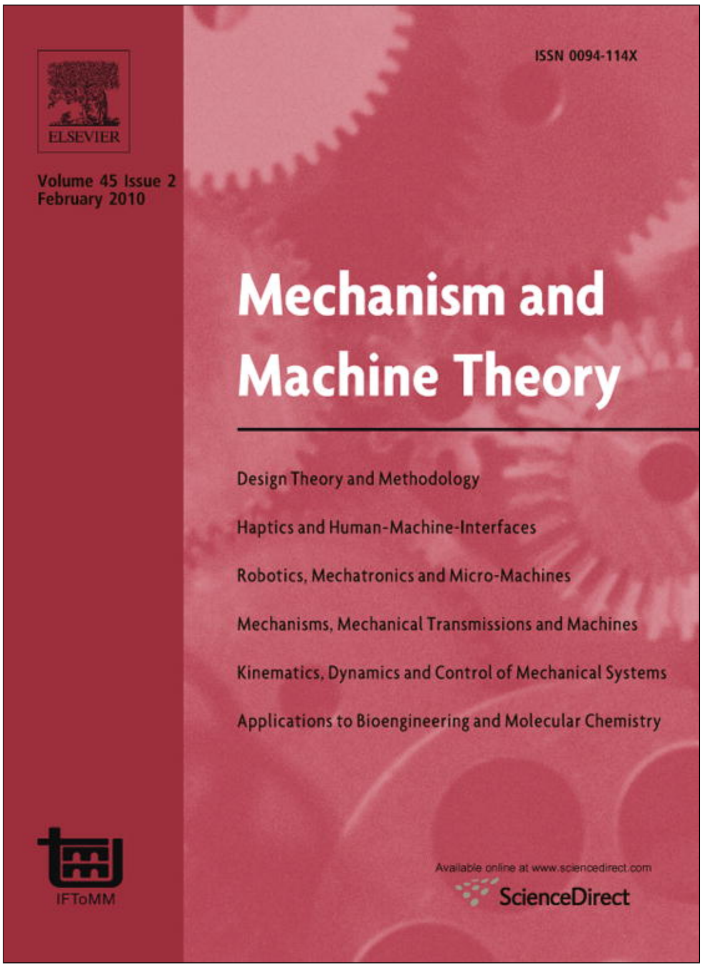
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Design of bistable compliant mechanisms using precision–position and rigid-body replacement methods

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A systematic approach to the synthesis of compliant mechanisms starting from the problem requirements is proposed. The method uses as topological solution space several atlases of compliant mechanisms specialized from rigid kinematic chains. Graph Theory is used to solve the type synthesis stage, while the rigid analytical synthesis combined with the well-known process of rigid-body replacement synthesis is used to synthesize the flexible members. Using these tools, we show that the initial and final equilibrium positions of partially compliant mechanisms can be designed from simple topological and kinematic considerations.

1. Introduction

The many advantages of compliant mechanisms compared to their rigid-body counterparts have produced a growing interest in *compliant mechanism synthesis* methods. This is currently a challenging area of development that has been addressed in several ways. Three main approaches can be highlighted:

(i) *Rigid-body synthesis*: where a *partially* compliant mechanism ("partially" referring to the presence of some rigid body or kinematic pair in the mechanism) is designed using rigid-body replacement starting from rigid-synthesis results obtained by the precision–position method [1]. The iterative analysis of Pseudo-Rigid-Body Models (PRBM) may also be used in combination with optimization techniques [2].

(ii) *Continuum synthesis*: where topology and shape optimization are combined with linear and nonlinear finite element analysis [3,4]. In Homogenization Methods the densities of elements are taken as design variables [5]; multiple materials were considered in Refs. [6,7], and recently, in Ref. [8] using Level Set Methods.

(iii) *Discrete synthesis*: the main difference with the previous approach is that the mechanism is represented by a network of truss or beam elements so that the topological synthesis can firstly be solved using discrete algorithms; then, several sizing methods are applied for each enumerated topology. Some examples are the following: the Load Path Representation [9], the use of the Spanning Tree Theory [10], and the Graph Theory approach [11]. In these works, the design of sectional areas of nonlinear beam elements is achieved in several ways, using different degrees of complexity: fixed coordinates of nodes [12] or moving coordinates (taking element lengths as variables) with avoidance of elements overlapping [9,13,10]; additionally, curved beams with variable thickness were considered in Ref. [11].

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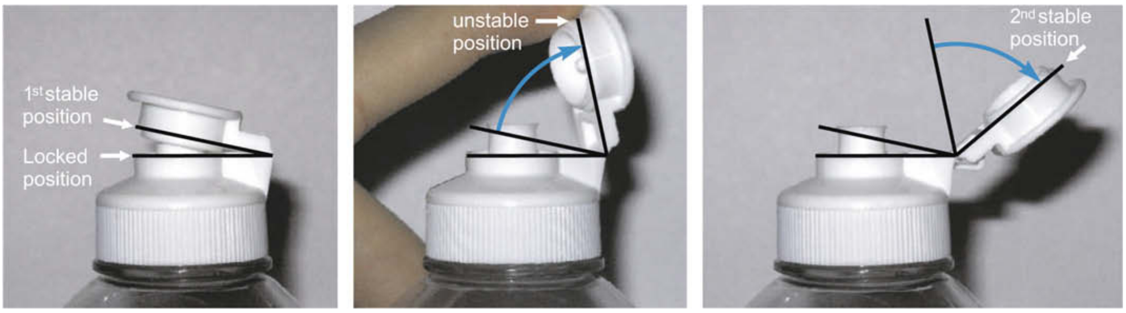

Fig. 1. Bistable mechanism of a bottle lid: left and middle photographs show two positions driven by a human; then, the lid is driven by releasing the internal strain energy until the second stable position is reached.

Since all methods consist in the sizing of a given topology, we can always consider the implicit existence of the two traditional stages used for synthesizing rigid mechanisms: the topological synthesis and the dimensional synthesis. Currently, the methods of categories (ii) and (iii) have the advantage of producing fully compliant mechanisms as solutions. These are the desired solutions when miniaturization, manufacturing from one layer of material, and uniform stress distribution are the main requirements, and are called *distributed compliance* mechanisms. However, in many cases, the optimal topology presents "de facto" hinges where compliance is concentrated and the replacement of this *concentrated compliance* by a revolute joint plus a torsional spring cannot be avoided. In mechanisms with heavy loads, large or full rotations, high energy storage, and severe space requirements, the evaluation of partially compliant mechanisms becomes of great importance and may even help to predict the behavior of a fully compliant solution. However, the atlas of partially compliant mechanisms could be too large to explore due to the combinatorial explosion.

The scope of this paper is limited to planar mechanisms of the linkage type. In previous works of the authors, an available solver for the type and dimensional synthesis of rigid linkage mechanisms was modified to computationally implement the kinematic synthesis of compliant mechanisms by means of *rigid-body replacement* [14,15]. The rigid-body replacement solver can be used in two kinds of synthesis applications:

(a) Replacement of rigid parts of a rigid mechanism by flexible members to obtain partially compliant mechanisms.
(b) Design of bistable mechanisms, i.e. mechanisms with two stable equilibrium positions; see, for example, the lid of a bottle in Fig. 1.

This paper presents results for a new approach for the design of bistable mechanisms. The method is based on the solution of the initial and final unstrained positions and one or more intermediate deformed positions of a compliant member composed of a flexible link clamped to another body. This concept simply consists in designing the positions for the neighboring bodies of a clamped joint. For instance, Fig. 2 illustrates three kinematic configurations (at times $t_{\mathrm{ini}}, t_{\mathrm{interm}}$ and t_{end}) of a clamped joint in a compliant mechanism which connects two bodies B_i and B_j, with at least one of them flexible. The clamped joint can be designed on the mechanism (pseudo) rigid version (Fig. 2a), given the proper motion constraints to satisfy equal initial and final relative angular positions θ_{rigid} between the bodies in such a way to obtain the same situation for the angle $\theta_{\mathrm{clamped}}$ in the replaced beam (Fig. 2b).

The paper is organized as follows: Section 2 continues the review of available compliant synthesis methods and introduces the scope of rigid-replacement synthesis. The design of bistable mechanisms is presented and illustrated in Section 3. The adopted method is introduced in Section 4; the type synthesis method is explained in Section 4.1, and the rigid-replacement rules are presented in Section 4.2. The design of a bistable actuator for a landing gear deployment/retraction is illustrated throughout Section 5. Finally, the advantages, disadvantages and concluding remarks are developed.

2. Compliant synthesis methods

Most of the topology optimization methods are based on the continuum approach in which the material is iteratively removed, either by reducing the density of an element or by eliminating the element completely. Lu and Kota [9] highlighted that *disconnection* and *gray areas* are the main drawbacks of continuum methods. They overcame these problems using their *load path representation* method: first, they enumerate topologies using a basic array of beam elements and retain those topologies for which there is always a path between the input, output and grounded members, passing through intermediate members; then, they size each alternative by changing positions of nodes and sections of beams. Santer and Pellegrino [13] also used this method with some improvements on the sizing by incorporating the avoidance of overlapping elements[1] (or link crossings) by penalization. Zhou and Ting [10] also proposed the enumeration of networks connecting input, output,

[1] The avoidance of overlapping elements is desired when the mechanisms must be manufactured in one layer of material.

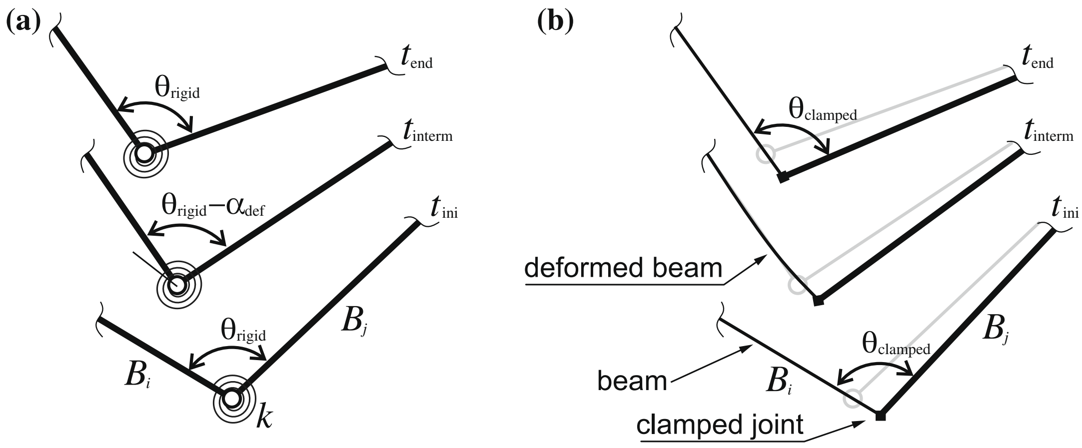

Fig. 2. Kinematic configurations of a bistable joint: (a) pseudo-rigid concept; (b) beam clamped to a rigid body.

support and intermediate nodes using the *spanning-tree theory*. Saxena and Ananthasuresh [7] proposed a number synthesis method by which an optimal topology obtained by means of discrete synthesis is converted into a pseudo-rigid one.

On the other hand, Murphy et al. [16] proposed the enumeration of compliant topologies from rigid kinematic chains in the form of atlases. The size of the data base of stored atlases is strongly dependent on the variety of links and joint types considered in the assignment procedure used to obtain the atlas. An atlas is a topological design space constituted only by *connected* topologies, but its use requires that the desired input and output members in the specifications be identified with members of each stored mechanism in all non-isomorphic ways. Several examples for visual exploration of compliant alternatives was presented by Murphy in Howell's book [2, App. G]. Also, there are available techniques for the automated exploration of rigid mechanisms [17–19]; however, the automated exploration of atlases of compliant mechanisms has not been addressed in the literature.

The use of rigid and compliant atlases of planar linkage mechanisms proposed by Pucheta and Cardona [14,18,20] allows the designer to define the prescribed parts, with fixations and the input/output relationship between their members, and then, automatically search them inside mechanisms of a selected atlas. This concept will be exploited in this paper in the topological synthesis stage. Then, the remaining problem is the sizing of each alternative.

Howell [2] presented the Pseudo-Rigid-Body Model (PRBM) as a very practical tool to simplify the analysis and synthesis of compliant mechanisms. By using rigid-body components, the PRBM allows the designer to model and analyze flexible members that undergo large nonlinear deflections. The inverse design situation, called *Rigid-Body Replacement Synthesis* [1], is also very easy to achieve by identifying rigid-body components as the PRBM of *flexible members to synthesize*. The approach is useful for designing mechanisms to perform a traditional task of kinematic synthesis – path following, function generation and rigid-body guidance – *without concern for the energy storage in the flexible members* [2, Chapter 8]. This paper extends the field of applications of that method to the design of bistable mechanisms satisfying null initial and final values for the energy characteristics.

3. Design of bistable mechanisms

A bistable mechanism has two stable equilibrium positions within its range of motion. Power requirements may be greatly reduced by using bistable mechanisms because they require energy only to switch the states, while requiring no energy to maintain the state [21,2,22,23]. The design of bistable mechanisms can be stated as a position synthesis problem where the energy storage characteristics are specified.

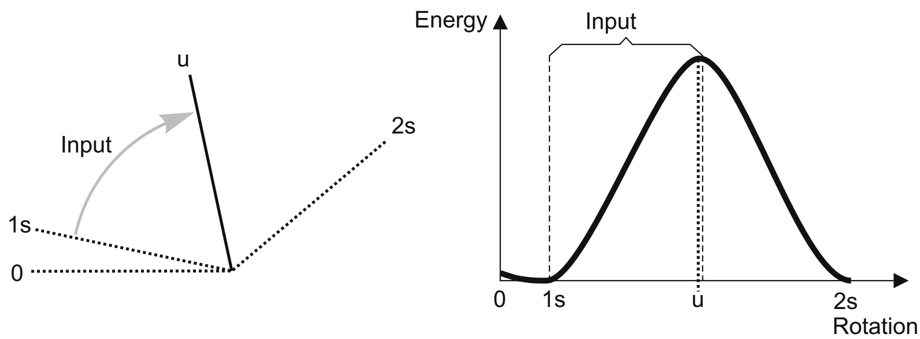

Fig. 3. Energy characteristics for the opening operation of the bottle lid shown in Fig. 1.

In Fig. 1, a sequence of photographs shows the actions required to open the lid of a bottle; Fig. 3 shows its energy characteristics where the angular positions are related to the energy stored in the compliant members. The mechanism is detailed in Figs. 4 and 5. It is a deformable triangle where one link is rigid, another link is the ground and the remaining member is a spring. The stored energy can be easily designed by choosing the shape of the triangle, since the summation of the lengths of the rigid links determines the deformed length of the spring. This triangle is widely used in several container cups and doors, among other applications.

Various flexible members, which have been used as parts of bistable mechanisms, have the *functionally equivalent* kinematic behavior of a spring; some of them are illustrated in Fig. 5: (a) a flexible body with living hinges [2, Fig. 11.1], (b) an initially curved pinned–pinned beam, and (c) a rigid-body segment clamped to an initially-straight pinned–clamped beam. Note that the, apparently unnecessary, complexity of the flexible member shown in Fig. 4does not have only an aesthetic reason but also overcomes the interference between links. A pinned–pinned beam has the advantage of supporting compression and extension loads on the line connecting the pins. A spring supports extension but needs a prismatic joint between pins to avoid buckling if the load is compressive; the spring can also be modeled as a body articulated in one end and connected to ground by a pinned slider joint, see for example, Ref. [2, Fig. 11.24]. A combination of (a) and (b), i.e. an initially-curved beam with living hinges can be found in Ref. [23, Fig. 13]. The last design concept (c) will be used in the examples given later.

Other bistable design for a shampoo lid is shown in Fig. 6. It consists in a single compliant hinge with bow shape [24]; the external borders of this bow suffer the maximum strains. It is said that the bow has two stable shapes rather than positions.

More complex energy specifications can be required. For instance, a non-null energy may be required at the stable positions. This requirement can be obtained with the aid of kinematic limitations or geometric constraints. For example, in the cellular phone [25] shown in Fig. 7, a little force has to be applied for opening the lid. In the second stable position, a remanent moment avoids an accidental closure (mechanisms with similar energy characteristics can be found in switches,

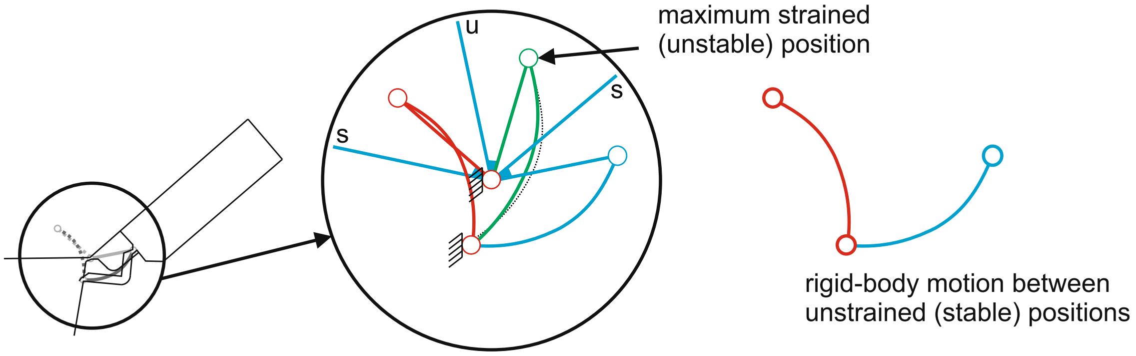

Fig. 4. Functioning principle: the flexible member of the mechanism is unstrained in the initial and final positions [21,2] while it suffers a maximum deformation in an intermediate position.

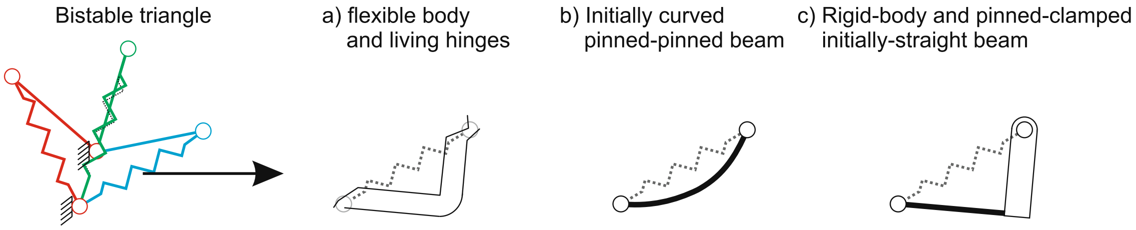

Fig. 5. A flexible triangle with one spring is the simplest bistable mechanism; the spring can be replaced by other equivalent members.

Fig. 6. Bistable hinge of a shampoo lid: the human hand actuates the lid from the first stable position up to the unstable position is surpassed.

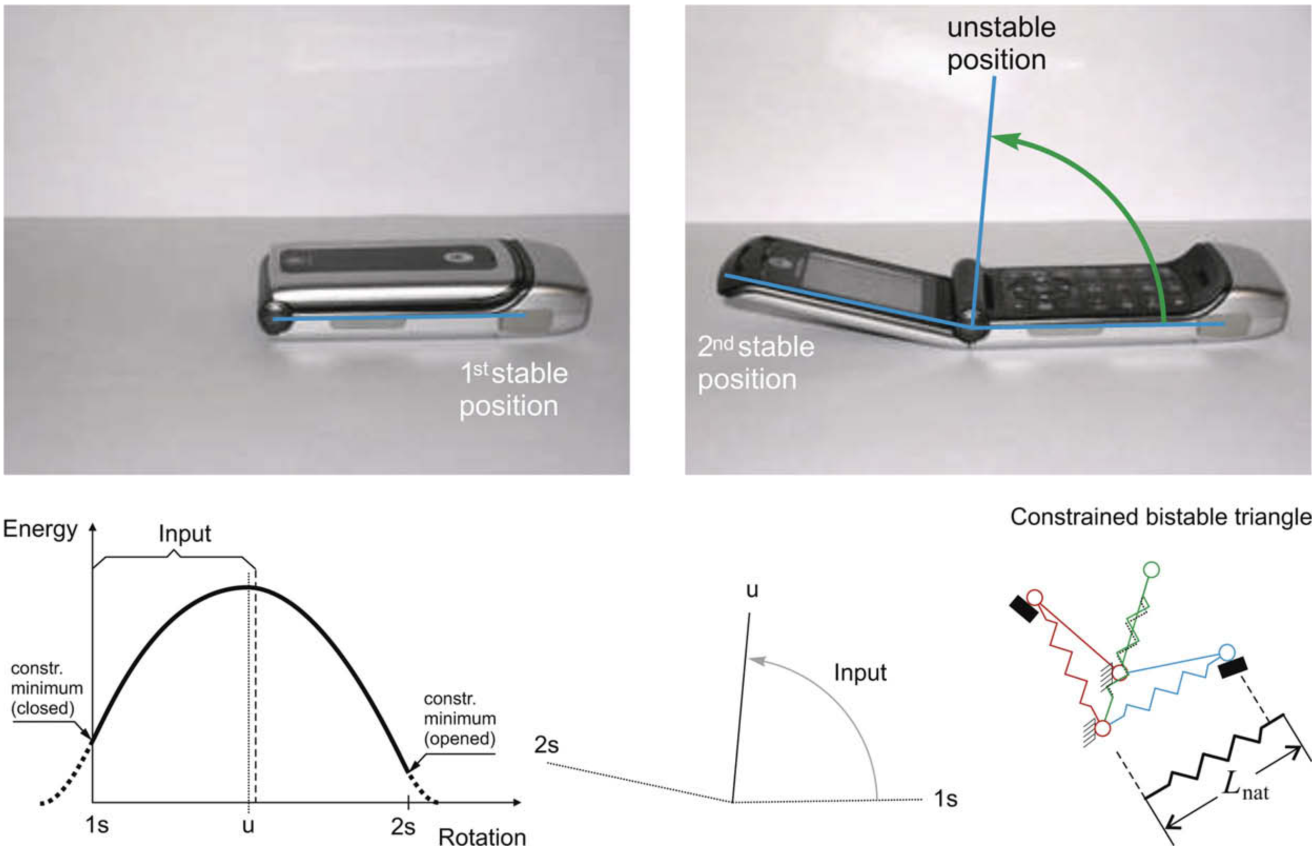

Fig. 7. Energy characteristics with non-null local minima in the stable positions: physical example (left) and functioning principle (right).

hinges for furniture doors, garage doors, and windshield wipers of vehicles). At the right of the same figure, we can see the functioning principle illustrated by a constrained bistable triangle where the rotating crank is limited by additional bodies and the spring is assembled in a compressed form, i.e., with a length shorter than its natural length L_{nat}. Thinking of energy as the "ball-on-the-hill analogy" [2,26], this is an example of two constrained equilibrium positions, see Fig. 8.

There are two typical approaches for designing bistable compliant mechanisms [2]:

Kinetostatic synthesis: based on solving analytical loop-closure position equations plus energy equations in terms of displacement variables, by sweeping the space of free parameters. This suits for the use of pseudo-rigid models; and
Analysis: based on intensive use of analyses and postprocessing of the simulated mechanisms. The stable equilibrium positions are identified by means of the force/torque vs. displacement/rotation functions. This suits for finite element analysis combined with optimization software.

Next, a new approach based on the synthesis of the initial and final unstrained positions of the compliant joints is exploited to design bistable mechanisms. The approach need not require solving energy equations nor performing dynamic analysis; however, the maximum energy requirement is not satisfied in a first stage, but is imposed in a second stage by optimization.

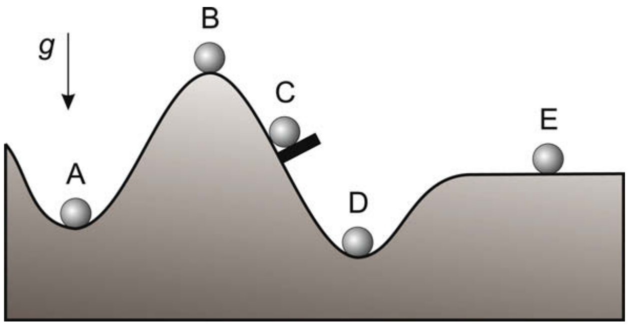

Fig. 8. Equilibrium positions of a ball subjected to a gravity field g [2,26]: (A) and (D) are stable; (B) is unstable; (C) is externally constrained stable; (E) is neutrally stable.

4. The proposed method

The prescribed value of the energy at precision position j is expressed as E_j. Howell remarked that "if all flexible members have their undeflected position at the same precision point, the energy stored at that position is zero, $E_j = 0$". Then, the bistable requirement presented in this paper for each mechanism is: $E_j = 0$ for $j = 1$ and $j = n_{pp}$, where n_{pp} is the number of prescribed positions; and $E_j > 0$ for $j = 2 \ldots n_{pp} - 1$. There is no restriction on the *number of clamped joints* in the mechanism, but all of them must develop a rigid-body motion between the initial and final position. The feasibility to develop these rigid-body motions depends on the motion constraints imposed on the bodies connected by each clamped joint; since the motions can be imposed by the initial problem or by a neighboring body (previously solved), they must be recursively evaluated.

The method implicitly involves a particular case of kinetostatic synthesis because the initial and final static forces/moments in the clamped beams are required to be null. The fundamentals shown in Fig. 2 are only based on kinematics and solid mechanics issues, so it is not necessary to compute additional energy equations to determine the stable equilibrium positions. In order to obtain a compliant bistable mechanism, two conditions must be fulfilled:

Topological condition: at least one member able to store energy (beam, spring or flexural hinge) must be present in the mechanism. Since replacement synthesis is applied for beams, the use of a clamped–revolute beam or a clamped–clamped beam is considered.[2]

Dimensional condition: the angle between two members connected by a clamped joint must be the same for the starting and final positions.

The proposed method consists of the following stages:

(i) Convert the kinematic compliant problem into a rigid one defined by precision positions.
(ii) Apply number synthesis methods to propose a valid topology.
(iii) Identify the clamped joints and motion constraints of the connected bodies to apply proper equality relationships (reject those over-constrained cases).
(iv) Solve the initial dimensions (link lengths and pivot positions) using the Precision–Position Method (PPM) [27].
(v) Identify the resultant parts as the Pseudo-Rigid-Body Models of the compliant members to be replaced.
(vi) Evaluate the kinematic performance of the solution.

These stages were implemented in a solver for type [18] and initial sizing of mechanisms [15, Chapter 8]. An additional optimization loop of *dimensional synthesis* using software for mechanism optimization can be applied to further refine the dimensions and, in this way, minimize the kinematic errors [28].

The type synthesis stage uses as topological solution space several atlases of (partially) compliant mechanisms specialized from rigid kinematic chains. These atlases – with rigid and flexible links and joints – configure a subspace of all possible compliant mechanisms. They are very useful concepts from the functional point of view, and additionally, they are well-suited for the systematization of the design process.

4.1. Type synthesis using atlases of (partially) compliant mechanisms

The type synthesis is the primitive stage of conceptual design where topology, link and joint types, must be chosen to form mechanisms in all valid forms, without repetitions, and satisfying the structural requirements of the problem. Graph Theory has been traditionally used to model and solve this problem. In the literature we can find fragmented developments on type synthesis. A lot of research was done on kinematic chains enumeration and atlas construction [29], but few studies explain how to make use of these atlases.

Pucheta and Cardona proposed the use of atlases in the form of a subgraph search problem [18]. The method is composed of four steps:

(**T1**) Give a graph description for the kinematic problem.
(**T2**) Choose an *atlas of mechanisms*, also represented by graphs, for the desired solutions.
(**T3**) Set the topological constraints to be fulfilled by the solutions.
(**T4**) Find a subgraph occurrence of the *initial graph* inside the atlas.

In step **T1** some rules are applied on the initial description of the problem to construct a graph denominated *initial graph*, denoted as G_{ini}. This graph has connectivity properties, and types of links and joints of the prescribed parts [20].

Step **T2** supposes two previously developed steps:

[2] The terms "revolute" and "pinned" are used in an interchangeable form for planar mechanisms. In pinned–pinned beams actuated at pins, compressive loads can produce buckling in initially-straight beams or store energy if the beam is initially curved. The *snap-through* effect of initially-curved pinned–pinned or initially-curved fixed–fixed beams is useful for many other engineering applications.

(**T2-a**) *Enumeration of all kinematic chains* for the desired degrees of freedom.
(**T2-b**) *Specialization of each kinematic chain* for the desired types of links and joints.

The kinematic chains enumeration was done following Tsai's proposal [30]. Each kinematic chain is represented by the adjacency matrix of its graph $G(E,V)$, where each link is represented by a vertex $v_i \in V$, and each joint $e_{ij} \in E$ is the connection between vertices v_i and v_j. The specialization of the link and joint types is formulated as an assignment problem where a colored graph representation is used. The adjacency matrix of the colored graph has integer entries representing link types on the diagonal entries and joint types on the outer diagonal ones. It is called *Type Adjacency matrix* $\boldsymbol{T}$ and is defined as follows:

$$T_{ii}(v_i) = \begin{cases} 0 & \text{if } v_i \text{ is the ground,} \\ 1 & \text{if } v_i \text{ is a rigid link,} \\ 2 & \text{if } v_i \text{ is a flexible link,} \end{cases} \qquad T_{ij}(e_{ij}) = \begin{cases} 0 & \text{no connection,} \\ 1 & \text{if } e_{ij} \text{ is a revolute joint,} \\ 2 & \text{if } e_{ij} \text{ is a prismatic joint,} \\ 3 & \text{if } e_{ij} \text{ is a flexible joint,} \\ 4 & \text{if } e_{ij} \text{ is a clamped joint.} \end{cases}$$

For example, the mechanism shown in Fig. 9 has a type adjacency matrix:

$$\boldsymbol{T} = \begin{array}{c} \\ 0 \\ 10 \\ 5 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 10 & 5 & 12 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 4 \\ 2 & 0 & 1 & 1 \\ 0 & 4 & 1 & 2 \end{bmatrix} \end{array}$$

Table 1 shows the number of specialized compliant 1DOF-mechanisms for the well-known four- and six-bar basic kinematic chains (BKC). An alphabet-like coloring is considered: the links alphabet is {0 = ground, 1 = rigid, 2 = flexible}, and the joints alphabet is {1 = revolute, 2 = prismatic, 3 = flexible_hinge, 4 = clamped(fixed)}.

The atlas obtained with ground, rigid and flexible links and revolute, flexible and clamped joints is denominated CompliantOneDofR. The rigid mechanisms are also included inside the atlas. All non-isomorphic assignments include mechanisms inversions, producing a very large number of mechanisms in the enumeration. When considering two types of kinematic pairs (revolute and prismatic), the atlas obtained is denominated CompliantOneDofRP. The number of alternatives is augmented almost 9 times due to the "combinatorial explosion" (see the last column of Table 1). Since typical kinematic problems often have only one prismatic joint, the atlas CompliantOneDofROneP is also built to accelerate the search.

In step **T3** the user can restrict the number of links or joints of a determined type to some predefined values, filtering the mechanisms taken from the atlas and thus reducing the number of explored mechanisms with undesirable characteristics. The purpose of filtering is to make the search more user-friendly, since the exploration within the space of solutions satisfying the topological specifications can easily lead to thousands of feasible mechanisms.

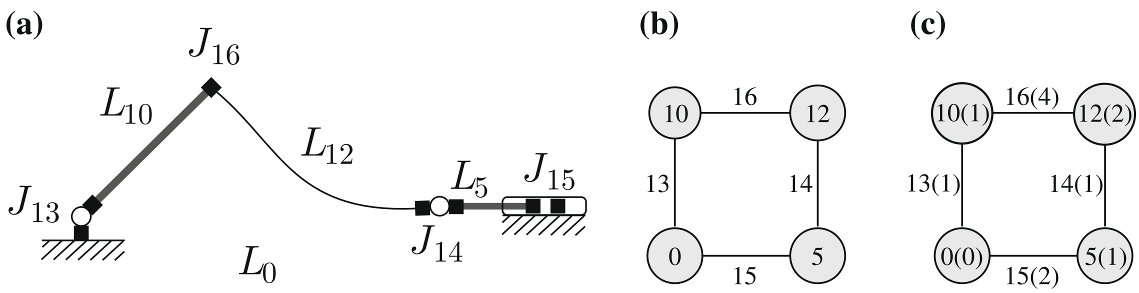

Fig. 9. Mathematical models for a four-bar mechanism: (a) FEM representation; (b) graph labeled with user's IDs; (c) graph with link and joint type colors (colored labeled graph).

Table 1
Compliant one-DOF linkage atlases (CompliantOneDof).

Suffix	BKC			Total
	Four bars	Watt	Stephenson	
R	211	50,267	52,507	102,985
RP	731	448,673	459,482	908,886
ROneP	506	178,845	183,623	362,974

Finally, step **T4** is automatically achieved by the software. The initial graph is exhaustively searched inside each graph taken from the atlas. All non-isomorphic subgraph occurrences are detected and enumerated as feasible topologies; more details can be found in [15, Chapter 3]. The search is exhaustive, and it finishes when a number of alternatives fixed by the user is reached.

After the type synthesis stage, an additional procedure decomposes each topology into single-open chains (SOCs), dyads and triads, passing through the number of prescribed positions. The terminology dyads and triads is of standard use in mechanism synthesis and refers to single open chains composed by two and three links, respectively [27]. These single-open chains are implemented as SOCs modules [15, Chapters 4 and 5]. Each SOC module has two functions: (i) evaluate the geometrical data to determine the multiplicity of solutions and the number of free parameters required to solve the geometrical loop-closure equations; and (ii) solve the equations once the geometrical data, free parameters and multiplicity are provided. Using the first function, all variables and their default bounds are determined and stored for each SOC of the alternative. The variables can be: (a) *free parameters*, which are also called "free choices" of the solution structure of the compatibility linkage used to solve the loop-closure equations expressed by means of complex numbers; (b) *missing motions* such as a rotation angle or a stretch factor of a complex number; and (c) *missing geometrical data* such as a coordinate of a new synthesized pivot or a component of the point of a desired trajectory.

After setting the equality relationships needed on the clamped joints, a second stage of initial sizing for synthesis is launched for each feasible alternative as described in the next section.

4.2. Rigid-body replacement synthesis

The design space of the sizing problem is defined by the variables identified in the sequence of single-open chains passing through a predefined number of prescribed positions. The user can change the values of their bounds, and a genetic algorithm is used to sweep the design space [31,15]. For every feasible individual computed as rigid by PPM, rigid links are replaced by flexible ones following the graph description and special rules; also, the motion of the flexible joints is constrained to certain limits. The fitness function consists in the minimization of the size of the mechanisms in combination with three weighted constraints: minimal length of link dimensions, non-inversion of transmission angle (which includes the limit angle for flexible joints), and allowed space. Eventually, instead of considering non-inversion of transmission angle as a constraint, a full kinematic analysis is made for each individual to compute the fitness function.

4.2.1. Short review of Pseudo-Rigid-Body Models

A beam can be modeled by an articulated rigid body with the beam *characteristic length* and by torsional springs located on its *characteristic pivots* to emulate the beam stiffness [2].

The *characteristic length* is computed as γL where γ is the *characteristic radius factor* and L is the beam length. The characteristic factor is determined as a function of the load case and boundary conditions. The spring stiffness $K(EI, L, \gamma)$ is used to reproduce the force–deflection relationship. It depends on the beam material properties E, the inertia of the cross-section I, the geometry L, and load case (direction of the applied force F and existence of end-moment loading M) through γ. The deflection is a function of the so-called *pseudo-rigid-body angle* Θ. See, for example, the parameterization of a beam of length L subjected to two different conditions in Figs. 10a and b. The boundary conditions as well as the load conditions are different. Both cases consider loads only at the end-points of the beams.

There is an error in the kinematic behavior of such simplified model. For example, in the cantilever beam, the error between the trajectories is shown in Fig. 11. The difference between the exact and the PRBM model increases as the rotated angle Θ is increased. By defining an admissible path error between the exact beam and the PRBM, Howell [2] computed a factor γ that maximized the deflection $\Theta_{\max}$ for which the admissible path error is verified, as function of the angle of the applied load F. The example shown in Fig. 11 corresponds to a load applied perpendicularly to the beam; a path error of 0.5% was obtained for a deflection angle $\Theta_{\max} = 64.3^\circ$, for $\gamma = 0.8517$. For angles of application of the load within the range $[63^\circ - 135^\circ]$, in order to get similar path errors the average value of γ_{ave} is 0.85 for a maximum deflection angle $\Theta_{\max} = 47^\circ$. For an end-moment loading, $\gamma_{\text{ave}} = 0.7346$ with $\Theta_{\max} = 124.4^\circ$. Also, for the fixed–fixed beam model shown in Fig. 10b, $\gamma_{\text{ave}} = 0.8517$ with $\Theta_{\max} = 64.3^\circ$.

When synthesizing a beam, the unknowns to be computed are its length and stiffness. To this end, we make the following assumptions:

Beam length: Starting from the geometry of a rigid mechanism, a beam can be designed by identifying revolute joints of the rigid mechanisms as the characteristic pivots of the PRBM of the beam; then the beam length can be roughly estimated by proposing $\gamma = 0.8517$ for all beams in the mechanism.

Beam stiffness: For an initial design, the Young modulus for steel, and the values of area $A = \left(\frac{L}{10}\right)^2$ and inertia $I = \frac{1}{12}\left(\frac{L}{10}\right)^4$ are heuristically assumed; these proposed values correspond to a square cross-sectional area with side $\frac{L}{10}$. These parameters can be later optimized once the initial mechanism is found.

After replacing the rigid link by a beam, the end-points of the beam would result in a load-case composed by transversal forces and end-moments which could be different from the pure bending hypothesis considered to get $\gamma = 0.8517$. This source of kinematic error is afterwards corrected by optimization once the initial topology is determined.

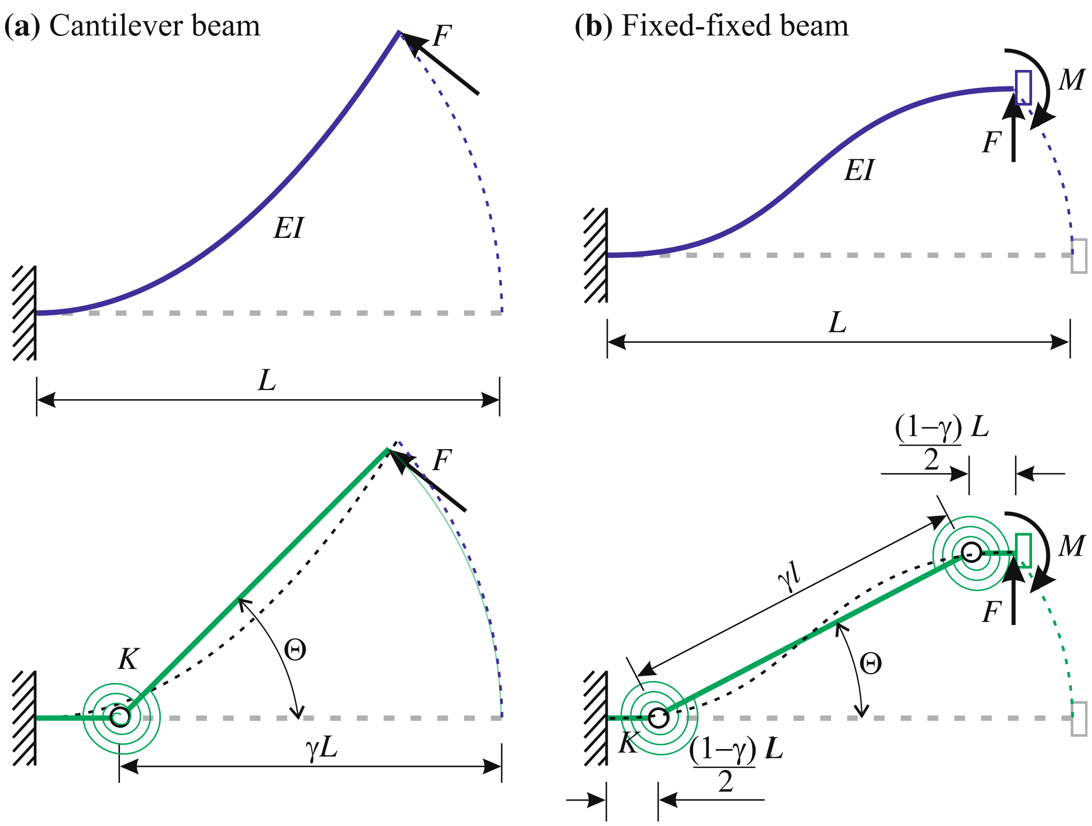

Fig. 10. PRBM models for two beams [2].

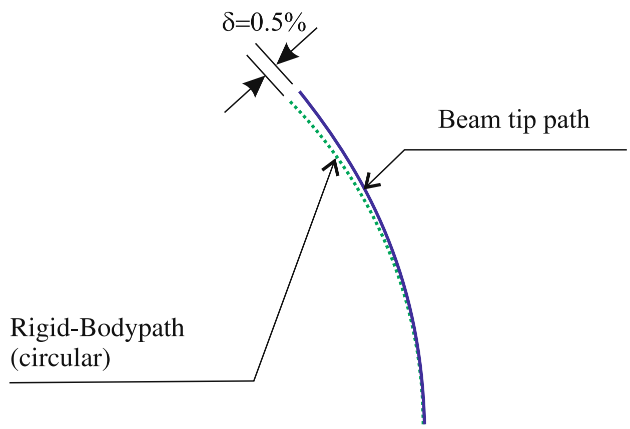

Fig. 11. Error between tip trajectories: exact beam vs. its PRBM model [2].

4.2.2. Replacement synthesis

For each feasible closed-loop topology, we will replace only binary rigid-bodies by beams. We consider that beams are structurally and functionally binary, initially straight and with constant cross-section and inertia. Another topological assumption is made: both (i) actuated links, and (ii) coupler links with imposed passing points, are considered rigid.

After computing the rigid dimensional synthesis, the set of minimal independent loops of a rigid mechanism topology is used to visit the links and analyze their feasibility to be transformed into flexible equivalents. Links are analyzed in groups of three consecutive ones. When a flexible link is found, decisions about changing the coordinates of its end-point nodes are taken considering not only the type of the terminal joints, but also the type of the terminal links; link and joint types are available from the colored labeled graph. Inside this analysis, ground and rigid links are equally considered as “rigid”.

Let l_i be the considered flexible link, and let l_{i-1} and l_{i+1} be the adjacent ones of rigid type and any connectivity degree, e.g. binary, ternary, and so on. Also, let R be the length of the rigid link, and L the length of its flexible equivalent. The *characteristic ratio* γ was taken to be fixed at a value $\gamma = 0.8517$. As it is shown in Fig. 12, we synthesize two typical cases in the following way:

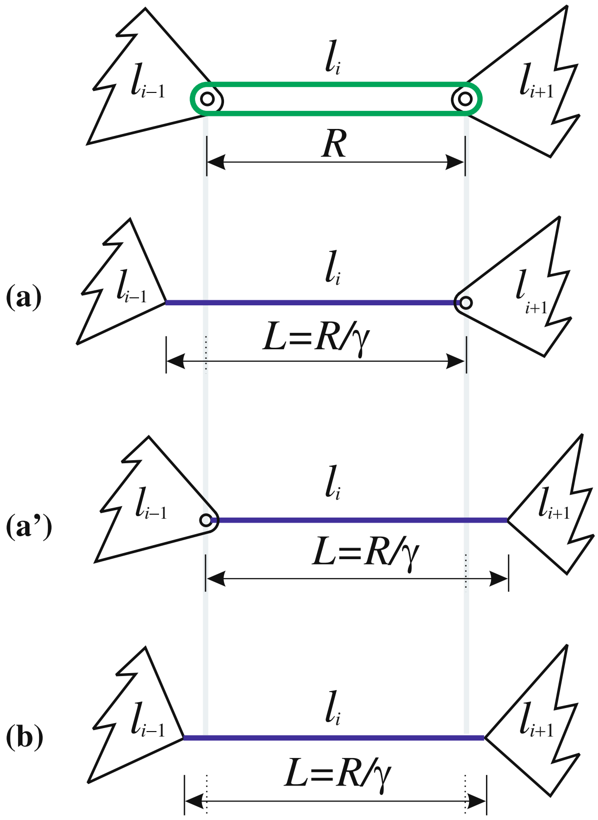

Fig. 12. Rigid-body replacements: (a) fixed–revolute, (a') revolute–fixed and (b) fixed–fixed.

(a) Fixed–revolute (revolute–fixed) flexible link: the fixed node is relocated at a distance

$$L = \frac{R}{\gamma}$$

from the pinned end, measured along the link axis.

(b) Fixed–fixed flexible link: both fixed nodes are relocated at a distance

$$L_{\frac{1}{2}} = \pm\frac{R}{2\gamma}$$

from the link midpoint along its axis.

With these simple modifications, the revolute joints are identified as the characteristic pivots of the PRBM model of the beam. Other cases are not replaced, for instance, if two consecutive links are flexible. Those cases are for now rejected from the solutions and still have to be investigated.

The rotation angle in the neighborhood of clamped joints must not exceed certain limits for practical reasons. After performing the rigid synthesis we have the information about rigid links rotations for a number of passing points; then we can measure the maximum rotation between two links on the revolute joint which will be the characteristic pivot of the PRBM of the flexible link. Although this measured rotation does not exactly reflect what occurs in flexible links, it is a useful approximation to develop a new constraint: the *limit angle for clamped joints* denoted as Θ_{max}, see Fig. 13b.

In Fig. 13a we can see the initial and final position for a rigid link to be replaced. The maximum rotation, θ_{rigid}, is measured on the revolute joint. True rotation at the clamped end of the flexible link, $\theta_{clamped}$, and the desired maximum angle, Θ_{max}, are shown in Fig. 13b. Note that the maximum rotation angle performed by the rigid link in the rigid mechanism is a rough but useful approximation of the end-points rotations of the flexible link in the transformed mechanism.

5. Example: bistable actuator for a landing gear mechanism

In previous works, a four-bar mechanism was synthesized for the deployment/retraction of a landing gear mechanism [31,15]. The simplest result is the folding linkage shown in Figs. 14 and 15.

A new study is made to analyze the feasibility of actuating the mechanism providing a bistable behavior without employing helical coil springs (thus, the bistable solutions showed in Figs. 5 and 7 are to be avoided). The description of the task is

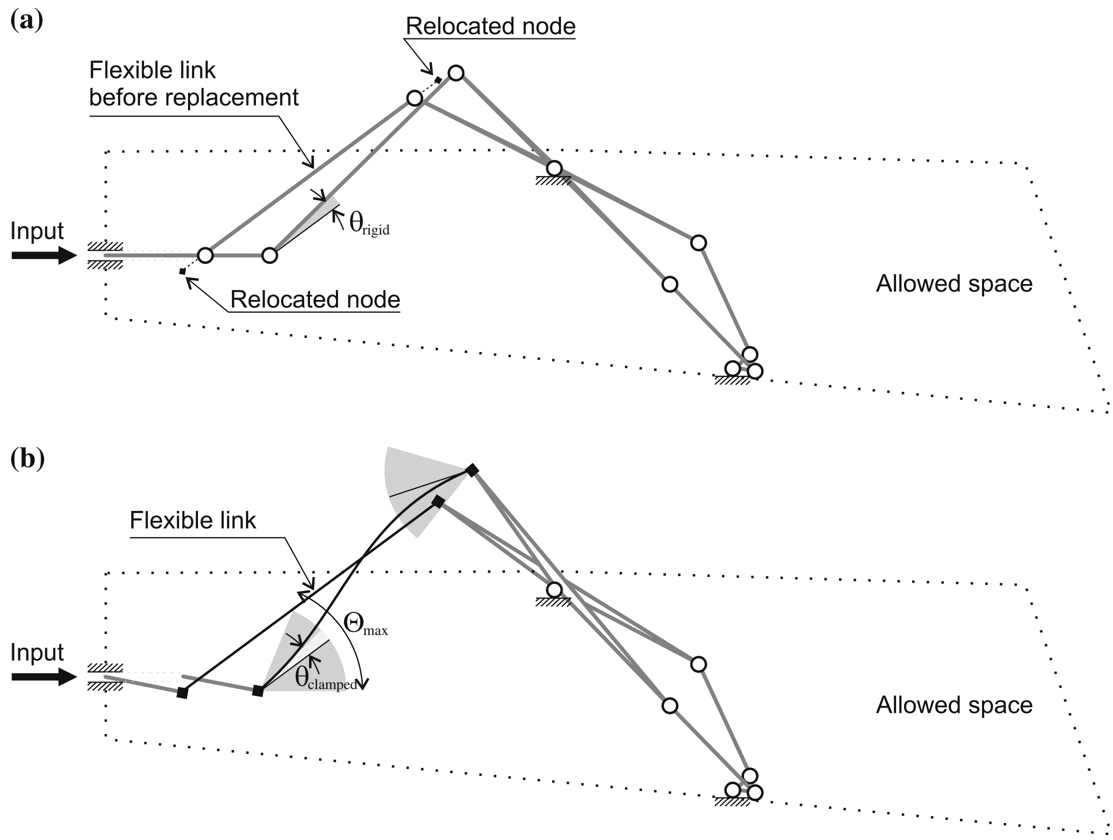

Fig. 13. Example of replacement of a flexible segment and its limit angle for clamped ends.

shown in Fig. 16. An *allowed space* constraint is defined to locate the actuator of the new mechanism. A rigid body is defined at a point connected with the ground by means of a hinge. This hinge will be the actuator and its parameters, rotation angles denoted by α's, must be computed by the dimensional synthesis procedure. Since the landing gear mechanism is rigid, the problem consists in finding all the feasible bistable mechanisms that are able to coordinate the motion between the rotational input and the existing four-bar mechanism. The rotation angle of link L_{23}, necessary to unfold -90° the leg L_{18}, was computed from previous analysis and found to be $\gamma_2 = 37.29^\circ$. An intermediate deformed configuration, for which approximately the deformation energy will reach a maximum, is proposed for a rotation angle $\gamma_1 = \gamma_2/2$. The CAD environment after the task definition is shown in Fig. 17.

5.1. Type synthesis

The atlas of compliant linkages CompliantOneDofR, which contains mechanisms with clamped and flexural joints, is selected as topological design space.

The following topological constraints are imposed (step **T3**, Section 4.1):

(i) Flexural hinges are not allowed.
(ii) The leg (L_{18}) and the intermediate body (L_{26}) of the landing gear are constrained to be binary. The ground (L_0) is constrained to be ternary to avoid the addition of new pivots.

After execution of the type synthesis solver, we get the alternatives shown in Fig. 18. New elements (joint J_{34}, the flexible link L_{33} and joint J_{35}) are added in four different ways without repetition. Although in this case the alternatives shown are a bit trivial and could easily be found by hand, we remark that the solver can enumerate solutions with higher orders of complexity, with more loops and pivots, where the combination of link types and joint types is not an easy work to be developed manually.

5.2. Dimensional synthesis

Each alternative proposed by the type synthesis is dimensioned independently.

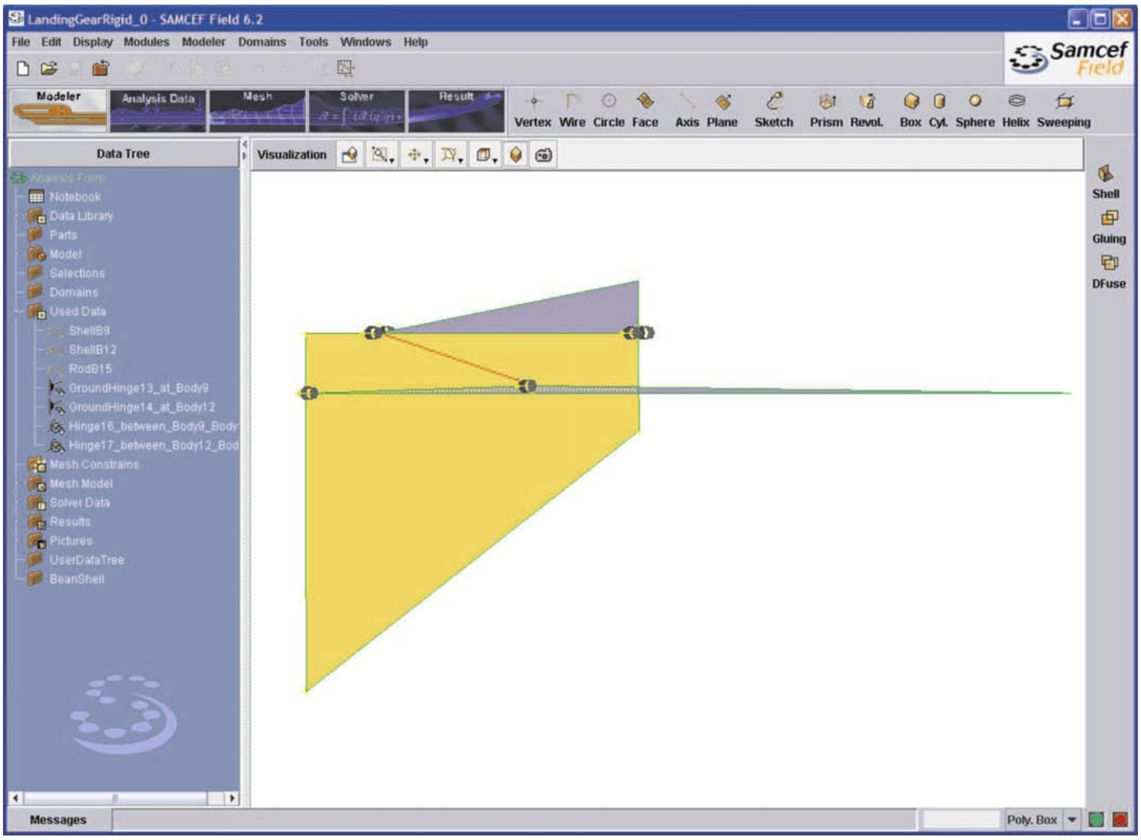

Fig. 14. Landing gear mechanism.

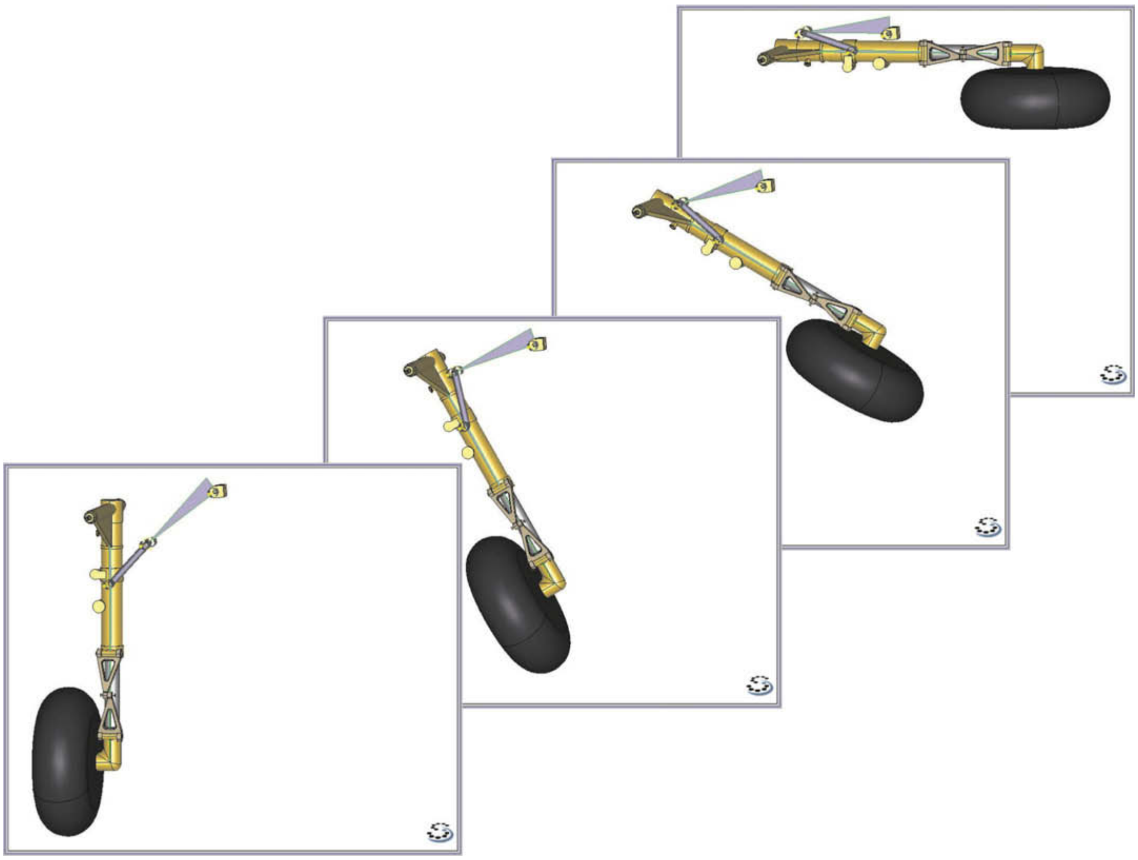

Fig. 15. Deployment of a landing gear mechanism passing through four precision positions.

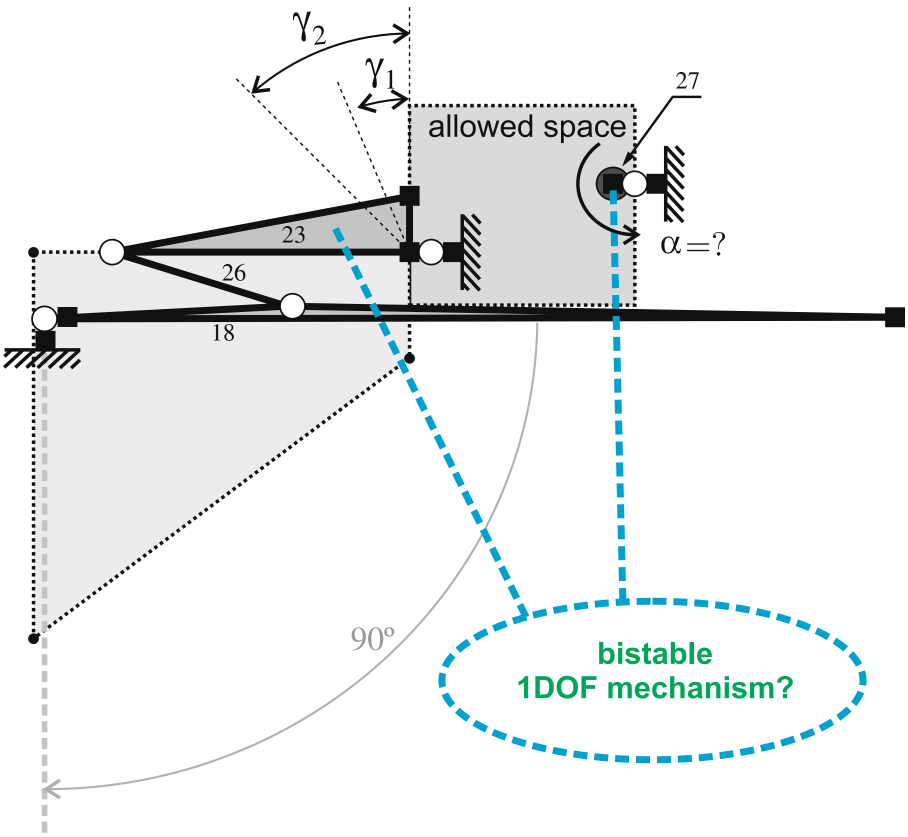

Fig. 16. Required bistable task.

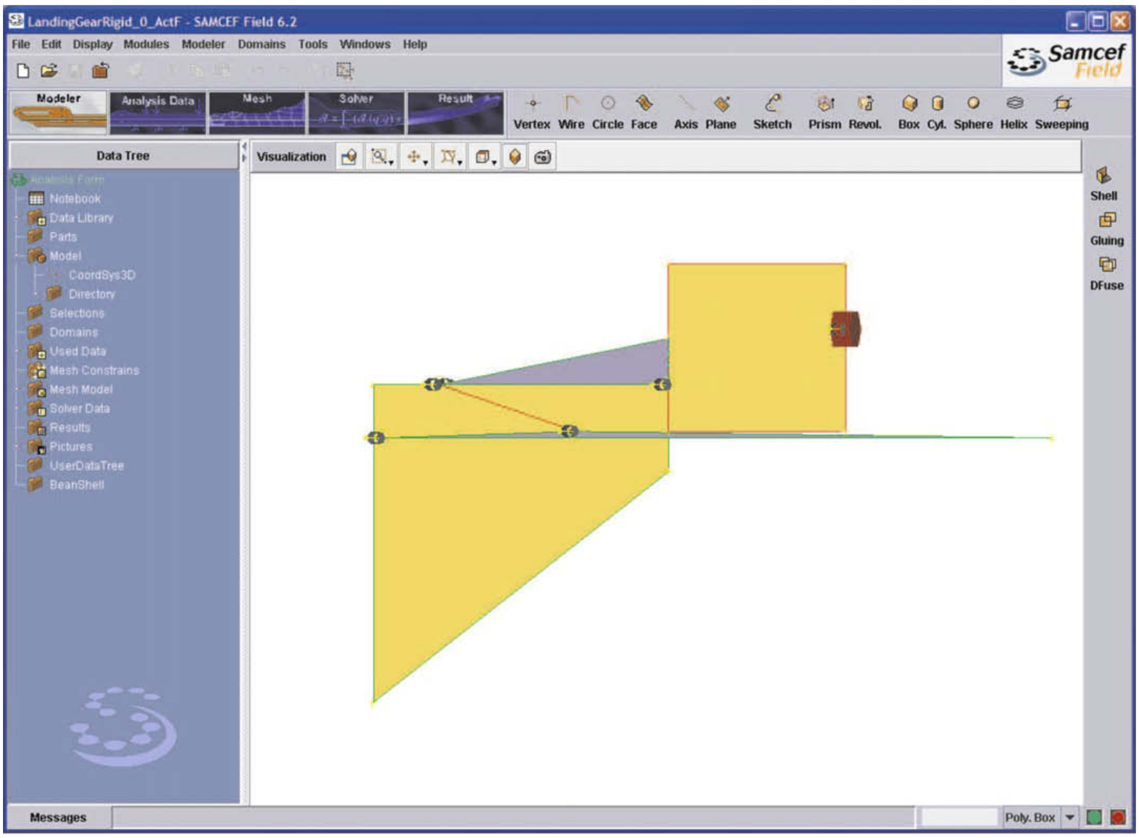

Fig. 17. Definition of the bistable task using a CAD environment.

5.2.1. Alternative 0

Alternative 0 has a pinned–pinned beam which does not store energy, since it moves as a rigid mechanism. The actuation of the joint J_{32} is computed so that the whole mechanism passes exactly through the three prescribed positions.

5.2.2. Alternative 1

Alternative 1 presents a flexible body L_{33} linked by a revolute joint J_{34} with body L_{23} and by a clamped joint J_{35} with link L_{27} (Fig. 18). The problem is solved using the precision–position method giving the rigid mechanism displayed in Fig. 19a. As shows Fig. 19b, the clamped joint J_{35} is inserted by changing the flexible link length following the replacement rules explained in Section 4.2.2.

The decomposition of the topology into single-open chains results in one triad passing through three positions. The identification of significant dimensions for links L_{27}, L_{33} and L_{23} results in three complex numbers denoted as $biZ_0, \boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$, respectively. These three complex numbers, which are used to represent the triad shown in Fig. 19a, rotate the angles α's, β's and γ's, respectively. Angles γ's are data of the previous synthesis problem, while the other angles, α's and β's, are identified as "free parameters" by the synthesis solver. The bounds for these parameters must be proposed by the user. We choose these bounds to obtain a bistable behavior.

In Fig. 19b we show the compliant mechanism after rigid-body replacement. Following the replacement rules, the size of the rigid link $|\boldsymbol{Z}_1|$ is stretched to be $|\boldsymbol{Z}_1|/\gamma$ in the flexible version.

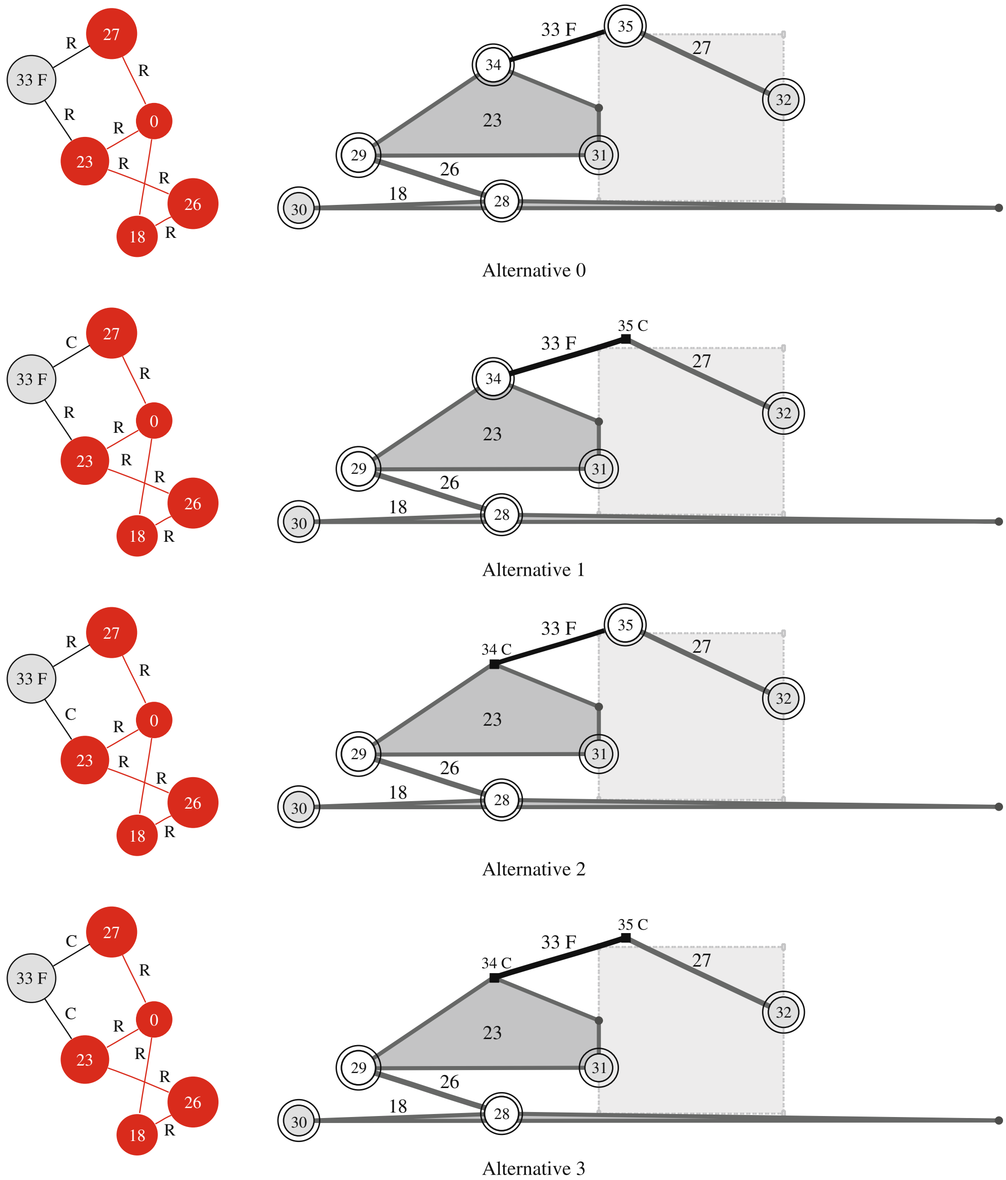

Fig. 18. Outputs of the type synthesis solver and their corresponding physical sketches for **Alternatives 0** to **3**. References for joint types in graphs are: R = revolute, C = clamped. In sketches, flexible links have a letter "F", other links are assumed to be rigid.

In order to obtain an unstrained final configuration, and bistable behavior, the relative angle at initial and final configurations between the clamped bodies must be the same; this angle is denoted as θ_{clamped}. Since the free parameters handle the design space for the rigid model, we must propose equal values for angles α_2 and β_2 to obtain θ_{rigid} in the starting and final positions; thus, links $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_1$ will develop a rigid-body motion between the initial and final positions. The deformation in the beam can be controlled by proposing the angles α_1 and β_1.

A triad passing through three positions with imposed offset has a nonhomogeneous system [15, Chapter 4]

$$\begin{bmatrix} (e^{i\alpha_1}-1) & (e^{i\beta_1}-1) & (e^{i\gamma_1}-1) \\ (e^{i\alpha_2}-1) & (e^{i\beta_2}-1) & (e^{i\gamma_2}-1) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{Z}_0 \\ \boldsymbol{Z}_1 \\ \boldsymbol{Z}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta}^1 \\ \boldsymbol{\delta}^2 \\ \boldsymbol{r} \end{bmatrix}, \tag{1}$$

or briefly as

$$\mathbb{C}^{\text{off}}\mathbb{Z} = \mathbb{D}^{\text{off}}, \tag{2}$$

where $\mathbb{Z}$ are the unknowns; $\boldsymbol{\delta}^1$ and $\boldsymbol{\delta}^2$ are the vector differences between the end-point displacements of the triad at precision positions 1 and 2, respectively (here, they are both null for each precision position); $\boldsymbol{r}$ is the complex number which closes the triad, i.e. a vector going from one pivot to the other one. This system of equations can be solved iff $\det(\mathbb{C}^{\text{off}}) \neq 0$.

The solution for Alternative 1 shown in Fig. 20 satisfies both energy and kinematic requirements. The rotation of the leg of the landing gear is equal to 90° for practical purposes (the difference being less than 6×10^{-4} degrees); see Fig. 21a.

After rigid-body replacement, the *stored energy vs. time* characteristics has the desired bistable behavior so that no further optimization is needed, see Fig. 21b. The maximum value reached by the stored energy is 7.826 J at time 1.013 s, while at the final time it is practically zero (0.014 J).

5.2.3. Alternative 2

Alternative 2 presents a flexible body L_{33} linked by a revolute joint J_{35} with body L_{27} and by means of a clamped joint J_{34} with link L_{23} (Fig. 18).

Fig. 22 shows the diagram for the identified triad. In order to obtain an unstrained final configuration of the flexible element, complex numbers $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ in the rigid design (a) must develop a rigid-body motion between the initial and final position, preserving the angle denoted as θ_{rigid}. This can be obtained by proposing a value for β_2 equal to γ_2. The deformation in the beam can be controlled by varying the angles α_1 and β_1.

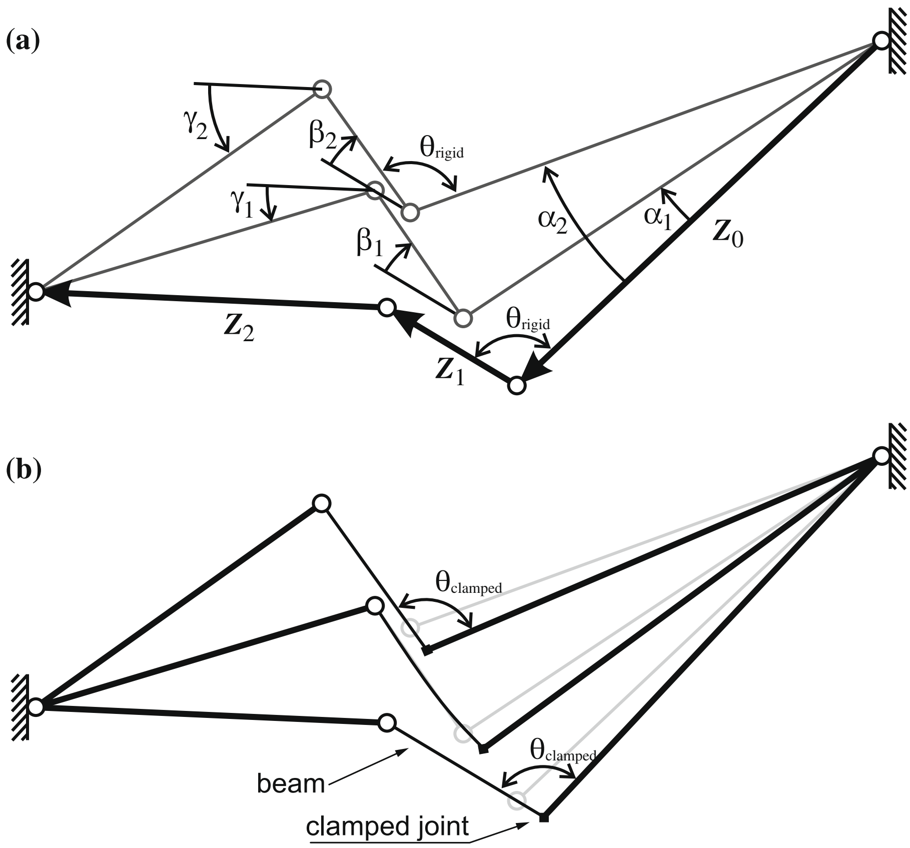

Fig. 19. Dimensional synthesis of Alternative 1 using rigid-body replacement.

After rigid-body replacement, the resulting model is simulated as shown in Fig. 23.

The difference with 90° in the leg rotation is 5×10^{-4} degrees, see Fig. 24a. The *stored energy vs. time* characteristics, shown in Fig. 24b, has the desired bistable behavior so that no further optimization is needed. The maximum energy reached has a value of 23.59 J and it is practically zero (0.0035 J) at 2 s.

5.2.4. Alternative 3

The flexible member in Alternative 3 has two clamped ends. If we call θ_{rigid} the angle between the flexible link and the driver link, and θ'_{rigid} the angle between the flexible link and the driven link of a four-bar mechanism like those shown in Figs. 19 and 22, it is easy to realize that we cannot set up conditions to preserve both angles at the final configuration, except for the trivial case $\gamma_2 = \alpha_2 = 0$, in which the allowed motion at driver and driven links can only be oscillatory.

Non-trivial motion conditions to obtain a bistable behavior of a clamped–clamped beam can however be defined on (i) a triad if one of the ends is not a pivot (by defining an intermediate deformed configuration), see Fig. 25; and (ii) on an single open-chain with four or more articulated complex numbers. Thus, the feasibility to obtain a bistable behavior is dependent on the mechanism type and motion constraints defined. Its complete discussion and application will be investigated in future research. Nevertheless, we show a synthesis result in Fig. 26 solved by setting an unstrained position in one of the clamped ends of the beam, which is interesting from the kinematic point of view.

The kinematic error in the leg rotation is of 0.425° so that the replacement synthesis helps again to find a good approximation of the required motion, see Fig. 27a. The stored energy, shown in Fig. 27b, increases monotonically with the leg rotation, so it is not able to match the final energy requirement.

5.3. Bistable mechanisms derived from change-point rigid mechanisms

A change-point mechanism is a mechanism that passes through a dead-center position, i.e., the driver and coupler links are collinear at an intermediate configuration (Fig. 28a). This configuration reached by the mechanism is also called a change-point or toggle position. From this position, even if the driver link returns to the initial position or continues its rotation, the driven link has an oscillatory motion. A change-point mechanism computed by the precision–position method and converted by means of replacement synthesis can be designed to provide a bistable behavior [2,22], see Figs. 28b and c. We

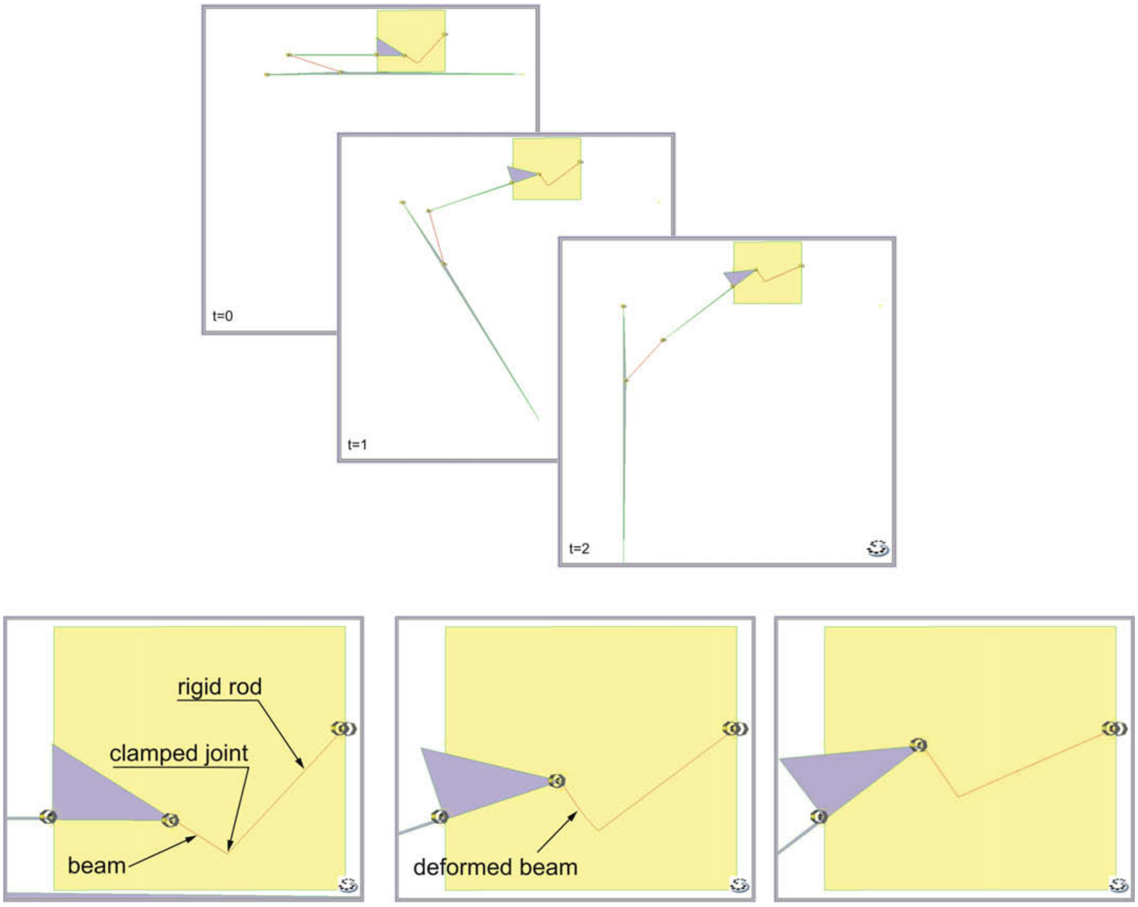

Fig. 20. Simulation for Alternative 1 at times 0, 1 and 2 s.

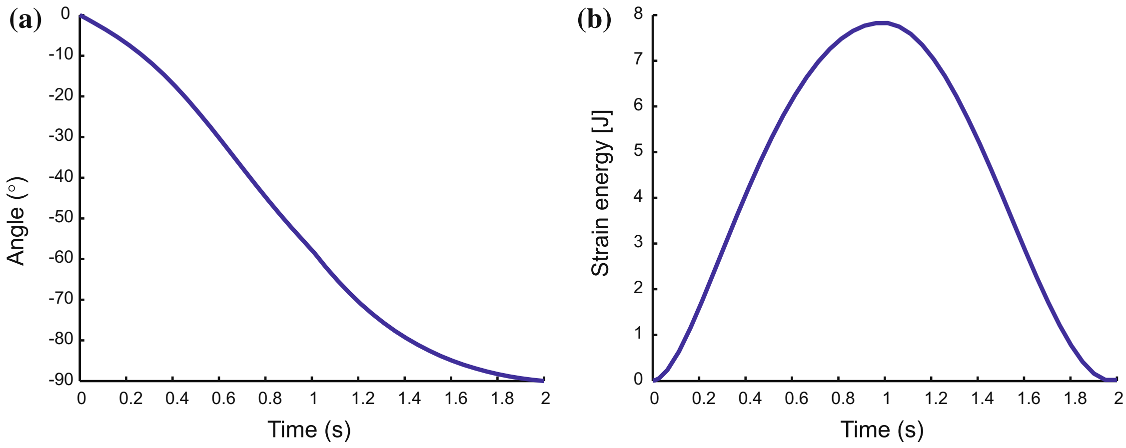

Fig. 21. Rotation of the leg (a) and stored strain energy vs. time (b) for Alternative 1.

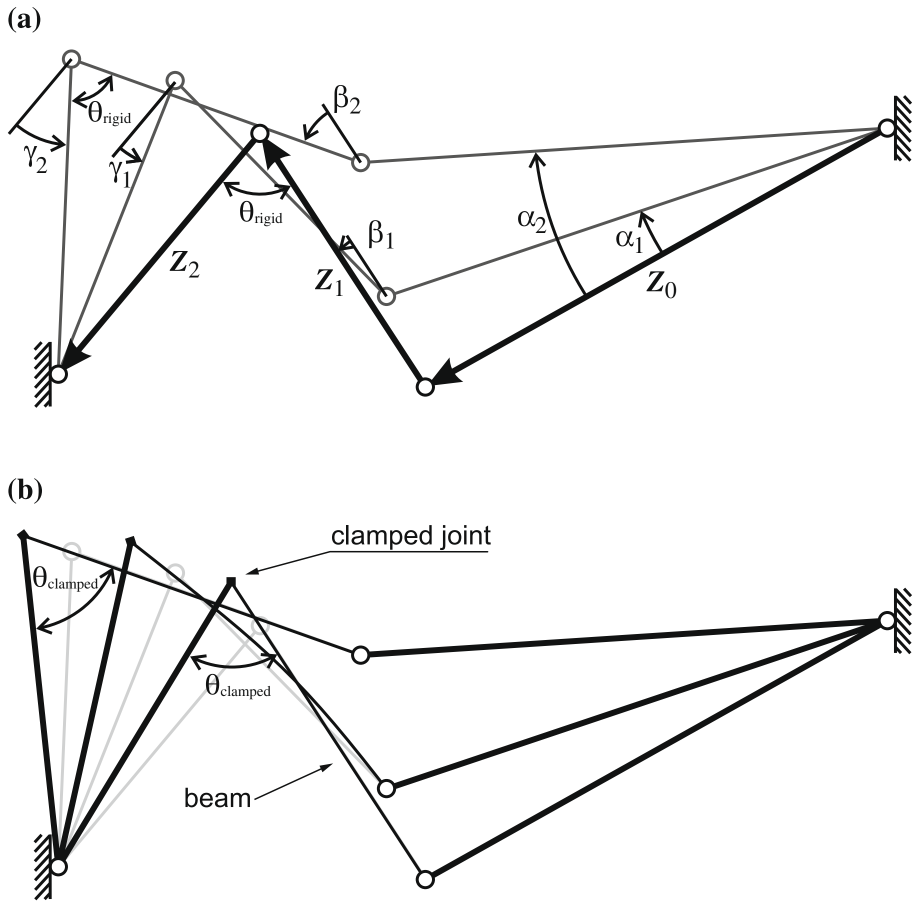

Fig. 22. Dimensional synthesis of Alternative 2 using rigid-body replacement.

have already seen that the bistable behavior can be obtained by simply proposing equal angular positions between the links connected by a clamped joint. In this case, it is equivalent to propose equal initial and final position for the driven link.

As we can see in Fig. 28c, in this mechanism we have a clamped joint connected to the ground. This situation was not obtained in the list of alternatives shown in Fig. 18 because a revolute joint J_{32} which attaches a one-node rigid body L_{27} to ground was specified in the initial description, see Fig. 16.

In order to get a change-point mechanism, we change the initial specifications of the landing gear problem in the following way: the mechanism is actuated on joint J_{31} of the folding mechanism and the rotational input J_{32} (Fig. 16) is replaced by a single clamped node. This allows, for example, a clamped connection between a flexible beam with ground, see Fig. 29.

The first feasible topological solution has a grounded beam (Fig. 29, above). In the dimensional synthesis, a three-position triad (Fig. 30) is solved for obtaining a change-point mechanism by setting the angles of the complex number $\boldsymbol{Z}_0$ in the

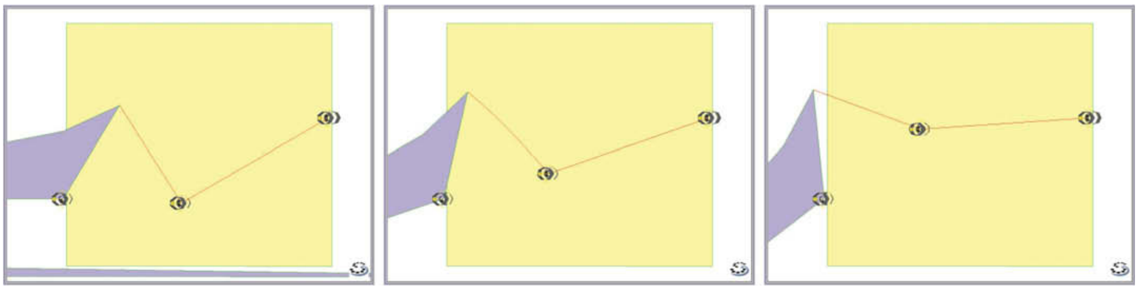

Fig. 23. Simulation for Alternative 2 at times 0, 1 and 2 s.

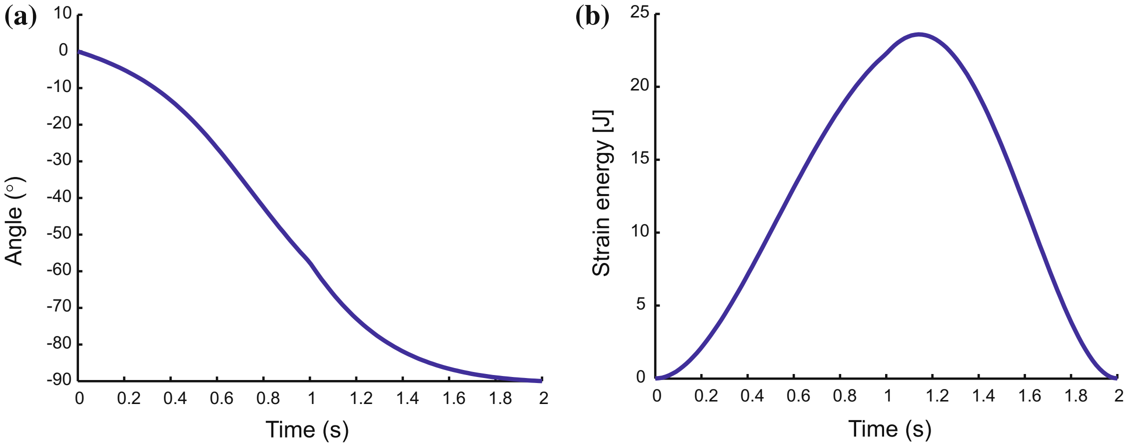

Fig. 24. Rotation of the leg (a) and stored strain energy vs. time (b) for Alternative 2.

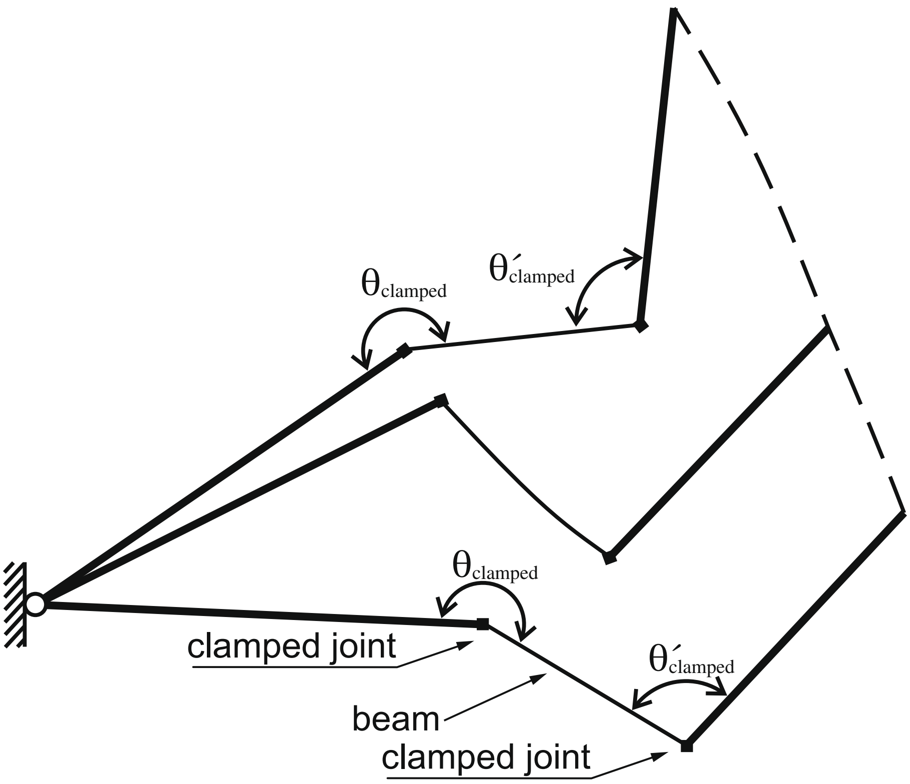

Fig. 25. Geometrical conditions required on the end-points of a clamped–clamped beam to develop a bistable behavior; example of a compatible motion.

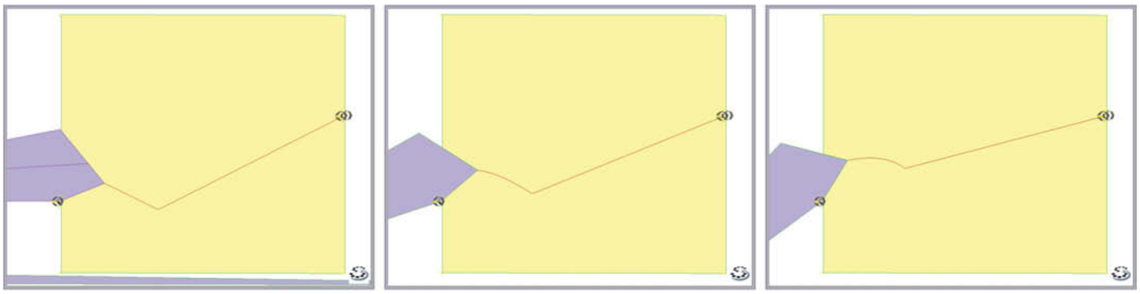

Fig. 26. Simulation for Alternative 3 at times 0, 1 and 2 s.

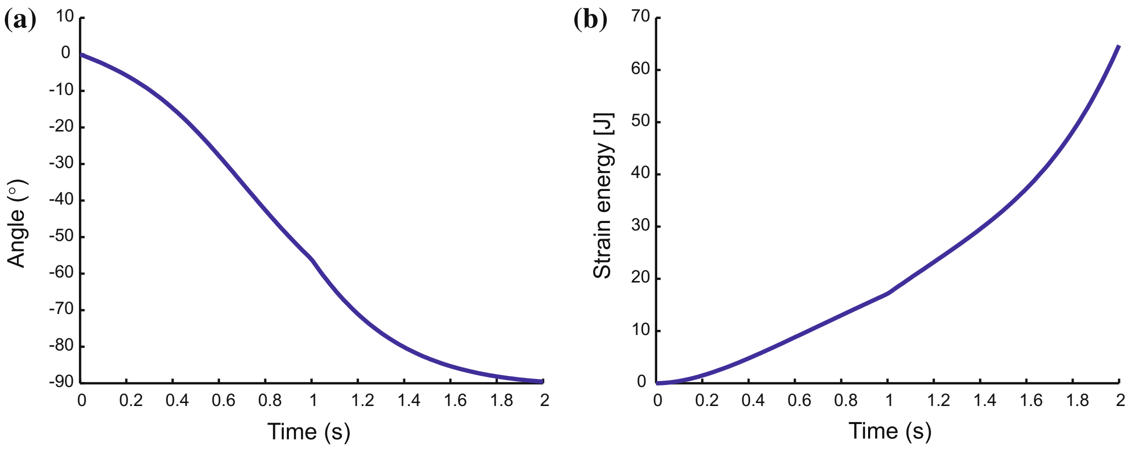

Fig. 27. Rotation of the leg (a) and stored strain energy vs. time (b) for Alternative 3.

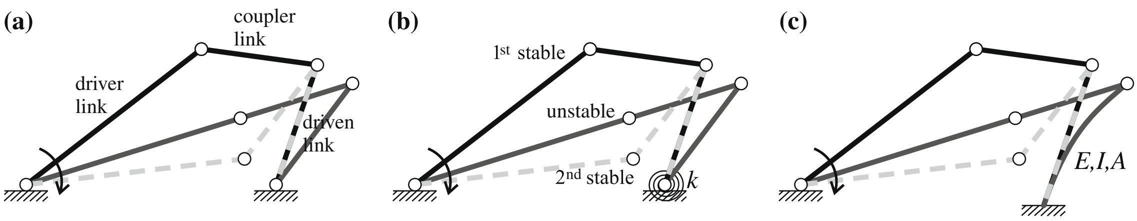

Fig. 28. Change-point mechanisms: (a) rigid, (b) bistable pseudo-rigid, (c) partially compliant obtained by replacement. The initial position is shown in black color, the second (change-point) position in dark grey color, and the final in a dashed line with grey color.

following way: $\alpha_1 = \frac{\pi}{6}$ rad and $\alpha_2 = 0$ rad. The angle α_1 limits the motion of the body represented by $\mathbf{Z}_0$ which is replaced by a clamped–revolute beam. The angles γ_1 and γ_2 are data and angles β_1 and β_2 are the free parameters.

The synthesis of the first alternative results in good agreement in both, the kinematic and the bistable behaviors; the resultant energy can be observed in Fig. 31a. The simulation of the bistable mechanism is shown in Fig. 32.

For the second alternative (Fig. 29, below) a replacement of a clamped–clamped beam was made. The synthesis showed a good kinematic behavior (Fig. 33), but was unable to fully verify the energy requirement, see Fig. 31b.

6. Discussion

The design of compliant mechanisms involves *motion*, *force* and *energy* requirements. Using the proposed method we firstly dealt with the fulfillment of *motion* and *energy* requirements for cases with null deformation energy imposed at the initial and the final positions, i.e., bistable mechanisms. From this concept and the numerical experiments developed, we can identify some advantages and disadvantages with respect to other approaches inspired on structural optimization.

6.1. Advantages

- The rigid-body replacement method combined with the exploration of atlases of compliant mechanisms enables the designer to find quick-design concepts for complex tasks.

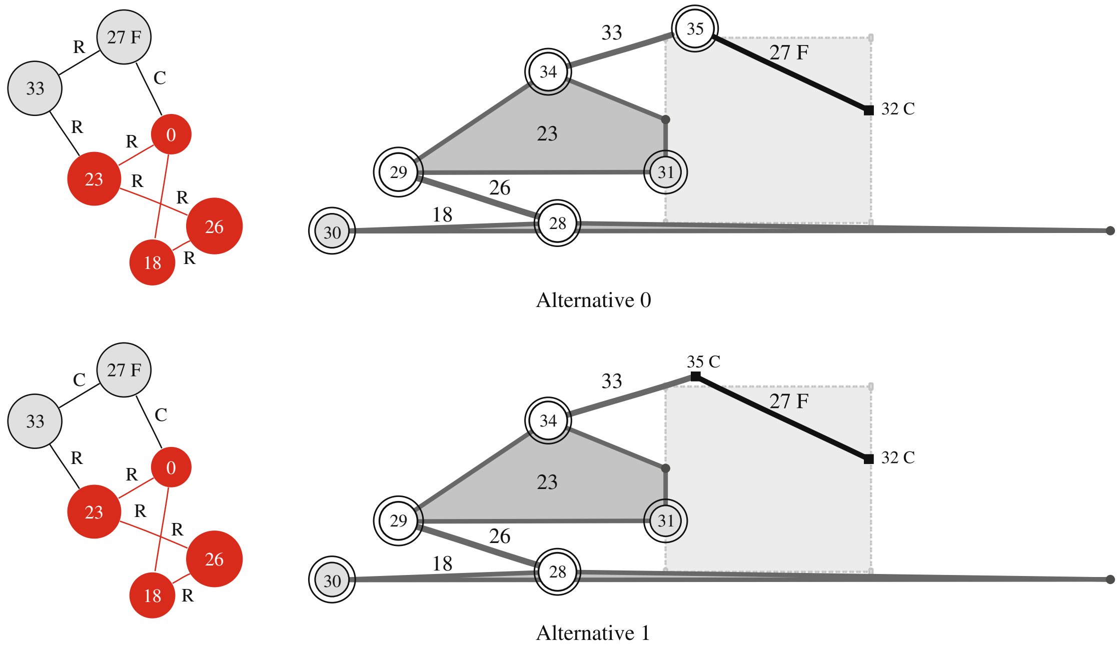

Fig. 29. Topological solutions for new specifications.

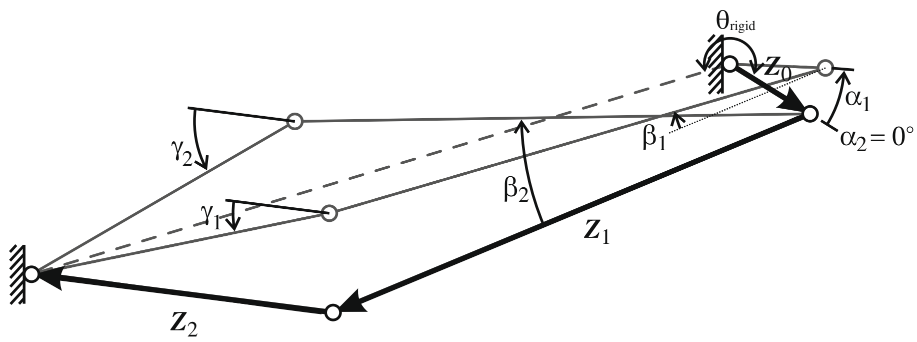

Fig. 30. Dimensional synthesis of Alternative 0 for obtaining a bistable change-point mechanism.

- For sizing each feasible topological alternative, precision-point synthesis provides a low number of variables which is combined with rigid-body replacement synthesis to find near optimum solutions.
- For bistable requirements, motion and energy can be designed conjointly using revolute–clamped or clamped–clamped beams and the precision-point method traditionally used for rigid synthesis. It is easy to set up bi-stability problems as a three-position kinematic task with control on the motion/deformation of clamped joints. All mechanisms for opening/closing operations are examples where the task can be adapted for three positions. Additionally, a given geometrical advantage could be defined between the input and output movements on the imposed parts at the problem description stage.
- Without applying the replacement rules, the design technique is useful for the design of bistable rigid-link mechanisms with springs.
- The bistable concept of designing the initial and final unstrained positions of clamped connections preserves validity in three-dimensional compliant mechanisms.
- Linearly or nonlinearly elastic materials can be used in the flexible members.
- Beams are preferred over other elastic members such as helical coil springs, for providing flexibility in micro-mechanisms because their viability to be manufactured in one layer of material.

6.2. Disadvantages

- Several difficulties found in the motion design of rigid mechanisms using precision–position synthesis such as poor behavior between passing points, and circuit and branch defects are translated to the replacement synthesis, thus the method is dependent of the availability of an efficient rigid synthesis solver.

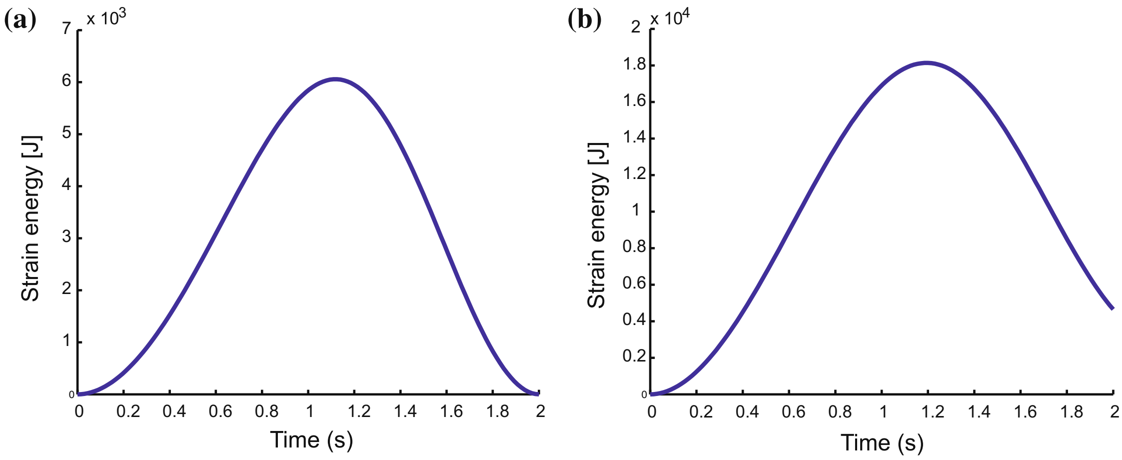

Fig. 31. Stored strain energy for a change-point mechanism with (a) a clamped–revolute beam and (b) a clamped–clamped beam.

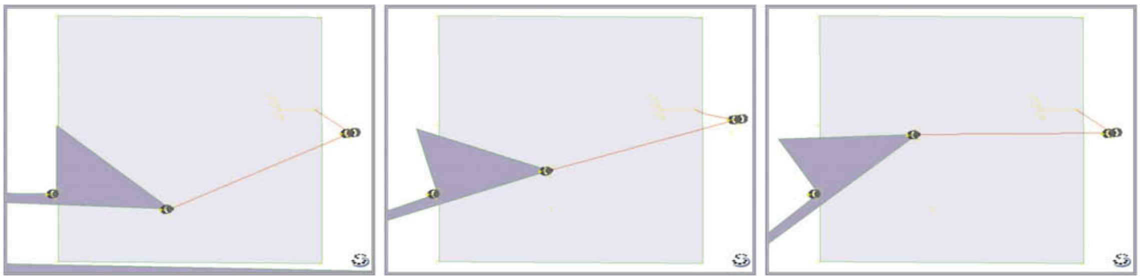

Fig. 32. Simulation for a bistable mechanism derived from a change-point mechanism with a clamped–revolute beam.

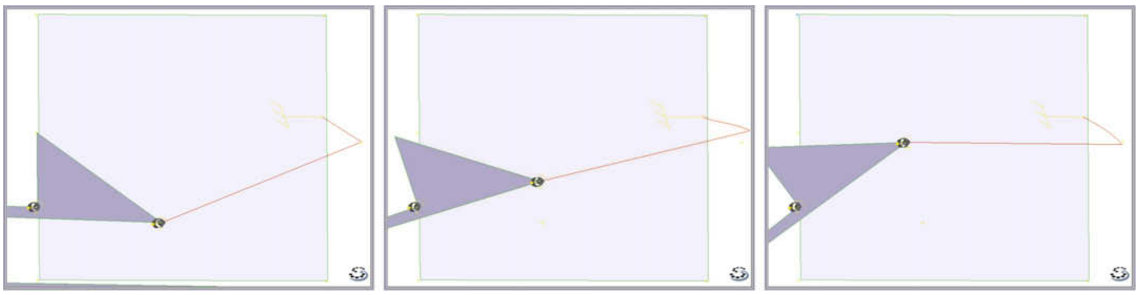

Fig. 33. Simulation for a bistable mechanism derived from a change-point mechanism with a clamped–clamped beam.

- In some situations the replacement produces a mechanism which changes its kinematic behavior drastically compared to its rigid version. Therefore, several approximations made between the rigid and the compliant mechanisms obtained by replacement synthesis lead to big kinematic errors and their causes must be investigated.
- Only a pure bending hypothesis is taken into account for beams design. Depending on the resultant load-case (forces and moments) in the solution, an undesired geometric nonlinearity, like beam's buckling, may occur between initial and final positions.

7. Conclusions

A new study of the kinematic design of bistable compliant mechanisms using the precision–position method and replacement synthesis was presented. The method helps to determine the best mechanism configuration to accomplish the desired task imposed on prescribed parts or sub mechanisms. The type synthesis part of the methodology enable the designer to easily explore atlases of compliant mechanisms derived from rigid kinematic chains to satisfy prescribed parts with imposed motion constraints. Then, the feasibility of getting a bistable mechanism can be evaluated from the kind of kinematic constraints imposed on the bodies connected by a clamped joint, but also considering the motion constraints imposed on them.

The study was limited to the replacement of binary initially-straight beams. In the example given, the use of a single characteristic ratio ($\gamma = 0.8571$) was useful – from the kinematic point of view – for designing the three replacement cases presented so that motion was guided with little deviation from the target value. These results need almost no further optimization. The problem was reformulated in order to design change-point mechanisms with a flexible link clamped to ground. The two new solutions found also fulfilled the kinematic task and one of them satisfied the required energy behavior. However, since the clamped joint cannot be actuated, that solution can only be used as passive compliant bistable loop.

The replacement rules and the satisfaction of bistable requirements by means of geometric considerations using precision–position synthesis are important contributions towards the automated conceptual design of compliant mechanisms.

8. Future research

In the bistable test problems shown, the bounds of variables were set "by hand" to obtain equalities and relationships between the angles. However, these relationships can be easily programmed to work automatically by implementing the proper rules.

Further research would incorporate synthesis rules for prescribing flexible segments as initial parts and for designing of initially curved segments. With respect to rigid kinematic pairs, the precision–position method is available for sizing mechanisms with multiple joints, e.g. straight and curved sliders. The design of flexible hinges such as *living hinges* and *small-length flexural pivots* [2] can be solved using rigid revolute joints and restricted in their motion, as for clamped joints, by incorporating of a limit angle of rotation. The *small-length flexural pivots* also need the development of dimensional rules for replacement.

The maximum amount of energy stored in the system as a function of the input can be prescribed. Note in the examples that the maximum energy reached is near the intermediate position. To approximate this requirement, one additional equation for the intermediate position may be added, that is to solve $E_{\text{interm}} = \frac{1}{2}k(\alpha_{\text{def}})^2 = E_{\text{imposed}}$. By assuming a torsional spring constant k for the pseudo-rigid body model of the clamped joint, the relative position α_{def} can be designed by varying the motion constraints; then with the resultant beam length L, the inertia property I can be computed from $k = \gamma K_\Theta \frac{EI}{L}$, where the stiffness coefficient can be $K_\Theta \approx \pi\gamma$.

The study can be extended to the design of multi-stable mechanisms and micro-mechanisms.

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References

[1] M.D. Berglund, S.P. Magleby, L.L. Howell, Design rules for selecting and designing compliant mechanisms for rigid-body replacement synthesis, in: Proceedings of the 26th Design Automation Conference, at the ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DETC2000/DAC-14225, Baltimore, Maryland, 2000, pp. 1–9.
[2] L. Howell, Compliant Mechanisms, John Wiley & Sons, New York, 2001.
[3] C.B.W. Pedersen, T. Buhl, O. Sigmund, Topology synthesis of large-displacement compliant mechanisms, International Journal for Numerical Methods in Engineering 50 (12) (2001) 2683–2750.
[4] O. Sigmund, Design of multiphysics actuators using topology optimization – Part I: One-material structures, Computer Methods in Applied Mechanics and Engineering 190 (49-50) (2001) 6577–6604.
[5] M.P. Bendsøe, N. Kikuchi, Generating optimal topologies in structural design using a homogenization method, Computer Methods in Applied Mechanics and Engineering 71 (2) (1988) 197–224.
[6] O. Sigmund, Design of multiphysics actuators using topology optimization – Part II: Two-material structures, Computer Methods in Applied Mechanics and Engineering 190 (49-50) (2001) 6605–6627.
[7] A. Saxena, G. Ananthasuresh, A computational approach to the number of synthesis of linkages, ASME Journal of Mechanical Design 125 (1) (2003) 110–118.
[8] M.Y. Wang, S.-K. Chen, X.-M. Wang, Y.-L. Mei, Design of multimaterial compliant mechanisms using level-set methods, ASME Journal of Mechanical Design 127 (5) (2005) 941–956.
[9] K.J. Lu, S. Kota, Topology and dimensional synthesis of compliant mechanisms using discrete optimization, Journal of Mechanical Design 128 (5) (2006) 1080–1091.
[10] H. Zhou, K. Ting, Topological synthesis of compliant mechanisms using spanning tree theory, ASME Journal of Mechanical Design 127 (4) (2005) 753–759.
[11] M. Sauter, G. Kress, M. Giger, P. Ermanni, Complex-shaped beam element and graph-based optimization of compliant mechanisms, Structural and Multidisciplinary Optimization 36 (4) (2008) 429–442.
[12] A. Saxena, G. Ananthasuresh, Topology synthesis of compliant mechanisms for nonlinear force–deflection and curved path specifications, ASME Journal of Mechanical Design 123 (1) (2001) 33–42.
[13] M.J. Santer, S. Pellegrino, Topology optimization of adaptive compliant aircraft wing leading edge, in: Collection of Technical Papers – 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, vol. 1, Honolulu, Hawaii, USA, 2007, pp. 142–154.
[14] M. Pucheta, A. Cardona, Kinematics synthesis of compliant mechanisms using rigid-body replacement, in: C.L. Bottasso, P. Masarati, L. Trainelli (Eds.), Multibody Dynamics 2007, ECCOMAS Thematic Conference, Politecnico di Milano, Milan, Italy, 2007, pp. 156–157.
[15] M. Pucheta, Computational methods for design and synthesis of planar mechanisms, Ph.D. Thesis, Universidad Nacional del Litoral, Santa Fe, Argentina, 2008.
[16] M. Murphy, A. Midha, L. Howell, The topological synthesis of compliant mechanisms, Mechanism and Machine Theory 31 (2) (1996) 185–199.

[17] D. Chen, W. Pai, A methodology for conceptual design of mechanisms by parsing design specifications, ASME Journal of Mechanical Design 127 (6) (2005) 1039–1044.
[18] M. Pucheta, A. Cardona, An automated method for type synthesis of planar linkages based on a constrained subgraph isomorphism detection, Multibody System Dynamics 18 (2) (2007) 233–258.
[19] Y. Liu, J. McPhee, Automated kinematic synthesis of planar mechanisms with revolute joints, Mechanics Based Design of Structures and Machines 35 (4) (2007) 405–445.
[20] M. Pucheta, A. Cardona, Synthesis of planar multi-loop linkages starting from existing parts or mechanisms: enumeration and initial sizing, Mechanics Based Design of Structures and Machines 36 (4) (2008) 364–391.
[21] B.D. Jensen, Identification of macro- and micro-compliant mechanism configurations resulting in bistable behavior, Master's Thesis, Brigham Young University, Provo, Utah, USA, 1998.
[22] B. Jensen, L. Howell, Identification of compliant pseudo-rigid-body four-link mechanism configurations resulting in bistable behavior, ASME Journal of Mechanical Design 125 (4) (2003) 701–708.
[23] B. Jensen, L. Howell, Bistable configurations of compliant mechanisms modeled using four links and translational joints, ASME Journal of Mechanical Design 126 (4) (2004) 657–666.
[24] J.M. Hess, N.J. Jelich, B.M. Mueller, Closure with multiple axis bistable hinge structure, US Patent 5,642,824, 1997.
[25] S.R. Wilcox, B.J. Hassemer, S.J. Gaynes, Radiotelephone having a self contained hinge, US Patent 5,628,089, 1997.
[26] J.K. Anderson, Piezoresistive sensing of bistable micro mechanism state, Master's Thesis, Brigham Young University, Provo, Utah, USA, 2005.
[27] G. Sandor, A.G. Erdman, Advanced Mechanism Design: Analysis and Synthesis, vol. 2, Prentice-Hall, New Jersey, 1984.
[28] F. Cugnon, A. Cardona, A. Selvi, C. Paleczny, M. Pucheta, in: Multibody Dynamics, Computational Methods and Applications, Computational Methods in Applied Sciences, Ch. Synthesis and Optimization of Flexible Mechanisms, vol. 12, Springer, Berlin, 2008, pp. 81–93.
[29] T.S. Mruthyunjaya, Kinematic structure of mechanisms revisited, Mechanism and Mechanism Theory 38 (4) (2003) 279–320.
[30] L. Tsai, Mechanism Design: Enumeration of Kinematic Structures According to Function, CRC Press, Boca Raton, 2001.
[31] M. Pucheta, A. Cardona, Type synthesis and initial sizing of planar linkages using graph theory and classic genetic algorithms starting from parts prescribed by user, in: J. Goicolea, J. Cuadrado, J.G. Orden (Eds.), Multibody Dynamics 2005, ECCOMAS Thematic Conference, Madrid, Spain, 2005.