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Mid-term planning optimization model with sales contracts under demand uncertainty

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ABSTRACT

Uncertainty modeling is a challenging topic in supply chain and operation management. When planning material purchase and stock levels, demand uncertainty could have an important impact on the plan results and its feasibility. Additionally, uncertainty could greatly affect customer satisfaction, inventory costs and company profits. From a modeling perspective, problems considering uncertainty are difficult to tackle and lead to complex optimization approaches. This work proposes a mid-term planning model dealing with sales contracts to diminish the effect of uncertainty. Another interesting feature is given by the selection of different price levels. Price elasticity functions are introduced for each customer in order to jointly decide demand targets and prices. A linear generalized disjunctive programming model is developed. Short execution time shows that this model can be applied to analyze several real scenarios to decide material purchase plan, inventory levels, sales strategies, prices and demand levels in a medium term horizon planning.

1. Introduction

Supply chain management deals with material, information and financial flows in a network consisting of vendors, manufacturers, distributors and customers (Anupindi & Bassok, 1999). The general purpose of a supply chain is to improve the overall efficiency to handle several sources of uncertainty, where product demand variability can be identified as one of the main sources (Gupta, Maranas, & McDonald, 2000; Rodriguez & Vecchietti, 2011). One alternative in order to mitigate the effect of demand uncertainty is to sign supply contracts between suppliers and customers. Supply contracts in a deterministic environment have been analyzed by Bansal, Karimi, and Srinivasan (2007) and Khalilpour and Karimi (2011), they remark that even assuming a deterministic model, contracts prevent from uncertainties in prices and amounts ordered. Under demand contracts a buyer commits a fraction of all his demand from a specific supplier with the aim of sharing demand uncertainty between both stakeholders (Anupindi & Bassok, 1999; Park, Park, Mele, & Grossmann, 2006). In this approach, customers beneficiate from better prices and financial conditions while companies assure certain demand level.

Modeling uncertainty is a key issue in production and operation management for several reasons. From the business perspective, uncertainty could have a significant impact on customer satisfaction, inventory costs and company profits. On the other hand, optimization models considering uncertainty are complex, involve non-linearities in probabilistic approaches and present large size, especially in these cases dealing with multi-stage stochastic scenarios. Due to those reasons, finding good models and methods to solve this type of problems has been a concern in this research field for the last decade. Several works can be found in the literature dealing with demand uncertainty. Gupta et al. (2000) present a two-stage stochastic model to solve the trade-off between inventory management and production cost under a context of demand uncertainty, which is handled applying a probabilistic framework through chance constraints. They handle three demand regimes: low, intermediate and high. The model is solved considering different levels of customer demand satisfaction for each product. They conclude that significant improvements can be achieved in terms of service levels to the customers with relatively small additional cost. This approach does not consider signing supply contracts with clients in order to reduce uncertainty impact. Regarding contract selection, Carrion, Conejo, and Arroyo (2007) propose a mathematical model to decide the forward contracts that an electricity retailer must sign in order to handle the demand uncertainty of end-users. The retailers face pool price uncertainties at the time of buying electricity via a contract; while at the time of selling it to clients they have to deal with the uncertainty in their demands. They propose a stochastic framework where uncertainty is modeled through time series and construct a scenario tree which is reduced using special reduction techniques. They maximize the expected profit

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Nomenclature

Sets

J	customers
K	products
P	materials
F	material families
T	period times
H	price levels
I	points of demand probability distribution
FP_{fp}	set that defines which materials p belong to each material family f

Parameters

ms	storage cost factor
$cost\_avg_{ft}$	average cost of material family f in period t
lc_{kj}	lost sales cost of product k from customer j
σ_{jkt}	standard deviation of product k and customer j in period t
$Loss_i$	cumulative probability of lost sales if level i of the demand probability distribution is selected
ε	service level values required applying the ε-constraint approach
CS	material stock capacity
Is_f	initial stock level for material family f
$Qmax_p$	maximal capacity from all suppliers to provide material p
α_{kf}	amount of material family f required to produce one unit of product k
Ir	interest rate
$cost_{pt}$	cost of material p in period t
μ_{jkh}	mean demand level of product in period t from customer j and price level h
$price_{hk}$	price of product k corresponding to price level h
δ_{hk}	contract discount for product k and price level h
ZP_i	values of the standard probabilistic variable in the point i of the demand probability distribution
ap_i	cumulative probability in point i of the demand probability distribution
NC	maximum number of contracts that can be signed in each period

Objective functions variables

NPV	net present value
sl	service level

Positive variables

m_{pt}	total cost of buying material p in period t
s_{ft}	stock level of material family f in period t
q_{pt}	amount of material p purchased in period t
qf_{ft}	amount of material family f purchased in period t
$cons_{ft}$	total amount of material family f required to satisfy selected demand level in period t
$savg_{ft}$	average material stock of material family f in period t
μ^*_{jkt}	mean demand level selected for product k, customer j in period t
OS_{jkt}	objective sale of product k for customer j in period t
$Income_{jkt}$	expected income of selling product k to customer j in period t
CI_{jkt}	expected income if contract is signed with customer j for product k in period t
NCI_{jkt}	expected income if no contract is signed with customer j for product k in period t
z_{jkt}	normal standard probabilistic variable used to describe

Boolean variables

Y_{jkt}	true if a contract is offered to customer j for product k in period t and false if no contract is offered to that customer, product and period

Binary variables

y_{jkt}	1 if a contract is offered to customer j for product k in period t and 0 otherwise
yh_{jkht}	1 if price and mean demand levels h are selected for customer j and product k in period t and 0 otherwise
yz_{jkit}	1 if expected demand level i is selected for customer j and product k in period t and 0 otherwise

for the retailer given a pre-specified risk level. Altintas, Erhun, and Tayur (2008) analyze the quantity discount contracts under demand uncertainty. They develop a multi-period model to capture the buyer's behavior under stochastic demand when the supplier provides an all-unit quantity discount. They analyze the special case when the transportation cost is high. These authors provide some guidelines and insights about the effectiveness of the discount parameters.

Most articles addressing this type of optimization problems consider an economic measure as the unique objective function. However, it is clear that it is not always possible to integrate all objectives in an economic metric. For that reason, some other approaches include multi-criteria programming because it is usual to have two or more objectives in conflict in this kind of problems. In these cases, there is no single global solution and the optimal set is given by Pareto curve (Pareto, 1971) where an increase in one objective means a decrease in the others. The challenging task is to find the best compromise solution within a set of possible optimal ones. The most extensive method used is given by the *ε-constraint* proposed by Haimes, Lasdon, and Wismer (1971). This approach has been mainly applied for supply chain design or enterprise-wide optimization, both characterized by a wide decisions scope and a large number of goals to satisfy (Guillén-Gosálbez & Grossmann, 2010; Guillén, Mele, Bagajewicz, Espuña, & Puigjaner, 2005; You & Grossmann, 2008).

In this approach, a stochastic model is proposed to deal with demand uncertainty. Customers demand is modeled as continuous random parameter with normal distribution and known mean and standard deviation (Zipkin, 2000). In order to avoid traditional non-linear terms involved in probabilistic approaches, the random normal variable is discretized in the formulation.

Sale contracts are also considered to mitigate the effect of unknown demand in the company profits. The model decides the set of customers to sign contracts according to their expected behavior to reduce demand uncertainty and decrease raw material safety stocks to satisfy unpredicted demand. In addition, the model also determines the portion of demand to be satisfied with traditional sales without using contracts. If contracts are not chosen, a target demand is considered which represents a level that the company is willing to satisfy in the horizon terms.

Another feature considered in this work is the selection of different price levels. Most planning optimization models from the literature consider that the product price is just one parameter that has previously been set by the company and a mean demand (and standard deviation) is known for that price. In this case, we introduce a demand elasticity function for each customer to model the relation between a set of possible prices and the corresponding

mean demand levels. Price elasticity of demand, measures the responsiveness of the quantity demanded of a good or service when there is a change in its price (Browning & Zupan, 2011). Then, prices are also decisions in this approach which gives a more real insight of the offer–demand relationship in the decision process. This demand model is part of a wider planning model. We propose a generalized disjunctive program to select material and quantities to purchase in order to satisfy uncertain demand in a multi-period horizon planning.

2. Problem statement

The problem consists of defining a mid-term tactical plan in order to face an uncertain environment. The main features of this problem are the following:

- Demand uncertainty is modeled using a probabilistic approach assuming that can be represented with normal probability distribution with known mean and standard deviation for each customer and product.
- Raw material purchase as well as material stock levels are considered, they are tactical decisions influenced by uncertain demand.
- "All-unit discount" contracts can be offered to customers in order to decrease the negative effect of demand uncertainty.
- Prices are also decision variables that are assigned to products in the model considering a demand elasticity function that defines the level of expected demand associated to each possible price.
- Two objective functions are considered. Net present value and service level are maximized. While the first one includes all incomes coming from expected sales and costs involved in the model decisions, the second one represents customer satisfaction and it is determined by the cumulative probability of satisfying demand. If service level is increased, purchase and stock costs are also higher.

As it was pointed out, one of the main decisions of this problem is to define what raw material purchase in each period in order to satisfy customer demand. It is assumed that raw materials are grouped into material families in order to give flexibility to purchase decisions, in this way a set of materials can be used for a given product.

Uncertain demand is modeled using a normal probability distribution with known mean (μ) and standard deviation (σ). This function is discretized in order to avoid non-linear terms in the formulation. For each term, a binary variable is introduced to select the level of demand the company is willing to satisfy. We use the standard random variable (z) to calculate the cumulative probability of each demand level (OS) and the expected lost sales (Loss) since we know that OS = $\mu + \sigma \times$ z. Further details regarding the values of these data please can be found in Stockburger (1996).

In order to decrease demand variability, the company can offer contracts to clients. If a contract is offered, it is assumed that this portion of demand is deterministic so the company will plan to purchase the exact required material to produce the amount of products forecasted (mean demand). This alternative shows a positive effect on purchase and stock costs but a negative effect on expected incomes since a discount is applied over all units sold if the contract is selected. On the other hand, if no contract is offered to a given customer and product it is assumed that this demand is uncertain. The service level objective is introduced in order to account for the maximum level of uncertain demand to satisfy. This service level is the probability that demand will be satisfied or, in other words, the probability that the company will not break stock.

Stock level decisions connect material purchase and demand decisions, thus in each period the expected inventory level is calculated considering the stock in the previous period plus the material purchase amount minus the consumption of material to satisfy the expected demand. The stock costs in the NPV are calculated considering the average level of materials in stock in each period and that unit storage cost is a proportion of the material family average cost. Stock capacity constraints are assumed as well as suppliers limitation for material purchase.

In this article we consider that if there is enough material in stock then the demand of products can be satisfied within the period term. This means that only stock for material is considered but not for final products. In this context, the definition of safety stock is an amount of material which is kept in stock in order to be able to satisfy an extra level of demand that exceeds the mean value. This extra demand level is determined by the service level as well as its standard deviation. The sum of this extra demand level and the mean expected level can be called a target demand or objective sales (OS_{jkt} variable in the model). In case no contract is selected for a given product k, customer j and period t, some safety stock must be determined because objective sales will be greater than mean demand.

Prices for final products are also decision variables in this problem. It is assumed that mean demand levels for each client and product are functions of the price selected. This means that a demand-elasticity function is introduced to model the relationship between price and mean demand level. A number of possible prices are considered for each client and product applying a discrete elasticity function.

Finally, we propose two objective functions in order to account for an economic target as well as another performance which measures the customer satisfaction. This second objective function is determined by the service level that represents the cumulative probability that demand will be satisfied. The main trade-off between these two objectives is given by stock costs. A higher service level means that more material stock is required to response to extra demand which, on the other hand, decreases the NPV of the company.

3. Problem formulation

As it was mentioned in the previous section, this problem presents two objectives to optimize. The first objective function is defined as the net present value during the planning horizon. Profits in each period are calculated according to the expected income, minus material purchases, lost sales costs and inventory holding costs. Materials p are grouped into families f in order to give flexibility to purchase decisions. In this way, different materials from a certain family can be used to satisfy product specifications.

$$Max \quad NPV = \sum_t \frac{\sum_k \sum_j Income_{jkt} - \sum_p m_{pt} - \sum_j \sum_k \sum_j Loss_i . yz_{jkit} . \sigma_{kj} . lc_{kj} - \sum_f savg_{ft} . ms . \cos t\_avg_{ft}}{(1+Ir)^t} \tag{1}$$

The NPV is calculated base on total incomes, $Income_{jkt}$, due to expected sales from each customer j, product k and period t minus the raw material purchase costs, m_{pt}, for each material p in period t. Loss sale costs are also considered in the next term, they are determined for each product k, customer j and period t. Since demand loss function ($Loss_i$) is discretized, each index i indicates one possible value for this function. Finally, inventory costs for each material family f in period t are also taken into account in the last term of this objective function.

The second objective is to maximize the service level which is given by the accumulative probability of satisfying demand. This

means that if for instance, service level is 50%, then only mean demand is satisfied. Any service level above this value supposes that the company is willing to satisfy a potentially higher demand. In this sense, the company must be prepared in terms of material purchase plan in order to be able to produce additional products if this extra demand level occurs. The increase in material purchase will also lead, naturally, to higher inventory costs.

The method used to formulate this two-objective problem is applying the ε-constraint approach. In this case, service level (sl) is considered the constrained objective, given by Eq. (2).

$$sl \geq \varepsilon \tag{2}$$

Eq. (3) shows that total initial stock of all families f in each time period t (s_{ft}) must be less than or equal to the material stock capacity CS. It is assumed that only raw materials are kept in stock, final products are manufactured according to customer orders.

$$\sum_f s_{ft} \leq CS, \quad \forall t \tag{3}$$

The stock at the beginning of each period for each raw material family f is given by Eq. (4) as the initial stock in the previous period plus the materials bought from suppliers in that period, $qf_{f(t-1)}$, minus the material consumption estimation $cons_{f(t-1)}$. This variable is determined by the maximum target demand the company aims to satisfy and the consumption of materials required to produce one unit of each product, as shown in Eq. (9).

$$s_{ft} = s_{f(t-1)} + qf_{f(t-1)} - cons_{f(t-1)}, \quad \forall f, \forall t > t1 \tag{4}$$

Eq. (5) establishes that in the first period, there is an initial stock given by Is_f:

$$s_{ft_1} = Is_f, \quad \forall f \tag{5}$$

Average stock level $savg_{ft}$ is estimated in Eq. (6) to determine the inventory costs in the NPV objective function. This level is estimated as half of: the initial stock in each period plus the quantity of material family ordered and the stock level at the beginning of next period.

$$savg_{ft} = \frac{s_{ft} + qf_{ft} + s_{f(t+1)}}{2}, \quad \forall f, \forall t \tag{6}$$

There is certain upper bound to each quantity q_{pt} that can be purchased of material p in period t. This limit is given by the suppliers' total capacity, $Qmax_p$, in Eq. (7). Note that in this case, materials are used instead of material families. Material families allows flexibility in the purchase plan since they consists of groups of material p that can be used indistinctly in the production process.

$$q_{pt} \leq Qmax_p, \quad \forall p, \forall t \tag{7}$$

Constraint of Eq. (8) determines the total amount bought of a material family qf_{ft} according to the materials p that belong to that family. This relationship is given by set FP_{fp}.

$$\sum_{p \in FP_{fp}} q_{pt} = qf_{ft}, \quad \forall f, \forall t \tag{8}$$

In order to calculate the consumption of material family in each period, given by variable $cons_{ft}$, it must be taken into account a target demand that the company wants to satisfy which can be greater than the mean demand forecasted. This corresponds to the objective sale OS_{jkt} for each product k, customer j in period t. The total raw material requirement for that OS_{jkt} is determined by Eq. (9) according to the unit material consumption α_{kf}.

$$\sum_j \sum_k OS_{jkt} . \alpha_{kf} = cons_{ft}, \quad \forall f, \forall t \tag{9}$$

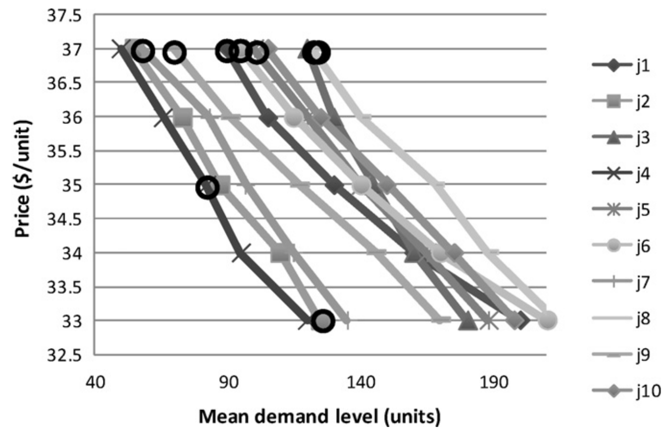

Fig. 1. Mean demand and price selected for product k_3 in period t_2.

Material purchase costs m_{pt} are calculated in the following Eq. (10) by multiplying the quantity ordered q_{pt} by the corresponding material cost $cost_{pt}$. Note that this parameter can vary during the horizon planning due to seasonal or inflation reasons.

$$m_{pt} \geq q_{pt} . cost_{pt}, \quad \forall p, \forall t \tag{10}$$

In order to take into account prices as decision variables in this model, a price–demand discrete function is assumed for each product and customer. Fig. 1 shows this situation for product k_3, which is a decreasing mathematical relationship between these two variables. In order to model this price–demand relationship, it is assumed that the company owns certain knowledge from their clients which could be obtained from marketing forecasts or previous negotiation processes with them. For each customer, the company can estimate the mean demand expected for each price offered. More elastic demand is shown when bigger changes in demand level are observed for each unit change in price. Once medium demand level μ^*_{jkt} is selected in Eq. (11), price is also determined. Note that μ^*_{jkt} is a decision variable while μ_{jkh} is a parameter indicating different mean values for demand of product k from client j when price level is h. From Eq. (11), yh_{jkht} is a binary variable which selects the price and demand level for each product and client. We have considered h possible demand and prices levels. This demand level is then used to calculate sales income in Eqs. (13) and (14).

$$\mu^*_{jkt} = \sum_h \mu_{jkh} . yh_{jkht}, \quad \forall j, \forall k, \forall t \tag{11}$$

The following constraint in Eq. (12) determines that one demand level h has to be selected for each product and customer in each time period. Note that more than one product can be ordered by the same customer j and different price levels can be selected for each of them.

$$\sum_h yh_{jkht} = 1, \quad \forall j, \forall k, \forall t \tag{12}$$

Disjunction (13) determines whether a contract will be offered to a customer for each product in each period time. If a contract is selected we assume demand uncertainty is negligible (i.e. $\sigma_{jkt} \sim 0$) so objective sales must satisfy at least mean demand level μ^*_{jkt}. Taking into account that $OS_{jkt_{kt}}$ is applied to determine the amount of raw material required, no safety stock is needed in this case. Otherwise, safety stock must be defined in order to address uncertain demand. This safety stock is determined indirectly by the additional demand ($z_{jkt}\,\sigma_{jkt}$) above the mean value selected in the second disjunctive term of Eq. (13) where variable z_{jkt} represents the standard

random variable with normal probability distribution. The amount of raw materials to satisfy this product demand is calculated in (8).

Expected income is determined in both cases. In the first one, we assume mean demand is served with the corresponding price and discount. In the second case, regular price is considered.

$$\begin{bmatrix} Y_{jkt} \\ OS_{jkt} \geq \mu^*_{jkt} \\ Income_{jkt} \leq CI_{jkt} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{jkt} \\ OS_{jkt} \geq z_{jkt}.\sigma_{jkt} + \mu^*_{jkt} \\ Income_{jkt} \leq NCI_{jkt} \end{bmatrix}, \quad \forall j, \forall k, \forall t \quad (13)$$

Contract income is determined in constraint (14) considering the mean demand selected, the corresponding price and the discount δ_{hk} offered to the customer. Note that since only one binary variable yh_{jkht} will be non-zero, then only one term on the right hand side (rhs) of Eq. (14) will be positive.

$$CI_{jkt} = \sum_h \mu_{jkh}.yh_{jkht}.price_{hk}.(1 - \delta_{hk}), \quad \forall j, \forall k, \forall t \quad (14)$$

Similarly, Eq. (15) establishes the expected income when no contract is selected to a given customer j and product k. As mentioned, only one term will be non-zero on the rhs of Eq. (15). In this case, since no contract is offered, no discount is considered and the normal price is applied to the expected demand level. Note also, that income is not overestimated since mean demand μ_{jkh} is used to calculate it. This means that even though the objective sale (OS_{jkt}) is higher, because represents the maximum potential demand level the company is willing to satisfy, the expected income only assumes that mean demand is expected to occur in order to allow a fair comparison between contract and no contract incomes.

$$NCI_{jkt} = \sum_h \mu_{jkh}.yh_{jkht}.price_{hk}, \quad \forall j, \forall k, \forall t \quad (15)$$

Uncertain demand distribution is discretized using binary variable yz_{jkit}. The normal standard variable z_{jkit} can assume i different values given by parameters ZP_i as shown in Eq. (16). This variable determines the additional demand for product k, customer j in period t that the company is willing to satisfy in case that no contract is signed.

$$z_{jkt} = \sum_i ZP_i.yz_{jkit}, \quad \forall j, \forall k, \forall t \quad (16)$$

Eq. (17) is a logical constraint establishing that if no contract is signed for a product k of customer j in period t, then one variable yz_{jkit} must be equal to 1 in order to determine variable z_{jkt} value. On the other hand, if a contract is selected ($y_{jkt} = 1$) no yz_{jkit} is positive. Note that binary variable y_{jkt} is equivalent to Boolean variable Y_{jkt} of disjunction (13).

$$\sum_i yz_{jkit} = 1 - y_{jkt}, \quad \forall j, \forall k, \forall t \quad (17)$$

Eq. (18) defines that certain service level sl must be satisfied if no contract is signed. This level is defined considering the cumulative probability ap_i associated to the value selected of z_{jkt} which is given by yz_{jkit}. Since sl is the second objective in this model, different values will be given in order to obtain several points from the Pareto curve.

$$\sum_i ap_i.yz_{jkit} \geq sl.(1 - y_{jkt}), \quad \forall j, \forall k, \forall t \quad (18)$$

The final equation in this formulation is given by an optional constraint that could be necessary if company policy restricts the amount of contracts they can offer to their clients (NC). In this case, Eq. (19) will be applied. In order to analyze the effect of this constraint in the formulation a special section will be considered in Section 4.

$$\sum_j \sum_k \sum_t y_{jkt} \leq NC \quad (19)$$

Table 1
Model size and performance.

Equations	Positive variables	Binary variables	CPUs
2381	1373	4600	116–471

Table 2
Pareto solutions obtained.

Difference % in NPV	NPV	Service Level	Difference % in Service Level	CPUs
23%	308579.1	**0.933**		165.75
17%	332782.6	0.89	5%	471.81
9%	364466.79	0.84	6%	190.66
5%	376874.92	0.77	9%	138.66
3%	387049.04	0.69	12%	116.33
1%	394086.4	0.59	17%	151.25
	398758.94	0.5	18%	148.23

4. Results

4.1. Multi-objective solutions

The formulation was posed in GAMS system using LogMIP (Vecchietti & Grossmann, 1999) to model disjunctive terms. The generalized disjunctive programming (GDP) problem is reformulated by LogMIP using convex hull relaxation which is solved as a MILP problem with CPLEX. The main advantage of this formulation is that contract decisions can naturally be posed in terms of discrete decision and generalized disjunctive programming. LogMIP allows the disjunction formulation in a natural way as they are presented in the original GDP formulation. Then it selects the relaxation (convex hull or big-M reformulation) according to the user input.

Several Pareto solutions are given according to the fluctuation of service level target (sl) and the resulting NPV. The examples consider 10 customers with 5 products to satisfy. For each product and customer, 5 price levels are evaluated with a price–demand discrete function as shown in Fig. 1. Uncertain demand function is discretized using 17 points. The horizon planning is given by 4 months where 4 material families are handled to group 13 raw materials. The model size and performance are presented in Table 1 considering all solutions obtained. According to Table 1, execution time in order to obtain Pareto solutions varies from 116 to 471 s. Table 2 shows the different solutions obtained and the percentage of variation in each objective comparing to the best possible solution for each objective. Note that higher service level implies additional inventory and material purchase costs. Furthermore, a higher service level has no positive effect on expected incomes (whether contract is offered or not) because they are calculated assuming the mean demand value and not the maximum potential demand. Then, when service level increases, the NPV diminishes. It can be considered that one compromised solution is the one highlighted in Table 2 since it is only 5% far from the best NPV value and involves a reasonable value for the service level (0.77), only 9% lower from the maximum value. This Pareto point will be further analyzed in this section.

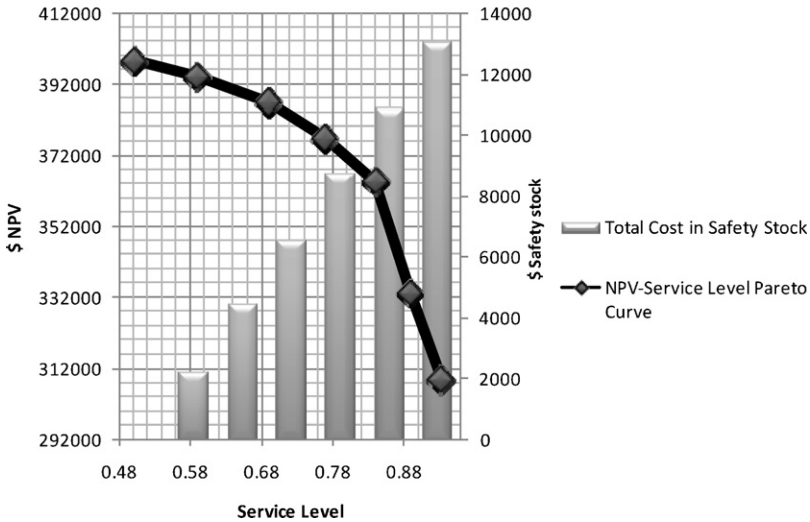

Fig. 2. Objective function and safety stock costs for Pareto solutions.

Some other interesting results come out from the multiple solutions. Fig. 2 shows the fluctuation of both objectives as well as the effect on safety stock costs. It is quite clear that in order to guarantee a higher service level the company needs to keep an increasing amount of material in storage as safety cost. The right vertical axis in Fig. 2 shows the values of this cost for the different solutions which also explains the decrease in the NPV value. Safety stock costs in Fig. 2 are calculating multiplying safety stocks by the average cost of material family ($cost\_avg_{ft}$) and the storage factor ms.

In addition, it is also important to analyze how costs structure changes from one solution to the other. Fig. 3 shows that even though raw material cost always plays a relevant role in total cost, stock cost are higher when service level is increased while lost sales cost diminishes in the same direction. From Table 3, it is noteworthy that total raw material cost also rises by increasing the service level since more material has to be bought in order to be able to satisfy a potential higher demand level (which means higher service level).

Regarding model decisions, the selected solution from the Pareto points obtained are analyzed. Fig. 1 shows mean demand and prices selected with black circles, for product k_3 for each customer j in period t_2. This decision is solved for all products and periods. In most cases, highest prices with lowest demand levels are preferred. The reason is not straightforward. Even though demand functions present high elasticity values, meaning that the percentage of increase in demand is higher than the percentage of decrease in price, average stock cost and material purchase costs also increase when mean demand does. In addition, there is a limitation in the number of contracts that can be ordered in each period. This also means that higher demand levels when no contract is offered imply an increase in safety stock cost in order to guarantee the service level assumed. So under some circumstances, many factors influence price and demand selection. It will be interesting to analyze the impact of no policy restriction regarding the number of contracts allowed on the solution.

Table 3
Present cost evolution in each Pareto solution.

Service level	Total costs	Material costs	Lost sales costs	Stock costs
0.5	277,110	213,098	37,133	27,157
0.59	281,415	226,539	26,172	28,704
0.69	279,254	232,060	17,872	29,322
0.77	274,643	233,447	11,535	29,661
0.84	281,785	243,463	7326	30,996
0.89	279,667	244,149	4475	31,043
0.933	283,956	249,598	2556	31,803

Table 4
Contract decision.

		j_1				j_2				j_3				j_4				j_5			
	y_{jkt}	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
Product	k_1	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
	k_2	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No
	k_3	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes	Yes
	k_4	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_5	No	No	No	No	No	No	No	No	No	No	No	Yes	No	No	No	No	No	No	No	No
		j_6				j_7				j_8				j_9				j_{10}			
Product	k_1	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
	k_2	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_3	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
	k_5	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No

No in grey and Yes in white.

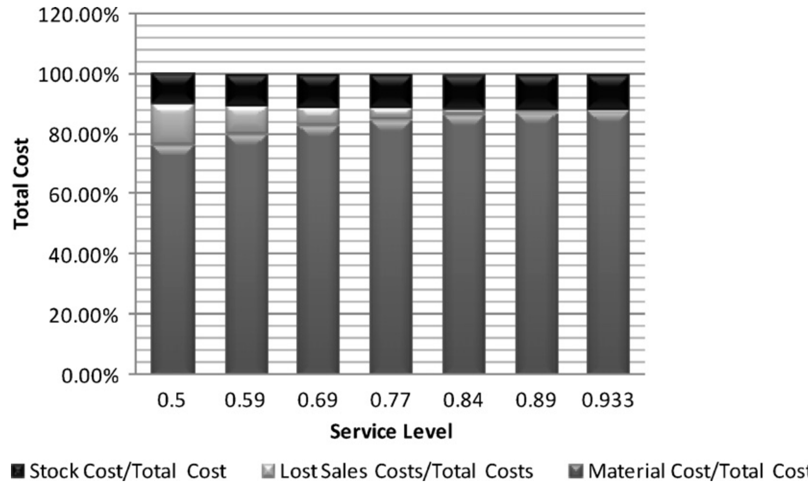

Fig. 3. Total cost structure depending on service level.

Table 5
Safety stock of material families.

Family	Periods			
	t_1	$t2$	t_3	$t4$
$f1$	1086	1086	1086	1100.25
$f2$	1719.98	1719.98	1719.98	1654.35
$f3$	2772.08	2772.08	2772.08	2772.08
$f4$	1104.25	1062.38	1062.38	1161.75

Another important result from the model is given by contract decisions. As it was mentioned, the number of contracts is restricted. In this example the limitation is 20 contracts per period. Table 4 shows to which customers j and for what products k the company should offer sales contract. Note that in all periods 20 contracts are selected meaning that this constraint is active in the solution. So, even though they offer a discount to the customer, profits are increased because no safety stock or lost sales have to be faced when contract is signed. The possibility of disregard demand uncertainty in these cases pays the decrease of income due to discount offered by the contract.

Due to the limitation in the number of contracts that can be offered in each period, the model has to choose in which cases contracts are more beneficial. A combination of factors plays a relevant role in this respect. On the one hand, discount offered to the customers can vary from one customer to the other as well as from one price level to another. In addition, customer standard deviation can be also different. If demand variability is high (which means high standard deviation) for a given customer and product, it is expected that offering a contract in that case is more beneficial than if a customer present a low variability. When a contract is offered, it means that the reduction of expected income due to the contract discount is better than the increase of purchase and stock cost due to uncertain demand.

For those cases where no contract is offered, safety stock must guarantee additional raw material availability in stock to cope with extra demand due to uncertain variations. These results are presented in Table 5. It is interesting to notice that safety stock level changes for each family and period to cope with demand variations when no contract is offered.

Average stock levels for material families fluctuate in the planning horizon as shown in Fig. 4. These value are calculated in the model according to material purchase, safety stock and consumption decisions. In order to compute safety stock of material families from Table 5 we calculate the total amount of material required to satisfy objective sales (variable OS_{jkt}) minus the total amount of

	t1	t2	t3	t4
f1	5346.5	5364.5	5346.5	5308.63
f2	4328.64	4328.64	4328.64	4258.33
f3	6856.74	6876.54	6856.74	6856.74
f4	6030.31	6010.94	5988.44	5988.63

Fig. 4. Average material stock evolution.

materials required to satisfy mean demand level (μ^*_{jkt}). Note that if we compare safety stock from Table 5 with the average stock level from Fig. 4 it can be clearly concluded that safety stock are actually a small proportion of this expected variable. This result shows that company decisions can definitely decrease demand uncertainty, which is one of the main positive effects of offering contracts to customers.

In this example for instance, since no contract was chosen in the case of product k_1 bought by customer j_1 in the first period, mean demand was selected with 200 units but target demand is defined in 271.25 units. For this additional demand of 71.25 units, raw material must be stored as safety stock. If demand exceeds this target, lost sales will occur so expected lost sales are determined by the right area under the curve in Fig. 5.

4.2. Contract limitation

In this section, we analyze the impact of changing the limitation in the number of contracts. As it was pointed out in the previous results shown, in every period this constraint is active in the solution meaning that the maximal number of contracts was always chosen.

Fig. 6 shows two relevant results. On the one hand, it is remarkable how the proportion of profits with and without contracts changes as the maximum quantity of contracts is increased. On the other, there is a clear improvement in the net present value which is the economic objective function in this model. From the first point in which only 10 contracts per period are allowed to the

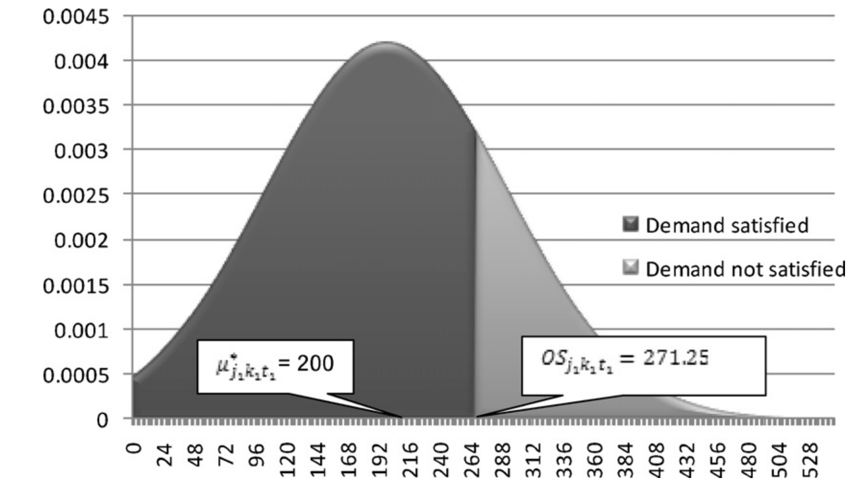

Fig. 5. Demand decisions for customer j_1, product k_1 in period t_1.

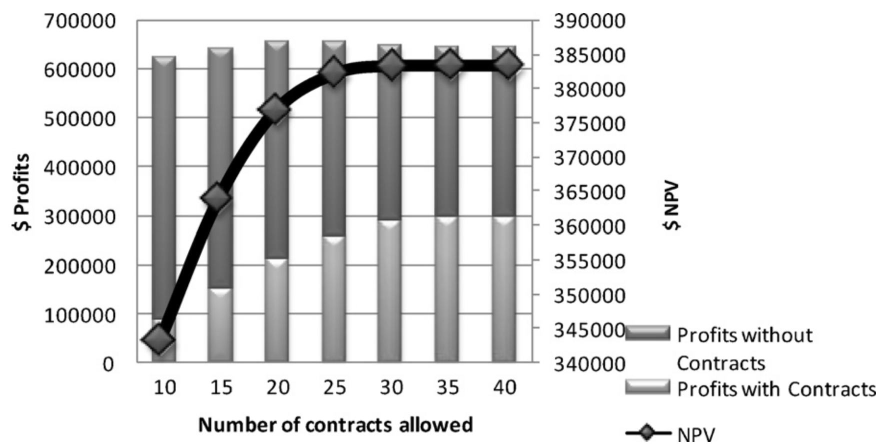

Fig. 6. Results sensitivity to the change in the maximal number of contracts.

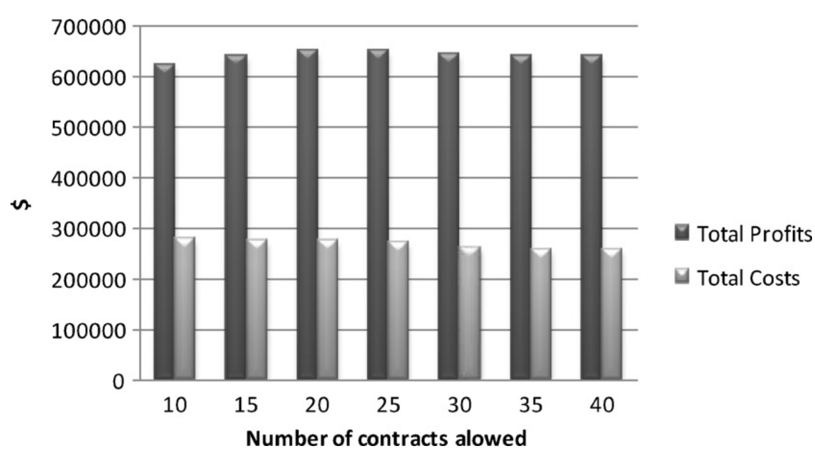

Fig. 7. Profits and costs evolution when changing the number of contracts allowed.

last one when 40 contracts can be selected, there is almost 12% increase in the NPV. In Fig. 7 the fluctuation of present costs and present profits is shown for each constraint level studied. It is also noteworthy that the optimal quantity of contracts is given between 30 and 35 since no change in the NPV value is observed from 35 to 40 contracts allowed. One interesting conclusion of this study is that this increase in the economic objective function does not compromise the demand satisfaction since service level is the same in all scenarios considered in this section. Then, a more flexible policy regarding the decision of offering contracts to customer can improve the complete set of Pareto-solutions.

4.3. Demand variability analysis

In order to study if offering contracts to customer can effectively decrease uncertainty effects on the company, the relationship between demand variability and contract decisions is presented in this section. Demand standard deviation was increased for all

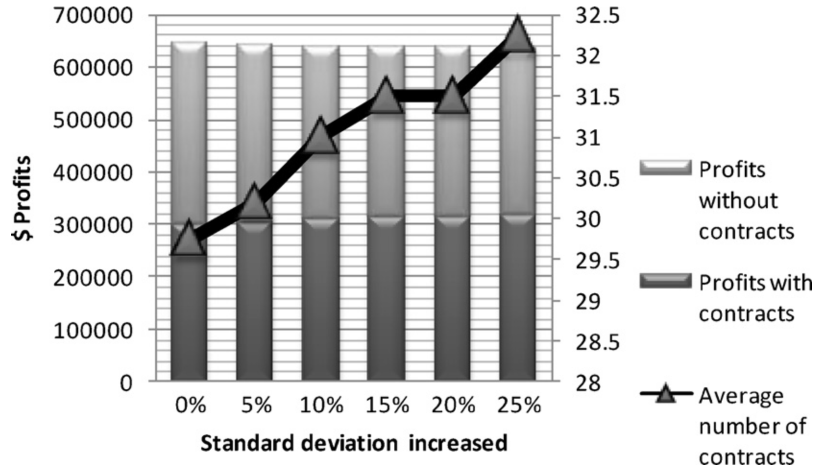

Fig. 8. Profits and number of contracts variations for different levels of demand standard deviation.

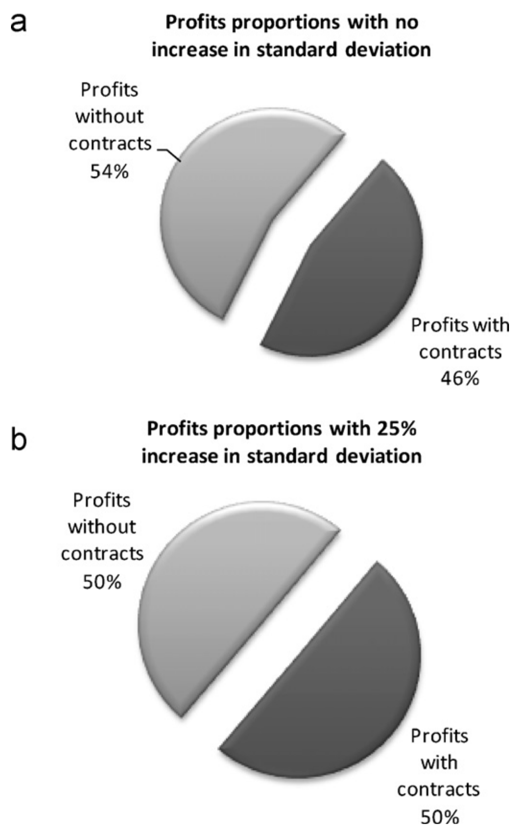

Fig. 9. (a) Profit proportions with no increase in standard deviation; (b) profit proportions with 25% increase in standard deviation.

customers and products and the number of average contracts selected in each time period is analyzed. From Fig. 8, it can be seen that the number of contracts grow from 29.5 to 32.5 in average, when there is a 25% higher standard deviation. In addition also the structure of profits from contracts changes. Fig. 9a and b presents that an additional 4% of profits come from contracts when standard deviation is increased in 25%. In conclusion offering contracts to customer can attenuate the effect of demand uncertainty and this result is even more relevant when demand variability is high. The negative effect of offering discounts to customer is paid by the fact that a certain proportion of demand can be guaranteed by the customer commitments.

5. Conclusions

The present work studies a multi-objective model in which service levels as well as net present value are both optimized in a problem where purchase plan, raw material levels in stock, safety stock and prices are decided in a mid-term planning horizon. In order to take into account the effect of prices decisions, a price-elasticity function is introduced in which different expected demand levels are associated with several prices. The model brings some interesting results that could help managers to make decisions regarding what level of demand is convenient according to price elasticity.

With the aim of mitigating the effect of demand uncertainty on company revenues, contracts with customers are also analyzed as decision variables in the model. In this respect, it is decided whether contract should be offered to customers in order to diminish the effect of uncertainty but offering the customer a discount over the regular price. When no contract is offered to a customer, it is considered that safety stock must be held to be able to satisfy the company demand target. Lost sales due to extra demand above that target level are also penalized in the objective function. Higher demand variability also stresses the importance of considering demand contracts as a tool against this uncertain variable.

It is also noticed that a flexible policy regarding the number of possible contracts to sign offer better economics results without decreasing demand service level. Finally, short execution time shows that this model can be applied to analyze several real scenarios to decide material purchase plan, inventory levels, prices and demand target in a medium term horizon planning.

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