

On Sparse Methods for Array Signal Processing in the Presence of Interference

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Abstract—We analyze the performance of several algorithms designed to solve the inverse sparse problem when they are applied to array signal processing. Specifically we study the error on the estimation of the complex envelope and the direction of arrival of signals of interest in the presence of interference sources using a uniform linear array. In particular, we compare the performance of the Enhanced Sparse Bayesian Learning (ESBL) algorithm against different algorithms tailored to this scenario. Since the former exploits interference information to diminish its unwanted effects, we find that it provides a reasonable tradeoff between runtime and estimation error.

Index Terms—Estimation, interference, sensor arrays, sparse models, sparsity.

I. INTRODUCTION

IN SEVERAL signal processing problems, observed data can be represented as the contribution of three phenomena, signal of interest, interference and noise. Given their often random nature, statistical tools and methods are required to remove interferences and noise. Different applications, from radar [1], through communications [2], to brain source localization [3] or in the present case of sensor arrays, data can be described using sparse linear models [4], [5]. This means that there exists a representation of the data wherein the signal of interest consists of a linear combination of a few components of an over-complete dictionary. The use of these models allows us to achieve better array resolution [6].

Looking for a sparse solution to a linear model is a non convex optimization problem, restricted to minimize the number of nonzero components, i.e., its ℓ_0 -norm. A common procedure to avoid this difficulty consists of using the ℓ_p -norm for $p \in (0, 1]$ as an alternate measure of sparsity. Examples of this type of algorithms are focal undetermined system solver (FOCUSS) [7], basis pursuit denoising (BPDN) [8], Dantzig Selector [9] and ℓ_1 -SVD [6]. Another class of algorithms use greedy pursuit, which iteratively approximate the support of the solution, such as orthogonal matching pursuit (OMP) [10], and its variants [11], [12]. Additionally, there exists another

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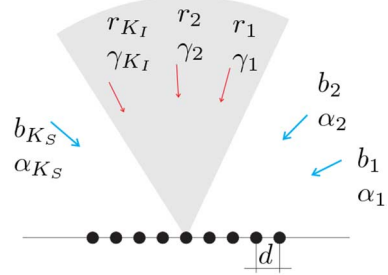


Fig. 1. M -element sensor array in the presence of K_S signals of interest and K_I interference signals.

kind of methods to solve the inverse problem that exploit statistical models and tools, such as Sparse Bayesian Learning (SBL) [13]. In a previous work we introduced a variant of the latter, named Enhanced Sparse Bayesian Learning (ESBL) [14]. This algorithm was originally designed to solve the radar problem of target detection in the presence of strong clutter. In this letter we analyze its performance when applied to general sensor arrays, comparing it to similar algorithms.

In the present array sensor application we wish to estimate the complex envelope of a signal and its direction of arrival (DOA). Unfortunately, there usually exist interfering signals impinging on the array resulting in erroneous estimation of the signal of interest. Assuming that there are few sources of signals, thus having signal energy in only a few directions of the space of possible DOAs, this scenario corresponds to signal estimation with spatial sparsity.

II. PROBLEM FORMULATION

A. Sensor arrays

We consider a uniform linear array (ULA) as depicted in Fig. 1, that consists of M collinear sensors separated by a distance d . There are K impinging narrow band signals, from which K_S are of interest, with DOAs $\alpha_1, \alpha_2, \dots, \alpha_{K_S}$, and K_I interfering signals, with DOAs $\gamma_1, \gamma_2, \dots, \gamma_{K_I}$. The latter are assumed to arrive from directions within the range from γ_I to γ_F . The output signal of the array is [15]

$$\mathbf{y} = \sum_{i=1}^{K_S} \mathbf{v}(\alpha_i) b_i + \sum_{j=1}^{K_I} \mathbf{v}(\gamma_j) r_j + \mathbf{w} \quad (1)$$

where \mathbf{y} is the M -vector of the complex envelope measurements, $\mathbf{v}(\alpha_i)$ and $\mathbf{v}(\gamma_j)$ are the steering M -vectors corresponding to directions α_i and γ_j , b_i and r_j are the complex amplitudes (scalars) containing the information transmitted by

the i^{th} signal of interest and the j^{th} interference source, and \mathbf{w} is Gaussian white noise. The structure of each ULA steering vector component for an angle α is

$$[\mathbf{v}(\alpha)]_m = e^{j\frac{2\pi d}{\lambda}m \cos(\alpha)}, \quad m = 0, \dots, M-1. \quad (2)$$

The measurements at the output of the sensors (1) can also be expressed in matrix form

$$\mathbf{y} = \mathbf{V}_s \mathbf{b} + \mathbf{V}_i \mathbf{r} + \mathbf{w} \quad (3)$$

where \mathbf{V}_s is a $M \times K_S$ matrix whose columns correspond to the steering vectors for each signal's DOA, \mathbf{b} is the K_S -vector with the transmitted information by all signals of interest, \mathbf{V}_i is the $M \times K_I$ matrix of steering vectors for the interference, and the K_I -vector \mathbf{r} contains the interference information.

Finally, when several snapshots of the measurement vector are available, the array output becomes

$$\mathbf{Y} = \mathbf{V}_s \mathbf{B} + \mathbf{V}_i \mathbf{R} + \mathbf{W} \quad (4)$$

where matrices \mathbf{Y} , \mathbf{B} , \mathbf{R} , and \mathbf{W} are built using the respective vectors from (3) as their columns. Note that a row of \mathbf{B} corresponds to time samples of the source signal's complex envelope.

B. Sparse Model

The sensor array scenario can be casted as a sparse model. We first discretize the space of angles of arrival in N components, $\theta_1, \theta_2, \dots, \theta_N$. This results in a $M \times N$ matrix $\tilde{\mathbf{V}}_s$ whose columns correspond to steering vectors of the array for each angle which are the atoms of the dictionary [8]

$$\tilde{\mathbf{V}}_s = (\mathbf{v}(\theta_1), \dots, \mathbf{v}(\theta_N)). \quad (5)$$

The steering vectors for the interference range correspond to some columns of matrix $\tilde{\mathbf{V}}_s$, those representing directions θ_i such that $\gamma_I \leq \theta_i \leq \gamma_F$, forming the $M \times P$ matrix

$$\tilde{\mathbf{V}}_i = (\mathbf{v}(\theta_I), \dots, \mathbf{v}(\theta_F)). \quad (6)$$

We note that this matrix represents all P possible interferences DOAs, since it contains the steering vectors of all the discretized angles within the range. However, the actual number of interference sources K_I is lower. In the proposed scenario the estimation algorithm distinguishes the signal from the interference using DOA information. Thus, the interference angle range, which we assume to be known, does not include the signal's DOA. Other problems may consider different features in order to separate the interference from the signal of interest, such as polarization or Doppler information, but that scenario exceeds the scope of this letter.

When we take D snapshots of the measurement vector, and they can be represented by the same atoms of the dictionary $\tilde{\mathbf{V}}_s$ sharing a common sparsity profile, we have a sparse representation of multiple measurements vectors (MMV) [16]. Then, the matrix \mathbf{B} has a small number of nonzero rows. This formulation is a realistic representation of multiple realizations of a signal contaminated by noise. The transmitted information is contained in the matrix \mathbf{B} whose i^{th} row is nonzero if $\theta_i = \alpha_i$. The goal is to estimate \mathbf{B} under the condition of common sparsity profile.

C. Algorithms for Sparse Models

We provide a brief description of the ESDL algorithm whose details can be found in [14]. This is an iterative algorithm that solves the previously described sparse inverse problem, where the algorithm not only computes a point estimate of \mathbf{B} through the posterior mean, but also estimates the statistical parameters of the model. Each iteration of the algorithm consists of two steps. In the first step the EM algorithm is used to find estimates of the model parameters, the covariances of the source signals, the interferences and the white noise. These results are then used to compute a point estimate of \mathbf{B} through the posterior mean which results in a non-sparse matrix. However, few of its components will be statistically significant. Thus, the second step of the algorithm implements a decision test to prune the spurious coefficients. The pruning threshold for the test is determined by a desired false alarm probability P_{FA} .

Some of the previously mentioned algorithms have extensions to handle multiple snapshots and are used to compare against ESDL. These are MOMP [17], MFOCUSS [18] and MSBL [19]. We remark that the ESDL algorithm does not need a priori information of the number of present signals, i.e. the sparsity profile of \mathbf{B} . Furthermore, this algorithm is the only one that considers the effect of the interference in the model and in the estimation process.

III. PERFORMANCE ANALYSIS

We illustrate the performance of the scenario described by Eq. (1) through the following example. The sensor array has $M = 20$ elements and inter-element distance $d = \lambda/2$. We set a grid of $N = 90$ possible angles of arrival between 0 and 180 degrees. We use $D = 10$ snapshots where the vector \mathbf{b} has a sparsity $s = 2$ with two arriving signals at $\alpha_1 = 120$ and $\alpha_2 = 160$ degrees. There are 10 interference sources arriving from random angles in the range $\theta_I = 20$ to $\theta_F = 100$ degrees, resulting in a range with $P = 40$ possible angles. This implies that one of the signals is close to the interferers and the other is practically clean. The power of the signals is $P_S = \|\mathbf{b}\|_2^2$ and the interference power is P_I . The noise power is $\sigma = P_I/10$. We run $MC = 500$ Monte Carlo simulations and estimate \mathbf{b} for different signal to interference ratios defined as $\text{SIR} = 10 \log_{10}(P_S/P_I)$. We applied the following algorithms, ESDL (for a false alarm rate of 10^{-4}), MSBL, MFOCUSS, MOMP, ℓ_1 -SVD and MUSIC. The last two cannot estimate by themselves the amplitude of the signal, only its spatial spectrum (and consequently its direction of arrival), thus we combine them with the Minimum Variance Distortionless Response (MVDR) estimator [15] to perform the estimation of the signal.

Fig. 2 shows the spatial spectrum for the 500 Monte Carlo runs with a $\text{SIR} = 30$ dB for the mentioned algorithms. Fig. 2(a) shows the original signals at 120 and 160 degrees and the randomly selected interference sources within the range 20 to 100 degrees. Fig. 2(b) shows the reconstruction using the ESDL algorithm which exploits the interference structure information and previous knowledge of its possible range of arrival. The rest of the methods cannot effectively reject the interference which affects the correct estimation of the signals of interest.

Fig. 3 shows a typical normalized spatial spectrum for some of the analyzed algorithms (ESDL, MSBL, MFOCUSS, ℓ_1 -SVD

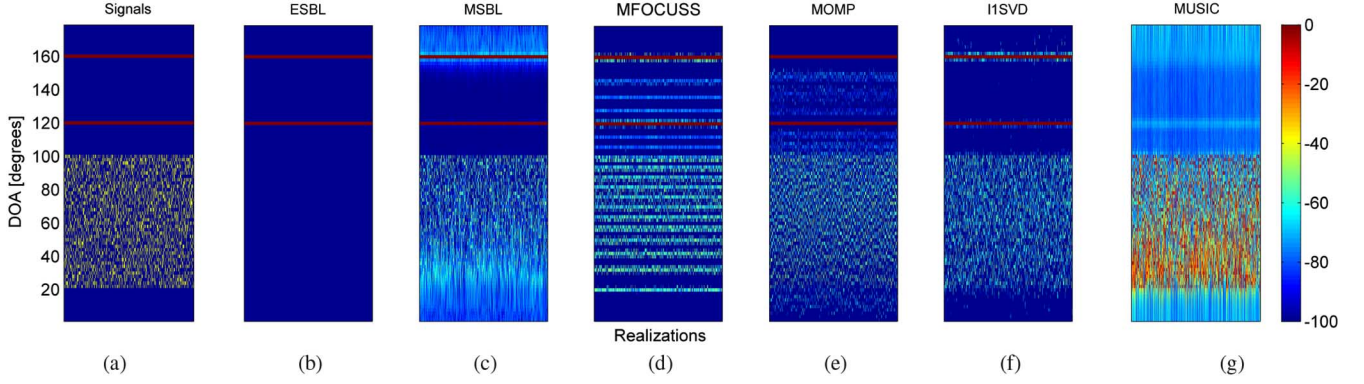


Fig. 2. 500 Monte Carlo runs of the spatial spectrum for two sources with DOA 120 and 160 immersed in 10 interference sources. (a) true value (b) ESBL, (c) MSBL (d) MFOCUSS (e) MOMP, (f) ℓ_1 -SVD and (g) MUSIC. (SIR = 30 dB, $N = 90$).

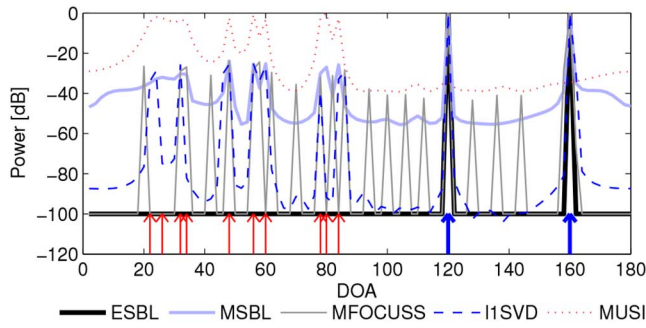


Fig. 3. Typical spatial spectrum for two sources (blue arrows) with DOAs of 120 and 160 degrees and 10 interference sources (red arrows) in the range 20 to 100 degrees using MUSIC, ESBL, MSBL, ℓ_1 -SVD and MFOCUSS with SIR = 30 dB.

and MUSIC) in the proposed scenario. The blue thick arrows denote the arrival directions of the signals of interest and the interferences are denoted by red thin arrows. We can clearly appreciate that the ESBL spectrum is null for all angles except those of the signals of interest, where achieves its maximum. MSBL also has maximums at these angles but the interference rejection is not as good. Similar behavior is obtained by MFOCUSS and ℓ_1 -SVD.

We analyze two ways of evaluating the performance of the algorithms. On one hand we want to asses the capacity of the algorithms to correctly recover the spatial spectrum. This is equivalent to computing the average over 500 runs of the estimation error of the matrix \mathbf{B} , i.e.,

$$\text{MSE}\{\mathbf{B}\} = \frac{1}{MC} \sum_{i=1}^{MC} \|\mathbf{B} - \hat{\mathbf{B}}_i\|_F^2. \quad (7)$$

Fig. 4 shows that using this measure, the ESBL algorithm outperforms the others. This result implies that the algorithm correctly recovers the direction of arrivals from the signals of interest, without interferences. For most applications this would be the most important and decisive measure of comparison.

In the present sensor array application we are interested in the behavior at the specific direction of arrival of each signal of interest, and in correctly recovering the information transmitted. Therefore, we also analyze the error in each of these directions. Fig. 5 shows the estimation error for the signal arriving from 120 degrees, $\sum_{i=1}^{MC} \|\mathbf{B}(\alpha_1) - \hat{\mathbf{B}}_i(\hat{\alpha}_1)\|_2^2 / MC$, and Fig. 6 shows the one from 160 degrees, $\sum_{i=1}^{MC} \|\mathbf{B}(\alpha_2) - \hat{\mathbf{B}}_i(\hat{\alpha}_2)\|_2^2 / MC$. The

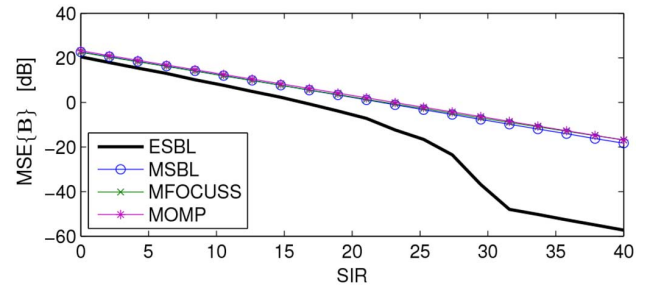


Fig. 4. Estimation error of the whole spatial spectrum.

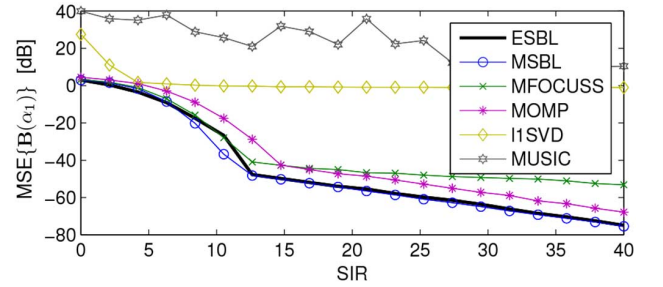


Fig. 5. Estimation error for the 120 degrees signal.

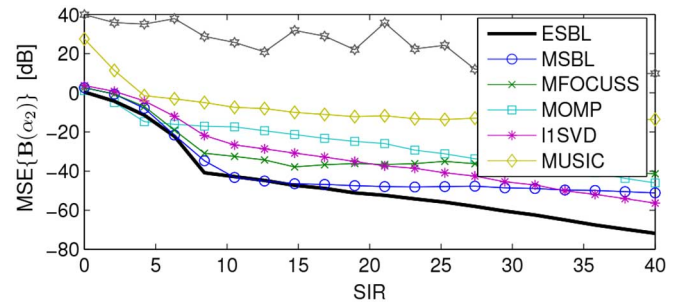


Fig. 6. Estimation error for the 160 degrees signal.

behavior of ESBL algorithm regarding the signal closer to the interference is similar to other algorithms, indicating that in this case there is no performance gain by ESBL. On the other hand, for the clean signal arriving from 160 degrees, the estimation error is much smaller for this algorithm. The performance of ESBL in these last two figures is closer to other algorithms because this performance measure does not show the error by selecting spurious components. Fig. 2 and 3 illustrate that ESBL has very low rate of falsely selected components.

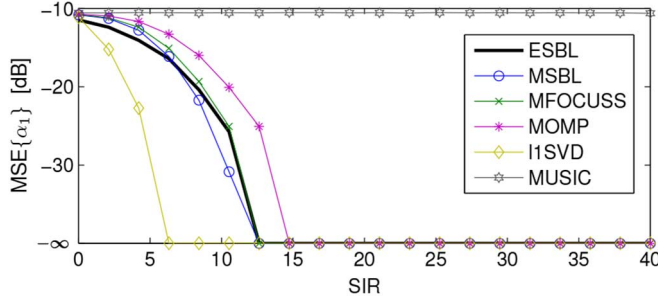


Fig. 7. Estimation error for the 120 degrees signal DOA.

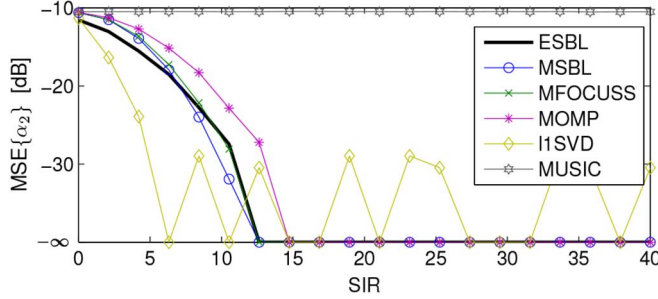


Fig. 8. Estimation error for the 160 degrees signal DOA.

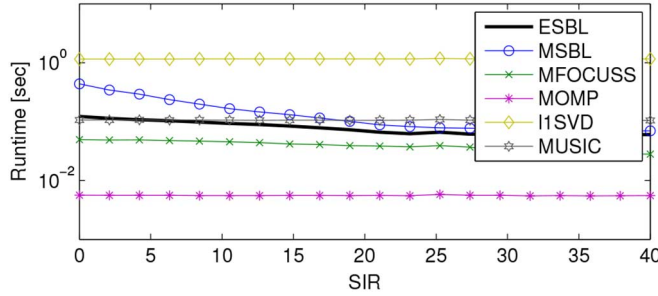


Fig. 9. Run time for the algorithms.

Another important measure is the estimation error in the direction of arrival of each signal of interest, $\|\hat{\alpha} - \alpha\|_2^2$. To evaluate this measure, we take the mean of error of 500 Monte Carlo runs. The result is shown in Fig. 7 and 8 for signals from 120 and 160 degrees respectively. For SIR values larger than 15 dB, the angles of arrival are detected without error, indicated by a virtual 0 in the logarithmic scales, by almost all algorithms except MUSIC. It can be appreciated that the ℓ_1 -SVD algorithm needs a smaller SIR to correctly estimate the directions of arrival, unfortunately its signal estimation is not reliable as shown previously.

We also evaluate the computing time for each algorithm for this sensor array application in particular. Their performance can be appreciated in Fig. 9. The MOMP algorithm is fastest but its performance in parameter estimation is not reliable. ESBL runtime is similar to MSBL, MFOCUSS and MUSIC. ℓ_1 -SVD has a larger computational complexity and is not viable for this application.

IV. CONCLUSION

We studied the performance of several sparse methods algorithms for sensor array applications. The goal is to estimate the

signal's complex envelope and its DOA, when immersed in interference. In particular, we analyzed the ESBL algorithm introduced in a previous work. This algorithm does not require information on the number of present sources since it nulls the directions where it considers no signals of interest present. In contrast, the other algorithms seek for peaks but they need to know a priori the number of sources, since they generate a significant number of small, spurious components. Using different performance measures we find that the ESBL algorithm has similar or better performance than the rest in several conditions, and uses same or less computing time.

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