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Research Paper

Quasi-bifurcation and Imperfection-sensitivity of Cylindrical Shells under Pressures due to an Explosion

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Abstract. The static and dynamic behavior of a horizontal cylindrical shell (as used to store fuels in tanks) is investigated in this work by means of computational modeling. Under a distributed pressure commonly used to model effects due to explosions, the geometrically nonlinear behavior is explored to identify bifurcation and limit points along the static equilibrium path, and the associated displacements. Critical load reductions due to imperfections are found in the order of 25%. The dynamic analysis is next presented to identify the possibility of reaching a quasi-bifurcation. It is found that the first peak in the transient response at which the displacement reaches the same value as in the limit static case occurs for a load which is about 3.5 times the static bifurcation load. The velocity is zero at this state and is identified as a quasi-bifurcation, at which the shell is expected to display a static instability. Imperfection-sensitivity of the quasi-bifurcation load is found to be of the same order as the static one. This is the first quasi-bifurcation study of a shell to identify dynamic buckling due to a nearby explosion.

Keywords: Dynamic buckling, Explosions, Imperfection-sensitivity, Quasi-bifurcations, Shell structures.

1. Introduction

This paper considers the dynamic buckling of horizontal cylindrical shells under blast pressures due to a nearby explosion. In this class of problems, the transient dynamic response of the "unperturbed motion" (known here as the "fundamental motion") shows an oscillatory pattern in which the first peak occurs at approximately the time when the impulsive load ceases to act. The response remains initially bounded and stable, but there is a load level at which the fundamental motion shows a divergent behavior. This phenomenon was identified in the literature as dynamic buckling, and Budiansky and Roth [1] proposed a stability criterion that quantifies a sudden growth in the maximum transient deflection for a small change in the load value. Kleiber et al. [2] identified limitations in this approach, including its focus on the fundamental motion thus neglecting the possibility of a new branching motion emerging from it.

Lawrence H. N. Lee (1923-2017), who was born in Shanghai and developed his academic career at the University of Notre Dame, was the first researcher to consider branching from the fundamental motion. Lee found that, at a critical time along the fundamental motion, a new mode may develop in what seems to be a bifurcation; this he named as "quasi-bifurcation" [3], here abbreviated as QB. Perturbed motions, in which there are changes in the initial conditions, were a key aspect in Lee's explanation of this phenomenon. At a QB, "there is at least one perturbed motion, which oscillates initially in the small neighborhood of the undisturbed trajectory, to depart monotonically from the neighborhood… This branching behavior develops in a mode that may exhibit small random oscillations at first, but a divergent motion is finally achieved" ([3], pp. 79).

The QB criterion initially adopted by Lee employed the critical time, t_c: "At a critical time… one of the components of the deviated motion… begins to grow" ([3], pp. 81). "Among all motions, the particular deviated motion satisfying the quasi-bifurcation criterion… branches off at an earliest time from the unperturbed motion" ([3], pp. 85). Lee found that the QB phenomenon had close links with the static buckling of the structure under the same load with both showing the same deflected shape and stresses at static and dynamic buckling: "The quasi-bifurcation mode of motion at $t = t_c$ is identical to the static buckling mode, which is independent of time" ([3], pp. 86).

For a column, Lee found that the dynamic buckling mode coincides with the static one, whereas the axial strain at QB was 2.5 larger than at the static buckling configuration. Further, at the critical time, the axial forces were the same as those obtained at static buckling. Finally, Lee postulated that initial disturbances that have a component in the static eigenmode are those that grow most rapidly in QB behavior. In several papers, Lee considered the QB of elastic-plastic continua [4] [5], inelastic columns [6], a finite strip supported by an elastic rigid foundation under axial impact [7], and elastic-plastic buckling of buried pipelines under seismic excitation [8].

The concept of QB was further explored by other researchers, including Michal Kleiber in Warsaw, Bernd Kroplin in Dortmund, Ekkehard Ramm in Stuttgart, Wilfred Kratzig (1932-2017) in Bochum, and George J. Simitses (1932-2016) in Georgia Tech and Cincinnati. Most researchers addressed this problem in order to extend QB to the field of computational mechanics.

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Kleiber and coworkers, stated that: “The concept of quasi-bifurcation seems to offer an attractive framework for working out numerical algorithms to deal with this class of problems” [2], and proposed the identification of singularities in the tangent stiffness matrix of a structure in order to evaluate QB loads. Their work was limited to the study of step loadings.

Kratzig and Eller [9] investigated QB under impulsive loading by solving a quadratic eigenvalue problem; however, their scheme was not simple to use in conjunction with general purpose finite element codes. Kratzig and Li [10] considered the implications of the dynamic stability conditions as employed by Lee and by Kleiber, and showed that this “only offers a sufficient condition for instability and a necessary condition for stability” ([10], pp. 97). Kratzig and Eller [9] explored Liapunov’s first approximation in the context of computational mechanics, in the form of the tangential equation of motion to develop numerical algorithms for QB behavior, thus developing a criterion to identify QB. This requires following the transient response of the structure and exploring the tangent stiffness matrix along this motion.

Burmeister and Ramm [11] used a freezing-in-time technique along the fundamental motion, and evaluated stability at each time-step (focusing on the first maximum in the motion) by means of an eigenvalue problem. Their study addressed step loading. Although the authors had in mind dynamic buckling as stated by Budiansky and not QB as in the work of Lee, their work would also serve to identify branching along the fundamental motion.

Dynamic buckling was investigated by Simitses [12] in a book based on previous papers [13][14]. Although he did not use Lee as a reference or the term QB in his text, his work deals with the same problem from an energy perspective. His interest focused on structures that show unstable behavior when they reach a static critical state (either unstable bifurcation or limit point), in which case a dynamic jump occurs at the critical state. At low levels of load the system oscillates close to an equilibrium state, and two or more possible static equilibrium positions are necessary to have a divergent behavior in the transient response. This divergent behavior occurs for a motion in the vicinity of the unstable critical state. Simitses employs the concept (also used by Kroplin and Dinkler) that when the initial kinetic energy T caused by an impulsive load is equal to the total potential energy V at an unstable static equilibrium position (if the same configuration is attained in the dynamic system), then a shift occurs from the fundamental motion to another state located far from the current state. The work due to an impulsive load induces an initial kinetic energy in an almost instantaneous way. At the beginning of the dynamic process, only kinetic energy is present and the energy V is zero; but as the system oscillates then V increases and T decreases until $T = 0$ (with zero velocity) occurs at a maximum in the fundamental motion (at which V is at a maximum). The system temporarily passes through a static configuration (under zero load). Thus, dynamic buckling (in the form of QB) occurs when the system receives an initial impulse that indices a maximum transient displacement at which V becomes the same value as in the critical state of the static problem.

Because impulse and kinetic energy need to be related in the procedure used by Simitses, this relation may be implemented in practice as explained by Doyle ([15], pp. 491).

Dinkler and Kroplin [16] proposed evaluating QB for an impulsive loading by using the “critical energy” approach, i.e., the strain energy computed at the static critical state. This critical energy represents the passage from an elastic pre-buckling to a post-buckling behavior. The strain energy is evaluated at the unstable static critical state, which may be a limit point or a bifurcation point. Next, the transient motion is considered, but instead of following the fundamental motion in the complete time domain, “only the initial part of the motion has to be investigated until the structure changes from the pre- to the post-buckling range…” [17]. For impulsive, short duration loads, as those considered by these authors, this initial part of the motion is generally limited to the first peak in the oscillation, at which the velocity becomes zero and, because the load has ceased to act, the energy reduces to the strain energy. It is the comparison between the critical (static) strain energy and the energy of the dynamic response that defines a stability criterion. These ideas follow the work done by Simitses (although not cited in their work) but the novelty was the computational mechanics perspective.

Further work was published by Lepik [18] in which the QB work of Lee for elastic-plastic structures was revisited. Lepik notice that “a shortcoming of the quasi-bifurcation approach is the fact that it allows us to analyze only the early time of buckling but does not give us any information about the post-buckling behavior of the structure” ([18], pp. 300). Lepik emphasized the role of traveling axial waves to analyze the buckling phenomenon, but his work was limited to the early buckling stages and could not follow the complete post-buckling process because a geometrically linear behavior was assumed.

The authors recently considered the QB behavior of simple shell-like structural systems [19], to explore the use of energy in the evaluation of dynamic QB. For a two degree-of-freedom system having both membrane and bending springs, an impulsive load was applied and the response was compared with the static unstable bifurcation behavior. Similar to what was observed by Dinkler and Kroplin [16][17], this work found that at a QB the strain energy in the first peak along the fundamental motion was the same as the critical strain energy in the unstable static bifurcation, and that the displacement configuration was the same in both cases. These results showed that the ratio of membrane to bending stiffness of the system dominates the QB response, whereas the system was moderately sensitive to geometric imperfections.

Most references considered in this review address step rather than impulsive loading, and focus on simple systems, but the quasi-bifurcation in shells impulsive loading and shells has received little attention at present. This paper extends previous works reviewed in this section to address problems of interest in the oil industry, namely the dynamic buckling behavior of fuel storage tanks and pipes under impulsive pressures due to an explosion. This dynamic buckling problem under impulsive loads in shells, has never been addressed in the literature.

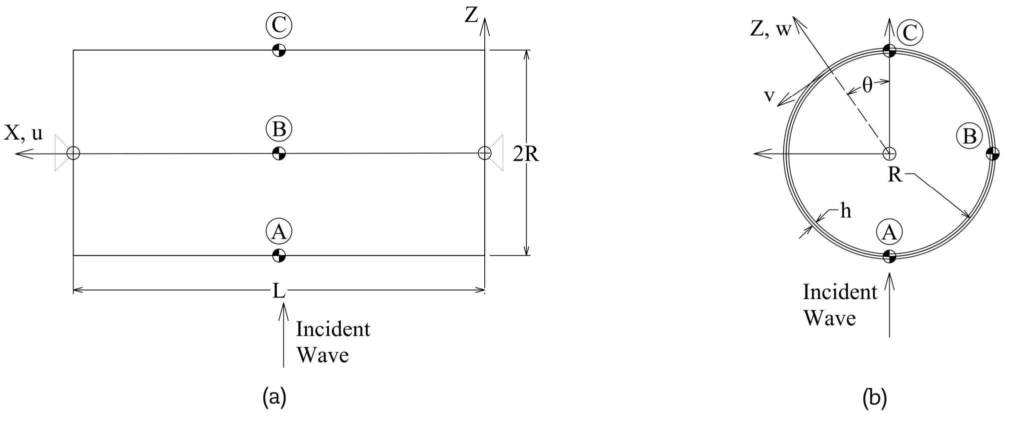


Fig. 1. Geometry of the horizontal tank considered and direction of incident pressure. (a) Side view, and (b) Cross-section.

2. Case studied and assumed blast load

The motivation of this research is the need to evaluate the consequences of explosions on fuel-storage tanks. Typical tanks employed in the oil industry are large vertical and smaller horizontal tanks. This work focuses on the horizontal tanks, which are less flexible than the vertical ones. Horizontal tanks are fabricated using cylindrical shells with various geometric configurations [20]. The specific geometry investigated in this work is shown in Fig. 1, in which the assumed boundary conditions at the edges represent a diaphragm restricting in-plane deformations but allowing rotations of the cross section.

The effect of an explosion as a blast load has been modeled in this work by means of a pressure pattern with assumed time and space variations. Tests on vertical cylindrical shells under a nearby explosion performed in France [21] and the United States [22] showed that the time-dependent pressure distribution around the circumference is almost uniform in elevation.

Because only the shell area most directly exposed to the explosion is affected by pressures, a convenient model due to Putelat and Triantafyllidis [23] was used in this work, in which a continuous function defines the pressures around the cylinder,

$$p(\theta,t) = p_0\left(1-\frac{t}{t_0}\right)e^{-tk_2/t_0}e^{-k_1^2(\theta-\pi)^2} \tag{1}$$

where p_0 is the peak overpressure; t_0 is the positive pulse duration; and k_1 and k_2 are parameters to adjust the pressure in time and space. Equation (1) assumes a uniform pressure along the longitudinal (horizontal) axis X shown in Fig. 1. Values of k_1=1.1, and k_2=2.0 were adopted here to follow a cosine square variation, which is commonly used in this field.

The applied pressures in space and time are shown in Fig. 2. In the present simplified model, all pressures are assumed to act simultaneously around the circumference. This is a simplification in the sense that in a real situation there is a time delay between pressures acting at ϑ=0° and those at other locations around θ; however, results for vertical tanks indicate that such delay has minor quantitative consequences on the dynamics of the shell [24].

A specific case of a horizontal tank has been investigated in this paper, with radius R = 2.0m, and length L = 16.0m between supports. A constant thickness of the shell h=0.01m is assumed in this case, leading to geometric relations L/R = 8 and R/h = 200. Modulus of elasticity E = 205MPa and Poisson's ratio 0.3 of steel were considered in the computations. Dependence of these moduli on strain rate and plasticity have not been taken into account in the present analyses. The shell is assumed to be empty and initially at rest, and dissipation has been neglected. The time of the positive phase of the impulse, identified as t_0 in Fig. 2(b), is taken as 0.025s. The geometry and load conditions represent a severe case in terms of shell slenderness and transient displacement amplitudes and this stringent condition has been chosen to validate the model.

3. Static analysis

Consider the shell under a static pressure that follows the same circumferential variation as that produced by the blast load. Results are presented in this section for Linear Bifurcation Analysis (LBA), Geometrically Nonlinear Analysis (GNA) assuming the perfect geometry, and Geometrically Nonlinear Analysis with Imperfections (GNIA).

Following convergence studies, the finite element mesh with 8904 elements was used in the analyses. These are quadrilateral elements, identified as S4R in the ABAQUS nomenclature. For the transient response of the shell using ABAQUS, an explicit algorithm was used. A peak pressure p_λ = 100kPa was adopted as a reference static pressure to make use of dimensionless load coefficients.

3.1 Bifurcation analysis

As a preliminary analysis, bifurcation buckling from a linearized primary equilibrium path has been investigated for this shell using LBA as implemented in the ABAQUS general purpose code [25]. This analysis provides a set of eigenvalues and eigenvectors to identify bifurcation buckling load (p_B) from a linear primary equilibrium path. The normalized load factor is $\lambda = p_B/p_\lambda$. The lowest eigenvalue leading to the bifurcation load was computed in this case for a maximum pressure of 68.6KPa, in which case the mode shape (eigenvector) is shown in Fig. 3. A central lobe towards the inside of the shell occurs at the meridian of maximum pressure, with side lobes having smaller amplitude and moving outwards. Large amplitude deflections occur in the zone affected by pressures, whereas negligible displacements are computed in the remaining of the circumference.

At the elastic critical state there is a bifurcation from a fundamental equilibrium path into a secondary path, so that there is a pre-buckling mode and an initial post-buckling mode. Due to energy conservation, the strain energy at the critical state computed as a state on the fundamental path (which is largely dominated by membrane action) is the same as the strain energy at the initial post-critical state (computed at an adjacent equilibrium state, in which there is also bending contribution to equilibrium).

3.2 Geometrically Nonlinear Analysis

A GNA study was carried out to compute the nonlinear static equilibrium path under increasing pressures. The equilibrium path based on Riks algorithm is shown in Fig. 4(a), in which the maximum pressure is plotted in the vertical direction versus the displacement at point A in Fig. 1 is in the horizontal axis. This is a nonlinear path with a maximum pressure of approximately p_{L0}=40.1 KPa (maximum pressure in the perfect shell), for which the displacement is w/h = 13. Following the maximum in pressure, the path drops and shows an unstable post-critical behavior. Although it is not shown in the figure, the path recovers stability at a much larger displacement, but for this to occur the shell should undergo large deflections, plasticity and damage.

(a) (b)

Fig. 2. Assumed pressure distribution in space and time. (a) Circumferential distribution computed from eq. (1) (solid line) and cos²(ϑ) (dashed line); and (b) maximum pressure variation in time.

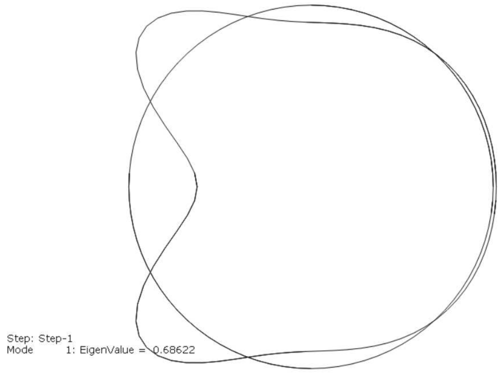
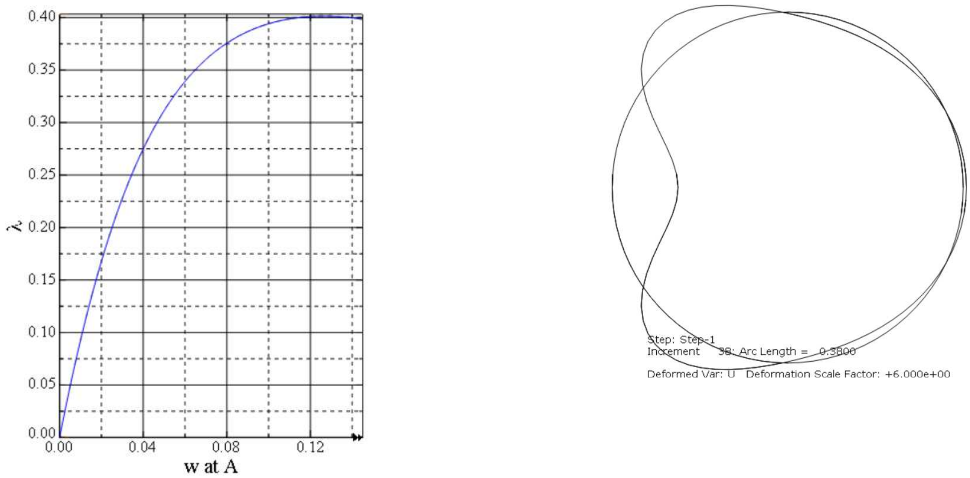


Fig. 3. Bifurcation mode associated with the lowest critical pressure 68.6 KPa, using LBA with ABAQUS. Displacements are plotted at the mid-span of the cylinder between supports, i.e., $x = L/2$.



(a) (b)

Fig. 4. Results for static pressure, using GNA with ABAQUS. (a) Equilibrium path for $\lambda = p_L/p_\lambda$; (b) out-of-plane displacements plotted at $x = L/2$.

The deflected shape of the shell at the state of maximum pressure is shown in Fig. 4(b), which is substantially the same as that given by the LBA. The shape shown in Fig. 3 is given by the eigenvector in the LBA study, whereas the mode plotted in Fig. 4(b) occurs along the fundamental nonlinear path and its amplitude is given at the point of maximum load that the system reaches, i.e., p_L.

Notice that there is a significant drop in maximum pressure between LBA and GNA studies, from 68.6 to 40.1 KPa, as a consequence of nonlinearity of the primary path. However, the mode shapes are almost identical in both cases.

3.3 Geometrically Nonlinear Analysis with Imperfections

The search for a relevant shape and amplitude of geometric imperfections in shells has been a subject of research for several decades. From a practical engineering perspective, tolerances have been specified for shells and surveys of real geometries are needed in order to verify if the construction remains within allowable bounds.

Maraveas et al. [26] compared the specifications of current US [27] and European [28] code provisions for vertical tanks under wind. The study was carried out on two large tanks using LBA, GNA and GNIA strategies within the ABAQUS environment. Of relevance to the present study on dynamic buckling under explosions, Ref. [26] highlights the shape and magnitude of imperfections that a designer should take into account. For real tanks in industry, the European recommendations [28] expect the designer to measure the geometry of the as-built shell and indicates the maximum amplitude of imperfection that would be acceptable. Three quality classes are identified, namely excellent, high and normal, and simple equations are given for each class based on the geometry of the imperfect shell.

In many situations it is not possible to perform measurements because the structure has not been fabricated or because average conditions are employed to characterize a population of tanks in a geographical region. In cases like that it is necessary to specify a credible imperfection shape and investigate the sensitivity of buckling loads with respect to the imperfection amplitude. Koiter [29] employed an imperfection with the shape of the eigenmode associated with the lowest eigenvalue in a LBA; this has been identified as an eigenvalue-affine imperfection. Such imperfection shape causes the maximum drop in the vicinity of a critical state but this may not be the case if larger displacements or imperfection amplitudes are considered. European recommendations specify that if the engineer does not have as-constructed data of a tank, then the most unfavorable geometry of imperfection should be assumed. This imperfection is usually taken as the eigenvalue-affine shape and it has been adopted by the shell buckling community [30]. For large amplitude imperfections it may be required to explore higher eigenmodes, whereas for structures that exhibit mode interaction it may be necessary to explore imperfection shapes by means of optimization algorithms. Bazzucchi et al. [31][32] recently explored imperfection-sensitivity in problems that show interaction between bifurcation and snap-through behavior; the authors emphasized cases where such interaction may be triggered by design parameters, such as slenderness and shallowness.

Table 1. Peak pressure and associated displacement amplitude for various imperfection amplitudes using GNIA.

ξ/h	0	0.5	1	1.5	2	2.5
p_L(KPa)	40.1	38.0	36.0	34.0	32.2	30.5
w_L/h	13	13	13	13	13	13

No specific tank was considered in the present investigation, so that an eigenvalue-affine imperfection was assumed. Following the European recommendations [28], the maximum amplitude of imperfection for the data considered in this paper and for normal quality of fabrication is in the order of 1.25 times the shell thickness. To illustrate the behavior, results have been computed for a maximum imperfection amplitude up to 2.5 times the shell thickness.

Riks algorithm was used in a GNIA study to investigate imperfection-sensitivity of the static case. The assumed geometric imperfections have the shape of the eigenvector in the LBA study (or the displacement configuration at the limit point in the GNA study), with a maximum amplitude scaled by a factor ξ that takes values in the range $0 < \xi/h < 2.5$.

Results are shown in Fig. 5. Under a given imperfection amplitude, the equilibrium path reaches a point of maximum pressure p_L. Then the path drops and the system has unstable behavior, with the consequence that it is not possible to find stable equilibrium states near the limit point in the path. The shape of the shell at p_L in each curve having an imperfection coincides with the eigenvector in Fig. 3 and is not shown here for reasons of brevity.

The maximum in each equilibrium curve in Fig. 5 is characterized by its pressure and the associated displacement; they are both listed in Table 1 for each imperfection amplitude investigated. The maximum pressure in this case reduces by 25% from 40.1 to 30.5 KPa. In the context of shell buckling, this is considered a moderate imperfection sensitivity.

The displacement in Table 1 remains the same (i.e. $w_L = 13h$) for all imperfection amplitudes considered; however, this may not occur in other problems.

4. Dynamic analysis

Investigation of the shell dynamic response under an impulsive pressure has been carried out using a special purpose program fully described by Ameijeiras and Godoy [33]. The implicit method IDA, available in the SUNDAILS [34] package, uses a variable time increment. The initial time steps are of the order of 4×10^{-7}s, and the mean time step during the analysis is of the order of 2×10^{-5}s. The impulsive pressure has been defined as given in eq. (1), which is represented in Fig. 2. It is assumed that the shell is empty.

4.1 Nonlinear dynamic response

The transient response is plotted as the maximum out-of-plane displacement at the meridian of maximum pressure, for a fixed pressure value. The plot is shown in Fig. 6, in terms of the transient response versus time. The fundamental mode is $T_n \approx 0.1$s. A pulse duration $t_0 = 0.025$s was adopted (t_0/T_n<0.3, impulsive), which is a reasonable value for pressures due to an explosion [24].

For $p_0 = 150$KPa, a maximum $w = 7.5h$ is obtained at time $t = 0.035$ s, which is higher than the time of duration of the positive pressure t_0. The maximum displacement w coincides with the first peak in the transient response for a given pressure p, and then the oscillations decrease in amplitude with time. In all cases, the deflected configuration at the first peak coincides with the static mode (either LBA or GNA).

For $p_0 = 250$KPa, $w = 15\ h$ is obtained and this occurs at $t = 0.035$ s. In fact, for all cases considered, the first peak occurs at the same time $t = 0.035$ s. The deflected shape of the shell at the first peak in Fig. 6 has the same shape as in the previous static studies, i.e., LBA, GNA, and GNIA. This is good news, because in this case, there is no need to compute the strain energy and one may infer the existence of a QB based on the amplitude of the displacement at the center.

4.2 Quasi-Bifurcation

Next, a QB criterion is employed following the works of Dinkler and Kroplin [16][17], and Simitses [12]: For a QB to occur, the strain energy at a first peak in the transient response should be the same as the critical strain energy in the static response at the unstable bifurcation. Notice that in the dynamic response the strain energy at the peak is the same as the kinetic energy at the initial time of the impulsive process.

Because the geometric configuration of the shell is the same in the static and dynamic configurations considered, then it is possible to take the displacement amplitude at the first maximum (rather than the strain energy) as a stability parameter.

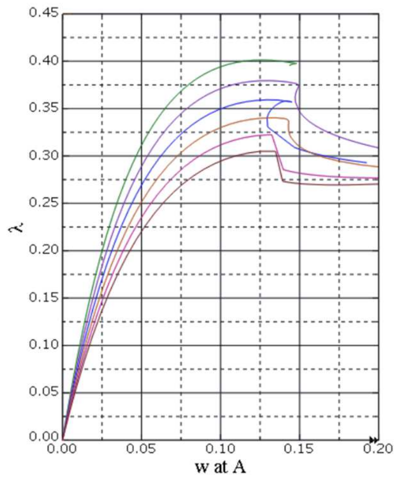


Fig. 5. Static GNIA results, for $l=p_L/p_l$ and six imperfection amplitudes (ξ/h = 0.0, 0.5, 1.0, 1.5, 2.0, and 2.5). The imperfection shape is shown in Fig. 3.

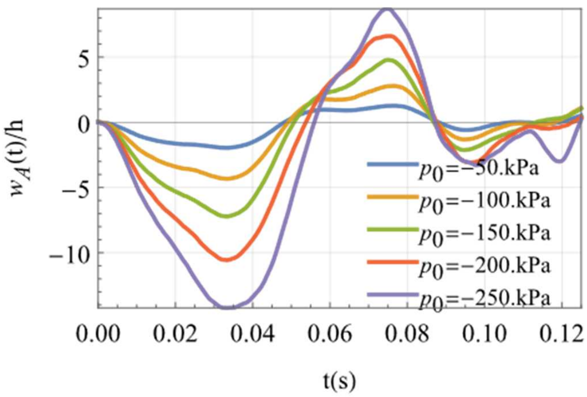


Fig. 6. Transient response for a horizontal cylindrical shell with L/R = 8 and R/h = 200 under impulsive pressure with t_0 = 0.025s.

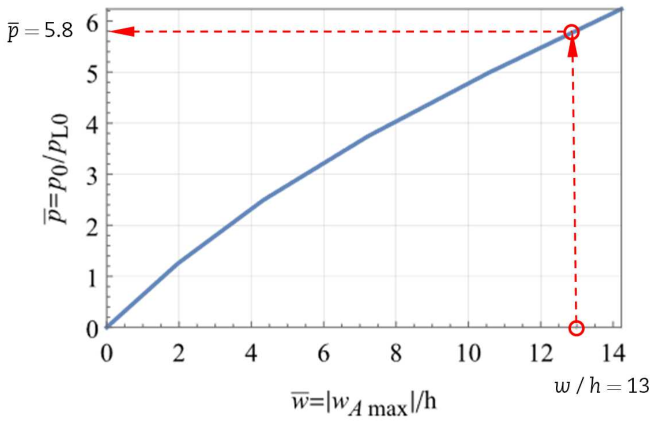


Fig. 7. Pseudo-equilibrium path for the cylindrical shell under impulsive pressure. Data as in Fig. 6.

A plot of dimensionless pressure versus maximum amplitude w is shown in Fig. 7, in which $\bar{p} = p_0/p_{L0}$ is the dynamic peak pressure normalized with respect to the maximum static p_{L0} pressure in the perfect shell. This plot is commonly known as a "pseudo-equilibrium path" ([35], p. 526) because of its similarity with an equilibrium path under static loads. The plot of Fig. 7 is useful to identify the pressure for which the maximum transient displacement w reaches the same level as in the static analysis at the critical state, in this case using GNA. The static displacement at the limit point in Fig. 4 was w/h = 13, and for this value of displacement in Fig. 7, the required pressure is $\bar{p}$ = 5.8 or $p = 5.8p_{L0}$ = 232kPa.

As stated before, the displacement configuration under p = 232kPa is the same (in both amplitude and shape) as the static configuration at the limit point, and because of that both states store the same strain energy. Further, because a peak is reached in the transient state, the velocity is locally zero and the system temporarily passes through a static condition. It has been identified that these conditions are the requirement to induce a QB in the transient response, i.e., the dynamic system passes through an unstable static state having the same strain energy (and deflected configuration) as in the unstable static state. For the present case, a QB occurs at p_0/p_{L0} = 5.8, i.e., dynamic buckling occurs at p_D = 5.8, p_{L0} = 232kPa. Notice that this value is lower than any instability that the system may attain using the criterion of large amplitude transient oscillations discussed by Budiansky [1], because the response of the system shown in Figs. 6 and 7 are still displaying a fairly linear relation between p and w. This shows that quasi-bifurcation occurs before dynamic buckling along the fundamental motion. This behavior was recently addressed by the authors for a shell-like structural system [19].

A condition for QB in the present case is that the system remains elastic in both the static and dynamic solutions for the states involved in the analyses. This condition is satisfied in the present case.

5. Influence of geometric imperfections on quasi-bifurcation response

Results for quasi-bifurcation states were identified in the previous section for a specific shell under impulsive pressures modelling an explosion. The influence of geometric imperfections is discussed in this section for the same shell, and the range of imperfection amplitudes is the same as in the static case. The authors were interested in the imperfection-sensitivity of QB because of its nature, in which the transient motion at an early time passes through the unstable configuration in the static problem. Thus, it seemed important to compare the imperfection-sensitivity of the static and the dynamic problems considered.

It may happen that each equilibrium path in an imperfect configuration has a different displacement w at the limit state; however, this is not the case in the present study, in which all static imperfect paths reached the same displacement at the limit point. This is shown in Fig. 8.

An imperfection-sensitivity plot is shown in Fig. 9. The results show that there is a 30% drop in QB pressure as a consequence of an imperfection in the form of the static eigenvalue of the same problem. This is slightly higher than the static sensitivity shown in Table 1. The plot is also useful to visualize zones for which stable and unstable behavior (as defined by QB) are expected.

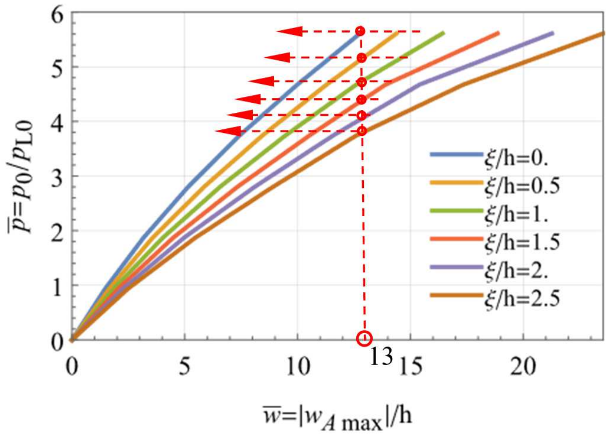


Fig. 8. Quasi-bifurcation pressures for the cylindrical shell with geometric imperfections in the range $0 < \xi/h < 2.5$. Data: $R/h = 200$, $L/R = 8$, $t_0 = 0.025$s.

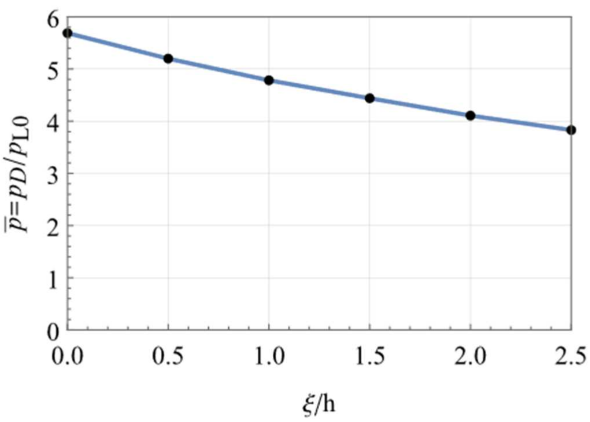


Fig. 9. Imperfection-sensitivity for quasi-bifurcation pressures. Data as in Fig. 8.

The sensitivity shown in Fig. 9 for QB is very similar to that obtained for the static problem in Table 1, and there are some differences because the static and dynamic loads are not identical. This is a reasonable result because the QB load is obtained based on the static one.

6. Conclusion

The dynamic quasi-bifurcation of shells employed in the oil industry was investigated for the first time to predict dynamic buckling under pressures due to an explosion. As such, the study should be important for engineers working in the oil industry in order to assess the vulnerability of existing structures and also to identify buckling in forensic investigations.

Quasi bifurcations, in the sense defined by Lee, were investigated in this case for horizontal cylindrical tanks. Both the static and the transient dynamic response were modeled, in the first case using LBA and Riks algorithm for GNA and GNIA. The dynamic response was modeled using geometric nonlinearity using a special purpose program developed by the authors [33]. The impulsive pressures due to an explosion were modeled as proposed in Ref. [23].

The main conclusions of the study may be summarized as follows:

- The static behavior of the cylindrical shell leads to different values of bifurcation pressure (LBA) and geometrically nonlinear pressure (GNA), but the deflected shape of the shell at the critical state remains the same. Thus, there is only one relevant mode regarding static buckling of the shell in this case.
- The dynamic behavior of the shell shows a transient response with displacements reaching a maximum at the first peak, for which the displacement configuration is the same as a configuration in the static case. This occurs for a response at the first peak, and changes in shape may only occur at higher times.
- Because of this coincidence between static and transient geometric configuration, there is a value of pressure at which the transient response passes through a state which is coincident with the static point of maximum pressure and having zero velocity. This state was identified as a quasi-bifurcation by Lee, and the strain energy is locally the same in both static and dynamic problems.
- At the quasi-bifurcation state, the system passes through conditions of static instability and loses stability under an impulsive load. As in the static case, a dynamic jump is expected to occur from the quasi-bifurcation state to another distant state having stable conditions. Along this dynamic jump, it is expected that the system will display large deflections, plasticity and damage. The quasi-bifurcation pressure thus obtained is more than three times higher than the static load obtained via LBA.
- This paper shows that the imperfection-sensitivity in the static problem is very similar to that in the QB problem; however, the two sensitivities are not identical because the loads (static and dynamic) at which instability occurs are different. Moderate imperfection-sensitivity was found in both the static and dynamic problems, with reductions in the order of 30% in maximum load for imperfection amplitudes of twice the shell thickness. Imperfection-sensitivity of the quasi-bifurcation load is slightly higher than in the static case, for the same shape and range of imperfection amplitudes.

The significance of the present studies to the oil industry should be stressed. Because failures of tanks due to effects from a nearby explosion are frequently reported, both designers and forensic engineers should understand the mechanisms of structural response and the path to failure under these sudden loads. This paper presents the first exploration of this problem for horizontal tanks, for which QB phenomena seems to occur.

The present case-study was limited to one shell geometry in order to illustrate the methodology to identify QB in shells under an impulsive pressure. Parametric studies should be performed before this constitutes a practical tool for use in the oil industry.

Author Contributions

M.P. Ameijeiras developed the computational model and theory validation, and performed all computations reported in this work. L.A. Godoy planned the research, suggested the cases to be computed, and wrote the present manuscript. Both authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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