

Bounded Rationality and Economic Evolution: some formal considerations

RESUMO: Diremos que evolução econômica tem lugar quando ocorre uma mudança estrutural. Neste trabalho se apresenta um modelo simples de como uma mudança evolutiva pode ocorrer em presença de agentes com racionalidade limitada. Cada agente tem em mente um modelo de funcionamento da economia que resulta de uma visão limitada do sistema. Esta restrição pode ser formalizada através de um sistema ESO, um modelo de autómatos celulares com regras endógenas de mudanças, o qual constitui uma extensão da noção de sistema criticamente auto-organizado (SOC).

PALAVRAS-CHAVE: Mudança estrutural, racionalidade limitada, evolução.

ABSTRACT: Economic evolution is said to occur when economic structures change. In this paper we present a simple model of how evolutionary change may be caused when the agents have bounded rationality. Each agent has a model of the internal (and external) workings of the economy. This model consists of a limited vision of the economy. This restricted point of view can be formalized through economic ESO systems, which are basically cellular automata with endogenous rules of change that extend the notion of critically self-organized systems (SOC).

KEY WORDS: Structural change, bounded rationality, evolution.

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1. Introduction

The analysis of economic evolution follows two, not necessarily disjoint, paths: one hand, it studies the transition from a mainly rural economy to one with a strong participation of industry and services (Chenery 1979). On the other hand it is possible to study evolution as "... qualitative process of change of the economic organization of a society" (Olivera 1959). In this sense, we can consider that the organization of a society is based on its constituent institutions. At the same time these institutions are strongly influenced by the characteristics of the agents (tastes, preferences and goals) as well as by the constraints faced by the economy (technology, resources, etc.). In other words, the structure of an economy can be defined by three parameters: the productive structure (firms and technology), the social-political structures (the "rules of the game") and the characteristics of the agents (preferences, rationality, endowments).

The qualitative changes alluded by Olivera can be represented as changes of the aforementioned parameters. However, the first obstacle faced in a research of this kind is to distinguish the truly *evolutionary* changes from those that arise from the internal dynamics of the system. Such difficulty has been usually ignored by economic theory: the parameters are usually conceived as fixed and change only as a result from exogenous changes. Here, instead, we will consider these parameters as mutable, but that not every variation will be of evolutionary nature.

We borrow from biology the main characteristic of a system in evolution: a system (organism) presents long stages of stability, followed by short periods of radical changes. These changes are called "punctuated equilibria". So, for example, changes of taste may arise because of fashion (I may buy more red clothing this year), because of (smooth) climate changes or for any other reason. These kinds of changes cannot be deemed as evolutionary. But if these changes conflict with other fundamental parameters, they force structural changes of a higher level. In general, possible sources of conflict reside in the technology, the deep

characteristics of the agents or of the socio-political structure (drastic changes of government, policy, etc.).

To distinguish smooth variations from evolutionary changes we can use models based on the formalism of cellular automata, in particular the model of a *sand pile*. As a first step, we define a state of the economy as a meta-stable state, represented as a critically self-organized system (SOC). A SOC system, when submitted to an exogenous perturbation responds with a dynamics represented by a power law ($1/f$ noise) where variations of all sizes occur (cascades). In this model there is **no** evolution in the sense of Olivera, because the fundamental parameters remain invariant, in particular, the critical parameter. This critical state differs in many ways from the notion of steady state usual in growth theory. This meta-stable state, instead remains "permanent" for a while, even if it generate variations of all sizes in all its components. This is, as said not the case of an evolutionary change.

In economics we assume that evolution happens when the organization of the society changes, that is, when its institutions change. In terms of the SOC model, it evolves when the rules of connection among components change. Then, the original SOC system is replaced by another (supported by the components and values of the previous one). The main analytical concern here is how this change arises and which conditions may induce it.

It may be asked why to use a SOC model. The answer is that open systems present several advantages over the conservative systems used normally in economics. Openness allows to incorporate nonlinearities, irreversibility and uncertainty, features distinctive of any real world economy (London 1996). Furthermore, these systems allow incorporating also all the main characteristics of models with agents with less than full rationality.

Methodological individualism indicates that "to explain the institutions and the social change is to prove how they arise as a result of the actions and interactions of the individuals" (Elster 1989). To explain why certain individual actions are performed we face multiple difficulties, the most salient arising from the underlying notion of

rationality involved. The assumption of the rational origin of decisions has been the foundation of economic theory (Tidsell 1996). This hypothesis contends that individuals decide over a set of options based on their desires and preferences. With fixed resources and technology, the aggregation of preferences yields the core of neoclassical economic theory. In this framework, any decision must pick out an element of an opportunity set derived from the constraints given by the scarcity of resources, the technology and the social norms. But in many cases, preferences and opportunities are not independent as assumed in that theoretical framework.

For the neoclassical approach, the beliefs of the agents about the market do not play any role. Moreover, the obstacles induced by transaction costs are ignored. Furthermore, the presence of social and economic friction as well as the diversity of behaviors, individual and collective, is ignored. Therefore, any limitation to full rationality cannot be incorporated without a change of focus. Economists have long ago realized that the main problem to be solved by the discipline is not only how belief systems and learning procedures influence behavior. It is also crucial to explain also why and how agents develop theories about the uncertain situations they face and how this in turn affects the decisions made by them (North 1993).

Herbert Simon pioneered the introduction of the notion of **bounded rationality** to explain the behavior of real world individuals. Agents have limited knowledge, data collection is costly and learning profits from previous mistakes. To sum up, results are generically just satisfactory rather than optimal (Tidsell 1996). With perfect information, information processing has no cost. With less than full information, there are uncertain alternatives that must be weighted (individually or by exchanges of messages among the agents) before making a decision. This induces to the existence of information asymmetries and transaction costs. This, in turn, explains the existence of **institutional frameworks** that regulate and modify behavior in that context. But the very existence of these frameworks is a cause of the persistence (or inertia) of economic behaviors.

Institutions arise to reduce the uncertainty of decision making with less than perfect information. In neoclassical theory, institutions do not play any substantial role. But in any study of economic evolution they become central. To understand the origin and change of institutions is a key to any explanation of evolutionary changes. In theories of evolution, the diversity of behaviors and their regulation by institutions induces political consequences and therefore adjustments (sometimes of a drastic nature) of the existing institutions.

The main sources of diversity in economic behavior are the motivations of the individuals, their perception of their possibility sets, their inference and calculation power, their history, etc. The diversity among groups is a consequence of the diversity among the individuals in each one. These differences (individual and collective) must in all matters of public concern be aggregated. The compromises among the different groups and individuals are reflected by the institutions of the society. But this, in turn, is sometimes a source of **uncertainty** in the system, since the transition to an institutional agreement is not determined.

The purpose of the line of research sketched here is to build a formal model capturing all the relevant facts discussed before. Bounded rational agents have to be represented in this model as a matter of necessity. A first step towards this goal is reported in this paper. In the next section we will describe how imperfect information is transmitted among agents and how this affects the prevailing institutions. This model is basically a sand pile that will allow us, in a further stage of research, carry out a simulation analysis. In section 3 we discuss the implications of our model.

2. Characterization of the Economic System

A self-organized system can be characterized as a linkage of sites plus the protocol that rules the communications among sites. The overall behavior of the system is a consequence of the interactions among the sites. If the system does not require an external control parameter in order

to generate critical behavior (i.e. to go through phase transitions when that parameter changes) we say that it is *critically self-organized*. In that case, the adjustment of the system to its environment is purely endogenous. This is a feature that is central for our representation of an economic system.

We define the sites of the system (a finite number I) as follows:

Sites i such that $i < n^*$ represent the consumers while those that verify $n^* < i < I$ represent the firms. The respective sets are S_c and S_p . Each site (agent) has an initial state S_i^0 . The rules that regulate the connections among agents constitute a consistent set R . Since the model is assumed open, it is prone to receive external shocks that propagate through the entire system. A condition on the shocks is that they have to be uniform both in their size and timing.

Even if some authors (Bak 1996) claim that SOC systems are enough to represent all the features of an economic system, this is not quite true. In fact, since a single set of rules governs the system, a SOC system cannot exhibit structural changes or a change in its dimensions. If, instead, this assumption is relaxed, we can represent evolution. First, we have to allow the possibility of shocks of any size and at any time. More important than that is that also the change of the rules must be allowed. According to the global state or the context, the connections and their strengths may change.

With these changes in mind we can introduce the concept of a self-organized economic system (ESO):

An economic system **ESO** is $\varepsilon = (T, R, M)$, where $T = (T_0, T_1, \dots)$ is a sequence of topologies, $R = (R_1, R_0, \dots)$, a sequence of consistent systems of rules, and M a set of meta-rules. A topology T_i is $T_i = (S_i, C_i)$, with S_i a finite set of sites (agents) and C_i the structure of connections among them. As an example consider the input-output matrix of an economy.

At each time period t the state of the economy is ε^t , where the consumers are collected into S_c , and the firms into S_p . For each site in S_c , a state $|s_c|$ is $|s_c| \in X^1 \times O_i^1 \times K_i^1 \times p^1$, where X^1 is the

space of resources. O^1 is a distribution of characteristics among agents (preferences and endowments). K^1 describes the knowledge of the agent, where k^1 is the model held by the agent about the behavior and structure of the economy. Finally, p^1 is the vector of current prices. For each element in $O^1 \times K^1 \times p^1$ it corresponds a unique c_d^1 , the desired demand of consumers. On the other hand, for a site in S_p , its state is $|s_p| \in X^1 \times Q^1$, such that for each vector of inputs ($w^1 \in X^1$) and its current prices p^1 , Q^1 is a compact set of outputs for the next period.

Finally, R^1 is an operator representing the social and political structure of the economy. It is such that for each $x^1 \in X^1$, $R^1(x^1, c_d^1) = (w^1, c^1, p^1)$. I.e., given a desired product and demand, it indicates which prices will prevail during the next period and how the product will be allocated among consumers (c^1) and firms (w^1).

In a critical self-organized system, the values of the neighborhood of the system at period t determine its future state at $t+1$. This is as a result of the application of the rules R_t . An external shock may slightly modify this situation (it makes it undetermined). If a shock D_s is applied in t it propagates until it converges to a definite pattern. It can be either periodic or aperiodic. In any case, this result is recorded and at period $t+1$ a new shock is applied to the system.

We can analyze the effect of a shock over the entire system. First we have to introduce to notion of a global state at each period t , $|e^t| \in X^1 \times O^1 \times Q^1 \times R^1$. A global state is a result of the aggregation of all the individual sites. So, if a shock D in period t is "in range", that is, $|e^t| + D_s \in \Omega$ (where Ω is the set of admissible states¹) then $T_{t+1} = T_t$, y $R_{t+1} = R_t$. If the shock is out of range, a meta-rule M_k is activated, generating a new topology and a new set of rules, depending on the previous structure and the magnitude of the shock. In other words: $(T_{t+1}, R_{t+1}) = M_k(T_t, R_t, D_s)$. Parsimonious behavior is also assumed, meaning in this context that S_t and S_{t+1} cannot be disjoint. This gives continuity to the system.

When shocks are in range, their propagation can be described by a function that represents the aggregate behavior of the

¹ The correct specification of Ω is a subtle issue. For our purposes it is enough to assume it as given, without discussing its exact form.

economy. If the strength of the shock is low nothing happens, but when it surpasses a certain critical limit a chain of changes is induced on the system changing its global state (albeit remaining in the admissible set).

Assume that the model has just one-dimensional. That is, that a single parameter determines represents all the relevant information about the behavior of the system. Moreover, assume that sites are homogeneous (in that they behave identically). Then, if a site presents a value $|s_i^t| > S^*$, where S^* is a critical threshold, the rule of transition is as follows:

$$|s_i^{t+1}| = |s_i^t| - F(S^* - |s_i^t|) + \sum F(S^* - |s_i^t|)$$

where $F(x)$ is null for $x = 0$ and has a positive value for $x > 0$. $F(\cdot)$ represents the propagation among sectors of the discrepancy between S^* and $|s_i^t|$.

The boundary conditions can adopt two different forms (L is a site on the boundary):

- I. Same rule for all sites $S_{L+1} = 0$.
- II. $S_L \rightarrow S_L - 1$, and $S_{L-1} \rightarrow S_{L-1} + 1$ for $S_L > S^*$

Condition I is not conservative (it represents intuitively a sand pile with barriers). Condition II, instead, preserves the sum of the S_i and can be thought as representing an open sand pile.

External shocks can be of two types:

- a. $S_i \rightarrow S_i + 1$
- b. $S_i \rightarrow S_i + 1$ and $S_{i+1} \rightarrow S_{i+1} - 1$

The second corresponds to a local increase of the slope of a sand pile. The first instead, just represents the addition of a single grain of sand to a site.

In our general (not restricted to a single dimension) conception of an economy we assume that the behavior of shocks is described by something close to Condition (a). In the same token, the behavior at the boundary is assumed as given by an analogue

to Condition I. That is, we conceive an economy as a non-conservative system submitted to shocks that reorder the local structures. In this more general setting, we replace the assumption of homogeneity of sites with the less restrictive assumption that the shock affects the *global* state of the economy (which is no more than the aggregation of the data in all sites). Recall that $|\varepsilon^t|$ represents a vector that arises as a consequence of the actions of the agents (both consumers and firms). This in turn results from the knowledge the agents have about the behavior and structure of the economy. The corresponding critical state of the system can be deduced comparing the behavior of the system with and without perfect information and rationality. In the former case, individuals process information at the light of the correct model of the economy. Even if they were initially mistaken, the dynamics of the system would end up correcting those errors. In that case, the aggregate knowledge is equivalent to the true state of the system:

$$k^t = \varepsilon^t$$

In other words, the agents know perfectly the topologies, the rules and the meta-rules that govern the economy.

If, instead, the individuals have bounded rationality, we have that:

$$k^t \neq \varepsilon^t$$

and the degree of error is:

$$\varepsilon^{t+1} - k^{t+1} = \mu^{t+1}(\varepsilon)$$

This error term is analogous to what in our one-dimensional model was represented by $S^* - |s_i^t|$. That is, the error term $\mu^t(\varepsilon)$ indicates how the actual state differs from the state it could have reached if agents had perfect information and rationality. Therefore, we define the critical state of the system, S^* , as the overall result of the aggregation of the states of sites $|s_c| \in X^t \times O^t \times K^t \times P^t$ and $|s_p| \in X^t \times Q^t$, which, together with $R^t(x^t, c_d^t) = (w^t, c^t, p^t)$ represent the state of the world in which agents have perfect information and rationality.

Since knowledge arises also from the past of the system and the adjustments suffered in its history we have to include a learning operator:

$$\gamma : \{\xi^{t-1}\} \rightarrow K$$

where $\{\xi^{t-1}\}$ is the set of possible histories of the economy ξ until period $t - 1$ and $\gamma : \{\xi^{t-1}\} = K^t$. The functional form of the learning operator depends on how individuals correct their misperceptions. Nelson and Winter (1982) indicate that firms follow routines more than maximizing behaviors. These routines are hardly corrected, unless the mistakes are of such a magnitude that force the firms to change drastically their behavioral patterns.

The system is submitted to external shocks that we identify as knowledge shocks. Of course, the dynamic path of knowledge obeys both endogenous and exogenous factors. The former case arises through phenomena like learning-by-doing, creation of human capital, etc. On the other hand, exogenous generation of knowledge occurs due mostly to the contact with other economies (imports often include added value from foreign technologies or cater different tastes). In any case, the endogenous variations can be of any of the following forms:

- Changes in the characteristics of the agents, due to a better knowledge:

$$\phi_1 : K \rightarrow O$$

- Changes in the space of allocations. A previously unknown good enters into the consumption basket of the agents:

$$\phi_2 : O \rightarrow X$$

then:

$$\phi_2 \circ \phi_1 : K \rightarrow X$$

- Changes in the production side of the economy, since new goods are demanded or introduced into the market. So:

$$\phi_3 : X \rightarrow Q$$

and accordingly:

$$\phi_3 \circ \phi_2 \circ \phi_1 : K \rightarrow Q$$

- Changes in the political structure. Agents force a change of the "rules of the game":

$$\phi_4 : O \rightarrow R$$

therefore:

$$\phi_4 \circ \phi_1 : K \rightarrow R$$

In this form knowledge propagates to all the relevant parameters of the economy.

Unless knowledge becomes stationary, all parameters will be submitted to permanent changes. In general, the rules will tend to avoid drastic changes: institutions exist to provide stability to the system, which is incompatible with permanent change. On the other hand, knowledge may change from one period to the other given the performance of the system and its history. But if the overall outcome remains admissible, the changes can be deemed as little.

But when the system is submitted to an external shock which is out of range, $|\varepsilon^t| + D_s \notin \Omega$, this opens the door for an evolutionary change. To describe how this may happen we define, a endogenous change of structure operator Φ_0 as:

$$\Phi_0 \equiv \langle \phi_1, \phi_2 \circ \phi_1, \phi_3 \circ \phi_2 \circ \phi_1, \phi_4 \circ \phi_1 \rangle \circ \gamma : \{\xi^{t-1}\} \rightarrow X^t \times O^t \times Q^t \times R^t$$

which indicates that the previous history of the economy determines the choice of feasible parameters. If

$$\Phi_0(\xi^t) \notin \Omega$$

this indicates that an endogenous driven transition occurs. That is, when the aggregate choices after a shock are inadmissible, the system has to evolve towards a new structure with new rules of behavior. In other words, a meta-rule is called into action. This is of course, an unintended consequence of the misperceptions and the ensuing uncertainty generated by the external shock.

In other words, the magnitude of the error is such that it generates a great internal tension in the the economy. This in turn leads to change the parameters. This is exactly what it is assumed to happen in ESO systems. On the other hand, since ESO systems are no more than a sequence of SOC systems, it is convenient to recall that we discussed two different kinds of changes. In the first place, **incremental** changes such that the structure remains generating admissible states. These changes do not modify the critical parameter

or the rules. Therefore these changes are those idiosyncratic in SOC systems. The other kind of changes is the one that induces drastic changes. We identify this kind of changes with those that lead to the change from a SOC system to another. That is, with **evolutionary transitions**.

In particular, an evolutionary transition occurs when the critical parameter changes. In the case of ESO systems, such a transition is generated by the activation of a meta-rule triggered by an inadmissible shock. Since the critical parameter is the state for perfect knowledge and rationality, to say that a shock is admissible is to say that the shock does not provide knowledge enough to force to a drastic revision of the previous assumptions and beliefs. Therefore the response will be smooth and no evolutionary transition will happen. However, if shocks are over the range of admissibility, a new meta-rule will generate a new topology and a new set of rules.

3. Final Reflections

The discussion in this paper constitutes a blueprint for a computational embodiment with functional characterizations. This further step is, however, far from straightforward. In fact, it is clearly impossible to expect to develop models useful to make real world predictions. This is due to several factors:

1. Individuals lack perfect rationality. The way in which they internalize and process information varies and only with very restrictive assumptions we can expect to come close to a functional form.
2. Real world processes are usually non-linear. Therefore, agents, even with perfect rationality, can make only short-term predictions.
3. Evolutionary transitions may arise in a multiplicity of ways, and no one can be a priori predicted (if it were predictable it would be an incremental change).
4. Consequently, there cannot exist a previous assessment of the set of meta-rules and the procedures for selection one.

The impossibility to predict is in the very nature of evolutionary processes. We can explain and taxonomize evolutionary changes that *already happened*. But we

cannot anticipate the direction or the magnitude of the change.

Even so, the possibility to exhibit experimentally patterns of evolution is very useful in order to find a general classification of evolutionary phenomena. But as said, this is not a small task. We have to specify the form in which a bounded rational agent learns or modifies her behavior. The relation among desires, beliefs and actions may well go **beyond** the standard notion of rationality. Apparently desires manifest themselves as the only independent element in the picture (Elster 1989), beliefs seem to be conditioned by desires, and both (desires and beliefs) determine actions. But on the other hand, the very set of possibilities may not only restrict actions but also the proper desires. This introduces a circularity that deserves a careful analysis.

Other difficulties may arise all over the place, but, as said, this research program may shed light over some of the most difficult and obscure points in economic theory. Therefore these humble beginnings cry out for a continuation.

² The issue of how to compare knowledge at different points of time is far from settled. It can be said that more is known if the overall welfare is higher with the new knowledge. This reflects a customary use of the notion of knowledge in contexts like learning-by-doing, the issuing of new patents, etc.

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