Radiation from black hole accretion in $f(R)$ gravity

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Abstract. We calculate the thermal radiation from a thin accretion disk around a rotating black hole taken into account strong field effects. We compare the resulting spectrum in general relativity (GR), with the spectra obtained when modified theories of gravity are considered. The spectrum of Cygnus X-1 in the high-soft state is used to impose constraints on the free parameters in these theories.

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INTRODUCTION

$f(R)$ gravity is a class of Extended Theories of Gravity, where the Lagrangian of the Hilbert-Einstein action is generalized to:

$$S[g] = \frac{1}{8\pi G} \int f(R)\sqrt{-g}\, d^4x, \tag{1}$$

where g is the determinant of the metric tensor, and $f(R)$ is an arbitrary function of the Ricci scalar.

Kerr-Newman f(R) black holes solutions with constant Ricci scalar were originally found by Carter [1, 2] and have been recently studied by Cembranos and collaborators [3]. In the present work we investigate the main features of relativistic accretion disks [4, 5] around Kerr-f(R) black holes with constant Ricci scalar.

We adopt for the radius of the outer edge of the disk [6]:

$$r_{\rm out} = 11 r_{\rm iso}, \tag{2}$$

where $r_{\rm iso}$ is the radius of the innermost stable circular orbit. In the next section we calculate the temperature and luminosity of an accretion disk around a Kerr black hole in $f(R)$ gravity with constant Ricci scalar, adopting the following values for the relevant parameters: $M = 14.8 M_{\odot}$, $\dot{M} = 0.472 \times 10^{19}\,{\rm g\,s^{-1}}$, and $\mathrm{a} = 0.99$, which are the best estimates available for the well-known galactic black hole Cygnus-X1 [7, 8].

KERR-$f(R)$ BLACK HOLES

The axisymmetric, stationary and constant R_0 Ricci scalar space-time metric that describes a black hole with mass M and angular momentum a, setting $\theta = \pi/2$, takes the form [1, 2, 3]:

$$ds^2 = -\frac{c^2}{r^2\Xi^2}\left(\Delta_r - a^2\right)dt^2 + \frac{r^2}{\Delta_r}dr^2 - \frac{2ac}{r^2\Xi^2}\left(r^2 + a^2 - \Delta_r\right)dt\,d\phi + \frac{d\phi^2}{r^2\Xi^2}\left[\left(r^2 + a^2\right)^2 - \Delta_r a^2\right]. \tag{3}$$

Here,

$$\Delta_r = \left(r^2 + a^2\right)\left(1 - \frac{R_0}{12}r^2\right) - \frac{2GMr}{c^2}, \tag{4}$$

$$\Xi = 1 + \frac{R_0}{12}a^2. \tag{5}$$

If $R_0 \to 0$, Eq. (3) represents the Kerr space-time metric in GR as expected.

Stable circular orbits are possible for R_0[1] $\in (-1.3 \times 10^{-1}, 1.45 \times 10^{-1})$. According to Eq. (2), if we take for the innermost stable circular orbit $r_{\rm iso} = 1.4545\, r_{\rm g}$, the outer edge of the disk yields approximately $16\, r_{\rm g}$. For positive values of the Ricci scalar, stable circular orbits are possible within a maximum radius. If $\mathsf{R}_0 > 6.67 \times 10^{-4}$ then $r_{\rm out} < 16\, r_{\rm g}$. We will then calculate the temperature and luminosity of accretion disks for $\mathsf{R}_0 \in [-1.25 \times 10^{-1}, 6.67 \times 10^{-4}]$.

For negative values of the Ricci scalar the results are displayed in Figures 1(a), 1(b), and Table 1. We see that the temperature of the disk increases for smaller values of R_0. The ratio of the maximum temperature between the GR and $f(R)$ cases, with $\mathsf{R}_0 = -1.25 \times 10^{-1}$, is 1.20. The peak of the emission rises a factor of 2, and the corresponding energy is shifted towards higher energies.

For $\mathsf{R}_0 \in (0, 6.67 \times 10^{-4}]$ we show in Table 2 the values of the location of the last stable circular orbit, maximum temperature, luminosity, and the energy of the peak of the emission. We conclude that for $\mathsf{R}_0 \in (0, 6.67 \times 10^{-4}]$ the temperature and energy distributions have no significant differences with Kerr's distributions in GR.

DISCUSSION AND CONCLUSIONS

The results presented in the last section can be compared with current observational data to introduce some constraints on a given $f(R)$ theory. In order to do this we shall consider Cygnus X-1, which is the most intensively studied black hole binary system in the Galaxy. A series of recent high-quality papers [7, 8, 9] have provided an unprecedented set of accurate measurements of the distance, the black hole mass, spin, orbital inclination and accretion rate of this source. This opens, by first time, the possibility to constrain modified theories of gravity with rather local precision observations of astrophysical objects in the Galaxy.

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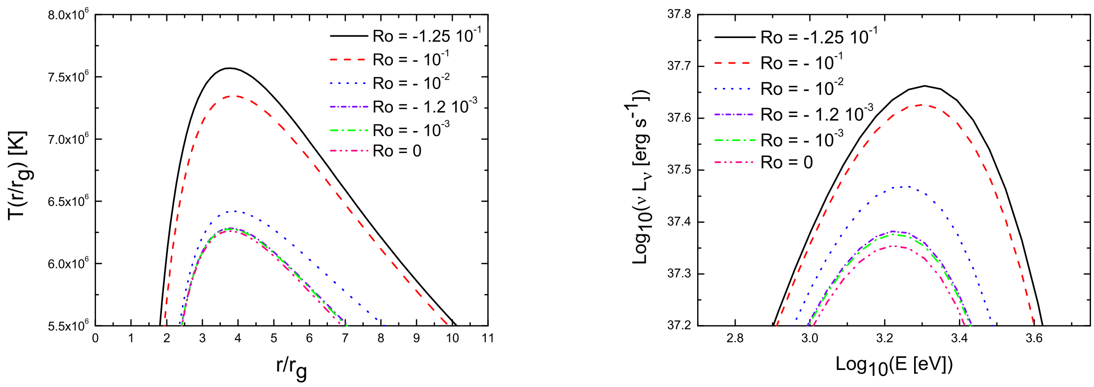
[1] Here R_0 is an adimensional quantity and is defined as $\mathsf{R}_0 \equiv R_0 r_{\rm g}^2$, where $r_{\rm g} = GM/c^2$.



(a) (b)

FIGURE 1. Plots of the temperature and luminosity distributions of a Kerr-$f(R)$ black hole for $R_0 < 0$. (a) Temperature as a function of the radial coordinate. (b) Luminosity as a function of the energy.

TABLE 1. Location of the last stable circular orbit and maximum temperature, maximum temperature, luminosity, and energy of the peak of the emission for an accretion disk around a Kerr-$f(R)$ black hole with $R_0 < 0$.

Kerr $f(R)$	$R_0 = 0$	$R_0 = -10^{-3}$	$R_0 = -1.2 \times 10^{-3}$
r_{iso}/r_g	1.4545	1.4523	1.4518
r_{Tmax}/r_g	3.79	3.79	3.79
T_{max}	0.539 keV	0.54119 keV	0.54148 keV
E_{max}	1659.4 eV	1659.4 eV	1659.4 eV
$L(E_{max})$	2.26×10^{37} erg s^{-1}	2.38×10^{37} erg s^{-1}	2.41×10^{37} erg s^{-1}
Kerr $f(R)$	$R_0 = -10^{-2}$	$R_0 = -10^{-1}$	$R_0 = -1.25 \times 10^{-1}$
r_{iso}/r_g	1.4325	1.2017	1.0419
r_{Tmax}/r_g	3.85	3.85	3.78
T_{max}	0.553 keV	0.663 keV	0.652 keV
E_{max}	1833.52 eV	2025.9 eV	2025.9 eV
$L(E_{max})$	2.94×10^{37} erg s^{-1}	4.23×10^{37} erg s^{-1}	4.60×10^{37} erg s^{-1}

The distance to Cygnus X-1 is currently estimated to be $1.86^{+0.12}_{-0.11}$ kpc [9]. This value was determined via trigonometric parallax using the Very Long Baseline Array (VLBA). At this distance, the mass of the black hole is [7] $M = 14.8 M_\odot$. This is the value adopted in all our calculations presented in the previous section. The accretion rate is $\sim 0.472 \times 10^{19}$ g s^{-1} and the spin parameter of the hole is 0.99 [8].

The spin parameter has been estimated using three different set of data (see [8] for details). Using these spectra, a maximum temperature for the inner disk of 0.539 ± 0.002 keV can be obtained (see [8], Sect. 7.5). Such a value immediately rules out all models of $f(R)$-gravity that display a constant negative scalar in the strong regime lower than $R_0 = -1.2 \times 10^{-3}$ (see Table 1). The value of $T_{max} = 0.539$ keV corresponds to zero Ricci scalar (i.e. GR). Models with small positive scalar in strong gravity remain viable. These models have an asymptotic behaviour consistent with a de Sitter space-time

TABLE 2. Values of the location of the last stable circular orbit, location in the radial coordinate of the maximum temperature, maximum temperature and luminosity, and the energy of the peak of the emission for an accretion disk around a Kerr-$f(R)$ black hole with $\mathsf{R}_0 > 0$.

Kerr $f(R)$	$\mathsf{R}_0 = 0$	$\mathsf{R}_0 = 10^{-4}$	$\mathsf{R}_0 = 6.67 \times 10^{-4}$
$r_{\rm circ}/r_{\rm g}$	1.4545	1.4547	1.4559
$r_{\rm Tmax}/r_{\rm g}$	3.79	3.79	3.79
$T_{\rm max}$	0.53942 keV	0.53927 keV	0.53843 keV
$E_{\rm max}$	1659.4 eV	1659.4 eV	1659.4 eV
$L(E_{\rm max})$	2.26×10^{37} erg s^{-1}	2.25×10^{37} erg s^{-1}	2.09×10^{37} erg s^{-1}

endowed of a small and positive value of the cosmological constant, in accordance with cosmological estimates.

The current observational uncertainties imply an upper limit $-1.2 \times 10^{-3} \leq \mathsf{R}_0 < 6.67 \times 10^{-4}$. Future high-precision determination of the parameters of other black hole candidates can be used to impose even more restrictive limits to extended theories of gravity.

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