d = 11$, five decision choices in each right-of-way $I = 5$, and only two decision periods $p+1 = 2$, the number of possible combinations are $5^{11 \cdot (1+1)} = 2.38 \cdot 10^{15}$.

In this work, the subject of optimization is the present value of the total expansion costs function $C(X,S)$, evaluated along a planning horizon T, for a proposed expansion strategy S. $C(X,S)$ is a non-linear function subject to a set of constraints, i.e. Kirchhoff's laws, generation plants capacity limits and transfer capacity of transmission lines, among others. Such constraints are represented by means of equality and inequality equations.

$$\min_{S_{opt} \in \overline{S}_f} \left\{ C|_S \right\} = \min_{S_{opt} \in \overline{S}_f} \left\{ C(X,S) \right\} \tag{4}$$

$$C(X,S) = \sum_{t=0}^{T} \left[\frac{C_I(X,S)}{(1+r)^t} + \frac{C_{Gen}(X,S)}{(1+r)^t} + \frac{C_{O\&M}(X,S)}{(1+r)^t} + \frac{C_{LoL}(X,S)}{(1+r)^t} \right] \tag{5}$$

subject to:

$$P_A(X,S) + P_B(X,S) + ... + P_R(X,S) = P_L(X,S)$$
$$b_{1\min} \le g_1(X,S) \le b_{2\max}$$
$$b_{1\min} \le g_2(X,S) \le b_{2\max}$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$b_{m\min} \le g_m(X,S) \le b_{m\max}$$

where

$C_I(X,S)$:   Investment costs of the new expansion decisions.

$C_{Gen}(X,S)$:   Production costs of the different generations units.

$C_{O\&M}(X,S)$: Annual O&M costs of the transmission network elements.

$C_{LoL}(X,S)$: Loss of load annual costs.

r : Annual discount rate.

3.3 Non-deterministic formulation

Basically non-deterministic formulations of the TEP problem are able to consider the possible events which could take place in the future by taking into account the uncertainty present in the information. In this category, the TEP problem can be solved either by means of a stochastic optimization-based formulation, where the objective function is typically formulated in term of an expected value or by means of a decision-making framework, which encompasses a deterministic optimization plus a decision tree analysis. Unfolding uncertainties are incorporated as branches and decisions are made on the evaluation of the consequences of deciding on the different expansion alternatives. In this sense, the decision-making framework allows the planners to gain insight into the risks involved in each expansion choice and could even suggest new and improved alternatives.

The dimension of the search space for the different TEP formulations depends on the number of decision choices, the number of decision variables and the number of periods. Additionally, the degree of detail of the model describing the temporal evolution of the PES along the planning horizon, namely demand discretization, time resolution and extent of the planning horizon is another important aspect to take into account since the computational effort for evaluating each combination depends on it.

To reasonably accomplish the challenging task of solving the TEP problem from a non-deterministic perspective, require incorporating and modeling a variety of data of diverse nature. Moreover, due to the large problem size, which is clearly defined by its stochastic, multi-period, multi-criteria and combinatorial nature, substantial efforts are required in order to sustain the viability of the proposed models. In this sense, an adequate treatment of the different types of the information is one of the most important stages before formulating the non-deterministic TEP model.

4. Handling information within the TEP

The process of solving actual planning problems requires handling a large amount of information from which only a small fraction is known with complete certainty. In this section, the major uncertainties affecting the TEP and referred to as variables that affect the outcomes of decisions and which are not known at time of planning, are analyzed and categorized from a descriptive viewpoint. Excluded here are the uncertainties originated in the model's user, i.e. what is not captured by the model but desired by the user, as well as uncertainties originated in the model (i.e. the "right" model structure, modelling techniques and tools).

4.1 Uncertainties present in the TEP

Data about the current state of the network is much more accurate than forecasted data. Furthermore, uncertainties present in forecasted data are very diverse in nature (Neimane,

2001). Therefore, it is recognized the importance of categorizing the uncertain information to be incorporated within TEP models.

In this work, it is assumed that forecasts and characterization of the forecast uncertainty are provided to the planning activity. Instead, the attention of this research work is posed in categorizing all the information to be handled within the TEP and proposing a systematic methodology for properly incorporating uncertain information of various source and nature within the TEP model.

4.2 Certain Information

Certain data are those parameters which can be defined explicitly (Neimane, 2001). This category includes the present network configuration, electrical parameters of the network components, possible expansion choices and their electrical parameters capacity limits of transmission lines, nominal voltages and voltage limits.

4.3 Information subjetc to stochastic uncertainty

Uncertainty in data mostly appears due to the inevitable errors incurred when forecasts are performed. When it is possible to objectively assess the magnitude of such errors with a satisfactory degree of confidence, then the uncertainty is said to be of random nature (Buygi, 2004). The uncertainty of such variables can be adequately represented by means of probability distribution functions. Demand, fuel prices and hydrologic resources evolution are typical examples belonging to this category. In (Vásquez *et al.*, 2008) a well-founded means for modelling random uncertainties is extensively presented.

4.4 Uncertain non-random information

When it is not possible to estimate with a satisfactory degree of confidence the errors incurred when forecasts are performed, information is deemed to be of a non-random nature (Buygi, 2004). Uncertainties in this group are related to human processes (e.g. investors decisions, changes in regulation, planners and managers investment strategies, beliefs or subjective judgments). In fact, the future does not appear to be predictable through extrapolation of historical trends applied to the current environment (Clemons & Barnett, 2003). Thus, non-random uncertainties assessment is derived from decision-makers perception, experience, expertise and reasoning. Inside this group there are two types of uncertainties.

The first type belongs to a large amount of valuable information that only can be expressed in linguistic form, e.g. "satisfactory", "considerable", "large", "small", "efficient", etc. Although this **vague information** has a very subjective nature and usually is based on expert judgment, it can be useful during the decision-making process. Fuzzy sets theory is a well-founded approach for modelling properly these kinds of uncertainties (Buygi, 2004).

The second type of non-random uncertain information is distinguished by holding uncertainties typical of dynamic environments that undergo severe and unexpected changes. This is the case with the TEP environment. According to the literature, these kinds of uncertainties are known as **strategic uncertainties** (Clemons & Barnett, 2003; Brañas et al., 2004; Detre et al., 2006). A specific feature of them is that they are gradually solved as new information arrives over time and, once enough information is known, the uncertainty is solved and disappears definitively (Dillon & Haimes, 1996; Clemons & Barnett, 2003).

Within the TEP problem, this uncertainty affects crucial events that could take place in the future, such as the generation expansion evolution or the delay on the expansion projects completion. Data with strategic uncertainties are considered the most important information to be handled within TEP since they are fundamental drivers of PES evolution and, therefore, of this decision-making problem. For further reading about this topic see (Detre *et al.*, 2006). On the other hand, within the PES planning environment, there are not much bibliographic references about modelling of strategic uncertainties in planning models. In (Neimane, 2001), this type of information has been designated as **truly uncertain information**[1]. Either discrete probability distribution functions or a scenarios technique are proposed for modelling information of this kind.

Taking into account the above mentioned, in this work it is proposed to model truly uncertain information by means of discrete probability distribution functions (PDF) where the probabilities assigned to the occurrence of different scenarios are assumed as known information. In this sense, a reasonable way for dealing with these two types of uncertainties is proposed in (Vásquez *et al.*, 2008).

5. The proposed flexibility-based TEP framework

The described TEP problem can be suitably faced by applying the decision tree technique, which basically consists in decomposing the whole problem into a number w of less complex sub-problems, each one concerned with solving a multi-period deterministic optimization as well as assessing the attributes of the expansion plans.

A sub-problem or complete path is represented by a number P of sequential discrete events. Such events are specified by the assumed discrete nature of strategic uncertainties. Under these conditions, each sub-problem handles only random uncertainties. Therefore, the different feasible expansion plans can be valued by applying a probabilistic analysis of the attributes of the objective function and decisions are made by applying a robustness-based risk management technique.

A master dynamic programming (DP) problem, by means of a backward induction of P sequential decisions, makes it possible to incorporate flexible options and, subsequently, rank the new flexible expansion strategies.

The entire proposed methodology, can be described as follows in five stages and illustrated in Fig. 1:

1. To decompose the TEP problem into w sub-problems.
2. To obtain a set of feasible expansion plans for each sub-problem w.
3. To assess the *OF* attributes of the different expansion plans for each path w.
4. To sequentially incorporate in the expansion plans, starting from the last decision period P, new flexible decisions for each path w.
5. To form flexible expansion strategies, by repeating 3 and 4 with backward induction until $P = 1$.

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[1] This term refers to relevant non-random uncertain variables, which convey strategic information.

INPUTS   *DECISION ANALYSIS*

Strategic
Uncertainties   **EVENTS TREE**
                                         FORMATION
                                         w **sub-problems**

                                              $i = 1$

                                         OPTIMIZATION

Expected Values of   **DETERMINISTIC**
random information   **OPTIMIZATION**
                                         Sub-problem i

                                         Set Si of feasible
                                         expansion plans $i = i + 1$

                                              $i = w$?   **NO**

                                              YES

                                    Sk = S1 U S2 U … U Sw

                                         PROBABILISTIC
Random   **ANALYSIS**
Uncertainties   **of Sk**

                                         SIMULATION

                                         Decoupled Attributes
                                         Assessment
                                         Ak,p and Af,p

                                         DECISION TREE
                                         FORMATION
                                         Opportunities for
                                         reducing risks

                                         DYNAMIC
Flexibility Technique: DG   **PROGRAMMING**
and Deferring options   **decisions based on**
                                         robustness
                                         (Sf formation)

                                         RISK ANALYSIS

OUTPUT   **Flexible Expansion**
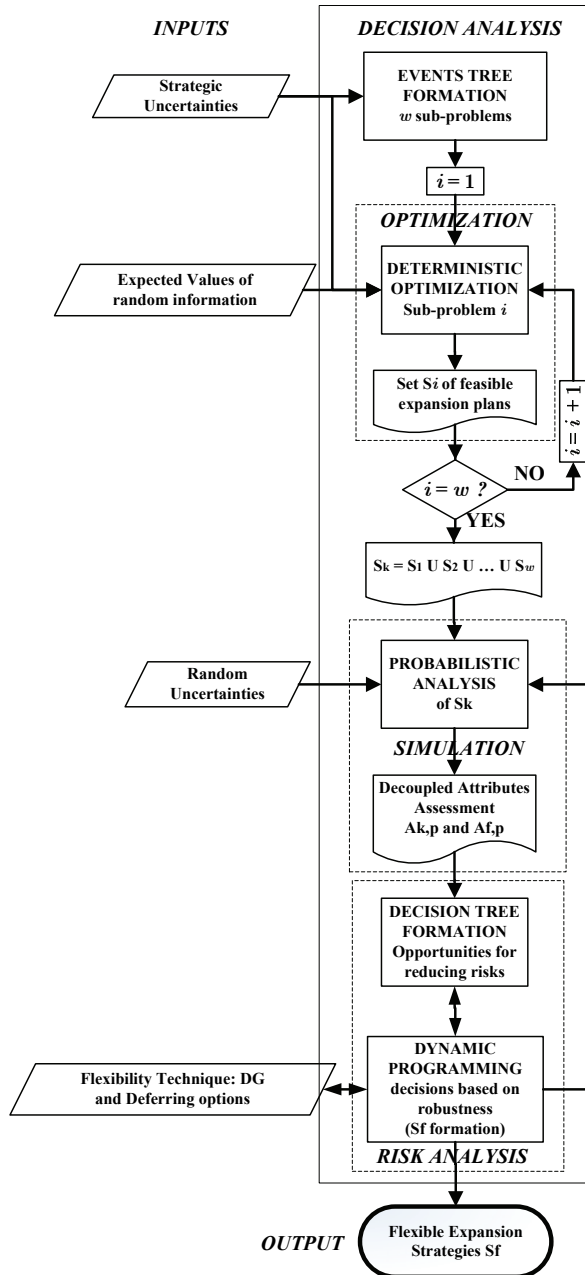   **Strategies Sf**

Fig. 1. Complete proposed framework for finding a flexible strategy

5.1 Decomposing the problem

The reason why optimization-based TEP models are inefficient is the presence of uncertainties. In fact, one of the most important concerns within the current TEP problem lies in suitably handling a large amount of uncertain information of diverse natures.

The traditional TEP formulations commonly reduce the future into an assumed probability-weighted certainty equivalent. This fact, in presence of strategic uncertainties implies averaging highly different scenarios. However, in practice equivalent scenarios will never take place since the future can unfold as either favourable or adverse. Therefore, stochastic optimization models formulated in terms of expected values are not suitable approaches for treating the TEP.

Event tree technique is a graphic tool that provides an effective structure for decomposing complex decision-making problems under the presence of uncertainties. The interested reader in decision tree analysis technique is further referred to Dillon & Haimes, 1996 and Majlender, 2003.
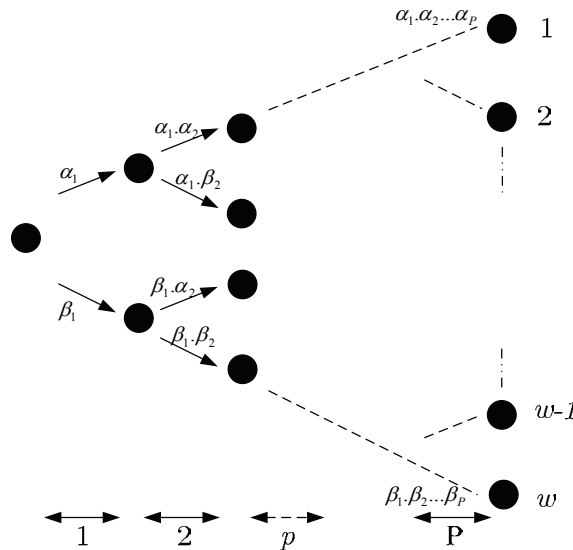
Fig. 2. Example of a binomial event tree

Fig. 2 depicts an example of a resulting events tree formed by assuming that the whole of the problem's strategic uncertainties can unfold into only two discrete scenarios. A complete event tree representing crucial states of the problem along the planning horizon allows getting insight about the diverse future circumstances, which candidate expansion plans should cope with.

Nodes of the event tree represent an explicit feasible scenario obtained as a result of combining all the possible discrete probability distributions of uncertain events along a discrete time p–decision periods. Each event is associated with composed occurrence probability which results from combining the discrete subjective probabilities assigned to the occurrence of a single uncertain event (α_p, β_p, …) and provided that the occurrence of such probabilities are independent of what happened in previous periods as shown in Fig. 2.

5.2 Obtaining a set of feasible expansion plans

The goal of this stage of the planning process lies in successfully reducing the dimension of the TEP by finding a set of feasible candidate expansion plans which fulfil fundamental constraints of the sub-problem. By reducing the search space, a rigorous economical and risk-based assessment of a reduced set of feasible expansion plans in subsequent stages turns practicable.

Under the scope of this work, it is assumed that the regulatory entity annually executes a centralized TEP task, in which a set of environmental, societal and political long-term energy policies must be achieved. In fact, the previous performance of environmental, societal and political feasibility assessments reduces the large number of decision alternatives to be considered by planners for searching candidate expansion plans for driving the expansion of the transmission grid. It is assumed that a number of possible transmission expansion alternatives have indentified. Despite this, the number of possible combinations of sequential decisions, i.e. the potential solutions, is still enormous. Since only a reduced number of combinations will meet the constraints of the TEP sub-problem, a technical efficiency-based assessment is a plausible means for reducing the search space and finding a set of technically feasible expansion plans.

The TEP sub-problems are formulated as a deterministic multi-period optimization and an evolutionary algorithm has been developed for properly solving such optimization problem (Vásquez, 2009)

Why is deterministic optimization the best choice?

The major foundations of this work for deciding on the deterministic choice lie in the nature of the TEP problem as well as in the problem decomposition proposed in the previous section. In fact, since only a reduced number of combinations will meet the TEP problem's constraints, and given that location, timing and type of the transmission expansion alternatives are discrete and limited in number, feasible candidate solutions are therefore also limited in number and noticeably different from one another. On the other hand, with the proposed decomposition of the TEP into sub-problems, strategic uncertainties have been removed temporarily. In this sense, the only presence of random information, which implies that uncertainties can be forecasted with a satisfactory degree of confidence, allows for a suitable technical assessment where the uncertain input variables are explicitly modelled by means of expected values.

5.3 Assessing the performance of expansion plans

The reduced number of candidate solutions allows a more detailed valuation of the expansion plans. This stage of the planning process entails performing a probabilistic technical-economical performance assessment of all the feasible expansion plans. The performance assessment of an expansion plan is achieved by accounting for a group of decoupled attributes of the objective function. Decoupled attributes A_k denote a measurement of the relative "goodness" of a specific transmission expansion plan S_k in every decision period p expressed by means of its probability distribution $F_{k,p}$. These p probability distribution functions represent the likelihood of the possible future values that the OF could acquire over time, characterizing the time-dependent risk profile of selecting the expansion plan S_k.

Stochastic simulation

Stochastic simulation techniques are applied for modelling the randomness of the objective function. In spite of the large computational effort demanded by Monte Carlo methods, the most significant advantage of the simulative approach over analytical probabilistic techniques is the accurate estimation of the tails of probability distribution $F_{k,p}$.

On the other hand, some planning engineers may worry about a possible conflict between the proposed deterministic optimization stage and the subsequent probabilistic and risk analysis stages. In fact, there is no conflict at all provided that all the feasible expansion plans have been found during the deterministic analysis stage. The probabilistic analysis stage is not intended to replace the deterministic TEP models, but to add better information on the merits of the expansion plan and its risk profile. This goal is achieved by assessing the time-decoupled attributes for every feasible expansion plan.

The total attributes of a specific expansion strategy S_k, A_k comprise all the information enclosed in the probability distributions $F_{k,p}$, which describe the possible future performance of S_k provided that all the problem uncertainties (random and strategic) have been taken into account during the simulative process (Neimane, 2001). If such resulting probability distribution function, defined in this work as F_1, can be fit to a Gaussian distribution, A_k can be expressed as follows:

$$A_k = F_{1,k}\left(\bar{C}_k, \sigma_k\right) \tag{6}$$

where,

$\bar{C}_k = \dfrac{1}{N_k}\sum\limits_{i=1}^{N}C_{k,i}$: OF's Expected Value for S_k

$C_{k,i}$: OF's value of the strategy S_k during the realization i. See Equation (5)

A_k : Total Attributes of the stratey S_k

$F_{1,k}$: OF's Probability Distribution for strategy S_k

N_k : Number of realizations until achieve the required
  confidence in determining $F_{1,k}$

σ_k : OF's standard deviation for strategy S_k

Although an assessment of A_k provides the information about the performance of an expansion strategy, the planner is unable to visualize the risk evolution over time and the effects on the OF's performance caused by the diverse type of uncertain variables. Nevertheless, having this information is a key issue for properly tackling the TEP. One of the major contributions of this work lies in successfully coping with these two concerns. In first place, Section 4.1 proposed to decompose the problem by applying the event tree technique. In second place, under the assumption that each node of the events tree represents one event unfolded by the combination of strategic uncertainties, a set of decoupled attributes where only random nature uncertainties are present needs to be evaluated. Under this perspective, by performing w Monte-Carlo realizations and then, by means of backward induction and considering the associated cumulative occurrence probabilities, the individual effects of the strategic uncertainties can properly be accounted for, from the last decision period until the first one. At the same time, the diverse time-

decoupled attributes of an expansion strategy F_p are assessed, step by step, until its total attributes F_1 are obtained.

At the end of this valuation process, F_1, which represents the total attributes of the analyzed expansion strategy, is obtained as follows:

$$\left. F_P\left(\overline{C}_P, \sigma_P\right)\right|_{w_{P-1}} = \left[f_{P-1,j}\left(\overline{c}_{P-1,j}, \nu_{P-1,j}\right) + \sum_{i=1}^{s_P} \alpha_{P,i} \cdot f_{P,i}\left(\overline{c}_{P,i}, \nu_{P,i}\right)\right]_{w_{P-1}}$$

$$\left. F_{P-1}\left(\overline{C}_{P-1}, \sigma_{P-1}\right)\right|_{w_{P-2}} = \left[f_{P-2,k}\left(\overline{c}_{P-2,k}, \nu_{P-2,k}\right) + \sum_{j=1}^{s_{P-1}} \alpha_{P-1,j} \cdot F_P\left(\overline{C}_P, \sigma_P\right)\right]_{w_{P-2}}$$

$$\vdots \qquad\qquad \vdots$$

$$F_1\left(\overline{C}_1, \sigma_1\right) = A\left(\overline{C}, \sigma\right) = f_{0,v}\left(\overline{c}_{0,v}, \nu_{0,v}\right) + \sum_{k=1}^{s_1} \alpha_{1,u} \cdot F_{2,u}\left(\overline{C}_{2,u}, \sigma_{2,u}\right) \qquad (7)$$

where,

F_p : *OF*'s decoupled attributes during the period p

$\overline{C}_{p,s}$: *OF*'s Expected Present Value for event s during p

$\sigma_{p,s}$: *OF*'s standard deviation for event s during p

$\alpha_{p,s}$: Ocurrence Probability of event s during the decision period p

s_p : Number of feasible discrete events during p

$f_{p,s}$: Partial *OF*'s Probability Distribution for event s during p

$\overline{c}_{p,s}$: Partial *OF*'s Expected Value for event s during p

$\nu_{p,s}$: Partial *OF*'s standard deviation for event s during p

Fig. 3 graphically shows the increasing uncertainty of the objective over time. As planning horizon extends in time, the risk grows accordingly. Provided that the probabilistic properties of expansion attributes are reasonably described by a Gaussian probability distribution, blue dots correspond to the annual expected values of the expansion costs and the vertical black segments represent the annual standard deviations of the objective function.
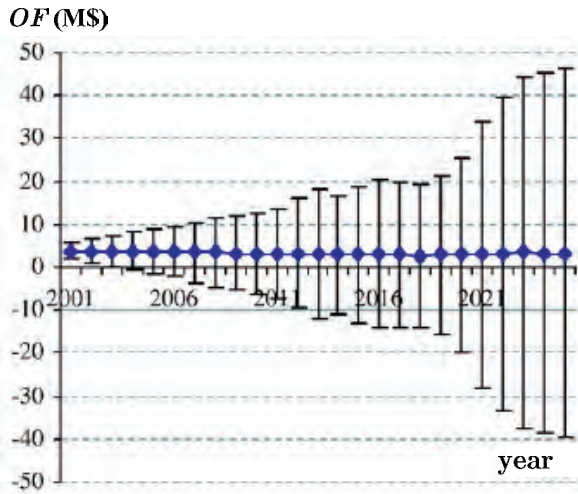
Fig. 3. Graphic representation of the time-decoupled attributes of an underlying asset

The idea of a decoupled assessment of the expansion plans' attributes can be rooted to the Bellman's Principle of Optimality since it allows applying dynamic programming for valuing the flexibility gained when embedded or contingent decision options are incorporated within the planning process (Dixit & Pindyck, 1994). In following sections, this process is explained in detail.

Ranking of expansion strategies and decision-making

Derived from the optimal portfolio selection theory, expansion plan attributes can be ranked based on their efficiency, by means of the Sharpe ratio r_{sharpe} (Nielsen & Vassalou, 2003). This index was proposed by Sharpe in 1966 as the ratio between the expected benefit and the risk, where risk is measured as a standard deviation of the benefit. According to static mean-variance portfolio theory, if investors face an exclusive choice among a number of alternatives, then they can unambiguously rank them on the basis of their robustness (Sharpe ratios). An expansion alternative with a higher Sharpe ratio will enable all investors to achieve a higher expected utility by accepted risk unit.

The inverse of r_{sharpe}, which is known as the coefficient of variation according to Ladoucette & Teugels (2004) and Feldman & Brown (2005) is a useful measure for comparing variability between positive distributions with different expected values. An alternative with a lower coefficient of variation will result in lower risk exposition per unit of expected benefit. In this work, the inverse of the r_{sharpe} will be used to measure the desirability of an expansion strategy.

In order to express in percentage the coefficient of variation, the use of a relative volatility, which is accounted for as the relation between the expected volatility of the underlying asset $\bar{\sigma}_k$ divided by the maximum expected volatility of all the evaluated strategies $\bar{\sigma}_{max}$, is proposed. See (8).

$$r_{sharpe}^{-1} = \frac{\overline{\sigma}_k}{\overline{B}_k} = \frac{\dfrac{\overline{\sigma}_k}{\overline{\sigma}_{max}}}{\dfrac{\overline{C}_{max}}{\overline{C}_k}} \cdot 100 \, (\%) \qquad (8)$$

where,

$\overline{B}_k$: Expected Benefit of the underlying asset S_k

$\overline{\sigma}_k$: Underlying asset expected volatility

5.4 Risk management by incorporating flexible Options

An important underlying assumption of the probabilistic optimization approach is the passive planner's attitude regarding the future. In fact, under this modelling paradigm, the diverse available choices that the planner has for reacting upon the occurrence of unexpected events are ignored. However, in practice planners have the ability for adapting their expansion decisions in response to undesired events (Gorenstin *et al.*, 1993; Dixit & Pindyck, 1994; Ku *et al.*, 2003; Vásquez & Olsina, 2007). A well-established way to systematically incorporate this fundamental aspect is the application of a complementary flexibility-based risk analysis stage.

New decision variables, new objective function

The flexibility-based risk analysis stage basically consists in solving an optimization problem of dynamic nature. The decision variables are the type, the timing and the location of the flexible -technical or managerial- options which are embedded in the previously obtained set of feasible expansion plans. Indeed, to solve this problem involves finding expansion strategies that are improved in performance in terms of their total attributes. Such expansion strategies are composed not only of large transmission projects D_p, but also of flexible decision options. Thus, flexible options O_p available in each decision period p, are planned for being advantageously incorporated if strategic uncertainties unfold as unfavourable scenarios.

Like previous stages, a total expansion costs-based objective function, which includes the new components of costs relative to the new flexible choices, is defined. This new *OF* is still subject to the same constraints of the original problem plus the constraints relative to the flexible options, e.g. generation capacity limits of DG plants and feasible locations of DG projects.

Visualizing opportunities for contingent decisions

A graphic illustration (see Fig. 4) of a complete event paths representing crucial states of the problem along the planning horizon together with the time-decoupled attributes information (F_P, F_{P-1}, …, F_1) suitably represents the dynamic process that this optimization problem involves. In fact, with this information the planner has an insight into the risks associated with the decisions as well as is able to determine the timing when it would be meaningful to incorporate flexible or contingent choices. The problem search space is therefore noticeably reduced.

Given that only one discrete probability function during each period is assumed, the nodes of events tree showed in Fig. 2 represents the planner's opportunity for incorporating flexible or contingent decisions.

5.5 Valuing flexibility and ranking expansion strategies

When an irreversible expenditure D_P is made, i.e. the investment option is exercised, not only the deferment choice disappears but also all the other investment choices (Kirschen & Strbac, 2004). The value of the lost option, analogous to a financial call option, is an opportunity cost, which depends on the project's irreversibility as well as on the existing risk and flexible embedded options at the decision time (Dixit & Pindyck, 1994; Ramanathan & Varadan, 2006). However, classical project appraisal methods overlook this interaction even though, in practice, it evidently affects the planner's decisions.

Since flexibility can only be assessed by comparison (Ku et al., 2003; Gorenstin et al., 1993), the value of a flexible option is assessed by comparing its coefficient of variation with the coefficient of variation of a feasible inflexible reference strategy (*flexibility* = 0) belonging to the set of feasible expansion plans S_k. This basic procedure can be systematically extended into a multi period strategies comparison problem and solved by using the dynamic programming formulation expressed in (9) and (10) (Dixit & Pindyck, 1994; Ramanathan & Varadan, 2006).
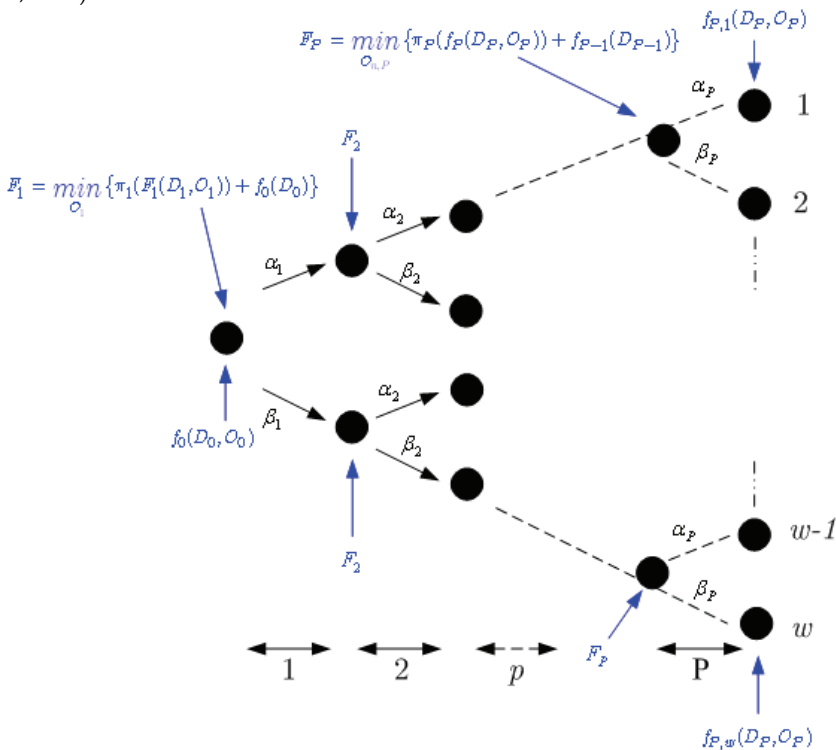

Fig. 4. Decoupled attributes of the objective function and decision tree representation

$$F_P = \min_{O_P}\{F_P(D_P,O_P)\} = \min_{O_{n,P}}\{\pi_P(f_P(D_P,O_P)) + f_{P-1}(D_{P-1})\}$$

$$F_{P-1} = \min_{O_{P-1}}\{\pi_{P-1}(F_P(D_{P-1},O_{P-1})) + f_{P-2}(D_{P-2})\}$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$F_1 = \min_{O_1}\{\pi_1(F_1(D_1,O_1)) + f_0(D_0)\}$$

where

D_p : Expansion decision to be defferred since period p

O_p : Flexible option incorporated in period p

π_p : Immediate costs valuation function during p

Going backward in dynamic programming allows decomposing a whole sequence of decisions into just two components: the immediate decision and a valuation function which encapsulates the consequences of all the subsequent future decisions.

As shown in Fig. 4, the process of incorporating flexible options starts at the last decision period ($p = P$), which is concerned with deciding for or against incorporating (min$\{F_P\}$) one of the available flexible choices O_P. This is a classic single-stage optimization problem under the presence of only random uncertainties. As was analyzed in Sections 4.3 and 4.4 of this chapter, this task is proposed to be solved by applying a robustness-based probabilistic decision approach. In fact, by assessing, on one side, the time-decoupled attributes of the static expansion plan F_P (D_P) and, on the other, the time-decoupled attributes of one or more new flexible expansion strategies composed by a flexible option F_P (O_P), the planner can decide about the incorporation or not of such a flexible option in $p = P$, by comparing the two coefficients of variation (r-1_{sharpe} (D_P), r-1_{sharpe} (O_P)).

This solution (min$\{F_P\}$) provides the information for the penultimate decision in P-1 which, in turn provides the information for deciding in P-2 and so on until $p = 1$ the moment in which a flexible strategy S_f is obtained. This procedure repeated for all the feasible expansion plans can be used for obtaining a set of flexible expansion strategies.

6. Numerical example: power supply capacity expansion planning problem of a the mining firm

In the following, a numerical planning problem built on an actual setting demonstrates the contribution of the proposed flexibility-based framework by enhancing the ability of making contingent expansion decisions along the planning horizon. Investing in DG projects and delaying a large transmission project are flexible options that the planner has available for reducing the planning risks.

Investment, energy procurement, and maintenance costs as well as expected unserved energy costs have been considered for computing the expected total costs of the diverse expansion strategies. Because of the short lead time, DG investment decisions are assumed to be made in the same time interval that the additional capacity is required. On the other hand, due to the large construction time, transmission projects are commenced one year before the additional capacity is required.

Let considers a mining firm which will operate over ten years located in a remote site without public service of electric energy supply. Daily production of the mine is assumed constant. It is known with certainty that the demand for the first to fifth year is 60 MW. Available information in year zero indicates that demand would increase to 120 MW depending on results of a current assessment of mineral reserves. The probability of the higher demand scenario is $p = 0.5$. The probability of power demand remaining at 60 MW is $1-p = 0.5$. Then, the expected value of demand along the second time period is 90 MW. Fig. 5 depicts the two possible demand paths along the planning horizon, which is set to 10 years.

The main question is: How the mining firm should meet, in an optimal way, its current and future requirements of electric energy under consideration of ongoing demand uncertainty? For successfully accomplishing this task, the proposed flexibility-based decision-making framework will be applied, which involves the development of the following stages:

- Identification of a set of feasible expansion strategies.
- Assessments of the corresponding objective function for each feasible expansion strategy.
- Incorporation of flexible decision options in order to conform new expansion strategies.
- Ranking the expansion strategies by properly valuing the flexibility of the options incorporated in 3.
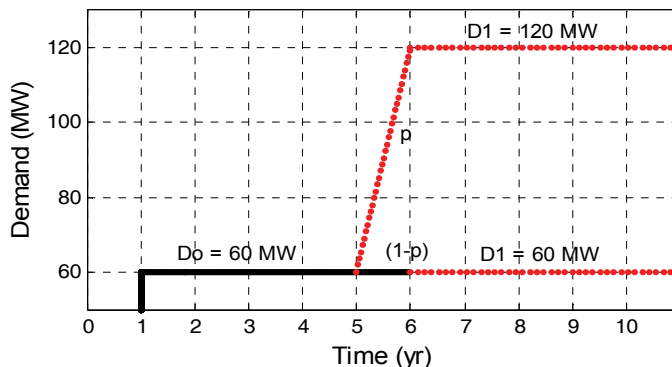

Fig. 5. Future demand scenarios along the operative lifetime of the mining firms

6.1 Obtaining Feasible Expansion Strategies

The large of economies of scale involved indicate that the most efficient expansion strategies have to deal with building 346 km of a new transmission line from the nearest system node instead of installing generation on site. Three technically feasible configuration of transmission lines are obtained as shown in Fig. 6 based admissible voltage limits.

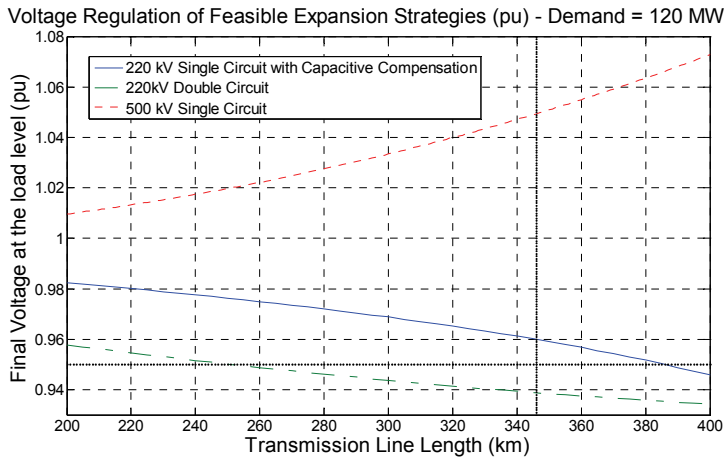Voltage Regulation of Feasible Expansion Strategies (pu) - Demand = 120 MW

Fig. 6. Convergence and voltage regulation of feasible expansion strategies

The identified strategies are listed as follows:

- To build in year zero a 220 kV single circuit radial transmission network from the nearest system node. In the sixth year a capacitive compensation in node B_2 is installed in order to improve voltage levels (see Fig. 8).
- To build in year zero a 220 kV double circuit radial transmission network from the nearest system node.
- To build in year zero a 500 kV single circuit radial transmission network from the nearest system node.

These three inflexible decision options allow the mining firm to purchase energy from the spot market and therefore meet its expected demand. The next stage in the decision-making process involves valuing and ranking, from a probabilistic viewpoint, the obtained expansion strategies.

6.2 Assessing the attributes of feasible expansion plans

The substantial economy of scales involved in the expansion of the processing plant of the mining firm leads to an increase of electrical demand in large discrete amounts. As remaining relevant variables are assumed to be known with absolute certainty, only the uncertainty affecting the load growth will be resolved over the time. For this reason, the risk profiles of expansions will have the same shape as the forecasted demand evolution (see Fig. 5). Therefore, the assessment of the attributes of the expansion alternatives along the planning horizon can be completely determined without applying the Monte-Carlo technique. The objective function (OF) of the constrained stochastic optimization problem is formulated as follows:

$$\min_{T}\left\{\text{E}[C_{T_i}(T)]\right\} = \min_{T}\left\{C_{I_{TL}}^{S_1} + \sum_{j=1}^{5}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_1}^{T} + \right.$$

$$+ (1-p)\cdot\sum_{j=6}^{10}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2}^{T} + $$

$$\left. + p\cdot\sum_{j=6}^{10}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2}^{T}\right\} = 302.87\,\text{M\$}$$

(11)

subject to:

$S_{G,j} - D_{i,j} - S_{L,j} - S_{NS,j} = 0\,(\text{MVA})$: Balance of power

$0.9 \leq V_3 \leq 1.1$: Voltage limits

$F_{12} \leq T$: Transmission capacity constraint

where

$\text{E}[C_T]$: Net Present Value (NPV) of the total expected costs.

$C_{I_{TL}}^{S_1}$: Investment cost in transmission network for strategy S_1.

$C_{A,j}$: Acquisition cost of energy in the spot market in year j.

$C_{O\&M,j}$: Operation and maintenance cost of lines and sub-stations incurred in year j. These costs are assumed to be 2% and 3% of the respective investment costs.

$C_{E[ENS],j}$: Expected costs of the energy not supplied in year j.

$S_{G,j}, D_{i,j}, S_{L,j}, S_{NS,j}$: Spot market power, power demand in i-th stage, power losses, and not supplied power in the j-th year.

The Value of Lost Load (VOLL) has been estimated at 500 \$/MWh and reflects the economic losses incurred when the mining firm stops its production. Discount rate is set to 12%/yr.

Because of the length, capacitive compensation is needed for the single circuit 220 kV choice. The fixed investment cost of compensation is 1 M\$ and capacity dependent costs are 17 000 \$/MW of incremental line capacity. The investment cost functions for 220 kV and 500 kV substations are depicted in Fig. 7.

Costs of transmission lines have been modelled as a linear function of the transmission capacity, as indicated in Table 2.

Table 3 provides the electrical line parameters needed for performing an AC power flow analysis on each alternative in order to verify voltage limits, line flows, losses, etc.

Table 2. Transmission lines investment costs

Voltage	Fixed costs \$/km	Capacity costs \$/(MW·km)
220 kV single circuit	90 000	800
220 kV double circuit	135 000	600

500 kV single circuit   200 000   300
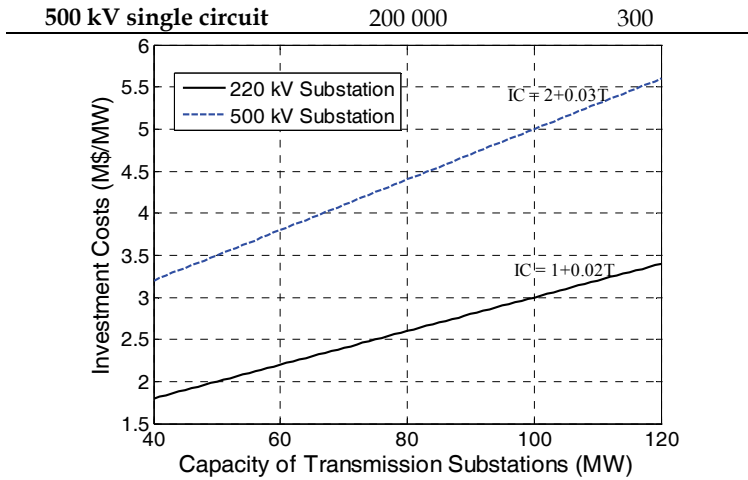
Fig. 7. Investment cost functions of transmission substations.

Voltage	R Ω/km	x Ω/km	B μS/km
220 kV single circuit	0.0481	0.385	2.341
220 kV double circuit	0.0241	0.192	4.682
500 kV single circuit	0.0234	0.279	4.169

Table 3. Electric parameters of transmission lines

In Table 4, reliability parameters of transmission components and DG plants are given, as they are necessary for computing the expected energy not supplied to the mining process. Stochastic behaviour of system components are modelled as two-state Markov reliability model (Billinton & Allan, 1996). Because of the small number of components, exhaustive state enumeration has been applied for the reliability evaluation.

Procurement costs of energy have been computed considering the long-term spot prices that would prevail in node B2 (see Fig. 8) provided that the transmission network was to be built with optimal capacity. The spot price duration curve in node B1 remains constant over the planning period and it is given in Table 5.

Parameter	Market	Line	Transformer	DG Plant
Pr(O)	0.99886	0.99545	0.99825	0.98000
Pr(F)	0.00114	0.00455	0.00174	0.01999

Table 4. Reliability parameters of system components

Duration (%)	6.96	13.87	38.64	32.46	8.33
Price ($/MWh)	82.29	75.7	61.7	57.6	37.03

Table 5. Spot prices during periods

According to (1), the power supply capacity optimization problem is solved when a transmission project, which satisfies technical and economic requirements for all anticipated demand scenarios, minimizes the total discounted expected expansion costs incurred along the planning horizon. In the valuation process, the occurrence probabilities of each demand scenario are considered and the remaining information is assumed to be known with certainty. Fig. 9 shows the performance of the three technically feasible expansion strategies identified before, which meet the uncertain power demand of the mining firm over the planning horizon.

Expansion strategies are ranked considering the minimization of the present value of total expansion costs. Under this perspective, the 220 kV single-circuit transmission line with a capacity of $T = 120$ MW and capacitive compensation in B_2, which is denominated S_1, would be the strategy that the planner would select under a classic risk-neutral probabilistic choice as it exhibits the lowest expected costs ($E[C_T]= 302.87$ M$).
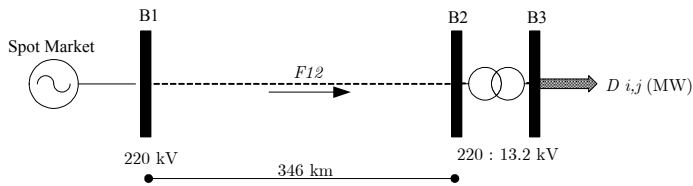

Fig. 8. Expansion strategy S_1, single circuit 220 kV- 120 MW transmission link
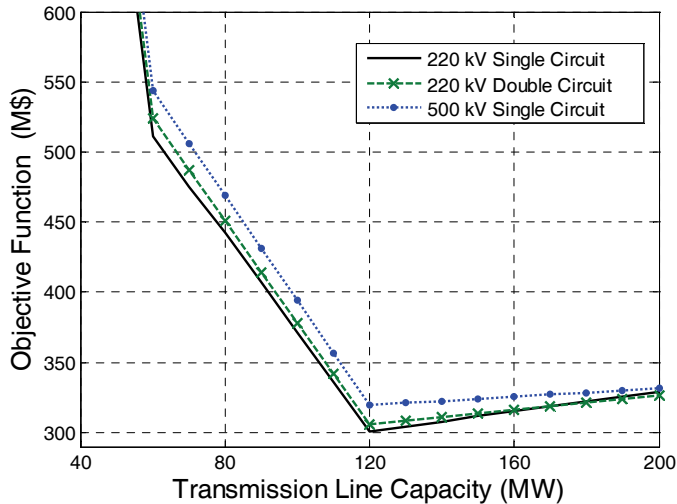

Fig. 9. Present value of expected expansion costs for all identified feasible expansion strategies

6.3 Flexible Expansion Strategies Conformation

For illustrative purposes the only feasible-inflexible expansion strategy considered during the next stages of the planning process is S_1. Unlike the classic probabilistic approach, the proposed valuation method accounts for contingent expansion choices, i.e. DG investments and delay of a large transmission project, that the planner has available in each demand

scenario. DG projects are based on installing diesel-fueled gensets. From manufacturer data, nameplate capacity of each DG diesel units is 15 MW. However the maximum power output is derated to 13 MW at the location altitude. Investment costs of diesel units are assumed to be 200 \$/kW. The DG plants fuel specific consumption is 217.98 l/MWh[2] and the fuel price is 0.3 \$/l. In Table 6 below, apart from S_1, are listed five additional expansion strategies with various degrees of flexibility, that planner could consider once better information about demand evolution arrives.

Real Options		
S	**Period 1**	**Period 2**
1	Single Circuit Trans Line 120 MW	
2	1st circuit trans. line 60 MW	2nd. circuit (expand to 120 MW)
3	Single circuit trans. line 60 MW	Single circuit trans. line 60 MW
4	DG 5x15 MVA	Single circuit trans. line 60 MW
5	DG 5x15 MVA	Single circuit trans. line 120 MW
6	DG 5x15 MVA	DG 5x15 MVA

Table 6. Description of possible expansion strategies to be valued

6.4 Decision-Making: Valuation and Ranking of Flexible Expansion Strategies
Valuation of flexible expansion strategies

Strategy S_2 consists of initially building the first circuit of a double circuit 220 kV transmission link with a capacity of 60 MW for satisfying the known demand from year 1 to 5. In the fifth year, if the power demand is increased to 120 MW, i.e. once uncertainty has been unfolded, the planner takes the decision of adding the second circuit, expanding the transfer capacity from 60 to 120 MW. Details of calculation of the discounted expected total expansion costs of S_2 are given by the following expression:

$$\mathrm{E}\left[C_{T2}\right] = C_{I_{TL}}^{60MW\,(1st\,circuit)} + \sum_{j=1}^{5}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_1=60MW}^{T=60\,MW} +$$

$$+(1-p)\cdot\sum_{j=6}^{10}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=60MW}^{T=60\,MW} +$$

$$+p\cdot\left[\frac{C_{I_{TL}}^{60\to120(2nd.circuit)}}{(1.12)^5} + \sum_{j=6}^{10}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=120MW}^{T=120\,MW}\right] \qquad (12)$$

$$\mathrm{E}\left[C_{T2}\right] = 297.1\mathrm{M}\$$$

---

[2] A specific fuel consumption of 217.98 l/MWh entails a combined efficiency of the genset (engine and generator) of 43.54%, assuming for diesel fuel an average Net Calorific Value (NCV) of 43 MJ/kg and a density of 0.883 kg/dm[3] at 15°C.

Strategy S_3 involves the construction of a 220 kV single circuit transmission line with a capacity 60 MW for satisfying the demand of the first period. In the fifth year, only if the power demand rises to 120 MW, the planner takes the decision of build another 220 kV single circuit transmission line with capacity 60 MW. The present value of the expected total expansion costs for S_3 is computed according to (13).

It is important to notice that the expected total costs of these more flexible strategies are lower than cost of strategy S_1. That is because the total expenditure has been separated into two stages. The second investment is committed in the fifth year only if it is actually needed.

$$E[C_{T3}] = C_{I_{TL}}^{60MW(single)} + \sum_{j=1}^{5} \left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j} \right)\Bigg|_{D_1=60MW}^{T=60\,MW} +$$

$$+(1-p)\cdot\sum_{j=6}^{10} \left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j} \right)\Bigg|_{D_2=60\,MW}^{T=60\,MW} +$$

$$+p\cdot\left[\frac{C_{I_{TL}}^{60MW(single)}}{(1.12)^5} + \sum_{j=6}^{10} \left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j} \right)\Bigg|_{D_2=120\,MW}^{T=120\,MW} \right]$$

$$E[C_{T3}] = 295.1M\$$$

(13)

In the following, some investment policies including DG projects are analyzed. Strategy S_4 comprise the installation of five 15 MW diesel generators for satisfying the power demand of the mining firm during the first period. Then, only if the demand effectively grows to 120 MW, a single circuit 220 kV transmission line with a capacity of 60 MW is built in year five, to meet the mining demand along the second period.

The investment cost of a diesel DG plant is 3 M\$. Additionally, costs fuel storage facilities are assumed to be 1.5 M\$. Maintenance costs are computed as a percentage of investment costs. For DG plants they are set as 5% and, for fuel storage facilities they are assumed to be 3% of its investment costs. Lifetime of DG generators is assumed to be 20 years. Linear depreciation has been used for assessing recovering value of DG equipment. In this case the mining firm sells off the five DG plants at the closing of the mining project. Assuming that 20% of the investments cost are required to uninstall the generation plant, the recovery net costs can be assessed and included. The present value of the expected total expansion costs for S_4 is computed according to (14).

$$E[C_{T4}] = \frac{C_{I_{DG}}^{60MW}}{(1.12)^1} + \sum_{j=1}^{5}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_1=60MW}^{DG=60\,MW} +$$

$$+(1-p)\cdot\left[-\frac{C_{recDG}^{60MW}}{(1.12)^{11}} + \sum_{j=6}^{10}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=60MW}^{DG=60\,MW}\right] +$$

$$+p\cdot\left[\frac{C_{I_{TL}}^{60MW(single)}}{(1.12)^5} - \frac{C_{recDG}^{60MW}}{(1.12)^{11}} + \sum_{j=6}^{10}\left(\frac{C_{G+A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=120MW}^{DG+T=120}\right] \quad (14)$$

$$E[C_{T4}] = 296.4\,\text{M\$}$$

where

$C_{I_{DG}}^{60MW}$: Investment cost of 60 MW DG power plants.

C_G : Annual generation costs of 60 MW DG power plants.

C_{G+A} : Generation and acquisition costs incurred when operate at the same time the DG plants and the trans. line.

C_{rec}^{60MW} : Recovery net value of DG equipments.

Similarly, expansion strategy S$_5$ consists in installing five 15 MVA diesel DG plants in the first year. If power demand escalates to 120 MW, a single 220 kV transmission line with capacity 120 MW is built to cover the energy needs during the second period. Under S$_5$, the five DG plants are sold off in the sixth year. The present value of the expected total expansion costs for S$_5$ is computed according to (15).

$$E[C_{T5}] = \frac{C_{I_{DG}}^{60MW}}{(1.12)^1} + \sum_{j=1}^{5}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_1=60MW}^{DG=60\,MW} +$$

$$+(1-p)\cdot\left[-\frac{C_{recDG}^{60MW}}{(1.12)^{11}} + \sum_{j=6}^{10}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=60MW}^{DG=60\,MW}\right] +$$

$$+p\cdot\left[\frac{C_{I_{TL}}^{120MW(single)}}{(1.12)^5} - \frac{C_{recDG}^{60MW}}{(1.12)^6} + \sum_{j=6}^{10}\left(\frac{C_{A,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=120MW}^{T=120\,MW}\right] \quad (15)$$

$$E[C_{T5}] = 294.04\,\text{M\$}$$

Similarly as with S$_4$ and S$_5$, strategy S$_6$ entails installing five 15 MVA diesel DG plants in the first period, and then, adding five new 15 MVA diesel DG for covering the mining peak load

in the second period only if it unfolds as the high demand scenario. The expected total costs of strategy S_6 can be evaluated according to (16).

$$E\left[C_{T6}\right] = \frac{C_{I_{DG}}^{60MW}}{(1.12)^1} + \sum_{j=1}^{5}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_1=60MW}^{DG=60\,MW} +$$

$$+(1-p)\cdot\left[-\frac{C_{recDG}^{60MW}}{(1.12)^{11}} + \sum_{j=6}^{10}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=60MW}^{DG=60\,MW}\right] +$$

$$+p\cdot\left[\frac{C_{I_{DG}}^{60MW}}{(1.12)^6} - \frac{C_{recDG}^{120MW}}{(1.12)^{11}} + \sum_{j=6}^{10}\left(\frac{C_{G,j}}{(1.12)^j} + \frac{C_{O\&M,j}}{(1.12)^j} + \frac{C_{E[ENS],j}}{(1.12)^j}\right)\Bigg|_{D_2=120MW}^{DG=120\,MW}\right] \tag{16}$$

$$E\left[C_{T6}\right] = 291.5\,M\$$$

Components of OF	$E[C_{T1}]$	$E[C_{T2}]$	$E[C_{T3}]$	$E[C_{T4}]$	$E[C_{T5}]$	$E[C_{T6}]$
Net Investment (M\$)	72.36	67.52	66.12	28.61	32.52	16.37
O&M (M\$)	8.46	7.59	7.03	5.57	5.16	5.25
Generation & Procurement (M\$)	221.9	221.9	221.9	262.2	256.3	269.83
E[ENS] (M\$)	0.163	0.067	0.087	0.020	0.034	0.006
TOTAL (M\$)	**302.88**	**297.07**	**295.13**	**296.40**	**294.01**	**291.45**
Standard Deviation (M\$)	50.2	56.5	71.6	68.9	65.5	61.8
Risk, r_{sharpe}^{-1} (%)	**70.11**	**77.40**	**97.44**	**94.17**	**88.80**	**83.06**

Table 7. Summary of decision variables for the proposed expansion strategies

Ranking of Flexible Expansion Strategies

Depending on the planner´s attitude regarding risk, the decision could be made either based on the minimum total expansion costs criterion (S_6) or by choosing the option with lowest risk (S_1). For instance, by comparing the present values of the expected present costs of the six alternatives it is concluded that flexible strategies S_4, S_5, and S_6, though much more expensive in terms of operating costs, are better than the economies of scale provided by S_1. A breakdown of the costs incurred by each alternative is provided in Table 7.

Fig. 10 illustrates the effect of uncertainty on the expected costs of each strategy. The economic efficiency of S_1 increases when the probability for the occurrence of the higher demand scenario is high. On the other hand, investing for retaining flexibility is more convenient if there is a low probability for the occurrence of the higher demand scenario.
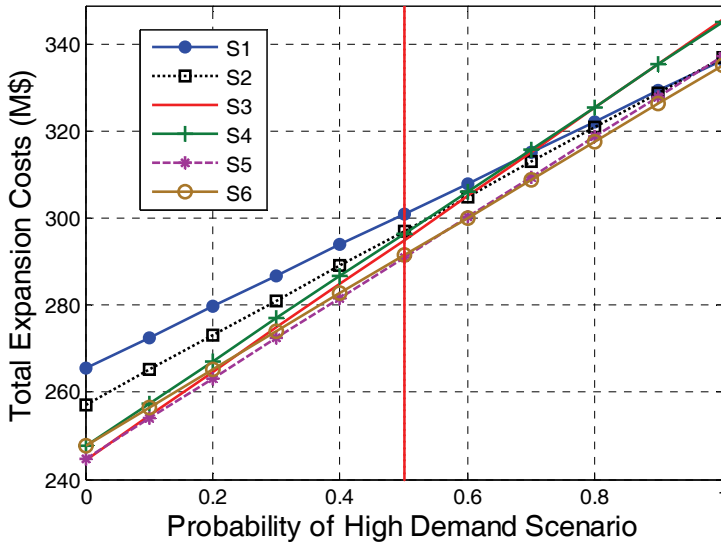
Fig. 10. Sensitivity analysis of the performance of the expansion choices to the probability of a high demand scenario

Valuing flexibility of the embedded options

Decomposing the problem based on the nature of the uncertainties allows the proposed framework to account for the value of the gained flexibility by comparison using discrete Sharpe ratios r_{sharpe}. Nevertheless, in this specific example, it has been assumed that the planner has made his decision based on the expected NPV of the total expansion costs. In fact, a suitable way for valuing the flexibility of strategies S_2, S_3, S_4, S_5, and S_6 with regard to S_1 is to ask how much lower should be the investment costs of S_1 (CI_{TL0}) to make this investment policy the preferred alternative. For instance, the flexibility of S_6 could be assessed by equating (11) and (16) and solving for transmission investment costs CI_{TL1}.

Fig. 11 shows that investment costs of the single-circuit 220 kV transmission line should be about 13% lower to prefer investment strategies S_1 to strategy S_6. Therefore, the flexibility value provided by S_6 can be computed as follows: $CI_{TL_1} = CI_{TL_0} + RO_6$; $RO_6 = Flexibility_{S_6} = 60.95 - 72.36 = 11.4 \text{M}\$$
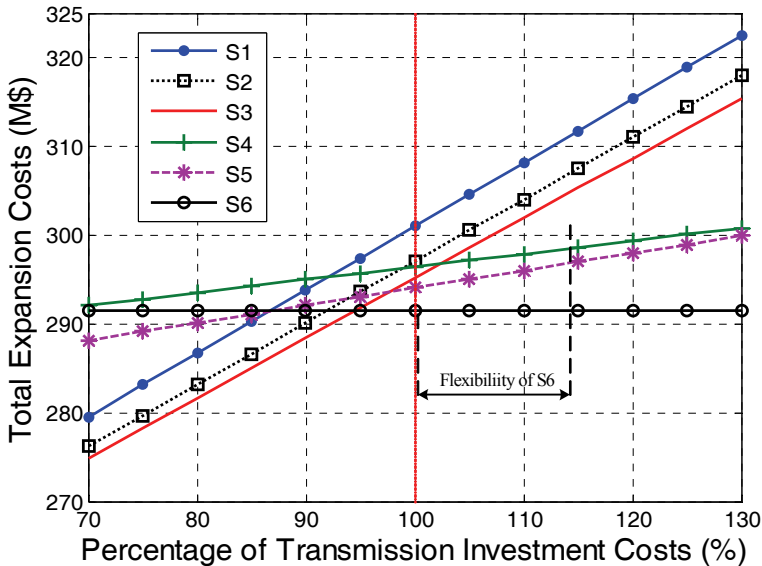
Fig. 11. Flexibility value for diverse transmission investment costs

7. Conclusion

Risk management techniques based on strategically incorporating flexible investment projects represent a tool for consistently dealing with risks present in transmission planning problems. The larger the irreversibility of the conventional expansion investments and the uncertainties affecting future conditions more efficiency can be achieved by the proposed planning approach.

In a numerical example, it has been verified that ignoring the value of flexible choices can lead to wrong investment decisions. Despite the large economies of scale of the traditional transmission expansion projects, it has been shown that the optimal investment strategy would be to preserve the wait option by investing in more expensive DG projects, while deferring the decision of building the transmission line until uncertainties are resolved.

In order to accomplish the goal of properly integrating DG investments and exploiting their potential benefits requires an efficient cost-based evaluation. A very important emerging issue is therefore the pricing and reward of system benefits provided by DG projects. In fact, electricity prices as seen by final consumers are substantially higher than prices at centralized generation levels. This difference is due to the added costs of T&D services to transport electricity from the point of production to consumption. Distribute generation, however, located close to demand, delivers electricity directly with limited requirement for use of T&D network (Djapic et al., 2007). Ignoring this particular feature in the valuation process could result a false DG non competitiveness perception regarding traditional expansion decisions.

Other important topic that calls for further investigation is the proper valuation of the planning flexibility provided by option of relocating DG facilities across large networks, as uncertainty on demand growth unfolds divergently among the different system zones.

8. References

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Acronyms

DG	Distributed Generation
DP	Dynamic Programming
EENS	Expected Value of Energy Not Supplied
FACTS	Flexible AC Transmission Systems
NPV	Net Present Value
O&M	Operation and Maintenance
PDF	Probability Distribution Function
PES	Power Electric System
TEP	Transmission Expansion Planning
T&D	Transmission and Distribution (Systems)
TS	Transmission System
VOLL	Value of Lost Load