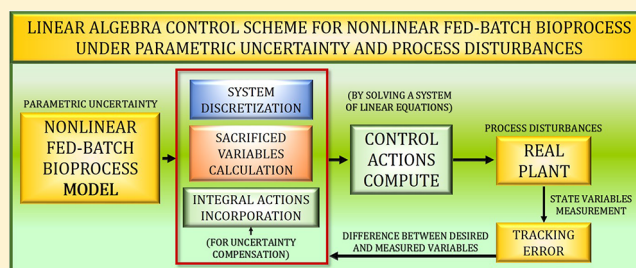


# Tracking Control of Optimal Profiles in a Nonlinear Fed-Batch Bioprocess under Parametric Uncertainty and Process Disturbances

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**ABSTRACT:** The problem of optimal profiles tracking control under uncertainties for a nonlinear fed-batch bioprocess is addressed in this paper. Based on the results reported by Pantano et al. [*Ind. Eng. Chem. Res.* 2017, 56, 6043], this work aims to improve the control system response against parametric uncertainty and process disturbances. The methodology is simple and easy to implement, but with excellent results. The design parameters are optimized by a randomized Monte Carlo algorithm. Besides, demonstration of the tracking error convergence to zero when the system is subjected to uncertainties is included in the article. The control system performance is tested through simulations, showing the improvement achieved.



## 1. INTRODUCTION

Bioreactor performance is determined not only by productivity but also by process quality, which is mainly affected by the disturbances in the process variables. Therefore, finding a way to control these distortions is the main task to guarantee quality.<sup>2</sup>

During the past decade, many researchers further investigated several control techniques applied to different bioprocesses,<sup>3–7</sup> especially after the implementation of “quality by design” for biopharmaceuticals by the U.S. Food and Drug Administration (FDA).<sup>8</sup> Particularly in fed-batch bioreactors many strategies were studied to improve the efficiency and reproducibility of bioprocesses.<sup>9–14</sup> However, the problem that arises is the gap between scientific research and the industry requirements. For example, usually research works on optimization and control strategies rarely consider model uncertainties, which are unavoidable in industrial processes;<sup>15–17</sup> this could lead to a poor real-life representation and, consequently, to a bad performance with severe risks.<sup>18–20</sup>

Unfortunately, the high degree of nonlinearity of the bioprocesses and the lack of high-quality experimental data make modeling the system a challenge. Therefore, the resulting model presents a high degree of uncertainty. As a consequence, there are several mathematical models for the same biological process, including different structures and parameters but concordant with the available information on said processes.

Uncertainties are one of the main obstacles in the development of advanced controllers for high-accuracy trajectory tracking control.<sup>21</sup> Not surprisingly, therefore, parametric uncertainty has remained high on the agenda of unsolved problems in control for the past three decades.<sup>22</sup> The main types of uncertainties that can be considered in a bioprocess are the uncertain values of parameters (most frequent), time-varying model parameters, uncertain non-

linearities, and unmodeled dynamics, among others. Besides that, there are external disturbances that are not modeled and affect the process variables, too.<sup>23–25</sup>

Many existing works focus on the model uncertainty quantification due to the lack of knowledge of model parameters and the underlying physics by combining the results from both computer simulations and physical experiments.<sup>26–31</sup> Generally, one of the most-used strategies for the model parameter identification and/or estimation involves an off-line optimization using a nominal model of the process.<sup>32–40</sup> The main disadvantage of this methodology is that the variability of microorganisms decreases the possibility of batch-to-batch repeatability.

Other techniques that are used for model parameter estimation, which try to improve the results obtained with nominal optimization methodologies, are those called “run-to-run” optimization, in which the information is extracted from previous runs and used to optimize the operation of subsequent ones.<sup>41–48</sup> However, the value of this improvement should be critically evaluated regarding the low-variability objective that is so important in the pharmaceutical and polymer industries.<sup>49</sup>

In the past two decades, several research projects have ventured into on-line optimization of the model parameters.<sup>50–54</sup> This kind of optimization is difficult to perform since the available models might only be locally valid and thus inappropriate for predicting final concentrations.<sup>49</sup>

On the other hand, several feedback control strategies are studied to deal with bioprocess uncertainties. Adaptive control

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80 techniques, for example, are an important option to control  
81 bioreactors under uncertainties.<sup>29,55–57</sup> Also, fuzzy control  
82 systems have been successfully applied to several bioreac-  
83 tors.<sup>58,59</sup> Alternative methodologies for controlling biopro-  
84 cesses under model uncertainties are trajectory-based con-  
85 trol,<sup>60</sup> model predictive control,<sup>61</sup> and hybrid control.<sup>62</sup>  
86 However, the application of these techniques in a real plant  
87 is quite difficult because of its complex design and  
88 implementation.

89 For the particular case of recombinant protein production  
90 with a double feed stream (Lee–Ramirez bioreactor<sup>63</sup>),  
91 Pantano et al.<sup>1</sup> developed a control strategy for tracking  
92 control of predefined optimal profiles. In that work, the control  
93 design includes a neural network state estimation for  
94 unmeasurable variables, enabling the closed loop implementa-  
95 tion. The methodology is characterized by its simplicity,  
96 versatility, and accuracy. Advanced mathematical knowledge is  
97 not required for design; only basic understanding of linear  
98 algebra is needed. The main technique advantage is that  
99 control actions computed are obtained solving linear system  
100 equations, ensuring the convergence to zero of tracking errors.  
101 Besides, that control technique has been applied successfully in  
102 several systems,<sup>5,64–69</sup> and therefore has a high potential for  
103 applicability in all bioprocess environments.

104 The objective of this work is to design an improved control  
105 strategy to achieve the tracking control of optimal protein  
106 concentration, cell density, and volume profiles obtained in ref  
107 70, with a minimum tracking error, when the system is  
108 subjected to parametric uncertainty and perturbations in the  
109 process. To achieve this goal, a term of uncertainty is  
110 incorporated into the original controller design, which is  
111 used to represent a wide range of model mismatches as well as  
112 perturbations in the process. Then, some integral terms of  
113 tracking error are incorporated into the controller design to  
114 compensate the uncertainty. In a similar way to the original  
115 methodology, the necessary and sufficient conditions are  
116 analyzed so that the system has an exact solution, but now  
117 taking into account the new terms. Finally, the control actions  
118 are found solving linear system equations.

119 The main contribution of this work is the extension of the  
120 proposed methodology in Pantano et al.,<sup>1</sup> to provide a positive  
121 answer to the challenging problem of tracking control in  
122 multivariable nonlinear systems with additive uncertainty and  
123 process disturbances. In this way, a more realistic problem can  
124 be solved taking into account that a complete and exact  
125 knowledge of the process model is never possible and, usually,  
126 the external perturbations in the real process dynamics are  
127 unavoidable. It is important to emphasize that this approach is  
128 achieved without significantly increasing the controller design  
129 complexity. Also, another important contribution is the  
130 demonstration of convergence to zero of the tracking error  
131 under parametric uncertainties and process disturbances.

132 The paper is organized as follows. First, a review of the  
133 original controller design is presented in section 2. Then, the  
134 extended controller methodology for contemplation of  
135 uncertainties developed in section 3. The results and  
136 discussion of the simulation tests, which include the adjust-  
137 ment of the parameters of the controller and the control  
138 system under parametric uncertainty and perturbations in the  
139 control actions, are shown in section 4. Finally, section 5  
140 outlines the main conclusions of this work.

## 2. ORIGINAL CONTROLLER DESIGN

The controller methodology proposed in ref 1 is mainly based  
on approximating the mathematical model (1) by employing  
Euler method, which, despite its simplicity, presents good  
results. The aim of this controller design is to find the control  
actions values that follow desired paths with minimal tracking  
errors.

The case study proposed in ref 1 for control is the Lee–  
Ramirez fed-batch bioreactor. The mathematical model is  
taken from Tholudur and Ramirez.<sup>71</sup>

The fed-batch bioreactor is described by the following  
model (1).

$$\begin{aligned}\dot{x}_1 &= u_1 + u_2 \\ \dot{x}_2 &= x_2\mu - \frac{u_1 + u_2}{x_1}x_2 \\ \dot{x}_3 &= \frac{u_1N}{x_1} - \frac{u_1 + u_2}{x_1}x_3 - \frac{\mu}{Y}x_2 \\ \dot{x}_4 &= x_2R - \frac{u_1 + u_2}{x_1}x_4 \\ \dot{x}_5 &= \frac{u_2I}{x_1} - \frac{u_1 + u_2}{x_1}x_5 \\ \dot{x}_6 &= -K_1x_6 \\ \dot{x}_7 &= K_2(1 - x_7)\end{aligned}\quad (1)$$

where

$$\mu = \left( \frac{\mu_{\max}x_3}{K_{CN} + x_3\left(1 + \frac{x_3}{K_S}\right)} \right) \left( x_6 + x_7 \frac{K_{CI}}{K_{CI} + x_5} \right) \quad (2)$$

$$R = \left( \frac{f_{\max}^0 x_3}{K_{CN} + x_3\left(1 + \frac{x_3}{K_S}\right)} \right) \left( \frac{f_I^0 + x_5}{K_I + x_5} \right) \quad (3)$$

$$K_1 = K_2 = \frac{k_{1X}x_5}{K_{IX} + x_5} \quad (4)$$

A more detailed description of the process can be found in  
ref 1. The nominal model parameter sets were validated and  
presented by Lee and Ramirez.<sup>72</sup> The glucose feeding rate,  $u_1$   
(L/h), and inducer feeding rate,  $u_2$  (L/h), are the two available  
sources as control actions for the fed-batch bioreactor. The  
reactor volume  $x_1$ , cell density  $x_2$ , and foreign protein  
concentration  $x_4$  are the variables whose optimal profiles are  
proposed to follow.

**Controller Methodology.** A differential equation system  
can be approximated according to the numerical integration  
rule of Euler as follow:

$$\dot{x}_i = \frac{x_{i,n+1} - x_{i,n}}{T_s} \quad (5)$$

where  $x_i$  is the  $i$  state variable in  $n$  and  $n + 1$  time instants, and  
 $T_s$  is the sampling time.

Although there exist general numerical computation  
algorithms in the literature for model approximation,<sup>47,73,74</sup>  
the Euler rule is a proper choice for this case since the  
approximation is only used to find the best manner to go from

175 the current state to the following one, and not to duplicate the  
 176 entire system evolution.  
 177 Taking into account eq 5, the mathematical model that  
 178 represents the bioprocess can be rewritten, as follows:

$$\begin{pmatrix} \mathbf{z}_{n+1} \\ x_{1,n+1} \\ x_{2,n+1} \\ x_{3,n+1} \\ x_{4,n+1} \\ x_{5,n+1} \end{pmatrix} = \begin{pmatrix} \mathbf{z}_n \\ x_{1,n} \\ x_{2,n} \\ x_{3,n} \\ x_{4,n} \\ x_{5,n} \end{pmatrix} + \dots$$

$$+ \begin{pmatrix} \xi_n(\mathbf{z}_n) \\ 0 \\ x_{2,n}\mu \\ -\frac{x_{2,n}\mu}{Y} \\ x_{2,n}R \\ 0 \end{pmatrix} + \begin{pmatrix} \beta_n(\mathbf{z}_n, \mathbf{u}_n) \\ 1 & 1 \\ -\frac{x_{2,n}}{x_{1,n}} & -\frac{x_{2,n}}{x_{1,n}} \\ N - x_{3,n} & -x_{3,n} \\ x_{1,n} & x_{1,n} \\ -\frac{x_{4,n}}{x_{1,n}} & -\frac{x_{4,n}}{x_{1,n}} \\ x_{1,n} & x_{1,n} \\ -\frac{x_{5,n}}{x_{1,n}} & I - x_{5,n} \\ x_{1,n} & x_{1,n} \end{pmatrix} \begin{pmatrix} \mathbf{u}_n \\ u_{1,n} \\ u_{2,n} \end{pmatrix} T_s \quad (6)$$

179  
 180 Denote by  $\mathbf{z}_n$  and  $\mathbf{z}_{n+1}$  the state vectors at the current time and  
 181 at the next one, respectively.  $\xi_n(\mathbf{z}_n)$  and  $\beta_n(\mathbf{z}_n, \mathbf{u}_n)$  are the input  
 182 matrices and  $\mathbf{u}_n$  is the control actions vector.  
 183 Then, in a generic form, the system can be expressed as  
 184 follows:

$$185 \quad \mathbf{z}_{n+1} = \mathbf{z}_n + [\xi(\mathbf{z}_n) + \beta(\mathbf{z}_n, \mathbf{u}_n)\mathbf{u}_n]T_s \quad (7)$$

186 Now, assuming a proportional approaching to the error:

$$187 \quad \mathbf{z}_{\text{ref},n+1} - \mathbf{z}_{n+1} = K(\mathbf{z}_{\text{ref},n} - \mathbf{z}_n) \quad (8)$$

188 with

$$\begin{pmatrix} \mathbf{z}_{\text{ref},n+1} \\ x_{1,\text{ref},n+1} \\ x_{2,\text{ref},n+1} \\ x_{3,\text{ref},n+1} \\ x_{4,\text{ref},n+1} \\ x_{5,\text{ref},n+1} \end{pmatrix} - \begin{pmatrix} \mathbf{z}_{n+1} \\ x_{1,n+1} \\ x_{2,n+1} \\ x_{3,n+1} \\ x_{4,n+1} \\ x_{5,n+1} \end{pmatrix} = K \left( \begin{pmatrix} \mathbf{z}_{\text{ref},n} \\ x_{1,\text{ref},n} \\ x_{2,\text{ref},n} \\ x_{3,\text{ref},n} \\ x_{4,\text{ref},n} \\ x_{5,\text{ref},n} \end{pmatrix} - \begin{pmatrix} \mathbf{z}_n \\ x_{1,n} \\ x_{2,n} \\ x_{3,n} \\ x_{4,n} \\ x_{5,n} \end{pmatrix} \right) \quad (9)$$

190 where  $x_{i,\text{ref},n}$  and  $x_{i,\text{ref},n+1}$  are the reference values obtained from  
 191 the optimal operating profiles in the  $n$  time and the next  
 192 sample time, respectively,  $k_i$  is the controller parameter for the  $i$   
 193 variable, and  $K$  is the control parameter matrix;  $\mathbf{e}_n$  and  $\mathbf{e}_{n+1}$  are

the tracking errors (difference between the reference and  
 actual profiles).

Then, the immediately reachable value for the state variables  
 is

$$\mathbf{z}_{n+1} = \mathbf{z}_{\text{ref},n+1} - K(\mathbf{z}_{\text{ref},n} - \mathbf{z}_n) \quad (10) \quad 198$$

It is important to remark that the optimal profiles to follow  
 were obtained by an open-loop simulation of the bioprocess  
 using the optimal feeding policies achieved by Balsa-Canto et  
 al.<sup>70</sup>

In this way, the actual state variables in the following  
 sampling time ( $x_{i,n+1}$ ) only depend on the reference profiles,  
 the actual state variables at the current time, and the controller  
 parameters (all values are known).

Consequently, substituting (10) in (6) and rearranging the  
 system equations:

$$\begin{pmatrix} \mathbf{A} \\ 1 & 1 \\ -1 & -1 \\ (N - x_{3,n}) & -x_{3,n} \\ -1 & -1 \\ -1 & (I - x_{5,n}) \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ u_{1,n} \\ u_{2,n} \end{pmatrix} + \begin{pmatrix} \mathbf{b} \\ \left( \frac{x_{1,\text{ref},n+1} - k_1(x_{1,\text{ref},n} - x_{1,n}) - x_{1,n}}{T_s} \right) \\ \left( \frac{x_{2,\text{ref},n+1} - k_2(x_{2,\text{ref},n} - x_{2,n}) - x_{2,n}}{T_s} \right) \frac{x_{1,n}}{x_{2,n}} - x_{2,n}\mu \\ \left( \frac{x_{3,\text{ref},n+1} - k_3(x_{3,\text{ref},n} - x_{3,n}) - x_{3,n}}{T_s} \right) x_{1,n} + \frac{\mu}{Y} x_{2,n} x_{1,n} \\ \left( \frac{x_{4,\text{ref},n+1} - k_4(x_{4,\text{ref},n} - x_{4,n}) - x_{4,n}}{T_s} \right) \frac{x_{1,n}}{x_{4,n}} - R x_{2,n} \frac{x_{1,n}}{x_{4,n}} \\ \left( \frac{x_{5,\text{ref},n+1} - k_5(x_{5,\text{ref},n} - x_{5,n}) - x_{5,n}}{T_s} \right) x_{1,n} \end{pmatrix} \quad (11) \quad 209$$

The optimal profiles to follow are those corresponding to  $x_1$ ,  
 $x_2$ , and  $x_4$ , which are known. The only unknown variables of  
 this system are defined as “sacrificed variables”  $x_{i,\text{ez}}$ ,  
 corresponding in this case to  $x_{3,\text{ez}}$  and  $x_{5,\text{ez}}$ . As can be seen in  
 eq 11, the system normally has no solution (five equations and  
 two unknowns). Therefore, the key of the control technique  
 proposed in ref 1 is that the values adopted by “sacrificed  
 variables” force the equation system (11) to have an exact  
 solution, which implies the tracking error is not only minimal,  
 but is equal to zero.

As mentioned above, the equation system (11) does not  
 have an exact solution; therefore, to accomplish the target of  
 this control methodology, that system must have an exact  
 solution. Then, a Gaussian elimination process is carried out to  
 find the necessary and sufficient condition for the system to  
 have an exact solution (the resultant expression can be seen in  
 ref 1). Therefore, the unknown variables,  $x_{3,\text{ez}}$  and  $x_{5,\text{ez}}$   
 (“sacrificed variables”) are computed for each sampling time.  
 Once the values of the sacrificed variables are found, the  
 system (11) has an exact solution and can be solved by the

230 least-squares method for the calculation of the control variables  
231 ( $u_{1,n}$  and  $u_{2,n}$ ,  $\mathbf{u}$  vector).

$$232 \quad (\mathbf{A}^T \mathbf{A}) \mathbf{u} = \mathbf{A}^T \mathbf{b} \Rightarrow \mathbf{u} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \quad (12)$$

233 The control actions ( $u_{1,n}$  and  $u_{2,n}$ ) applied at time  $n$  allow  
234 following the desired trajectories with a minimal error.

235 **Tracking Error.** The tracking error is defined as the value  
236 of the difference between the reference and real trajectories. In  
237 a generic way, the tracking error for each state variable is  
238 defined as

$$239 \quad e_{i,n} = x_{i,\text{ref},n} - x_{i,n}, \quad \text{for } i = 1, 2, 3, 4, 5 \quad (13)$$

240 The dimensionless tracking error (for desired variables) at  $n$   
241 instant of time is calculated as

$$242 \quad \|e_{\text{ad},n}\| = \sqrt{\left(\frac{e_{1,n}}{x_{1,\text{ref},\text{max}}}\right)^2 + \left(\frac{e_{2,n}}{x_{2,\text{ref},\text{max}}}\right)^2 + \left(\frac{e_{4,n}}{x_{4,\text{ref},\text{max}}}\right)^2} \quad (14)$$

243 and the total tracking error

$$244 \quad E_T = T_s \sum_n \|e_{\text{ad},n}\| \quad (15)$$

245 The reference final values for the desired variables are  $x_{1,\text{ref},\text{max}}$   
246 = 1.9 L,  $x_{2,\text{ref},\text{max}} = 13.92$  g/L, and  $x_{4,\text{ref},\text{max}} = 3.1$  g/L.

247 Note that the total tracking error (TTE, eq 15) is  
248 dimensionless.

249 Taking into account eqs 8 and 11, the original control  
250 system proposed in ref 1 (that did not take into account  
251 parametric uncertainties or process perturbations for the  
252 controller design) can be denoted as

$$253 \quad \begin{matrix} \mathbf{e}_{n+1} \\ \begin{pmatrix} e_{1,n+1} \\ e_{2,n+1} \\ e_{3,n+1} \\ e_{4,n+1} \\ e_{5,n+1} \end{pmatrix} \end{matrix} = \begin{matrix} \text{bounded linearity} \rightarrow 0 \\ \begin{pmatrix} k_1 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_5 \end{pmatrix} \end{matrix} \begin{matrix} \mathbf{e}_n \\ \begin{pmatrix} e_{1,n} \\ e_{2,n} \\ e_{3,n} \\ e_{4,n} \\ e_{5,n} \end{pmatrix} \end{matrix} + \dots + T_s \begin{matrix} \text{bounded nonlinearity} \rightarrow 0 \\ \begin{pmatrix} 0 & 0 \\ -x_2 e_{3,\text{ez},n} \mu_{\lambda} & -x_2 e_{5,\text{ez},n} \mu_{\theta} \\ 0 & 0 \\ -x_2 e_{3,\text{ez},n} R_{\lambda} & -x_2 e_{5,\text{ez},n} R_{\theta} \\ 0 & 0 \end{pmatrix} \end{matrix} \begin{matrix} \mathbf{E}_n \\ \begin{pmatrix} E_{1,n} \\ E_{2,n} \\ E_{3,n} \\ E_{4,n} \\ E_{5,n} \end{pmatrix} \end{matrix} \quad (16)$$

254 Equation 16 demonstrates that the tracking errors for all  
255 variables tend to zero when  $0 < k_i < 1$ ,  $i = 1, 2, 3, 4, 5$ , and  $n \rightarrow$   
256  $\infty$ . A demonstration can be seen in ref 1.

### 3. CONTROLLER DESIGN UNDER PARAMETRIC UNCERTAINTY AND PROCESS DISTURBANCES

258 In this section, a methodology for the parametric uncertainties  
259 and process disturbance handling is presented.

260 In order to quantify the model mismatch and process  
261 disturbances, an additive uncertainty is incorporated into the  
262 original controller design. According to eq 7, the real  
263 bioprocess model is assumed:

$$\begin{matrix} \text{real system} \\ \text{model} \\ \mathbf{z}_{n+1} = \mathbf{z}_n + [\xi(\mathbf{z}_n) + \beta(\mathbf{z}_n, \mathbf{u}_n)] T_s + \mathbf{E}_n \end{matrix} \quad (17) \quad 264$$

where  $\mathbf{E}_n$  quantifies the uncertainty. Note that this term of  
265 uncertainty can be used to model a wide class of model  
266 mismatches as well as perturbed systems. 267

The mismatch model and external perturbations might  
268 depend on the state variables or the system input. Therefore,  
269 considering a real plant:  $\mathbf{z}_{n+1} = g(\mathbf{z}_n, \mathbf{u}_n)$ , the additive  
270 uncertainty can be expressed as  $\mathbf{E}_n = g(\mathbf{z}_n, \mathbf{u}_n) - \hat{g}(\mathbf{z}_n, \mathbf{u}_n)$ ,  
271 where  $\hat{g}(\mathbf{z}_n, \mathbf{u}_n)$  is the nonlinear system model in discrete time.  
272 Now, note that if  $\mathbf{z}$  and  $\mathbf{u}$  are bounded and  $g$  is Lipschitz, as it  
273 will be assumed, then  $\mathbf{E}_n$  can be modeled as a bounded  
274 uncertainty.<sup>75,76</sup> 275

Analogously to the procedure followed for eq 16, but now  
276 taking into account the additive uncertainty, the next  
277 concluding expression is obtained: 278

$$\begin{matrix} \begin{pmatrix} e_{1,n+1} \\ e_{2,n+1} \\ e_{3,n+1} \\ e_{4,n+1} \\ e_{5,n+1} \end{pmatrix} = \begin{pmatrix} k_1 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_5 \end{pmatrix} \begin{pmatrix} e_{1,n} \\ e_{2,n} \\ e_{3,n} \\ e_{4,n} \\ e_{5,n} \end{pmatrix} + \dots \\ + T_s \begin{matrix} \text{bounded nonlinearity} \rightarrow 0 \\ \begin{pmatrix} 0 & 0 \\ -x_2 e_{3,\text{ez},n} \mu_{\lambda} & -x_2 e_{5,\text{ez},n} \mu_{\theta} \\ 0 & 0 \\ -x_2 e_{3,\text{ez},n} R_{\lambda} & -x_2 e_{5,\text{ez},n} R_{\theta} \\ 0 & 0 \end{pmatrix} \end{matrix} \begin{matrix} \mathbf{E}_n \\ \begin{pmatrix} E_{1,n} \\ E_{2,n} \\ E_{3,n} \\ E_{4,n} \\ E_{5,n} \end{pmatrix} \end{matrix} \end{matrix} \quad (18) \quad 279$$

Looking at eq 18, it can be seen how the additive uncertainty  
280 directly affects the tracking error. Anyway, the presence of the  
281 term  $\mathbf{E}_n$  makes the tracking error not converge to zero. 282

The main contribution of this work is to compensate the  
283 uncertainty and achieve the tracking error convergence to zero  
284 when the process moves forward. Therefore, the objective is to  
285 compensate the uncertainty and achieve the tracking error  
286 convergence to zero when the process moves forward. 287

**Integral Action.** In order to deal with the additive  
288 uncertainty effect on the tracking error, the incorporation of  
289 an integral action in the state variable system is proposed.  
290 Therefore, depending on the supposition of the time variation  
291 of  $\mathbf{E}_n$ , a series of the tracking error integrators are added in the  
292 control actions calculation. 293

In a real system, it is assumed that the effects of additive  
294 uncertainties on tracking errors are unknown and each  
295 component of  $E_{\phi,n}$  can be represented by a polynomial of  $m$   
296 order. 297

As definitions, the first order difference for additive  
298 uncertainty is  $\delta \mathbf{E}_n = \mathbf{E}_{n+1} - \mathbf{E}_n$ , the second order difference  
299  $\delta^2 \mathbf{E}_n = \delta(\delta \mathbf{E}_n) = \delta(\mathbf{E}_{n+1} - \mathbf{E}_n) = \mathbf{E}_{n+2} - 2\mathbf{E}_{n+1} + \mathbf{E}_n$ ;  
300 analogously, the  $q$ th order difference can be expressed as  $\delta^q \mathbf{E}_n$   
301 =  $\delta(\delta^{q-1} \mathbf{E}_n)$ .<sup>77</sup> Note that the  $q$ th order difference of a  $q - 1$   
302 polynomial order is zero. 303



304 **Constant Uncertainty.** If uncertainties remain constant  
 305 throughout the process, i.e.,  $\mathbf{E}_n = \text{const}$ , then the first order  
 306 difference is equal to zero:  $\delta E_n = \mathbf{E}_{n+1} - \mathbf{E}_n = 0$ .  
 307 The integral action proposed is defined as

$$308 \quad \mathbf{U}_{1,n+1} = \mathbf{U}_{1,n} + \int_{nT_s}^{(n+1)T_s} \mathbf{e}(t) dt \cong \mathbf{U}_{1,n} + \mathbf{e}_n T_s \quad (19)$$

309 where  $\mathbf{e}(t)$  is the continuous time error in the state vector and  
 310  $\mathbf{U}_{1,n+1}$  is the integral of the error. The subscript "1" means the  
 311 first integral of error, and  $n$  is the instant of time.

$$319 \quad \begin{matrix} \mathbf{z}_{\text{ref},n+1} \\ \begin{pmatrix} x_{1,\text{ref},n+1} \\ x_{2,\text{ref},n+1} \\ x_{3,\text{ref},n+1} \\ x_{4,\text{ref},n+1} \\ x_{5,\text{ref},n+1} \end{pmatrix} \end{matrix} - \begin{matrix} \mathbf{z}_{n+1} \\ \begin{pmatrix} x_{1,n+1} \\ x_{2,n+1} \\ x_{3,n+1} \\ x_{4,n+1} \\ x_{5,n+1} \end{pmatrix} \end{matrix} = \begin{matrix} \mathbf{K} \\ \begin{pmatrix} k_1 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_5 \end{pmatrix} \end{matrix} \begin{matrix} \mathbf{z}_{\text{ref},n} \\ \begin{pmatrix} x_{1,\text{ref},n} \\ x_{2,\text{ref},n} \\ x_{3,\text{ref},n} \\ x_{4,\text{ref},n} \\ x_{5,\text{ref},n} \end{pmatrix} \end{matrix} - \begin{matrix} \mathbf{z}_n \\ \begin{pmatrix} x_{1,n} \\ x_{2,n} \\ x_{3,n} \\ x_{4,n} \\ x_{5,n} \end{pmatrix} \end{matrix} + \dots + \begin{matrix} \mathbf{L}_1 \\ \begin{pmatrix} L_{1,1} & 0 & 0 & 0 & 0 \\ 0 & L_{1,2} & 0 & 0 & 0 \\ 0 & 0 & L_{1,3} & 0 & 0 \\ 0 & 0 & 0 & L_{1,4} & 0 \\ 0 & 0 & 0 & 0 & L_{1,5} \end{pmatrix} \end{matrix} \begin{matrix} \mathbf{U}_{1,n} \\ \begin{pmatrix} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ U_{1,5} \end{pmatrix} \end{matrix} \quad (21)$$

constant integral action

320  
 321 Now, following the procedure carried out for the original  
 322 controller, eq 21 is replaced in the mathematical model of the  
 323 system represented in eq 6:

$$324 \quad \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ (N - x_{3,n}) & -x_{3,n} \\ -1 & -1 \\ -1 & (I - x_{5,n}) \end{pmatrix} \begin{pmatrix} u_{1,n} \\ u_{2,n} \end{pmatrix} = \dots = \begin{pmatrix} \left( \frac{x_{1,\text{ref},n+1} - k_1(x_{1,\text{ref},n} - x_{1,n}) + L_{1,1}U_{1,1,n+1} - x_{1,n}}{T_s} \right) \\ \left( \frac{x_{2,\text{ref},n+1} - k_2(x_{2,\text{ref},n} - x_{2,n}) + L_{1,2}U_{1,2,n+1} - x_{2,n}}{T_s} \right) x_{1,n} - x_{2,n} \mu \\ \left( \frac{x_{3,\text{ref},n+1} - k_3(x_{3,\text{ref},n} - x_{3,n}) + L_{1,3}U_{1,3,n+1} - x_{3,n}}{T_s} \right) x_{1,n} + \frac{\mu}{Y} x_{2,n} x_{1,n} \\ \left( \frac{x_{4,\text{ref},n+1} - k_4(x_{4,\text{ref},n} - x_{4,n}) + L_{1,4}U_{1,4,n+1} - x_{4,n}}{T_s} \right) x_{1,n} - R x_{2,n} x_{4,n} \\ \left( \frac{x_{5,\text{ref},n+1} - k_5(x_{5,\text{ref},n} - x_{5,n}) + L_{1,5}U_{1,5,n+1} - x_{5,n}}{T_s} \right) x_{1,n} \end{pmatrix} \quad (22)$$

326 The next step is to find the necessary condition for the  
 327 system (eq 22) to have an exact solution, which is achieved  
 328 through a Gaussian elimination process similar to that  
 329 presented in ref 1. In this way, the values of the variables  
 330 sacrificed for each sampling time are found. Then, the control  
 331 actions can be calculated using eq 12.

332 Now, to demonstrate that the tracking error of the real  
 333 system under perturbations converges to zero, simple  
 334 mathematical operations are carried out:

335 Expressing eq 18 in the following summarized way:

$$336 \quad \mathbf{e}_{n+1} = -\mathbf{K}\mathbf{e}_n + \mathbf{NL}_n - \mathbf{E}_{1,n+1} \quad (23)$$

337 Adding the integral term

$$338 \quad \mathbf{e}_{n+1} = -\mathbf{K}\mathbf{e}_n + \mathbf{NL}_n + \mathbf{L}_1\mathbf{U}_{1,n+1} - \mathbf{E}_{1,n+1} \quad (24)$$

339 Replacing eq 19 in eq 24:

$$340 \quad \mathbf{e}_{n+1} = \mathbf{K}\mathbf{e}_n + \mathbf{NL}_n + \mathbf{L}_1\mathbf{U}_{1,n} + \mathbf{L}_1\mathbf{e}_n T_s + \mathbf{E}_{1,n+1} \quad (25)$$

341 Increasing eq 25 in one sampling time:

According to eq 10, but now considering parametric 312  
 uncertainties and process disturbances, the control action can 313  
 be computed taking into account the new terms: 314

$$315 \quad \mathbf{z}_{n+1} = \mathbf{z}_{\text{ref},n+1} - \mathbf{K}(\overbrace{\mathbf{z}_{\text{ref},n} - \mathbf{z}_n}^{\mathbf{e}_n}) + \mathbf{L}_1\mathbf{U}_{1,n+1} \quad (20)$$

where  $\mathbf{L}_1$  is the matrix corresponding to integral action with a 316  
 constant uncertainty. 317

Then, for each variable: 318

$$\mathbf{e}_{n+2} = \mathbf{K}\mathbf{e}_{n+1} + \mathbf{NL}_{n+1} + \mathbf{L}_1(\mathbf{U}_{1,n+1} + \mathbf{e}_{n+1}T_s) + \mathbf{E}_{1,n+2} \quad (26) \quad 342$$

343 Clearing the integral term from eq 24, replacing in eq 26, and  
 rearranging: 344

$$345 \quad \mathbf{e}_{n+2} = -\mathbf{e}_{n+1}(-1 - \mathbf{K} + \mathbf{L}_1T_s) - \mathbf{K}\mathbf{e}_n - (\mathbf{NL}_{n+1} - \mathbf{NL}_n) \\ \delta E_{2,n} = 0 \\ + (\overbrace{E_{2,n+1} - E_{2,n}}) \quad (27)$$

Therefore, the sets of parameters  $\mathbf{K}$  and  $\mathbf{L}_1$  are optimized for 346  
 the stability assurance of eq 27. The nonlinearity term ( $\mathbf{NL}_{n+1}$  347  
 $- \mathbf{NL}_n$ ) tends to zero because it depends on the error in the 348  
 sacrificed variables, which tends to zero, too. Then  $\mathbf{e}_n \rightarrow 0$  as  $n$  349  
 $\rightarrow \infty$  despite the uncertainties, if they are considered constant. 350

**Linear Uncertainty.** If uncertainties can be represented by 351  
 a linear function, where the second order difference is equal to 352  
 zero:  $\delta^2 E_n = \delta(\delta E_n) = \delta(\mathbf{E}_{n+1} - \mathbf{E}_n) = \mathbf{E}_{n+2} - 2\mathbf{E}_{n+1} + \mathbf{E}_n = 0$ , 353  
 then a new integrator must be considered. In a similar way to 354

355 the procedure before, but now introducing two integral actions  
356 defined by  $\mathbf{U}_1$  and  $\mathbf{U}_2$ :

$$357 \quad U_{\varphi,2,n+1} = U_{\varphi,2,n} + \int_{nT_s}^{(n+1)T_s} \mathbf{U}_{\varphi,1}(t) dt$$

$$\cong U_{\varphi,2,n} + \mathbf{U}_{\varphi,1,n+1}T_s \quad (28)$$

358 where the subscript “2” means an integrator for a linear  
359 perturbation.

360 Then, a new term is added to eq 20:

$$361 \quad \mathbf{z}_{n+1} = \mathbf{z}_{\text{ref},n+1} - \mathbf{K}(\overbrace{\mathbf{z}_{\text{ref},n} - \mathbf{z}_n}^{\mathbf{e}_n}) + \mathbf{L}_1\mathbf{U}_{1,n+1} + \mathbf{L}_2\mathbf{U}_{2,n+1} \quad (29)$$

362 The new parameter  $\mathbf{L}_2$  corresponds to double integral action.  
363 In a similar way as simple integral action, the demonstration of  
364 convergence to zero of the tracking error for the double  
365 integral action is operating as before, and taking into account  
366 that  $\delta^2 E_n = 0$ , the final expression for tracking error is

$$367 \quad \mathbf{e}_{n+3} = -\mathbf{e}_{n+2}(-\mathbf{K} + T_s(\mathbf{L}_1 + \mathbf{L}_2T_s) - 2)$$

$$- \underbrace{\mathbf{e}_{n+1}(2\mathbf{K} - \mathbf{L}_1T_s + 1)}_{\text{bounded nonlinearity}} + \dots + \mathbf{K}\mathbf{e}_n$$

$$+ \underbrace{(\mathbf{NL}_{n+2} - \mathbf{NL}_{n+1} + \mathbf{NL}_n)}_{\delta^2 E_{2,n} = 0} + \underbrace{(\mathbf{E}_{n+2} - 2\mathbf{E}_{n+1} + \mathbf{E}_{2,n})}_{\delta^2 E_{2,n} = 0} \quad (30)$$

368 Now, as can be seen in (30), under constant or linear  
369 varying uncertainty, this has no influence on the error  
370 dynamics. The parameters  $\mathbf{K}$ ,  $\mathbf{L}_1$ , and  $\mathbf{L}_2$  are optimized for  
371 the stability assurance of eq 30, as shown in the previous case.

372 Now, in a generic way, if the uncertainties can be  
373 approximated by  $q - 1$  order polynomial, the influence of  $\mathbf{E}_n$   
374 on  $\mathbf{e}_n$  will be eliminated by introducing  $q$  integrators.

375 **Optimization of Controller Parameters.** There is a  
376 recurrent problem for this kind of controllers, and it is how to  
377 define the best tuning parameters to achieve a good closed-  
378 loop response. In this subsection an algorithm based on Monte  
379 Carlo randomized experiment is proposed to find the optimal  
380 controller parameters.

381 In the original controller presented in ref 1, the controller  
382 parameters are represented by  $\mathbf{K} = \{k_1, k_2, k_3, k_4, k_5\}$ , and the  
383 conditions for the tracking error tending to zero according to  
384 eq 27 are that  $0 < k_i < 1$ ,  $i = \{1, 2, 3, 4, 5\}$ . But now, integral  
385 actions are added; therefore the amount of parameters to select  
386 increases and the conditions change:

$$\mathbf{K} \text{ (original controller)}$$

$$\underbrace{k_1, k_2, k_3, k_4, k_5}_{\mathbf{L}_1 \text{ (integral action, constant uncertainty)}}$$

$$\underbrace{L_{1,1}, L_{1,2}, L_{1,3}, L_{1,4}, L_{1,5}}_{\mathbf{L}_2 \text{ (integral action, linear uncertainty)}}$$

$$\underbrace{L_{2,1}, L_{2,2}, L_{2,3}, L_{2,4}, L_{2,5}}$$

387 In a generic form, for the integral action, the parameters are  
388 denoted by  $L_{m,i}$  where  $m$  is the number of integrators and  $i$   
389 corresponds to each state variable.

390 To choose the optimization criteria of the parameters  
391 considering the integral actions, resort to the characteristic  
392 equations (27) and (30) (simple and double integral actions,

respectively). In the case of the controller without integral  
action, the characteristic equation can be rewritten as

$$r - k_i = 0 \quad (31) \quad 395$$

Therefore, the parameters are directly the roots of polynomial.

For a simple integral action (one integrator, constant  
uncertainty), the characteristic equation is (27) and can be  
rewritten as

$$r^2 + r(-1 - k_i + L_{1,i}T_s) + k_i = 0 \quad (32) \quad 400$$

Thus, clearing the parameters according to the roots:

$$k_i = r_1r_2$$

$$L_{1,i} = (-r_1 - r_2 + r_1r_2 + 1)/T_s \quad (33) \quad 402$$

For a double integral action (two integrators, linear  
uncertainty), the characteristic equation is (30) and can be  
rewritten as

$$r^3 + r^2(-k_i + T_s(L_{1,i} + L_{2,i}T_s) - 2) + r(2k_i - L_{1,i}T_s + 1) - k_i = 0 \quad (34) \quad 406$$

Clearing the parameters

$$k_i = r_1r_2r_3$$

$$L_{1,i} = (2r_1r_2r_3 + 1 - r_1r_2 - r_1r_3 - r_2r_3)/T_s$$

$$L_{2,i} = (-r_1 - r_2 - r_3 - r_1r_2r_3 + 1 + r_1r_2 + r_1r_3 + r_2r_3)/T_s^2 \quad (35) \quad 408$$

Thus, for the tracking error to tend to zero and to ensure the  
system stability, the roots of the polynomial must be between 0  
and 1.

The performance index used to optimize the controller  
parameters is to minimize the total tracking error (15):

$$\mathbf{C} = \min_{\mathbf{K}, \mathbf{L}}(E_T) \quad (36) \quad 414$$

## 4. RESULTS AND DISCUSSION

In this section, the behavior of the controller against  
parametric uncertainty and perturbations in the process is  
evaluated through simulations. First, a Monte Carlo algorithm  
to tune the optimal controller parameters is carried out. Then,  
the system is tested under parametric uncertainty and process  
disturbances, comparing the system responses with and  
without integral action.

**Controller Tuning.** In order to determine the optimal  
controller parameters, a Monte Carlo algorithm (MCRA) is  
carried out. This methodology has been recently employed for  
controller tuning<sup>1,64,78</sup> with satisfactory results.

According to the criteria for parameter selection discussed in  
section 3, the procedure for the simulation is detailed below.

**Controller Tuning without Integral Action ( $m = 0$ ).** In this  
case, the methodology employed is the same as in ref 1, where  
1000 simulations are performed. Considering eq 31, for each  
simulation a random value  $r_i \in (0,1)$  is assigned for each state  
variable, then the control actions and the performance index  
(36) are computed. Once the simulations are finished, the  
optimal controller parameter set is corresponding to the  
minimum  $\mathbf{C}$ .

436 **Controller Tuning with an Integral Action ( $m = 1$ ).** Again,  
437 1000 simulations are performed, but now the parameters are  
438 calculated according to eq 33.

439 **Controller tuning with a double integral action ( $m = 2$ ).**  
440 Once the 1000 simulations are performed, the controller  
441 parameters are computed according to eq 35.

442 The necessary information to carry out the simulations is  
443 taken from ref 1. The final time and sampling time are  $T_f = 10$   
444 h and  $T_s = 0.1$  h, respectively.

445 Table 1 shows the optimized values for the controller  
446 parameters. Depending on the value of  $m$ , there will be 5, 10,  
447 or 15 parameters.

**Table 1. Optimal Controller Parameters**

param	$m = 0$	$m = 1$	$m = 2$
$k_1$	0.2805	0.0038	0.0404
$k_2$	0.6761	0.9412	0.9412
$k_3$	0.6520	0.7941	0.6589
$k_4$	0.1694	0.2032	0.3842
$k_5$	0.1266	0.5382	0.5382
$L_{1,1}$	–	8.8102	0.0851
$L_{1,2}$	–	0.0088	2.3546
$L_{1,3}$	–	1.1183	1.2538
$L_{1,4}$	–	1.4755	2.4587
$L_{1,5}$	–	0.6504	1.0085
$L_{2,1}$	–	–	0.2588
$L_{2,2}$	–	–	1.5285
$L_{2,3}$	–	–	3.0497
$L_{2,4}$	–	–	1.9246
$L_{2,5}$	–	–	2.7533

448 **Disturbances and Uncertainties Handling.** For the  
449 quantification of uncertainties and disturbances in the  
450 simulation work, an approach consists of specifying only the  
451 upper and lower limits within which the real disturbance or  
452 uncertainty is assumed to evolve. The idea of this approach is  
453 thus to replace the knowledge of exact values of unknown  
454 inputs by a known range of the values within which they  
455 evolve.<sup>79</sup>

456 **Parametric Uncertainty.** In the bioprocesses, the model  
457 parameters vary in an unpredictable manner.<sup>80</sup> This can lead to

a structural instability in the dynamic behavior of the system.<sup>20</sup> 458

Therefore, a strict and efficient control system is necessary. 459

In several research fields, probabilistic methods have been 460  
found to be useful for dealing with problems related to 461  
robustness of systems affected by uncertainties.<sup>81</sup> In particular, 462  
the Monte Carlo randomized algorithm has been used for 463  
uncertainty quantification in many applications such as the 464  
Rothermel model,<sup>48</sup> river flow rate forecast,<sup>82</sup> radioactive 465  
decay, power system generation, and traffic on roads,<sup>83</sup> among 466  
others. From the point of view of process control, Monte Carlo 467  
methods are effective tools for the analysis of probabilistically 468  
robust control schemes.<sup>65,81</sup> In this subsection the system is 469  
simulated considering errors on modeling using the MCRA 470  
method. The procedure consists of replacing the exact model 471  
parameters values by a range of them within which they can 472  
vary. The defined range for model parameters is  $\pm 20\%$  of the 473  
nominal values. Then,  $N = 1000$  simulations are executed using 474  
the optimized controller parameters set. 475

Taking into account the worst case of uncertainty, all the 476  
model parameters are varied. In the first instance, the control 477  
system is tested without integral action. Then one integrator is 478  
added and finally, two integrators are added. The results are 479  
compared below. 480

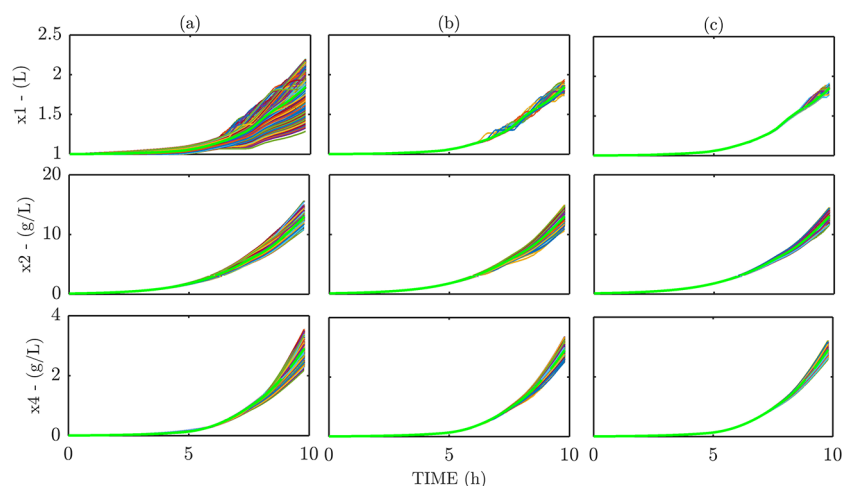
The tracking of optimal profiles for the three cases is shown 481  
in Figure 1. 482 f1

Figure 2 shows the TTE for 1000 simulations to evaluate the 483 f2  
controller performance under parametric uncertainty for three 484  
cases: no integrator (a), one integrator (b), and two integrators 485  
(c). As can be seen, the total tracking error range is visibly 486  
reduced when integral actions are added. 487

In Table 2, the total tracking errors for the three cases are 488 f3  
shown. Now, taking the average value of TTE for each case 489  
and translating it to percentage, the improvement of the 490  
controller with integral action can be easily quantified (see 491  
Figure 3). Note that the error is reduced by 48% by adding one 492 f4  
integral action and 61% with two integrators. 493

The above figures show that the integral actions 494  
incorporated into the controller are effective and give 495  
robustness to the control system. 496

**Perturbation in the Control Actions.** In all processes 497  
there are also disturbances acting on the plant in addition to 498  
parametric uncertainties, modifying the expected values of the 499



**Figure 1.** Tracking of optimal profiles for 1000 simulations under parametric uncertainty ( $\pm 20\%$ ). (a) Nonintegral action ( $m = 0$ ), (b) one integral action ( $m = 1$ ), and (c) two integral actions. Green lines show the reference values.

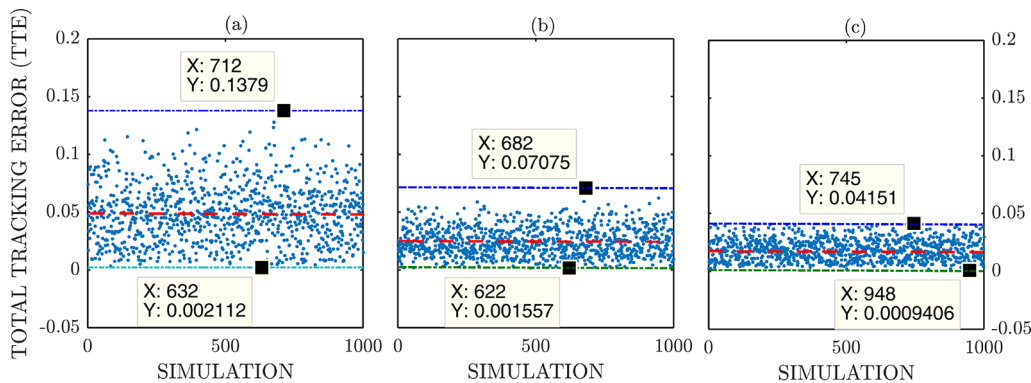


Figure 2. Total tracking error for 1000 simulations under parametric uncertainty ( $\pm 20\%$ ). (a) Nonintegral action, proposed by Pantano et al.<sup>1</sup> ( $m = 0$ ), (b) one integral action ( $m = 1$ ), and (c) two integral actions ( $m = 2$ ). Central dashed lines show the mean values.

Table 2. Reduction of TTE by Adding Integrators under Parametric Uncertainty

	$m = 0$	$m = 1$	$m = 2$
TTE	0.0472	0.0245	0.0186
%	100	52	39

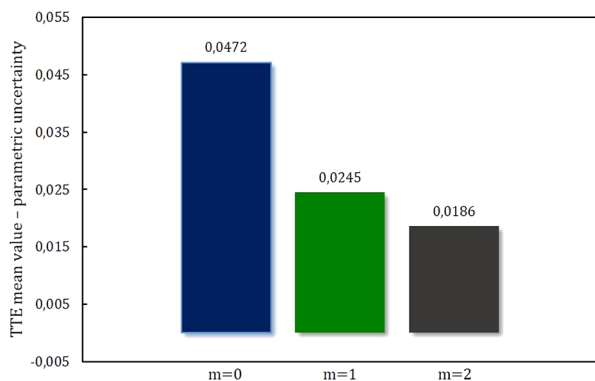


Figure 3. TTE mean value for the three cases. Process simulation under parametric uncertainty (20%).

Figure 4 shows the controller adjustment to hold the tracking error in a minimal value spite of perturbations in the control actions and parametric uncertainties.

Table 3 shows the TTE for the controllers. Note how, again, by adding only one integral action the tracking error is reduced by 68%. The response improves by 75% with two integrators.

Table 3. TTE for Perturbed System with Parametric Uncertainty

	$m = 0$	$m = 1$	$m = 2$
TTE	0.1036	0.0335	0.0253
%	100	32	24

Figure 5 shows a bar diagram of the cumulative error (TTE) for the real system under parametric uncertainties and

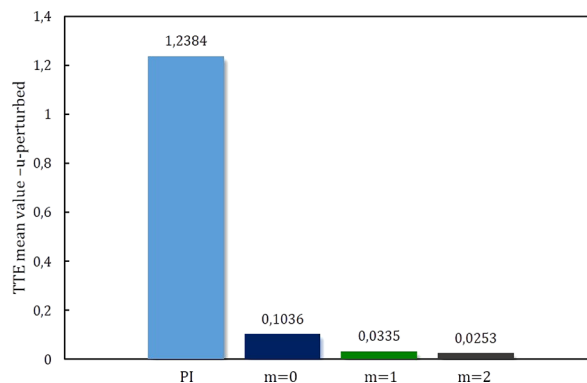


Figure 5. TTE for the three cases. Process simulation under parametric uncertainty and perturbations in the control actions.

system variables. Therefore, a random perturbation in the control actions taking into account parametric uncertainties is simulated for the controller evaluation. The two control actions are perturbed by 30% of calculated values:

$$u_{1,perturbed} = u_{1,unperturbed}(1 + \text{random}(\text{unif}, 0, 0.3) + 1)$$

$$u_{2,perturbed} = u_{2,unperturbed}(1 + \text{random}(\text{unif}, 0, 0.3) + 1)$$

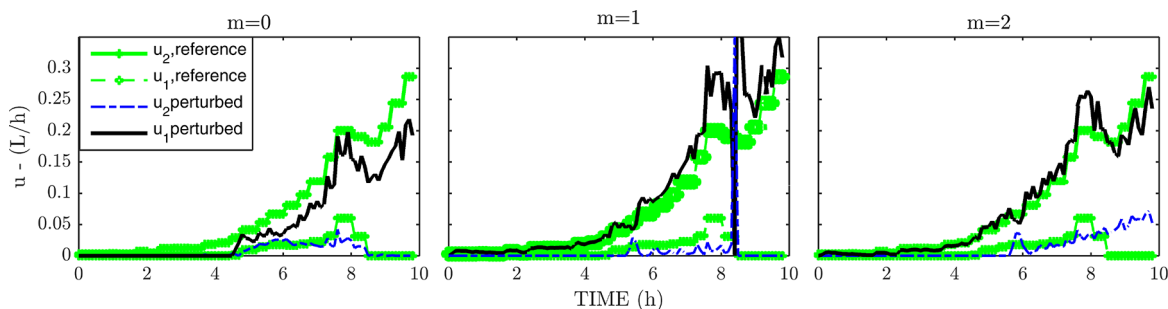


Figure 4. Perturbed control actions considering parametric uncertainty.



512 perturbation in the control actions. This is presented for the  
513 original controller proposed in Pantano et al.<sup>1</sup> ( $m = 0$ ), and for  
514 the two controllers proposed in this paper: one integrator ( $m =$   
515 1) and two integrators ( $m = 2$ ). In addition, as a comparison,  
516 TTE is shown for a classic PI controller, which was  
517 automatically tuned by a Matlab autotuning function.

518 As can be seen in this section, the simulation work  
519 demonstrates the effectiveness of the controller proposed in  
520 this work, by adding integral actions to the design, in order to  
521 solve the problems of control under parametric uncertainty  
522 and process disturbances.

## 5. CONCLUSION

523 This paper proposes an improved control system for fed-batch  
524 production of recombinant protein, a multivariable and  
525 nonlinear bioprocess which is highly susceptible to model  
526 and process disturbances due to its biological nature. The  
527 original controller proposed in ref 1 achieved the tracking of  
528 optimal profiles of volume, cell density, and protein  
529 concentration (main product) with a minimum error through  
530 the development of an effective control law based on linear  
531 algebra. The technique proposed in this work extends the  
532 original controller design to take into account parametric  
533 uncertainty and process disturbances. Several simulations were  
534 carried out to test the controller performance. The system was  
535 evaluated under parametric uncertainties ( $\pm 20\%$ ) and random  
536 perturbations in the control actions ( $\pm 30\%$ ), improving the  
537 system response up to 70% by adding some integral actions of  
538 the tracking error in the control actions computation. The  
539 optimal controller parameters (with and without integrators)  
540 were successfully found through a Monte Carlo experiment.  
541 Although this method has low complexity, the results are  
542 reliable and the method solves a real problem for bioprocess  
543 control.

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### Notes

550 The authors declare no competing financial interest.

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