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DIFFUSIVE METRICS INDUCED BY MULTIAFFINITIES. THE COVID-19 SETTING FOR BUENOS AIRES (AMBA)

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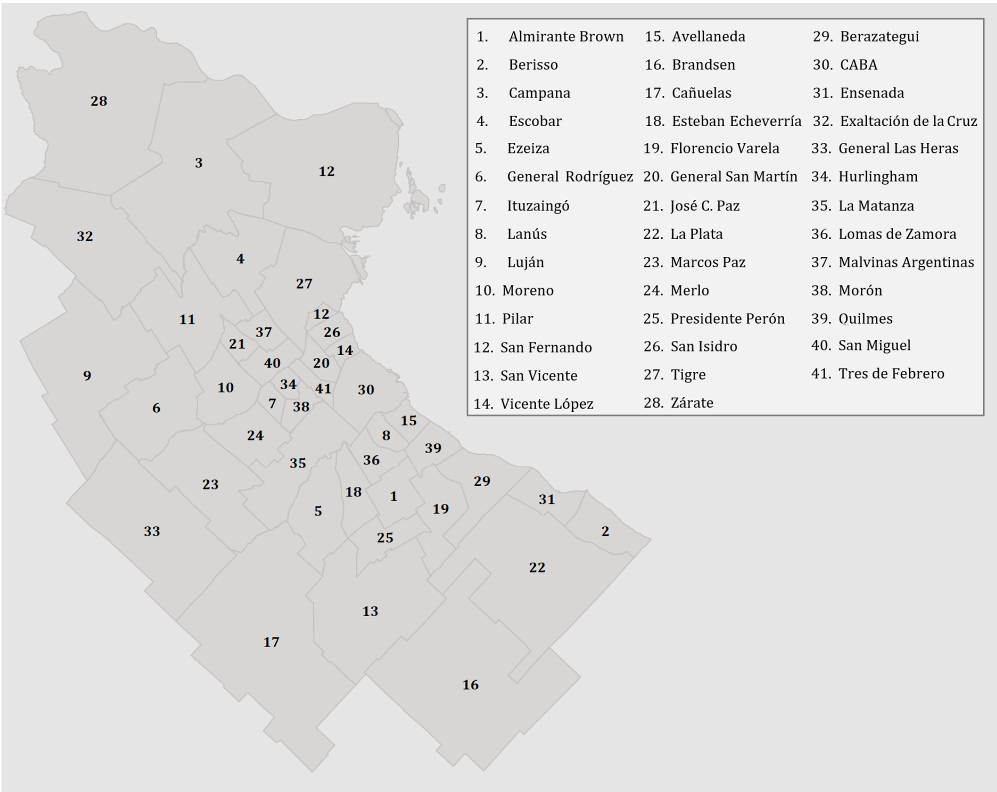
Abstract: In this work we aim to use tools of discrete harmonic analysis in order to provide a metric in the set of the 41 cities belonging to the largest urban concentration in Argentina based on public transport and neighborhood. The results can be applied to predict and control the spread of COVID-19 and other pandemic diseases in such a setting.

Keywords: *weighted graphs, diffusion, graph Laplacian, metrization, COVID-19, AMBA-Argentina*
2000 AMS Subject Classification: 90C35, 60J60, 54E35

1 INTRODUCTION

The acronym AMBA is used to name the 41 cities that concentrate one third of the total population of Argentina and is spatially concentrated around Buenos Aires City. The following map depicts their distribution.



Aside from the geographical distance between locations i and j in the map there is a valuable information given by the public transport system in AMBA. The system SUBE keeps a great amount of information that allows to have another geometry provided by a connectivity distance built on this big data source. With the idea of considering at once a diversity of affinities between two cities i and j, such as euclidean distance, neighborhood, public transport, private transport, etcetera, we introduce a diffusive metrization of the graph with 41 vertices that takes into account these diverse factors which all together contribute to the motion of people inside AMBA.

In Section 2 we introduce the metric based on the construction of Coifman-Lafon (see [1]) for a multiweighted undirected graph, through the spectral analysis of the Laplace operator determined by a convex combination of the affinities. Section 3 is devoted to apply the metric to the case of AMBA, by showing the families of metric balls computed using some of the data provided by the system SUBE.

2 Metrization of multiweighted graphs

Let $G_k = \left(V, E, \vec{a}, W^k\right)$, $k = 1, \ldots, K$ be a finite sequence of undirected weighted graphs with the same set of vertices V, the same set of edges E and the same weight in each vertex $\vec{a} = (a_1, \ldots, a_n)$, $a_i > 0$, $i = 1, \ldots, n = \#(V)$. For each k, $W^k = \left(w^k_{i,j} : i, j = 1, \ldots, n\right)$ determines the affinity of each pair i, j of vertices with respect to the feature k. We shall assume that $\vec{a}$ and each W^k are normalized to probabilities, i.e. $\sum_{i=1}^{n} a_i = 1$, $\sum_{i,j=1}^{n} w^k_{ij} = 1$, $k = 1, \ldots, K$. Let $\vec{\theta} = (\theta_1, \ldots, \theta_K)$ be a vector of length K with $\theta_k \geq 0$ for every k and $\sum_{k=1}^{K} \theta_k = 1$. Set $w_{i,j} = \sum_{k=1}^{K} \theta_k w^k_{i,j}$. The parameters θ_k can be chosen according to our perception of the relevance of the k-th feature for the construction of the metric.

The main result of this section is contained in the following statement and makes use of the Coifman-Lafon diffusive metrization scheme. See [1] and [2], for a different approach to metrization see [3].

Proposition 1 *Let G_k be as before, $k = 1, \ldots, K$ and $\theta_k \geq 0$ with $\sum_{k=1}^{K} \theta_k = 1$. Then for $t > 0$ the function $d_t(i,j) = \sqrt{\sum_{\ell \geq 0} e^{2t\lambda_\ell} |\phi_\ell(i) - \phi_\ell(j)|^2}$ is a metric on V, where $(\lambda_\ell, \phi_\ell)$ is the spectral resolution of the Laplacian $\Delta f(i) = \frac{1}{a_i} \sum_{j=1}^{n} w_{ij}\big(f(j) - f(i)\big)$ where $w_{i,j} = \sum_{k=1}^{K} \theta_k w^k_{i,j}$.*

3 The case of AMBA with neighborhood and public transport

With the above notation, $V = \{1, \ldots, 41\}$ represents the 41 cities that constitute AMBA, with W^1 reflecting the data provided by SUBE and W^2 the fact that two cities are neighbor to each other. Here we show only a small part of the unnormaized matrix W^2.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
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1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	1

Figure 1: Unnormalized submatrix of W^2 (20×41)

In the next page we exhibit the full unnormalized form of W^1.

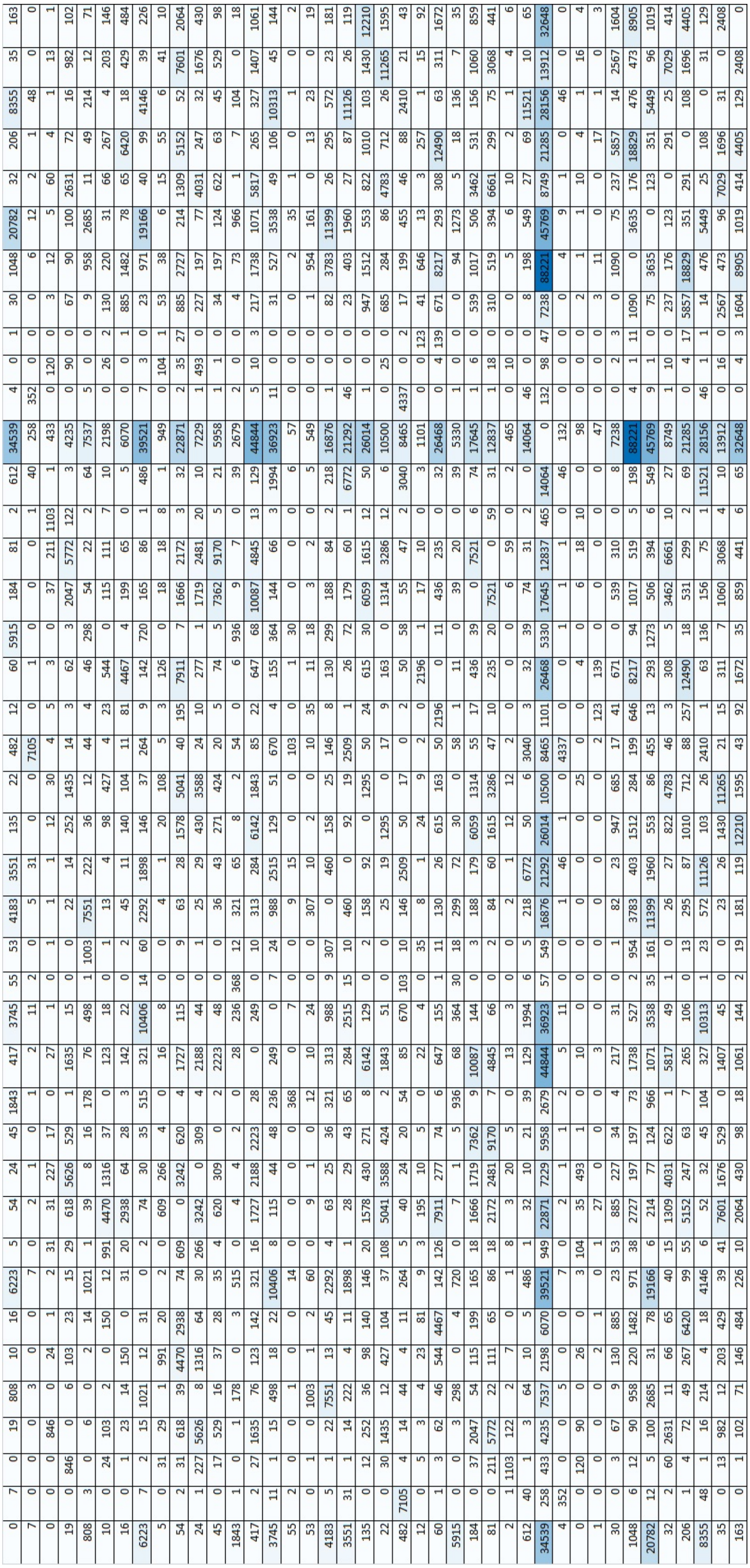

Figure 2: Matrix that reflects the data provided by SUBE before normalizing to probability

Let us only illustrate some d_t-balls for $t = 0.25$ with $\theta_1 = \theta_2 = \frac{1}{2}$ and two different weights $\vec{a}$: the uniform $\vec{a}_u = \left(\frac{1}{41}, \ldots, \frac{1}{41}\right)$ and

$$\begin{aligned}\vec{a}_d = (&0.0023, 0.0009, 0.0004, 0.0014, 0.0015, 0.0009, 0.0012, 0.0030, 0.0007, 0.0009, 0.0011, 0.0015,\\ &0.0008, 0.0016, 0.0049, 0.0005, 0.0006, 0.0018, 0.0015, 0.0031, 0.0013, 0.0008, 0.0012, 0.0010,\\ &0.0019, 0.0022, 0.0014, 0.0006, 0.0019, 0.0095, 0.0011, 0.0004, 0.0015, 0.0018, 0.0018, 0.0026,\\ &0.0013, 0.0018, 0.0029, 0.0018, 0.0034)\end{aligned}$$

which is a normalization of the density of the disease in each location (total number of active infections over population) by July 2020. The algorithms is implemented in Python.

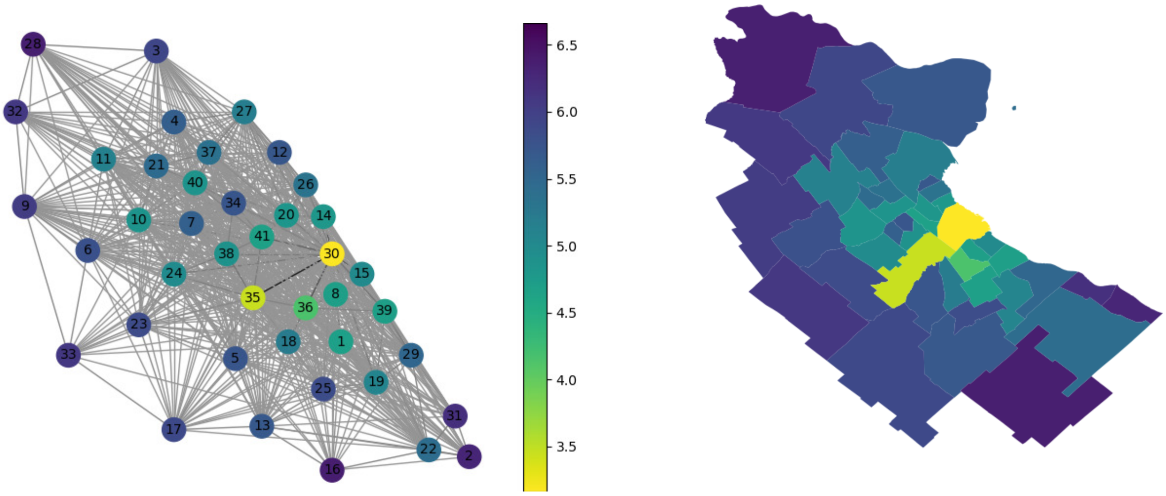

Figure 3: Weight $\vec{a_u}$. Balls centred at CABA. Growing radii according to the scale of colors. Left: Graph, Right: map.

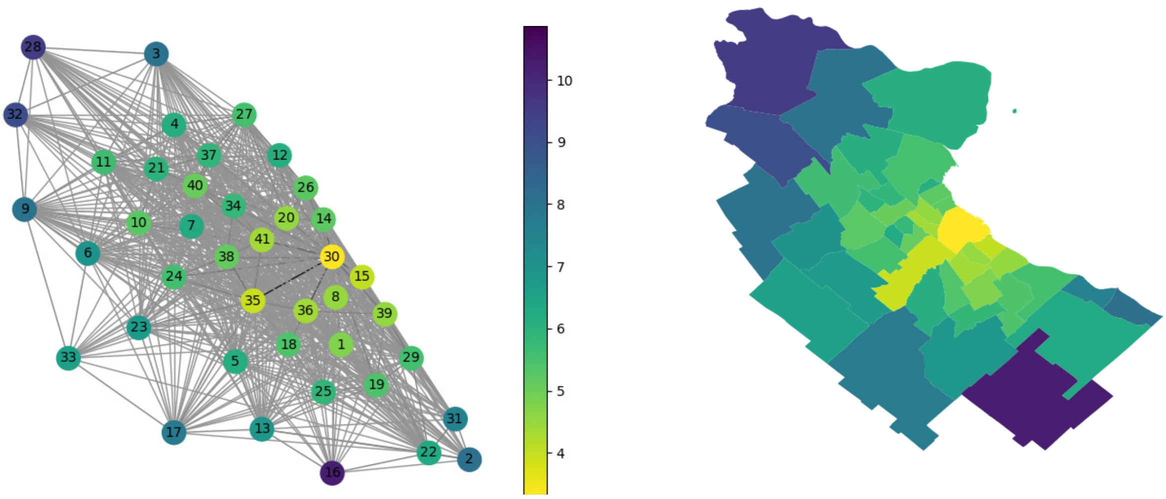

Figure 4: Weight $\vec{a_d}$. Balls centred at CABA. Growing radii according to the scale of colors. Left: Graph, Right: map.

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