BAAA, Vol. 58, 2016
P. Benaglia, D. D. Carpintero, R. Gamen & M. Lares, eds.

Asociación Argentina de Astronomía
Boletín de artículos científicos

Scalar-Tensor-Vector Gravity: solutions with matter content

F.G. Lopez Armengol[1] & G.E. Romero[1,2]

[1] *Instituto Argentino de Radioastronomía, CCT-La Plata, CONICET, Argentina*
[2] *Facultad de Ciencias Astronómicas y Geofísicas, UNLP, Argentina*

Contact / flopezar@iar-conicet.gov.ar

Resumen / La teoría Escalar-Tensorial-Vectorial es una teoría alternativa para la interacción gravitatoria, fomulada en el año 2006 por John Moffat. El límite de campo débil de la misma ha descrito exitosamente observaciones del Sistema Solar, curvas de rotación galáctica, la dinámica de cúmulos de galaxias y observaciones cosmológicas, sin la necesidad de imponer componentes oscuras. Estudiamos las soluciones con contenido de materia de esta teoría y las aplicamos a la construcción de modelos estelares sencillos. Específicamente, derivamos la ecuación modificada de Tolman-Oppenheimer-Volkoff y la integramos para distintas ecuaciones de estado politrópicas. Encontramos que estos modelos admiten masas mayores a los correspondientes modelos relativistas. Los valores máximos de las masas totales dependen del factor $\alpha \in [0, 1)$ que cuantifica la desviación de la teoría general de la relatividad.

Abstract / Scalar-Tensor-Vector Gravity (STVG) is an alternative theory of the gravitational interaction. Its weak field approximation has successfully described Solar System observations, galaxy rotation curves, dynamics of galaxy clusters, and cosmological data, without the imposition of dark components. The theory was formulated by John Moffat in 2006. We explore non-vacuum solutions of STVG and apply them to some stellar toy-models. Specifically, we derive the modified Tolman-Oppenheimer-Volkoff equation in STVG and integrate it for different polytropic equations of state. We find that stellar models in STVG admit larger masses than in general relativity (GR). Maximum masses depend on the factor $\alpha \in [0, 1)$ that quantifies the deviation from GR.

Keywords / gravitation — equation of state — stars: interiors

1. Introduction

The Scalar-Tensor-Vector Gravity theory (STVG), also referred as MOdified Gravity (MOG), is an alternative theory for the gravitational interaction formulated by John Moffat (2006). In STVG, the gravitational coupling constant G is replaced by a scalar field whose numerical value usually exceeds Newton constant G_{N}. This assumption allows to describe correctly galaxy rotation curves (Brownstein & Moffat, 2006), dynamics of galaxy clusters (Moffat & Rahvar, 2014), phenomena associated with the Bullet Cluster (Brownstein & Moffat, 2007), and cosmological data (Moffat & Toth, 2007). All this without requiring the existence of dark components. In order to counterpart the enhanced gravitational constant at Solar System scales, Moffat proposed a gravitational repulsive Yukawa-like vector field ϕ^{μ}. In this way, Newton gravitational constant can be retrieved and STVG coincides with GR in all Solar System predictions.

In this work, we study spherically symmetric, static, and non-vacuum solutions of STVG field equations, and obtain the modified Tolman-Oppenheimer-Volkoff equation (TOV). These results are used to construct three stellar toy models: Sun-type stars, withe dwarfs (WD), and neutron stars (NS). We integrate numerically the modified TOV equation for different polytropic equations of state (EoS) and compare the outcomes with general relativity results.

Our work is organized as follows. In Sec. 2. we present the STVG field equations and derive the spacetime metric components. Then, in Sec. 3., we show the modified TOV equation and the EoS used for numerical integration. Sec. 4. is devoted to our main results and conclusions.

2. STVG static, spherically symmetric, matter sourced solution

A simplified version of Moffat's original action is (2006):

$$S = S_{\mathrm{GR}} + S_{\phi} + S_{\mathrm{S}} + S_{\mathrm{M}}, \tag{1}$$

$$S_{\mathrm{GR}} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}R, \tag{2}$$

$$S_{\phi} = \omega \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi^{\mu}\phi_{\mu}\right), \tag{3}$$

$$S_{\mathrm{S}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{G^3}\left(\frac{1}{2}g^{\mu\nu}\nabla_{\mu}G\nabla_{\nu}G - V(G)\right) + \frac{1}{G\mu^2}\left(\frac{1}{2}g^{\mu\nu}\nabla_{\mu}\mu\nabla_{\nu}\mu - V(\mu)\right)\right]. \tag{4}$$

Here, $g_{\mu\nu}$ denotes the spacetime metric, R is the corresponding Ricci scalar, and ∇_{μ} is the covariant derivative; ϕ^μ denotes a Proca-type massive vector field, μ is the mass of the field, $B_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$, and $\omega = 1/\sqrt{12}$; $V(G)$, $V(\mu)$ denote the potentials of the scalar fields $G(x)$, $\mu(x)$, respectivly. We adopt the metric signature $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, and choose units with $c = 1$ (velocity of light in vacuum). The term S_M refers to possible matter sources.

In this work, we neglect the mass of the vector field because its effects manifest at large distances from the source and we are interested in the local stellar structure (see Moffat, 2015). Also, we ignore the contributions of the scalar fields to the field equations because we approximate G as a constant. Numerical values for G are chosen in accordance with Moffat's previous works.

Varying the action with respect to $g^{\mu\nu}$ and taking the previous simplifications into account, we get the metric field equations:

$$G_{\mu\nu} = 8\pi G\left(T^\mathrm{M}_{\mu\nu} + T^\phi_{\mu\nu}\right), \tag{5}$$

where $G_{\mu\nu}$ denotes the Einstein tensor, and $T^\mathrm{M}_{\mu\nu}$, $T^\phi_{\mu\nu}$ are the matter and vector field energy-momentum tensors, respectively. We take for the enhanced gravitational coupling constant the same prescription as Moffat (2006):

$$G = G_\mathrm{N}(1+\alpha), \tag{6}$$

where G_N denotes the Newton gravitational constant, and α a free parameter whose value we sample. Within the adopted approximations, STVG coincides with GR for $\alpha = 0$.

Variation of the simplified action with respect to ϕ_μ yields:

$$\nabla_\nu B^{\mu\nu} = -\frac{\sqrt{\alpha G_\mathrm{N}}}{\omega} J^\mu, \tag{7}$$

where J^μ denotes the four-current matter density, and the constant $\sqrt{\alpha G_\mathrm{N}}$ is determined to adjust phenomenology.

We model spacetime with a static, spherically symmetric geometry:

$$\mathrm{d}s^2 = \mathrm{e}^{\nu(r)}\mathrm{d}t^2 - \mathrm{e}^{\lambda(r)}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \tag{8}$$

Regarding the stellar matter, we model it with a static, spherically symmetric, perfect fluid energy-momentum tensor:

$$T^{\mathrm{M}\mu}{}_\nu = [p(r) + \rho(r)]u^\mu u_\nu - p(r)g^\mu{}_\nu, \tag{9}$$

where $p(r)$ and $\rho(r)$ denotes the pressure and density of the fluid r-shell, respectively; and $u^\mu \to \left(\mathrm{e}^{-\nu/2}, 0, 0, 0\right)$ denotes the four-velocity of a mass element with coordinate r. The corresponding four-current matter density is:

$$J^\mu = 4\pi\rho u^\mu = \frac{4\pi\rho}{\sqrt{g_{00}}}\frac{\mathrm{d}x^\mu}{\mathrm{d}x^0} \longrightarrow \left(\mathrm{e}^{-\nu/2}4\pi\rho, 0, 0, 0\right). \tag{10}$$

Replacing the latter expression for J^μ in Eq. (7) we obtain the components of $B^{\mu\nu}$. Then, we solve the field Equation 5 and obtain the metric components:

$$\mathrm{e}^{-\lambda(r)} = 1 - \frac{2GM(r)}{c^2 r} - \frac{1}{r}\frac{4\pi G}{c^4\omega}\int \mathrm{d}r\frac{Q^2(r)}{r^2}, \tag{11}$$

$$\nu(r) = -\lambda(r) + \frac{8\pi G}{c^4}\int \mathrm{d}r\mathrm{e}^{\lambda(r)} r\left(c^2\rho(r) + p(r)\right), \tag{12}$$

where we have reintroduced the speed of light factors, and have defined the quantity:

$$Q(r) \equiv \int \mathrm{d}r e^{\lambda(r)/2}\sqrt{\alpha G_\mathrm{N}}\rho(r)4\pi r^2. \tag{13}$$

Notice that, for a point mass source, we retrieve Moffat's spherically symmetric black hole solution (Moffat, 2015).

3. Modified Tolman-Oppenheimer-Volkoff equation

From the conservation equation:

$$\nabla_\mu\left(T^\mathrm{M}_{\mu\nu} + T^\phi_{\mu\nu}\right) = 0, \tag{14}$$

we isolate the derivative of the fluid pressure with respect to the radial coordinate. Using expressions (11) and (12) for the metric components, we obtain the modified Tolman-Oppenheimer-Volkoff equation for STVG:

$$\begin{aligned}\frac{\mathrm{d}P(r)}{\mathrm{d}r} = &-\frac{\mathrm{e}^{\lambda(r)}}{r^2}\left(\frac{4\pi G}{c^4}p(r)r^3 - \frac{2GQ^2(r)}{\omega c^4 r} + \right.\\ &\left.+\frac{GM(r)}{c^2} + \frac{2\pi G}{\omega c^4}\int \mathrm{d}r\frac{Q^2(r)}{r^2}\right)\left(\rho(r)c^2 + p(r)\right) + \\ &+\frac{Q(r)}{wr^4}\frac{\mathrm{d}Q(r)}{\mathrm{d}r}.\end{aligned} \tag{15}$$

Setting $\alpha = 0$ nullifies every Q-term and the classic relativistic TOV equation is retrieved.

In order to integrate Eq. (15), we need to determine an EoS that relates the stellar density and pressure. We consider three polytropic stellar models with different central densities:

- Solar-type star (Padmanabhan, 2000):
$$P = 3.1367\times 10^{14}\rho^{4/3}, \quad \rho_c = 150\ \mathrm{g\,cm^{-3}}, \tag{16}$$
- Withe dwarf (Padmanabhan, 2000):
$$P = 4.881\times 10^{14}\rho^{4/3}, \ 10^5\ \mathrm{g\,cm^{-3}} < \rho_c < 10^6\ \mathrm{g\,cm^{-3}}, \tag{17}$$
- Neutron star (Orellana et al., 2013):
$$P = 10^{5.29355}\rho^2, \ 10^{14.6}\ \mathrm{g\,cm^{-3}} < \rho_c < 10^{15.9}\ \mathrm{g\,cm^{-3}}. \tag{18}$$

Deviations from GR are expected for non-vanishing α (see Eq. (6)). From Solar System observations, Moffat (2006) determined the upper limit:

$$\alpha_\odot < 1. \tag{19}$$

The stars considered in the models have a few solar masses. Then, we expect α to be similar to its Solar System value. We sample the theory with three values of α given by $\alpha = 0$, $\alpha = 10^{-3}$ and $\alpha = 10^{-2}$.

We proceed to integrate Eq. (15) numerically applying a fourth-order Runge-Kutta method (Press et al., 1992).

4. Results and conclusions

The integration of Eq. (15) for the EoSs and for the α values mentioned above yields the density profiles shown in Fig. 1. As was expected, the repulsive behavior of gravity slows down the radial decrease of the density. Smaller values for α do not produce significant deviations from GR and major values yield unrealistic results.

The repulsive behavior of gravity entails larger stellar masses than GR models. In Fig. 2 we plot the final masses as a function of the stellar radii for NS and WD with different central densities. As can be seen from the graph, STVG allows NS and WD masses up to 2.8 $\mathrm{M}_\odot$, with non-exotic EoSs.

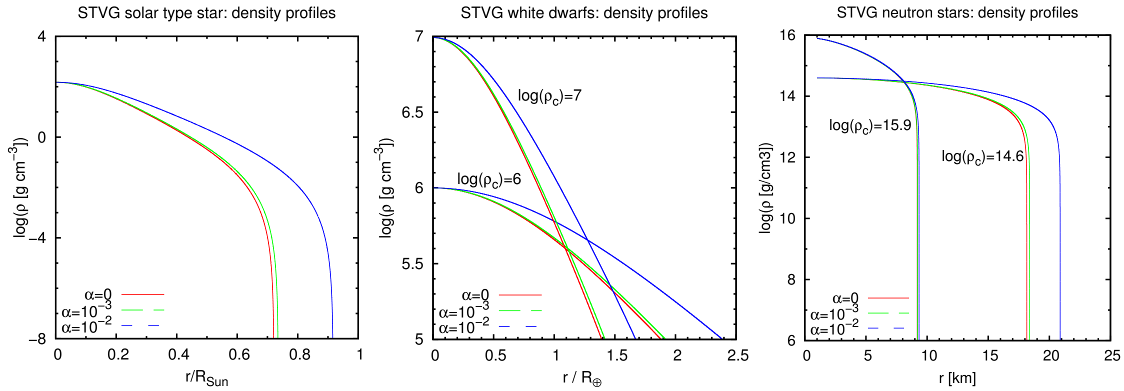

Figure 1: From left to right, density profiles for STVG Sun-type, WD, and NS stellar models. We sample the theory with three values of the free α parameter. For $\alpha = 0$ we retrieve GR results. For $\alpha = 10^{-3}$ we do not obtain significant deviations from GR. However, for $\alpha = 10^{-2}$ the effects of repulsive gravity become evident. In the case of WD and NS, we show profiles for different central densities.

We conclude that STVG admits realistic matter-sourced solutions that can be used to construct stellar models. The main difference of STVG stellar models and relativistic ones is the slower decrease of the density profile and therefore larger final masses.

Recent estimations of NS masses exceeds the maximum predicted by GR realistic models (Antoniadis et al., 2013; Demorest et al., 2010; Kiziltan et al., 2013). This fact and our results make STVG a theory worthy of attention and further tests. We expect to include the contributions of the scalar fields to the field equations and dynamics in the near future. Also, we expect to construct STVG realistic models appealing to more sophisticated EoSs.

Acknowledgements: We thank Federico García, Santiago del Palacio and Daniela Pérez for valuable discussions. This work is supported by ANPCyT under grant PICT 00878. GER is also supported by grant AYA2013-47447-C3-1-P (Spain).

References

Antoniadis J., et al., 2013, Science, 340, 448
Brownstein J. R., Moffat J. W., 2006, ApJ, 636, 721
Brownstein J. R., Moffat J. W., 2007, MNRAS, 382, 29
Demorest P. B., et al., 2010, Nature, 467, 1081
Kiziltan B., et al., 2013, ArXiv e-prints
Moffat J. W., 2006, JCAP, 3, 4
Moffat J. W., 2015, European Physical Journal C, 75, 175
Moffat J. W., Rahvar S., 2014, MNRAS, 441, 3724
Moffat J. W., Toth V. T., 2007, ArXiv e-prints
Orellana M., et al., 2013, General Relativity and Gravitation, 45, 771
Padmanabhan T., 2000, Theoretical Astrophysics: Volume 2, Stars and Stellar Systems. Theoretical Astrophysics, Cambridge University Press
Press W., et al., 1992, Numerical Recipes in FORTRAN 77: Volume 1, Volume 1 of Fortran Numerical Recipes: The Art of Scientific Computing. No. v. 2 in Numerical Recipes in FORTRAN: The Art of Scientific Computing, Cambridge University Press

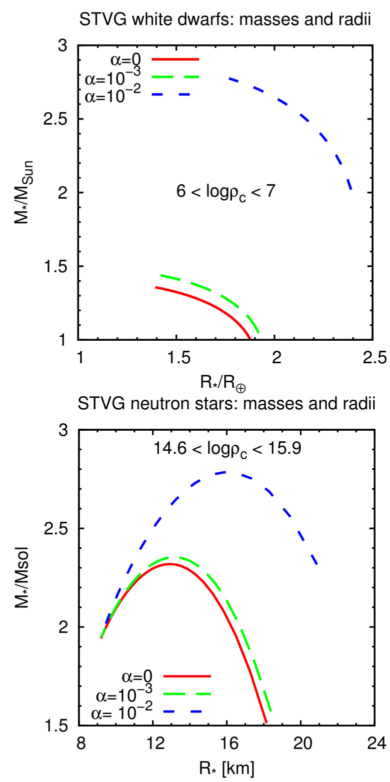

Figure 2: Total masses and radii for STVG WD (top) and NS (bottom). We retrieve GR results for $\alpha = 0$, and study the STVG deviations for $\alpha = 10^{-2}$ and $\alpha = 10^{-3}$. Both graphs show that STVG admits larger masses than GR.