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Structure of Compact Stars in Palatini $f(R)$ Gravity

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Abstract. The study of the structure of compact objects in modified theories of gravity can be useful to constrain $f(R)$-theories in the strong gravitational regime. In particular, the structure of compact stars in the theory with Langrangian density $f(R) = R + \alpha R^2$ have been recently explored using the metric formalism.

In this work we analyze configurations of neutron stars in *squared*-gravity using the Palatini formalism, in which the field equations are of second order, and the modified Tolman-Oppenheimer-Volkoff equations for a spherically-symmetric and static metric can be derived without approximation, as in General Relativity.

The numerical integration of the structure equations allows us to study the mass-radius configurations and the characteristics of internal profiles. We compare our results with those obtained using General Relativity.

1. Analytical Formulation

The so-called $f(R)$ theories of gravity are obtained when the Ricci curvature scalar, R, is replaced, in the Einstein-Hilbert action, by a function of it. In particular, the simplest choice $f(R) = R + \alpha R^2$, also called *squared*-gravity, has been shown to be a viable alternative to General Relativity (GR), which satisfies the current Solar System tests for gravity (Sotiriou & Faraoni, 2010). However, gravity in the strong gravitational regime is largely unconstrained by observations. Hence, the study of the properties of Neutron Stars (NSs) and Quark Stars (QSs) in different gravitational frameworks may help in setting constraints (and eventually discard) alternative theories.

The modified Hilbert-Einstein action for *squared*-gravity in the Palatini formalism is

$$S[g, \Gamma, \psi_{\rm m}] = \frac{1}{16\pi} \int {\rm d}^4 x \sqrt{-g} f(R) + S_{\rm m}[g_{\mu\nu}, \psi_{\rm m}], \tag{1}$$

where $f(R) = R + \alpha R^2$, $R \equiv g^{\mu\nu} R_{\mu\nu}(\Gamma)$, $R_{\mu\nu}(\Gamma) = -\partial_\mu \Gamma^\lambda_{\mu\nu} + \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\nu\lambda} - \Gamma^\lambda_{\nu\rho}\Gamma^\rho_{\mu\lambda}$ and $\Gamma^\lambda_{\mu\nu}$ is the connection. The matter action, $S_{\rm m}$, depends on the matter field, $\psi_{\rm m}$, and the metric, $g_{\mu\nu}$, but is independent of the Christoffel symbols. Here α is a free parameter of the theory which must be positive due to stability considerations (Sotiriou & Faraoni, 2010). The scalar curvature R can be solved as an algebraic function of the trace T of the energy-momentum tensor as $R = -8\pi T$.

We assume a spherically-symmetric and static metric, $\mathrm{d}s^2 = -e^{A(r)}\mathrm{d}t^2 + e^{B(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$, and we consider a perfect-fluid matter with energy-momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$ (where ρ is the energy density and p is the pressure). With these considerations, the modified Tolman-Oppenheimer-Volkoff (TOV) equations are:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{1}{1+\gamma_0}\,\frac{\rho + p}{r(r-2m)}\left(m + \frac{4\pi r^3 p}{1+2\alpha R} - \frac{\alpha_0}{2}(r-2m)\right), \qquad (2)$$

$$\frac{dm}{dr} = \frac{1}{1+\gamma_0}\left(\frac{4\pi r^2 \rho}{1+2\alpha R} + \frac{\alpha_0 + \beta_0}{2} - \frac{m}{r}(\alpha_0 + \beta_0 - \gamma_0)\right), \qquad (3)$$

where $m(r) \equiv r(1 - e^{-B(r)})/2$. The quantities α_0, β_0 and γ_0 are functions of $f(R)$ and the derivatives of $f(R)$ with respect to R and the coordinate r (Olmo, 2008; Barausse et al., 2008).

The above system of differential equations can be solved if an Equation of State (EoS) $p = p(\rho)$ is given. Through this work we use three different EoS: PLY, which is a polytropic EoS of adiabatic index 2; SLY, which is a realistic EoS for nuclear matter (Haensel & Potekhin, 2004); and SQM, which is a simple linear EoS for quark matter (Degrand et al., 1975). The derivatives of $f(R)$ with respect to the coordinate r, hidden in the functions α_0, β_0 and γ_0, are expressed in terms of the first and second derivatives of p with respect to ρ, by means of the EoS, as R is related to T, contrary to GR, where no derivatives of the EoS are involved. Thus, analytical approximations instead of tabular EoS are needed in order to achieve enough precision.

2. Results

In Figure 1 we show the family of static configurations for the PLY, SLY and SQM EoS considering several values of the α parameter (see the legend). Whereas for simple polytropic EoS (PLY and SQM), the static configurations are almost indistinguishable from GR ($\alpha = 0$), in the case of the realistic and more complex SLY EoS, relevant differences arise. The maximum mass, $M_{\rm max}$, grows up to $\gtrsim 10\%$ when $\alpha_8 = 10$. Moreover, contrary to what is found by means of a perturbative approach in the metric formalism (Arapoğlu et al., 2011; Orellana et al., 2013), $M_{\rm max}$ increases for $\alpha > 0$.

In Figure 2 we show the internal mass profiles $m(\rho)$ obtained for the SLY EoS using the same values of the α parameter (solid lines, left axis),

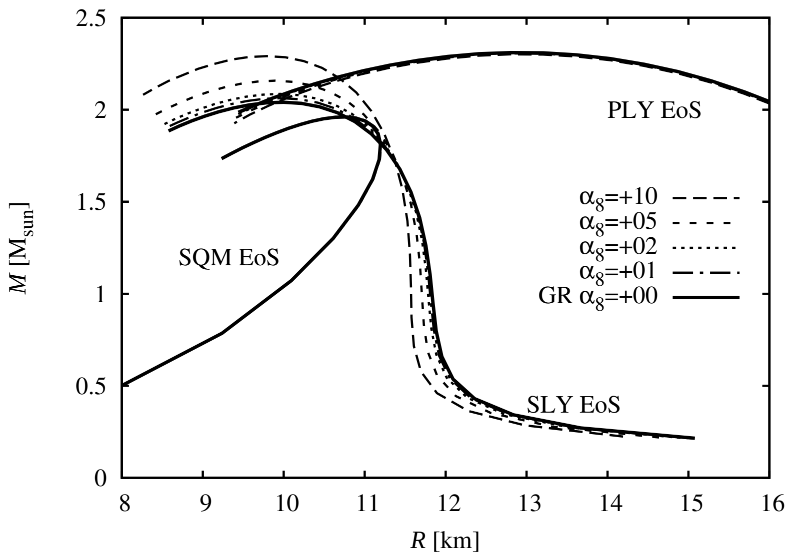

Figure 1. Mass-Radius relations obtained for three EoS considering $\alpha_8 = \alpha/10^8$ cm^2 = 10, 5, 2, 1, 0 (GR case).

considering $\rho_c = 2.5 \times 10^{15}$ gr cm^{-3}, which corresponds to $\sim M_{\rm max}$. Strong differences with the GR case (thick-solid line) can be noted: (i) a huge difference in the total mass, M, of the static configurations arises at densities $\sim 10^{14}$ gr cm^{-3}, and (ii) a counter-intuitive feature in the mass profile occurs at densities $\sim 5 \times 10^{11}$ gr cm^{-3}, which is seen as a dip in the profile ($dm/d\rho > 0 \rightarrow dm/dr < 0$). The latest behaviour was also pointed out in the metric formalism in Orellana et al. (2013) and it is independent of the value of α. In Figure 2, we also plot the logarithmic first and second-order derivatives of the SLY EoS in gray-dashed lines (see right axis), which makes explicit the correlation between this particular behaviour of the profiles and the complexity of the EoS. On the other hand, we found neither counter-intuitive features nor huge differences in the total mass in the internal profiles obtained with PLY and SQM EoS.

3. Discussion

We have shown how mass-radius configurations of compact stars are modified under *squared*-gravity in the Palatini formalism. The field equations depend not only on the explicit $p - \rho$ relation (the EoS), but also on the first and second-order derivatives of p with respecto to ρ. If a simple polytropic or linear equation is adopted, differences between compact stars in *squared*-gravity and GR are negligible. However, if a realistic EoS, as SLY, is used, the maximum mass achievable can be significantly larger than its value in GR, becoming an observable signature of modified gravity in the strong gravitational regime. Moreover, we also found huge differences in

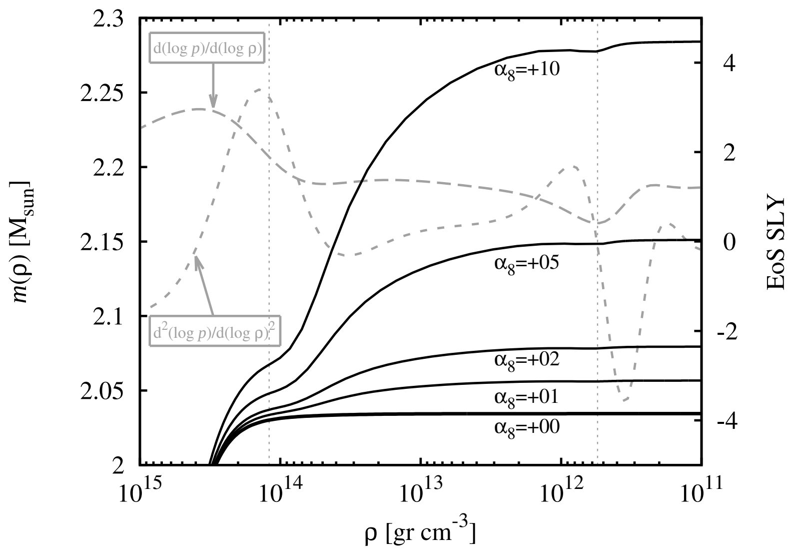

Figure 2. Mass internal profile (black-solid lines, left axis) and logarithmic derivatives of the SLY EoS (gray-dashed lines, right axis) for NS configurations with $\rho_c = 2.5 \times 10^{15}$ gr cm^{-3} and α_8 =10, 5, 2, 1, 0 (GR case). Notice the correlation between features on the mass profiles and the high-order derivatives of the realistic EoS, for $\alpha > 0$, at densities 5.5×10^{11} and 1.2×10^{14} gr cm^{-3} (gray-dotted lines).

the internal mass profiles, which present a counter-intuitive characteristic ($dm/dr < 0$) in the outer layers of the compact stars.

We shall continue our research by exploring the impact of differentiability constraints on the stellar structure calculations in modified gravity.

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