Economic growth, human capital and negative externalities: stability and chaos

Abstract: In this work we analyze the importance of the accumulation of human capital through the formal educational system for economic growth. We base our analysis on two previous models (London: 2005, London: 2006) in which, starting with the framework in (Lucas: 1988) we add a parameter of distortion on the formal educational sector.
The mathematical model is built upon a logistic equation. Since it is not analytically solvable, we run simulations for different scenarios, using data from Argentina. The main conclusion we reach is that the stability of the growth process depends critically on a fine tuned combination of the main variables. This suggests very specific policies of development that albeit being intuitively sound, can be inefficient

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Silvia London[1]

[1] Universidad Nacional del Sur – CONICET.
e-mail address slondon@uns.edu.ar,
slondon@criba.edu.ar

Introduction

The empirical literature suggests that the differences of income across different economies of the world are related to strong discrepancies among the rates of productivity. These rates, in turn, are determined by the stocks of capital by worker and the levels of education attained by the labor force. The accumulation of capital per worker is the main feature that characterizes a process of economic growth. The acceleration in accumulation of physical and human capital is the distinguishing trait of the economies during their initial years of development Ros (2001). On the other hand, these economies tend to exhibit moderate levels of education at the beginning of the process. Later they improve in this aspect, and the resulting fast growth of the ratio capital product is strongly associated with the increase of the product by worker, as well as with a relatively high level of education and a fast industrialization. In contrast, the non-developed countries show low levels of formal education and a slow process of industrialization. Azariadis and Drazen (1990) suggest that a critical level of human capital is a necessary condition for fast growth. But it is not a sufficient condition. Several countries of Latin America had high levels of education at the beginning of the 1960s and, nevertheless, their rates of growth have been very small since then Ros (2001).

In any case, the production of human capital can explain the persistence of growth and development in the long term. It provides the potential of generating rent both on the basis of innate capacities and acquired qualifications. While the former are given, the latter can be improved through a formal system of education.

Even though it is evidently so important in this matter, human capital was not incorporated in the first analyses of economic growth. The first approaches that introduced human capital into growth models were those of Uzawa (1965) and Phelps (1966) (in Barro – Sala-i-Martín 1995). Uzawa presented two fundamental assumptions: 1) Work can be of two classes: productive and educational. The first is used in the production sectors of the economy while the second includes both "people who teaches", permanently separated of the productive sector, and "people who learns", that will join the productive ranks: 2) the rate of increase of the augmentative technical progress of work is a concave and increasing function of the proportion of the educational sector in the total workforce.

The contribution of Phelps is essentially an effort to find a satisfactory expression for the technological function to be incorporated into the simple Solow's model. The term of technical progress A(t) should, for Phelps, be a function of the workforce devoted to research, since more progress can be obtained intensifying the efforts in scientific and technological research, although the law of the decreasing yields applies both to the activities of research and production.

The high complexity of the mathematical models used and a certain aloofness from the real world problems lead to the stagnation of growth theory until in the 1980, the theories of the endogenous growth presented fresh perspectives on the subject. The new generation of economists gave greater relevance to the works of empirical character, which in turn lead to improved analytical treatments of real world phenomena.

Retaking the idea of Uzawa, Lucas (1988) postulates two sectors, where the sector that produces human capital (education) is more intensive in the use of this factor than the other, the goods producing sector. In the latter sector, the production is given by a Cobb-Douglas function, where the arguments are physical and human capital. It is affected by an externality: the average stock of human capital. The important assumption of the model is that human capital is produced, with increasing returns, using human capital as the single input. The main conclusion is that education becomes the main engine of growth.

In this work we develop a model of growth based on the accumulation of human capital accumulation along the lines of Lucas'model (1988). But in this case the emphasis is on the public financing of the educational system and the existence of a negative externality on education. One important variant with respect to Lucas' model is the assumption of a maturation delay of the investment in education, which leads to a representation by means of a logistic equation. This, in turn, opens the possibility of generating irregular behaviors. Since this is not a desirable result, in the second part of the work we perform a *qualitative analysis* of the model, up from simulations run on data of the Argentinean economy. Given these results, the third part concentrates on the general recommendations of policy suggested by them as well as on future lines of investigation.

I A model of growth with chaotic orbits

In this section we present a simple model of economic growth, incorporating the effect of the Education, (more concretely the investment in education as a percentage of the GNP) on the macroeconomic performance of an economy. For this we will briefly present the formal arguments of the Lucas model, which will be adopted for developing our model.

The Lucas'model

Lucas considers three different models: a) one emphasizing the accumulation of physical capital and technological change, b) another in which human capital accumulates by an increase of formal education, c) a third one, in which the accumulation of human capital is done by means of a process of *"learning by doing"*. The model that this author considers to be the main engine of economic growth and will be considered here is the second.

The model starts from the *microeconomic foundations* of macroeconomics: the maximizing behavior of producers and consumers. The accumulation of one of the factors (human capital) decided endogenously yields a persistent growth of the per-capita output, contrary to the one that obtains in the neoclassic model of growth (SOLOW, 1992) due to exogenous technical progress. This conclusion is of course sought by Lucas, since like most of the authors of the endogenous growth, he looks forward to get beyond the weaknesses of the neoclassic growth theory, in particular the not convergence and the not equalization of the prices of the factors that arises even with unrestricted international trade.

With this goal in mind, Lucas incorporates to the Solow's model human capital as an additional input. He intends this factor to represent the general level of ability of the individuals. Expressing the variables in efficiency units he postulates the following production function:

$$Y = F\,(N^e, K) \qquad (1)$$

Where N^e it is the effective force of work, and K capital. On the other hand, the accumulation of human capital is:

$$\dot{h}_t = \delta(1-u)h_t \qquad (2)$$

Where δ it represents the productivity of the investment in human capital, and $1-u$ the amount of hours assigned to the accumulation of human capital.

The linearity in the accumulation of h guarantees a persistent growth. The effort dedicated to the accumulation of human capital fully affects the rate of change of its own level.

The general production function that relates these variables is a Cobb-Douglas type

$$Y(K, N^e, h) = AK_t^{\alpha}\,(u_t . N_t^e . h_t)^{1-\alpha}\, h_{at}^{\gamma} \qquad (3)$$

Where h_{at}^{γ} are the external effects of human capital, which appear because all the individuals benefit from its average level in the economy. A is the technology level, and u the hours assigned to the production of the final good.

From these equations and from the utility function of the individuals, Lucas finds the rate of growth of the economy. In this presentation we will concentrate on the basic equations, about which we will make new assumptions, and then we will study the dynamics of the system.

Human capital and investment in education

We will keep the relations described by equations (1)-(3), but expressing the model in discrete time. We will also give an alternative meaning to some of the variables. In particular, we will see human capital in terms of its financing sources, either public or private.

We assume that the stock of human capital at every period depends basically on the investment made in the previous period in the educational sector, being the lag between periods due to the maturation time of the investment. The amount invested is described as a proportion of the product of the past period, $eY_{(t-1)}$, where e represents the proportion of the public and private expenditures on education. Besides this influence, human capital at each period will be affected by a proportion π of the product at $t-1$, summarizing both the historical conditions and the initial productivity of human capital. Finally, it will also be determined by the number of hours destined to its accumulation, $(t-1)$, where u is the number of hours assigned to work, as well as by the efficiency of the educational system δ.

The functional expression is as follows:

$$h_t = (\delta(1-u)e + \pi)Y_{t-1} \quad (4)$$

With δ and e between 0 and 1, and u constant. Note that the accumulation of human capital depends on the historical macroeconomics conditions (summarized in π), under the assumption that the investment in education generates an "accelerator effect". This is one of the mayor differences between our approach and Lucas'.

On the other hand we consider the existence of a tax on capital equivalent to the amount of investment in education. As in the case of human capital accumulation, we assume that there exists a time lag between an investment and its maturation. Then:

$$K_t - K_{t-1} = s(1-e)Y_{t-1} \quad (5)$$

In this version we will not incorporate physical depreciation neither of capital nor of human capital.

The equation of the Product is now:

$$Y(K, N^e, h) = AK_t^{\alpha}(uN_t^e.(\delta(1-u)e+\pi)Y_{t-1})^{1-\alpha} h_{at}^{\gamma} \quad (6)$$

We will further assume the existence of two externalities: on the one hand, the educative average as represented by Lucas (1988) in its original model. On the other, we incorporate a negative externality represented by the educational average multiplied by **dispersion parameter** λ. This parameter does not have relation with the efficiency of the formal educational system, reflected in δ, and normally approximated by the quality in the educational system, the existing infrastructure, etc. This dispersion parameter arises even in highly efficient systems, when the efforts to educate dissipate as a consequence of the features of the social system. That is, it summarizes those hardships for the educations system like the existence of extra-age students, unfavourable educative familiar climates, scholastic desertion, etc.[1] In an empirical reading of the formalism, the parameter λ allows to represent the outcomes from the investment on education: two economies may have equally efficient educational systems and invest the same amounts in education but nevertheless yield different values of λ. The fact is that social issues that affect the performance of students may induce wide differences in the yields of the investment on education. So, for instance, if kids have to drop out of school, or if they are undernourished, etc. an increase of investment in schools will not translate into an increase of the quality of human capital.

$$h_{at}^{\gamma} = \left[(1-\lambda)\frac{h_t}{N}\right]^{\gamma} \quad (7)$$

$$h_{at}^{\gamma} = \left[(1-\lambda)\frac{(\delta(1-u)e+\pi)Y_{t-1}}{N}\right]^{\gamma} \quad (8)$$

If we assumed N constant and equal to 1 we obtain:

$$Y_t(K,h) = AK_t^{\alpha}(u(\delta(1-u)e+\pi)Y_{t-1})^{1-\alpha} \left[(1-\lambda)(\delta(1-u)e+\pi)Y_{t-1}\right]^{\gamma} \quad (9)$$

If we assume ***K/Y*** to be constant, and taking the equation (5), we can rewrite the expression for the Product as follows:

$$Y_t(Y) = A\left[s(1-e)Y_{t-1} + \nu Y_{t-1}\right]^{\alpha}(u(\delta(1-u)e+\pi)Y_{t-1})^{1-\alpha} \left[(1-\lambda)(\delta(1-u)e+\pi)Y_{t-1}\right]^{\gamma} \quad (10)$$

In (10), the output of each period depends now on the level of product in the previous period. The rest of the variables are constants.

This last equation shows **the actual trajectory of the output**, which may differ from the equilibrium path. In the ideal case these two coincide and investment and savings will be equal ex-ante, for both physical and human capital.

As Lucas (1988), the level of product at every period is a function of the accumulation of physical and human capital. The difference with the original model is that this last variable requires of certain investment that depends on the level of economic activity at the previous period, and that this investment reflects the general characteristics of the system, in particular the values of the dispersion and efficiency parameters. In other words, a negative externality is introduced (in addition to the standard one) (λ) being the amount of resources that must be destined to palliate the effects of lack of prevention in health, bad feeding habits, and the existence of opportunity costs of getting education, among others, that cause actions (conscious and unconscious) that affect the educational system.

If we concentrate the analysis on the effect on the product from the negative externality, and doing a little algebra, equation (10) can be rewritten as:

$$Y_t(Y) = aY_{t-1}(1 - bY_{t-1})^{\gamma} \qquad (11)$$

With $a = A\left[s(1-e)+v\right]^{\alpha}(u(\delta(1-u)e+\pi))^{1-\alpha}$, $b = \lambda(\delta(1-u)e+\pi)$

Even with this assumption, the solution still cannot be given in an analytical form. When the traditional methods of resolution are not possible, a graphic-qualitative approach or, alternatively, a numerical approach can be applied. The qualitative or topological theory analyzes the **properties of the solution** without in fact knowing it. An equation whose properties extensively were analyzed in Chaos Theory is the logistic equation[2], that in generic form is expressed like:

$$x_t = a x_{t-1}(1 - x_{t-1}) \qquad (12)$$

Actually, equation (11) of our model is a logistic equation. In it the value of parameter a (that determines the stability or instability of the system) obtains from a set of interrelated variables. On the other hand, there is a slope of the equation given by b.

Given all this, it becomes necessary to analyze the behaviour of the system, for which we must resort to computational simulations. The following section will be dedicated to this point.

II Behavior of the System

Let us consider again equation (11). In order to run this set of simulations we must give values to the parameters and variables. We will always start with an initial level of $Y = 1$, and will consider the following approximated values of the other parameters, obtained according to Argentinean data corresponding to years 2000 and 2003 (see statistical appendix):

$v = 2{,}65$

$e = 0{,}095$

$\lambda = 0{,}20$

$\alpha = 0{,}50$

$s = 0{,}20$

$\delta = 0{,}65$

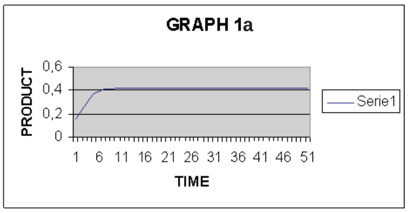
A as well as π will take qualitative values (fuzzy) distinguished like "high", ""low" and "medium". Finally, $0 < \gamma < 1$.

Once established the values of the main parameters, we run simulations to establish the conditions under which the system is stable. The **analysis is of qualitative type**, and even though real data for certain values of parameters are being considered, the rest of the values are not related to their real world counterparts.

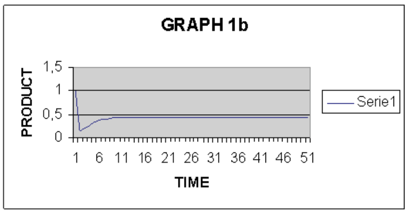
The simulations of the behavior of the system are performed under different scenarios. This tool of analysis is quite different from the traditional method, based on the use of dynamic optimization, in which the objective function of a representative agent is maximized. The line of work followed here, does not intend to be a substitute for the traditional method, since we do not incorporate a utility function, nor do we seek general conclusions. Instead, we look for results that may suggest some conjectures about the behavior of the system.

Numerical experiments

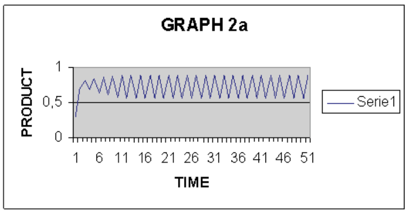
The first set of simulations assume a low value of A (the parameter that reflects how technological and institutional conditions impact on the productivity to the production's factors), and a low productivity of human capital. Graph 1 describes the behavior of the system: it stabilizes at a low level of product, when γ is maximum (=1).

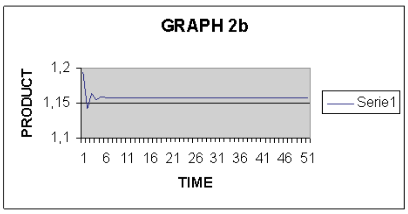


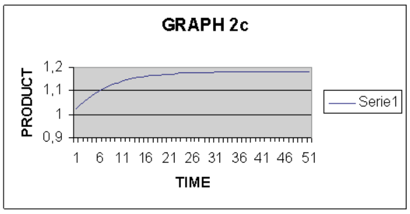
The final values do not change if we consider a much smaller externality (0,20). The path followed by the system differs only for the transient (the first 6 periods) :



The stability remains if we increase the productivity of human capital, without varying A. Increases in A yield higher results in terms of the levels that can be reached, but the path becomes unstable if we keep a low productivity of h (graphical 2a). The system attains stability for mean levels of π (graphical 2b) and it sensibly reaches higher and stable levels for high levels of π (graphical 2c):



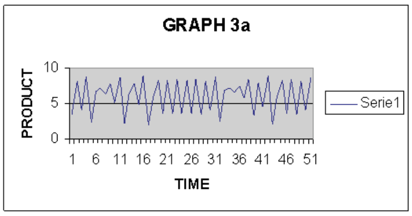




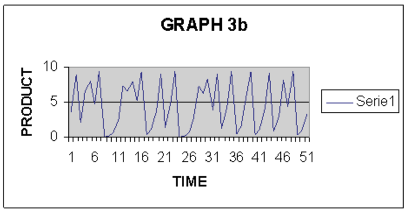
Once again, modifications of γ only affect the results of the first periods, without modifying the final state.

The previous graphs indicate the existence of an important relation between the degree of general technology A and the effect that the productivity of the human capital has on the system: when the latter is low, the trajectory of the productive system fluctuates, exhibiting a cyclical behavior. This does not happen when A is small: the system is stable but gets trapped in a low level. We could interpret this result as an indication of the existence of a **poverty trap**.

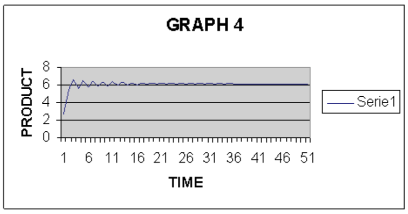
The stability shown in graph 2c changes when we increase the level of A: for very high levels of A the system becomes again unstable, until it reaches the threshold of chaotic behavior:



This behavior can be interpreted in relation to the previous results: the inability of the system to adapt its technology to the amount of human capital that has been generated causes a reaction against the status of the previous period (running an endless process of test and error). This unsuccessful adaptation process becomes even more unstable when the technology requirements are greater. Thus, the amplitude of the oscillations increases in parallel to the values of A, as it can be seen in graph 3b, that displays a variation in A of 7% with respect to graph 3a:

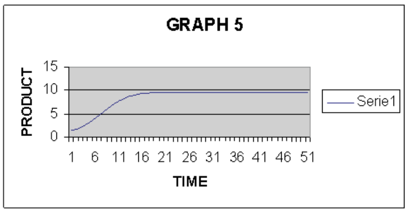


Recall that in these simulations the values of the cost of investment on education and the participation of human capital in the production function are constant. If in this situation (high levels both the productivity of human capital and technology, Graph 3b) we assume an increase in the investment on education e, we do not reach a stable situation. The source of this instability is the technology. For that reason, a change in the participation of the factors stabilizes the system. Taking $\alpha = 0.4$ instead 0.5, we obtain:



This last result is interesting for the analysis of some current proposals by the Argentinean government: according to the Law of Educational Financing, article 3, the goal is to increase the educational budget from the current 3% of the GNP to a 6% in 2010. If we use this value as an input in our first simulation, we find a downfall of the aggregate product: the conditions of productivity and technology do not provide incentives to invest in human capital, and the only effect is a reduction (because of the tax on capital) of the product. The trap of the poverty not only persists but worsens.

For average values (simulation 2) the effect is similar, that is, the system starts to oscillate and even may get a chaotic character. Therefore, an increase of the public investment in education results in an unstable behavior unless the relative participation of human capital in the production function, α, and/or the technological-institutional conditions A become higher, and the dispersion parameter remains at its lowest level:



With these values an attainable maximum is reached and remains as a stable state. The higher the value of the variables and the historical productivity of human capital, the higher is the steady state attained by the system.

A result that seems to be of little interest is the rather insignificant effect of the intensity of the externality on the final result. But in fact, this is quite relevant due to the fact that modifications in γ alter the transition. This may become a central issue in further analyses.

Final considerations

Education and more generally the formation of human capital are fundamental for the economic growth. In this sense, the role of public investment is critical, since it translates into higher rates of scholastic attendance, lower rates of desertion and extra-age levels, etc.

Nevertheless, from a macroeconomic point of view, the cost of education cannot be treated as if it were an isolated phenomenon. Its effectiveness is conditioned by the other variables of the system. The technology, the capital/product relation, as well as the overall institutional conditions have an impact on the effectiveness of the human accumulation of capital. It follows that a public policy aimed to increase the levels of activity of a country (GNP) should not be concentrated in a single source of growth, but it must contemplate to the complementarities and other features of the system.

These conclusions seem to be backed by our qualitative analysis, based on a system of equations with the potential to yield unstable and even chaotic solutions. But this is just the first step, since the final goal it is to establish a complete model of growth, incorporating a refined variant of the dispersion parameter as well as the effects of the income distribution on itself and on the participation of the fragment of society deprived of education.

Statistical appendix

We consider a constant relation K/Y, v, according to the values we obtain from the data in table 1:

Table 1 – Capital/product relation (K/Y) as percentage of the GNP

Year	K/Y
2000	2.5
2003	2.8

Source: elaboration on the basis of www data.cai.org.ar

In order to quantify the cost or investment in education, we must distinguish between public and private investment. The data on public cost are reflected in table 2:

Table 2 – Consolidated public cost of Education (CPE), Scientific and Technological Activities (S&T) and Research and Development (R&D) as percentage of the GIP[1]

Year	CPE	S&T	R&D	TOTAL
2000	4.5	0.52	0.44	5.46
2003	3.6	0.46	0.41	4.47

Source: elaboration on the basis of data of www.me.gov.ar and HTTP://www.secyt.gov.ar

As far as data on the investment of private education, for this work we use information from FLACSO (2003) that indicates that it similar to public investment. Then, we consider the total investment =2(CPE+ACyT+I&D).

We based the efficiency data on several reports of CLACSO, in which the quality of the Argentinean system is drawn from the results of standard tests of mathematics and language taken to students of primary and secondary schools. In average, the results indicate that the efficiency is around 60%.

The data that should be used to obtain the value of the dispersion parameter must be carefully chosen. It is necessary to consider that education is not only provided by a formal system, but that there exists sources of informal education (at home, in other institutions, the cultural environment, etc.) that condition and affect the way in which the formal training provided by schools is finally internalized. On the other hand, the system as a whole is affected by the amount of students that are older than what is required by the scholastic years they attend (extra-age). This kind of scholastic delay increases the actual costs of education[2].

Finally, important pieces of data that indicate the degree of success of educational efforts are the number of individuals that finished the secondary cycle, the number of college students, and the net rate of illiteracy[3], as shown in tables 3 and 4 (SITEAL, 2006).

Table 3 – Rates of scholarship, scholastic delay and extra-age

	2000	2003
Rate of scholarship (Sc)	76.3	78.7
% of students with 2 years or more of scholastic delay (ED)	11.9	10.2
Rate of extra-age (EA)	30	24

Source: elaboration upon SITEAL data

Tabla 4 – Educational successes (ES)

	2000	2003
Net rate of primary schooling (TNEP)	94.7	96.5
Rate of illiteracy (TA)	1.5	1.4
% of population with 20 years or + with sec. Complete (SC)	18.1	19.5
% of population with 25 years or + with finished college studies (UC)	8	8.2

Source: elaboration on data from SITEAL

Based on these variables, we define the parameter of dispersion as an Educational Dispersion scale indicator (EDI):

$$EDI = \lambda = \frac{a(100 - TA) + b(ED) + c(EA)}{ES}$$

With a, b and c are weights verifying $a + b + c = 1$.

This equation indicates that the dispersion will be higher for a lower Educative Rate (TA), a higher percentage of educative delay, (ED) a higher percentage of extra-age (EA) and lower educational successes, ES. The values of to, b and c are 0.30, 0.35 and 0.35 respectively, giving the highest weights to the problems of delay and extra-age, which jointly constitute a 0.70 of the EDI.

The results are shown in Table 4.

Table 5 – Value of the dispersion parameter

	According to the educational successes:	
Λ	TA	TNEP
2000	0.221	0.229
2003	0.186	0.190

Source: own elaboration

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