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ORIGINAL ARTICLE

Pre-service teachers' experiences within modelling scenarios enriched by digital technologies

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Abstract
This paper presents issues related to the use of technology by secondary school pre-service mathematics teachers engaged in open mathematical modelling projects. Pre-service teachers developed these projects within a regular mathematics education course during the teacher education program at the university concerned. This environment, called a modelling scenario, is the context for our study, which is based on the analysis of projects produced by seven consecutive cohorts of pre-service teachers beginning in 2010. The research questions guiding our study seek to determine which technologies pre-service teachers chose, for which modelling purposes they chose them, and in which phases of the modelling process they were significantly used. During the modelling process, pre-service teachers used the Internet, spreadsheets, mathematical software and programming languages. The Internet, the most utilized technology, was used to find information or data, to select variables, or to formulate problems. The other three technologies significantly influenced the processes of mathematical solution and validation. The in-depth analysis of a single project shows an original use of visual affordances of technologies and a dialectical relationship between problem posing and technology use. The study enables us to conclude the necessity for coordinated action between mathematics educators and mathematicians to generate a deeper understanding of modelling as a pedagogical proposal and mathematical activity.

Keywords Digital technologies · Pre-service mathematics teachers · Mathematical modelling scenario

1 Introduction

The development of mathematical modelling[1] activities and the applications of mathematical models to solve real world problems in mathematics classes at different educational levels are trends in the mathematics education international context (Stillman et al. 2013, 2015; Kaiser 2014). At the same time, the use of digital technologies[2] accompanying modelling processes is becoming more evident and more widely reported in educational contexts (Confrey et al. 2010). Generally, based on the assumption that access to technologies expands opportunities for mathematical modelling in classrooms, research around this connection has increased.

In Argentina, and in line with the previously mentioned trends, the official curricular documents for secondary schools recommend the introduction of modelling for the teaching of mathematics in order to solve problems external as well as internal to mathematics, promoting the study of the limitations of a mathematical model to explain a phenomenon of interest. These documents also propose the development of activities for mathematics classes, including the use of "graphic software as a medium to enrich problem understanding since it empowers graphic representation, calculation speed and modelling" (Ministerio de Educación 2012, p. 22).

The inclusion of modelling and technologies in the local mathematics curriculum for secondary schools signals challenges for teacher education, in particular,

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[1] To avoid repetition, at times we will use "modelling" in place of "mathematical modelling".

[2] To avoid repetition, at times we will use "technologies" in place of "digital technologies".

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for pre-service teacher education. Though not yet implemented, new national standards for initial mathematics teacher education at the university level are under discussion. In such discussions, we can find statements promoting the necessity for introducing modelling activities and the use of technologies in regular mathematics courses during initial teacher education. These challenges for teacher education are also present at the international level, and many researchers, such as Blum (2015), Lingefjärd (2013), Doerr (2007) and Niss et al. (2007), express their concerns, proposals, or recommendations regarding the necessity for modelling activities, or modelling activities with technology, in pre-service teacher education. Based on the recommendations expressed in the national standards and in line with these authors' ideas, we agree with the necessity for providing future teachers with opportunities to experience a technology-enhanced modelling process during their prospective education.

Yet, the reality is that our university teacher education program is far from heeding such requirements. On one hand, mathematical courses usually provide little (or no) room for active modelling. On the other hand, although technologies are gaining ground in our teacher education program, they usually take a supplementary role, thus lessening their potential for enhancing mathematical thinking and learning.

In order to reverse this situation locally, in 2010 we decided to create a *mathematical modelling scenario* within a regular mathematics education course, which is part of the teacher education program at our university. It has continued since then, and today it is well established in the course. For our work, we use the notion of *modelling scenario* as defined by Esteley (2014):

> ...[it is] characterized by the presence of a set of spaces, situations, circumstances, materials, actions and interactions that give meaning to the process [of mathematical modelling] and with that, it transforms that set into an experience whose purpose is to bring modelling into the classroom as a pedagogical approach (p. 99).

In the modelling scenario that we created, pre-service teachers are invited to develop their own modelling projects, freely using digital technologies. Since there is no previewed mathematical content to be learned through the modelling projects, the focus is on modelling as a mathematical activity that deserves to be taught for its own sake. This approach is compatible with the perspective of *modelling-as-content* as characterised by Julie and Mudaly (2007).

This particular scenario, created with educational aims, has also been researched (Esteley 2014; Villarreal et al. 2010, 2015). In this paper, we concentrate on particular experiences of pre-service teachers working with modelling projects mediated by the use of technologies, and we set out to answer the following questions:

> Q1: What technologies do pre-service teachers select to use for their modelling projects, and for which modelling purposes?
> Q2: In which phases of the modelling process, does the use of technologies become significant?

2 Pedagogical and epistemological background

In order to contextualize our study, we describe here the pedagogical and epistemological background that supports our actions as teacher educators and researchers. Two main theoretical assumptions constitute this background: considering and understanding teacher education by taking into account the relationship between *experience* and *sense-making* (Larrosa 2003), and assuming that the production of mathematical knowledge in school is mediated by different types of media (Borba and Villarreal 2005). Both assumptions are closely related to the notion of *modelling scenario* previously presented.

The notion of modelling scenario (Esteley 2014) is linked to the concept of *context* in the sense of Lave (1988), who represents the context as conformed by two components, namely, a fixed arena and a setting or scenario, and conceives the scenario as "a *relation* between acting persons and the arenas in relation with which they act" (p. 150). The scenario refers to what is created by subjects who develop their activities in interaction with the arena and others. The activities and the experiences are "dialectically constituted in relation to the setting" (p. 151). The scenario generates the activities and these, in turn, generate the scenario. In this way, our descriptions of modelling activities reveal arenas, spaces, actions, subjects, technologies, and knowledge, and show the experiences in which future mathematics teachers made sense of what was experienced in a modelling scenario.

In a modelling scenario, the relationship between *experience* and *sense-making* acquires a special significance that opens a new educational perspective. Such a perspective, as it has been addressed by the Spanish educational philosopher Larrosa, intends to expand the educational visions that mainly prioritise the relationships between *science* and *technique* or *theory* and *practice* (Larrosa 2003). It can also be associated with ideas proposed by Britzman (2003), who points out that:

> ...academic separation between theory and practice is another manifestation of the fragmentation between knowledge and lived experience. However, this dualism can be transformed dialectically if we

raise questions about theory that include voices and experiences of practices (p. 64).

For Britzman (2003), "the experience is lived rather than chosen or acquired" (p. 13). Similarly, for Larrosa (2002), experience is "what happens to us," and not "what happens around us." Experience includes not only what is lived by the subject, but also the reflection upon it; such reflection leads to the production of meaning or sense for what was experienced (Larrosa 2002). The experience is unique for each subject; each experience involves interaction with "others", and the result of the experience implies a transformation for the subject related to it. Within the notion of modelling scenario, "others" can be humans, but also any type of media, including digital technologies.

Many theoretical perspectives exist regarding the integration of technology into mathematics classrooms, including mathematical modelling scenarios. We have adopted the perspective that assumes knowledge is produced by collectives of *humans-with-media* (Borba and Villarreal 2005). In this perspective, cognition is not considered as an individual enterprise, but rather as a social one. In addition, cognition includes tools, media by which knowledge is produced; these components are essential, not auxiliary or complementary. When discussing media, these authors mean any kind of tool, device, equipment, instrument, artefact, or material resulting from technological developments. For the purposes of this article, we are interested specifically in digital technologies, which include the Internet, any kind of software, and programming languages.

According to Borba and Villarreal (2005), media transform practices, content, and ways of knowing. Thus, different types of media can reorganise ways of thinking and producing knowledge. This reorganisation may imply changes or transformations in educational environments, for example, in the kind of problems that can be formulated, in the ways of solving them, in the ways of validating and communicating knowledge. Particularly, in mathematics classrooms, digital technologies are understood as actors that call for new pedagogical approaches. Meanwhile, if the actors involved in a classroom do not accept the invitation to experience new ways of knowing, teaching, or learning with technologies, the transformational impact on students' or teachers' actions will not necessarily occur.

In the next section, we first review some seminal work that features aspects of the relationship between modelling and technologies in educational contexts. Next, we review work about pre-service teachers and modelling, and finally we discuss papers addressing aspects of pre-service teachers working with technology in modelling scenarios.

3 Mathematical modelling, technology, and pre-service teachers: a review

3.1 Mathematical modelling and technology

The role of technology in modelling has been analysed by authors such as Greefrath (2011), Doerr et al. (2017), or Borba and Villarreal (2005), among many others. Greefrath (2011) recognises that technology may play a central role in the modelling process by broadening possibilities of solving certain mathematical models which would be inaccessible if digital tools were not available. He emphasises that technologies may influence each phase of a modelling cycle and not simply be an appendix to it. In line with the work of Greefrath, Doerr et al. (2017) point out that the role of computational media in modelling is an important aspect that is not well captured by the usual representations of the modelling cycle. They emphasise that computational media empower the mathematical processes involved in modelling activities. These authors recognise technologies as important actors for modelling and the existence of a synergic relationship between modelling and technologies. Borba and Villarreal (2005) have already explored this synergic relationship and assert that in their association, technology and modelling resonate naturally.

According to the authors previously reviewed, we maintain that modelling scenarios are fertile ground for integrating technologies into the teaching and learning of mathematics. As evidence we describe several studies in which the students or teachers take advantage (or not) of the technologies at their disposal in modelling scenarios. Brown (2015), for example, observed a modelling experience with 16-17-year-old Australian students in a technologically rich teaching and learning environment. She reports that some students did not take advantage of the affordances of the technologies for the purpose of visualizing different representations of the same model or comparing different models.

Other authors show that technologies make sophisticated models accessible for students. Such is the work presented by Soares (2015), who used the software Modellus with Brazilian biology students to discuss the role and effects of the parameters in a system of non-linear ordinary differential equations that model malaria transmission, even though the students were not familiar with the concept of derivative. Rodríguez and Quiroz (2015) studied the impact of technology in the promotion of specific modelling competencies among engineering students in Mexico. Diverse technologies were used to assemble electric circuits, collect data, and generate graphs. These uses promoted modelling competencies such as understanding and analysing the real problem, manipulating variables, identifying and structuring problems, analysing mathematical results in physical terms, and interpreting the model.

Borba et al. (2016) highlighted some possibilities offered by using digital media in the development of a modelling project in which 12–13-year-old Argentinean secondary school students freely selected a theme of their interest to investigate and formulated problems related to it. The authors reflected on the influence of the Internet on the kinds of problems formulated and solved, and on the role of the Internet as a data source.

Villarreal et al. (2010) explored aspects of the relationship between technology and students'/teachers' modelling activities in different educational contexts in Argentina. The paper provides evidence of modelling scenarios in which teachers and students attributed new meanings to the use of technologies: as media to think-with and solve modelling problems.

3.2 Mathematical modelling (with technology) and pre-service teachers

As previously stated, the development of modelling activities in mathematics classrooms implies challenges for teacher education. These challenges are not new and many researchers have expressed their recommendations for teacher educators. Doerr (2007) asserts that, "Pre-service teachers need to encounter modelling experiences that provide for a range of contexts and tools and that engage them in meta-level analyses of their modelling activity" (p. 77). Niss et al. (2007) emphasise that if we want applications and modelling to be efficiently, successfully, and reflectively included in the teaching agenda of mathematics teachers, pre-service teachers need opportunities to develop that capacity. More recently, Blum (2015) refers to the necessity of providing future teachers with the professional knowledge needed to carry out modelling activities and to develop teaching experiences with modelling. Gastón and Lawrence (2015) state that teacher training for pre-service teachers should include knowledge about what mathematical modelling is, how it can be incorporated in teachers' lessons, and how to assess modelling activities. They assert that it is desirable that future teachers gain meaningful experience with modelling and applications of mathematics, either through modelling activities in mathematics courses or through specific mathematical modelling courses.

All these reviewed authors state the need for pre-service teachers to experience mathematical modelling themselves in order to develop knowledge about modelling and its role in teaching and learning. Different ways of accounting for such a need have emerged around the world. A variety of mathematical modelling courses, or modules inserted in mathematics education courses, have been developed. All this movement has motivated researchers to study pre-service teachers' modelling activities.

Anhalt and Cortez (2016) worked with pre-service teachers attending a modelling module within a mathematics pedagogy course in their teacher preparation program in the USA. The study reveals that during the module, pre-service teachers broadened and deepened their conceptual understanding of modelling. Villarreal et al. (2015) characterised pre-service mathematics teachers' experiences while developing free modelling projects in the context of a mathematics education course in Argentina. The study revealed that the free selection of non-mathematical themes while starting a modelling project was an obstacle for the pre-service teachers. However, it became clear that the creation of modelling scenarios in a mathematics education course supported reflections related to the role as teachers in such scenarios. Widjaja (2013) developed an exploratory case study with a small group of Indonesian pre-service teachers to build awareness of mathematical modelling. Modelling was not part of their teacher education program. They worked with a single open-ended assigned modelling task related to their daily lives, and then obtained and analysed several models. The study identified pre-service teachers' difficulties in recognizing factors that caused limitations to their models and limitations in validating them.

Considering pre-service teachers working with modelling and technology, Lingefjärd (2007) claims that the "use of modelling and technology in instruction with the purpose of enriching students' mathematical learning is valued by future teachers if they are convinced of their impact on their own learning of the content" (p. 477). According to this author, future teachers' experiences with modelling during their mathematics learning contributed to the construction of an image of modelling, thus enhancing mathematics teaching and learning. In a different publication, Lingefjärd (2013) analysed a set of mathematical modelling tasks for prospective teachers in which the use of technology was central. He reminded readers that modelling is "one part of the mathematical curriculum that may be broadened and enhanced through the use of technology" (p. 59). Daher and Shahbari (2015) investigated, from a cognitive perspective, the phases of modelling processes that 30 middle school pre-service mathematics teachers went through when using spreadsheets in a model-eliciting activity. The researchers reported that technology assisted pre-service teachers' work in flexible and creative ways, and concluded that proper utilization of technology provokes meaningful mathematical learning for learners engaged in modelling real-world situations.

In this review we presented a sample of research studies referring to the importance of modelling experiences for pre-service teacher education. Some also include the use of technologies as a synergic partner accompanying the modelling process. Several studies analyse actions of pre-service

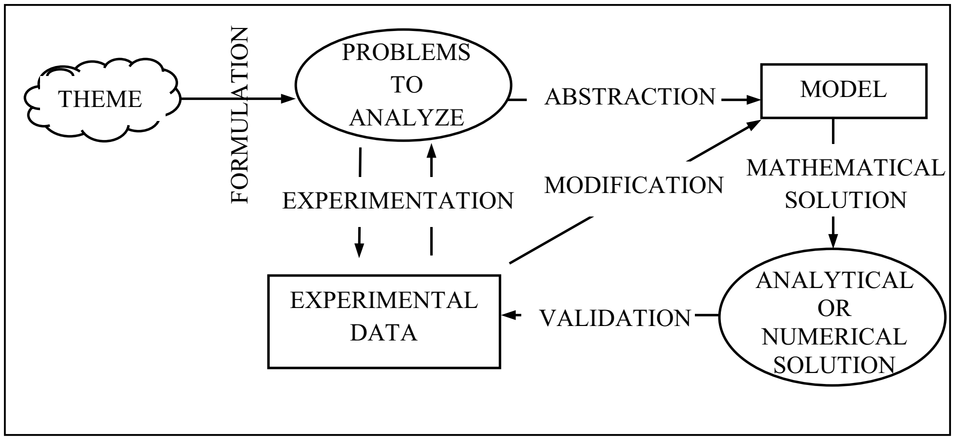

Fig. 1 The modelling cycle (adapted with permission from Bassanezi 2002, p. 27)

teachers as modellers in different modelling scenarios, generally in mathematics education courses. Studies into how the use of technologies impacts the learning of mathematical modelling among pre-service teachers are less prevalent. Thus, in this sense, our work contributes to this knowledge base, considering experiences of pre-service mathematics teachers working in open modelling scenarios.

The next section presents the research design, including details of our modelling scenario as a research setting, the collected data, and the analytical procedures.

4 Research design

4.1 Our modelling scenario and research setting

Our research was conducted with pre-service teachers enrolled in the mathematics teacher education program of the Faculty of Mathematics, Astronomy, Physics and Computation at the University of Córdoba (Argentina). The program lasts 4 years, with mathematics courses taught by mathematicians comprising 66% of the curriculum, and 34% of the remaining courses dealing with educational issues taught by pedagogues or mathematics educators.

Pre-service teachers studying at our university go through their teacher education program experiencing few modelling activities. This was our arena, in the sense of Lave (1988), where we intervened in 2010 in order to create a new scenario, a modelling scenario, within a regular mathematics education course. Each year since 2010, at least one of the authors of this paper has acted as teacher in this course. This course is part of the third year of the teacher education program; it runs for 30 weeks with two 4-h classes per week. In this course, several trends in mathematics education are studied, among them, mathematical modelling. The notions of model, mathematical model, and the mathematical modelling process are discussed. The six phases of the modelling process (formulation, experimentation, abstraction, mathematical solution, validation, and modification) are carefully described and discussed as well; a diagram of the modelling cycle is shown in Fig. 1.

Students discuss modelling activities produced in different educational contexts and also solve several modelling problems. Finally, the pre-service teachers are invited to develop their own modelling projects. For this, the pre-service teachers form small groups and select a real-world theme of their interest, formulate problems related to this theme, select variables, raise hypotheses, design experiments (if necessary), search for information, collect and process data, solve the problem, and work on a validation phase (see Fig. 1). Some of these activities need to be carried out outside of class. The teachers act as guides to help formulate or reformulate problems, recommend possible sources of data, and suggest new questions in order to engage students in more complex modelling processes. At the end of the process, each group writes a report and makes an oral presentation for the entire class, accompanied by a slide presentation using presentation software of choice. During these presentations, the class asks questions and makes comments about the projects. In many cases, discussions arise regarding modelling as part of the students' future task at school, or reflections about the role of technology for the modelling process.

The modelling activities described above require approximately 6 weeks. The created mathematical modelling scenario offers students[3] the opportunity to experience a complete open modelling process, following the phases of the modelling cycle shown in Fig. 1. These experiences are the source that allowed us to conduct a seven-year study, from 2010 to 2016.

[3] From now on, if nothing else is specified, "students" will refer to "pre-service teachers".

4.2 The data

From 2010 to 2016, 108 students produced 32 projects. After the realization of the modelling projects, the available data were the groups' written reports, files produced with software when creating the models (spreadsheets, GeoGebra, Python, etc.), slide presentations for students' oral presentations, and our field notes. With the exception of 2010 and 2012, the oral presentations were videotaped; thus we have 25 of the 32 oral presentations on video.

4.3 Data analysis

In order to answer our research questions, we divided our data analysis into two stages. In the first stage, we read the written report of each project and we watched the corresponding videotape, if available. We also read our field notes in which we had registered details about each group's modelling process. Once we finished this first reading of the data, we compiled, for each project, a list of the types of technologies that had been used and the purposes for their use during the modelling process. *Types of technologies* and *modelling purposes of use* were our dimensions of analysis at this stage of the study. For the two most frequently used technologies (Internet and spreadsheets), we developed a refined classification of the purposes of use and their relation to the phases of the modelling cycle presented in Fig. 1. During this first stage, our purpose was to answer our first research question.

As a result of this first analysis, we realised there was an appreciable difference regarding the role of technologies in the projects produced in 2016, compared to those carried out previously. For this reason, and as a second stage of our analysis, we concentrated on the relationships between uses of technologies and phases of the modelling cycle we observed in those modelling projects.

During this stage, in which our purpose was to answer our second research question, the analytical procedure follows an emergent design. First, we detected, the use (or not) of technologies in the six phases of the modelling cycle for each of the 2016 projects. Then, we analysed the projects more deeply in order to determine significant uses of technologies. Due to space limitations, in this paper we report on only one project in depth.

5 Main results and analysis

In this section we answer the research questions:

> Q1: What technologies do pre-service teachers select to use for their modelling projects, and for which modelling purposes?
>
> Q2: In which phases of the modelling process does the use of technologies become significant?

In order to organise the results, we consider two subsections corresponding to Q1 and Q2, respectively. We display the data in tables according to the selected dimensions of analysis and we analyse them, recognising patterns in the information presented in the tables. Finally, we offer empirical evidence that supports our analysis.

5.1 Uses of technologies and their purposes in pre-service teachers' modelling projects

To answer Q1, we consider two dimensions of analysis: *types of technologies* students used in their modelling projects and *modelling purposes of use*. After a first analysis of the 32 projects, we observed that in 30 of them, some type of technology had been used for modelling purposes. We distinguished four types of technology: *Internet, spreadsheet, mathematical software*, and *programming languages*. In the cases involving Internet and spreadsheet, we created categories of purposes of use related to the phases of the modelling cycle that were enriched by such use. These categories and their relation to the modelling phases are presented in Table 1. Mathematical software was used only in two projects, while programming languages were used to program codes that gave an account of phases of the modelling process in the particular context of their problems, so we did not establish categories for either of the two.

Even though the Internet was mainly used as a vast source of information, we distinguished five categories of purposes (coded as I1–I5 in Table 1) associated with different phases of the modelling cycle; for the spreadsheet, we distinguished three categories of purposes (coded as S1–S3 in Table 1), all of them associated with the mathematical solution and validation phases. The purposes of use of the mathematical software were different in the two projects which resorted to using such software, though in both cases they were associated with the mathematical solution phase. These purposes of use, as well as those of programming languages, are specified in Table 2. This table shows the distribution of the modelling projects per year, the themes of the 32 projects and the number of students participating in each, the identification (ID) code assigned to each project, the technology used in each project, and the students' purposes for these technologies. These purposes use the codes from Table 1.

The Internet was utilised in 75% of the projects, and in 67% of these, it was used for finding information or data related to the student-selected theme (I1). For example in project P6 about trash and recycling, students used the Internet as a source of data. They obtained the number of inhabitants in Córdoba city from the 2010 census, the amount of trash produced per day there, the percentage of

Table 1 Categories of purposes of use for Internet and spreadsheet and phases of the modelling cycle

Types of technologies	Modelling purposes of use	Phases of the modelling cycle
Internet	I1 Search for data or information to start the construction of the model (numerical data as well as information that helps to understand the phenomenon in study and formulate problems related to it)	Formulation Experimentation
	I2 Select variables	Abstraction
	I3 Formulate or reformulate the problem	Formulation Modification
	I4 Generate data using online applications	Experimentation
	I5 Validate the model	Validation
Spreadsheet	S1 Display data using tables or different types of graphical representation to communicate results from the modelling process	Mathematical solution
	S2 Make simple calculations using the automatic affordances of the spreadsheet	Mathematical solution
	S3 Program custom functions using the native functions from Excel for making calculations in a more efficient way, running simulations or applying the created model	Mathematical solution Validation

the population that sorts the trash, etc. In some projects (P1, P20, P22), the Internet information helped to select variables (I2). For instance, in P20 the students looked for information about anorexia nervosa and selected some variables used to diagnose it (weight, body mass index, etc.). These variables were then used to construct the model. The Internet also played an important role in projects P14, P23, P27, and P32 by helping students formulate or reformulate their problems (I3). In project P27, which we describe in detail in the next subsection, the students were concerned about the need for public secondary schools in the city of Córdoba. Initially, they had asked themselves: How many schools will be needed to reach the entire population of Córdoba city, and how long will it take to reach that number? From the data found on the Internet, the students became aware of the magnitude of their questions. Some hypotheses, restrictions, and limits were established, and the initial questions went through a process of transformation and reformulation until reaching this final form: How many schools are necessary to guarantee access to the public secondary education in the southeast region of the city of Córdoba? Internet use for generating data (I4) was detected in two projects (P24, P27). In P24 regarding teacher salary negotiation, students accessed a local teacher union website and found a simulator to calculate teacher salaries. This enabled them to generate data necessary for the modelling project. Figure 2 shows the salary simulator displaying the calculation of a primary school teacher's monthly salary at the beginning of her career. In two projects (P18, P24) students turned to the Internet to validate the model (I5). In P18, students were concerned about the alcohol consumption by young people between 12 and 18 years, as well as its possible causes, and the age of the onset of drinking. To answer their concerns, they designed a survey, and then they analysed the results statistically. For validation purposes, they compared the results produced by the model with data from the local ministry of health obtained on the Internet.

The students that developed project P31 about expenses for a tourist trip, wrote an interesting reflection in their final report about the use of the Internet, which mirrors what we were observing as university teachers: "Access to the Internet was necessary for developing the process. We can say that connectivity may transform educational practices". Students thus recognise the importance of the Internet for their modelling process, as well as its transformational power in education.

The second most utilised technology was the spreadsheet (56% of the projects). It was used mainly for making pie or bar charts, and tables with the purpose of presenting and systematizing data (S1). This classic purpose of use was shown in 61% of the projects using spreadsheets. Another classic purpose for using spreadsheets was to make simple calculations using their automatic affordances (S2). For example, to complete a table or calculate the successive terms of a sequence in a recursive way, the students used the auto-fill feature to fill cells with data that followed a pattern. This was the case in project P32, in which relationships between paper consumption, recycling, and deforestation were studied. The students posed several questions: How much paper do we get after it is recycled a certain number of times? How many trees will be saved from logging by the recycling of paper? Knowing that paper can be recycled six times at most, that 90% of the paper can be recycled, and that 17 trees are necessary to produce 1 ton of paper, the students fixed an initial amount of 10,000 tons, and used a spreadsheet to answer their questions. Finally, the programming affordances of spreadsheets, through use of their native functions, enabled the programming of new custom functions for making calculations, running simulations, or applying a created model (S3). Projects P29 and P31 are examples of this use. In P31, in order to calculate the costs of a tourist trip, students constructed a program using the logical function *If* in nested cycles. This program made it possible to simulate the final cost of a tourist package. In their written report, they

Table 2 Technologies and purposes of use

Modelling projects and number of students	ID code	Types of technologies	Modelling purpose of use
2010			
Household water consumption (5)	P1	Internet Spreadsheet	I2 S1
Human genes-Interactions (3)	P2	Internet Spreadsheet	I1 S1
Travel costs for end-of-year school trip (3)	P3	Spreadsheet	S1–S2
2011			
Rainwater harvesting for human consumption in a dry zone of Córdoba (4)	P4	GeoGebra	Display graphs relating the area of a rainwater collector roof to the average precipitation in the zone
Household electric energy consumption (4)	P5	Internet Spreadsheet	I1 S1
Trash and recycling (2)	P6	Internet	I1
Lottery games (4)	P7	Internet Spreadsheet	I1 S3
2012			
Waiting time at the university dining hall (4)	P8	Spreadsheet	S1
Bottled gas supply in a countryside area (4)	P9	No use of technology	
Soy consumption (4)	P10	Internet	I1
Investment recovery for a photocopy shop (4)	P11	Internet C++	I1 Program a function to calculate the time needed to amortize the investment
2013			
Teacher retirement (4)	P12	Spreadsheet	S3
School rucksack overweight (4)	P13	Internet	I1
Effect of the HPV vaccine (4)	P14	Internet	I3
Provision of food and drinks for a social event (4)	P15	Internet	I1
2014			
Recycling of trash (4)	P16	Internet Spreadsheet GeoGebra	I1 S1 Solve graphically a linear programming problem to minimise waste generation
Time spent waiting for the bus (4)	P17	No use of technology	
Teenagers' alcohol consumption in Córdoba (3)	P18	Internet Spreadsheet	I5 S1
Contraceptive methods and teen pregnancy (4)	P19	Internet	I1
Anorexia nervosa (4)	P20	Internet Matlab	I2 Program the created model for updating daily metabolism data of a person who suffers from nervous anorexia, to predict the time needed to achieve the desired weight
2015			
Backpacker tourism by Latin America (3)	P21	Internet Spreadsheet	I1 S3
Suicide (2)	P22	Internet	I2
Deforestation and reforestation in Córdoba mountains (3)	P23	Internet IPython	I3 Calculate recursively the quantity of trees of a certain species that germinated in a period of time
Teacher's salary negotiation (4)	P24	Internet Spreadsheet	I4–I5 S1
Probability of Argentina winning the America Cup (2)	P25	Internet Spreadsheet	I1 S1–S2

Table 2 (continued)

Modelling projects and number of students	ID code	Types of technologies	Modelling purpose of use
2016			
Expenditures on cell phone use versus expenditures on staple foods (3)	P26	Internet Spreadsheet Python	I1 S1–S2 Program a multi-objective method to decide which cell phone company offers the most convenient service, considering costs and quality
Need for secondary public schools in the southeast region of the city of Córdoba (3)	P27	Internet Spreadsheet	I1–I3–I4 S3
Optimising family route to school and work (2)	P28	Octave	Program a function to optimise, in terms of monetary expenses, the itinerary of a family that moves to school and to work by car and/or bus
Probability of completing school homework requiring the use of magazines. For example "find five words containing *br*"(4)	P29	Spreadsheet Python	S3 Program the counting of combinations of two prefixed letters contained in a given text
Buying a new car (1)	P30	Internet Spreadsheet	I1 S2
Calculation of expenses for a tourist trip (2)	P31	Internet Spreadsheet	I1 S3
Relationship between paper consumption, recycling and deforestation (3)	P32	Internet Spreadsheet	I1–I3 S2

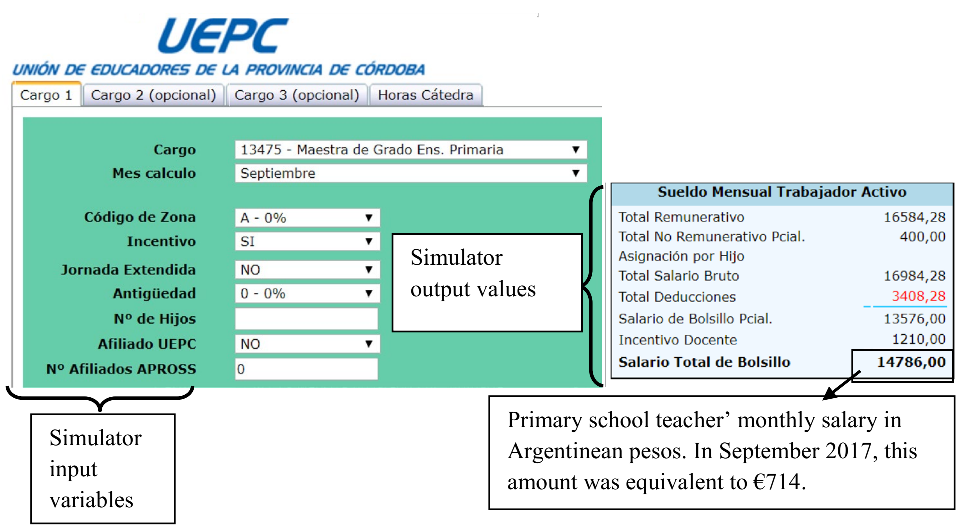

Fig. 2 Result of a simulation for a primary school teacher's monthly salary

said: "The use of the logical operator *If* determined that we had to invest time to learn its operation, which we did not know, but at the same time [such a process] allowed us to understand how a model can be made more complex by the use of digital technologies". In this case, the students had to study the use of the function in order to produce a model.

In addition to the use of the programming functions of spreadsheets, other software or programming languages were also used to program: C++ in P11, Matlab in P20, IPython in P23, Python in P26 and P29, and Octave in P28. In these projects, at least one of the students in the group had had previous knowledge of programming languages. In projects P11, P20, P28 and P29, the students dared to make their original models more complex, proposing new problems or increasing the number of variables involved, because they knew that they could develop a program that would account for large routine calculations or for handling a vast amount of data. In these cases, the use of programming languages was essential, while in P23 and P26 their use was dispensable. The specific modelling purposes these programming languages were used for are detailed in Table 2. The generic modelling purposes were to experiment (P29), to find the mathematical solution (P26, P28), and to validate (P11, P20, P23).

Table 3 Phases of the modelling cycle in which technologies were used

Projects Phases	P26	P27	P28	P29	P30	P31	P32
Formulation		X	X	X	X		X
Experimentation	X	X		X		X	X
Abstraction		X	X	X	X		X
Mathematical solution	X	X	X	X	X	X	X
Validation					X	X	X
Modification				X	X	X	

In summary, we found that while developing their modelling projects, students appealed to different types of technology with diverse purposes. The Internet and the spreadsheet were the most employed. The Internet made it possible to search for information mainly used to formulate or reformulate the problem, build the model, and select variables. The spreadsheet was basically a means to communicate results through tables or graphs. However, it was also used to perform calculations, either through native functions or by programming custom functions. When the students had a background in programming languages, they dared to pose questions whose response may have required extensive calculations. The use both of the spreadsheet and the programming of languages occurred in the mathematical solution and validation phases.

In the next section, we address the second research question with respect to the phases of the modelling process where technology use is of significance.

5.2 Relationships between technologies and modelling

In this section we deepen our analysis of relationships between the use of technologies and the phases of the modelling cycle. Table 2 shows that in 2015 and 2016, the use of technologies increased in diversity of uses and purposes. This could be related to the fact that the teachers in charge of the mathematics education course were convinced that using technologies might enhance the modelling process, and consequently, they explicitly encouraged it. Because of this, and in order to answer Q2, we decided to concentrate our analysis on the seven projects produced in 2016 in particular. Here we first conduct a general analysis considering the seven projects, and then we concentrate our analysis more deeply on project P27.

Table 3 shows in which phases of the modelling cycle we recognised the use of technologies for each project. One can notice that the use of technologies was more frequent in the first four phases, especially in the mathematical solution phase.

On deeper analysis of each project, we observed that although students used technologies in two or more phases of the modelling cycle in all 2016 projects, such uses were significant in five of the seven projects (P27, P28, P29, P31, P32). We consider that the use of technologies is significant when it was essential to developing the following aspects of the modelling process: influencing the problems posed, the search of information, the decisions made, or the creation of the model. In summary, technologies are significant for modelling when it would have been impossible without them to develop the project in the way it was done.

Most of the projects in which a significant use of technology was made, present dense mathematical, phenomenological, or technological work; thus it would be difficult to grasp their essence in a few lines without losing the richness of the work. For this reason, and due to space limitations, we describe just one particular modelling project in detail: P27. This project shows an original and unconventional use of digital technologies within the created mathematical modelling scenario, which exploits the power of visualisation. Additionally, it demonstrates the dialectical relationship between problem posing and uses of technologies, as well as the socio-critical modelling perspective.

Project P27 shows a significant use of technologies in the first four phases of the modelling cycle. The students were concerned with the equitable distribution of secondary schools in Córdoba city and the need for new public schools in this city. Based on these concerns, they investigated such distribution and necessity. To this end, they began searching for data on the Internet and found statistical data from the official 2010 local census. They looked for information about the location of the secondary schools within the city and read online news referencing the necessity of new schools to cover the demands in some social and economically disadvantaged sectors of the city. Taking into account the information gathered from the Internet, the interactions with their teachers, and the magnitude of their initial concerns, the students finally formulated the following question: How many schools are necessary to guarantee access to public secondary education in the southeast region of the city of Córdoba?

The students decided to focus their attention on this region, which is a densely populated district of the city, and thereafter, they were able to visualise an unequal distribution

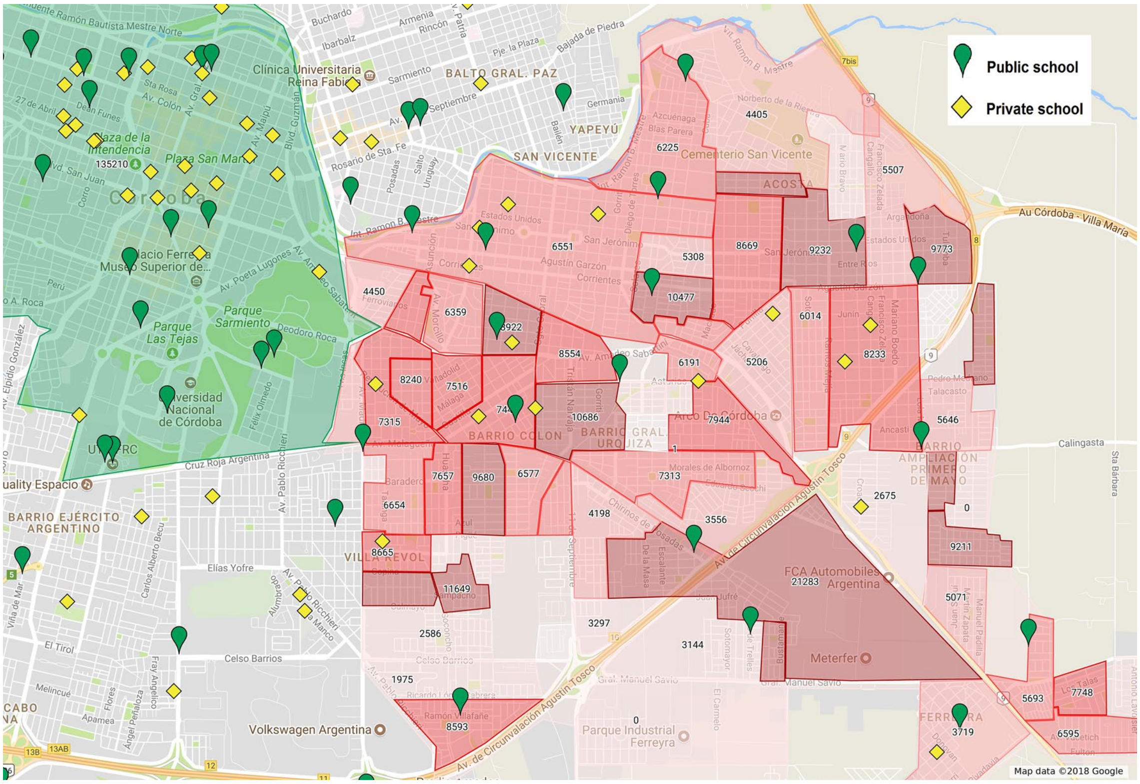

Fig. 3 Distribution of public and private schools in Córdoba

of the schools on a map they created. A Google application called My Maps enabled the students to create a map on which they located all the public and private schools in Córdoba. To do this, they first created a database containing the addresses of all the schools there; second, they used a spreadsheet to construct and organise those data in a table, and finally, they loaded that database into My Maps. For studying the distribution of secondary schools in the southeast region, they demarcated it using the Polygon tool offered by My Maps. Completing this task allowed the students to reflect on the unequal distribution of schools in the city: "Seeing all the schools at the same time and demarcating the region enables us to witness the unequal situation in which the inhabitants of this area are".

To advance the construction of the model, the students delimited the neighbourhoods of the southeast region using the Polygon tool again and added the number of inhabitants in each neighbourhood to the map. In their report, the students wrote: "All this work was empowering and made sense when we annexed, on the map, numerical data coming from a large table of data. For this, we used again digital software". Figure 3 shows a partial map of Córdoba city in which the students located all the schools and inserted the number of inhabitants in each neighbourhood of the southeast region. The southeast and central regions were coloured to highlight the difference in density of schools.

The students recognised the importance of their map, since it enabled them to visualise the complexity of the problem, and it was also a medium to denounce the inequality of the distribution of schools in the city. The students took advantage of the visual affordances of the application My Maps as a strategy to communicate their results in an impactful way.

The next step in the students' project was to answer the question about the number of public schools needed in the southeast region. To this end, they searched new data regarding the number of secondary school pupils in the region; they made some assumptions about the ideal number of pupils per classroom and the distribution of the school population in the region. The use of technologies in this step was not significant.

In their written report, as well as in the oral presentation of their work, the students recognised the central role of technologies in the phases of formulation, abstraction, and mathematical solution. They specifically mentioned the selection of variables, the search and treatment of data, and the construction of the map as a model that shows the distribution of schools in Córdoba.

This work brings into evidence the synergic relationship between modelling and technologies. It also shows a collective of *students-with-technologies* in action. Without the technologies, the realization of the project would not have been possible.

6 Discussion

Looking back on our study through the lenses of our pedagogical and epistemological background as well as the literature review, allows us to deepen the analysis and open discussions on some of the results presented.

Authors such as Doerr (2007), Niss et al. (2007), Blum (2015) and Gastón and Lawrence (2015) refer to the necessity of providing future teachers with opportunities to experience the modelling process during their prospective education in order to place modelling on the agenda of their future teaching activities. In line with these authors, Lingefjärd (2007, 2013) asserts that technology may broaden and enhance pre-service teachers' experiences with modelling processes. Considering these perspectives, and in our role of teacher educators, we offered future teachers the opportunity to experience the modelling processes within a regular course on mathematics education.

We have described our educational context, as conformed by an arena and a (modelling) scenario, and we have analysed the students' modelling processes mediated by technologies. In our literature review, although we found some studies about pre-service teachers using technology in modelling (Daher and Shahbari 2015; Lingefjärd 2007, 2013), we observed a scarcity of literature in this area. We recognise that the openness of our modelling scenario is a feature that differentiates our work from that of Widjaja (2013), Daher and Shahbari (2015) and Anhalt and Cortez (2016). In this sense, our study contributes to understanding of the impact of technology on students' open modelling activities.

Our results bring evidence of the actions, learning, decisions, and reflections produced by students during their modelling projects, as well as the purposes of using technologies in such work. Some students had to learn to use specific programming functions in spreadsheets or to look for an Internet application that met their needs for constructing their model. Especially in 2016, students used technologies in a significant way, encouraged in many cases by their teachers. The authors of project P31 recognised this encouragement, noting:

> Motivated by the suggestion of the teachers, we accepted the challenge to redefine it [the model]... Thus, the formulation of the problem and also the mathematical model were modified, evidencing the cyclical aspect of any modelling process.

The interaction between students and teachers, and the consequent student decision of learning to use programming functions to improve their models are actions of a collective of *humans-with-media* (Borba and Villarreal 2005), a particular collective of teachers, students, and technologies used for producing a mathematical model. In most projects, the access to technological tools transformed the students' mathematical production and revealed possibilities of discovering significant ways of using technology for producing mathematical models. Evidence of this transformation is embodied in the words that two students wrote in their final report when they reflected on the use of Excel:

> Technologies shaped our work; other skills were put into play. Technology allowed us to generate other knowledge; we did not think less using it; instead we thought differently; it acted as a reorganiser of our thinking.

We could also see students taking advantage of visual affordances of digital technologies. Brown (2015) highlights the potential for technology to have a transformational impact on students' modelling activities. Although in her work with secondary school students she reported that such potential was not necessarily being realised, the case of the students carrying out project P27 was different. These students used the strategy of identifying neighbourhoods of the southeast region of Córdoba city with distinct colours to show differences in their population density and an unequal distribution of schools in that region as compared to the central area. They were able to capitalise on visualisation for representing data and communicating an unfair situation.

Our results give an idea of how the technologies the students used supported their modelling projects. The openness of our pedagogical proposal enables us to examine technology use in students' projects, in choosing a theme of interest to model and in formulating their own questions, two aspects of a modelling process that have been rarely studied. We offer evidence of the influence of technologies, with varied levels of significance in the modelling process. For example, the Internet has been a main actor, especially in the phases of formulation and experimentation. As in the study with secondary students of Borba et al. (2016), our work contributes to the importance of the Internet for modelling processes, though in our case, with university students. Spreadsheets, mathematical software or programming languages significantly influence the phases of mathematical solution and validation. We offered evidence of technology uses in all the phases of the modelling process as articulated by Bassanezi (2002), recognising the synergy between modelling and technology, in line with other literature (see, for example, Greefrath 2011; Doerr et al. 2017).

Media reorganised the ways of thinking, the production of models, and the activities generated in our educational context. In a similar way, it is also worth noting that the kinds of activities proposed for the modelling scenario favoured the incorporation of technologies, and most of the students accepted the challenge of experiencing a modelling process mediated by technologies. Such reorganizations made evident a dialectical relationship (Lave 1988) between the scenario and the activities mediated by technologies.

Some of the modelling projects carried out by students, and their reflections, expressed during oral presentations or in a written final report, are evidence of transformational experiences in the sense of Larrosa (2002) and Britzman (2003). In particular, some students made sense of the experience by envisioning possible implications for their future as teachers:

> We consider it is very important to have experienced the modelling process, since our personal experiences will influence the way we will be as teachers in the future. By experiencing it, we feel and live the process as our future students would, and this fact convinced us that mathematical modelling can be implemented as a pedagogical strategy.

It seems that the experience lived by the pre-service teachers permitted them to envision that modelling could be a possible pedagogical proposal in their future as teachers. In fact, in the following year, some of the students who had participated in our 2016 study produced modelling activities involving technologies and applied them during their teaching practices in secondary schools. This fact inspires us to a future study on how the previous immersion of students in modelling scenarios with technologies might impact their later teaching practices.

Meanwhile, it is also important to recognise that not all students in our 7-year study lived a transformational experience in the modelling scenario. In these cases, the students did not engage in their modelling projects or did not use technology in a significant way. Different reasons may explain such situations: lack of time, difficulty selecting a theme of interest and posing problems related to it, difficulty working collaboratively, among others. We suspect that many students were not convinced of the relevance of modelling and the importance of technologies for mathematics teaching and learning, or maybe we (as teachers) were not capable of making the essence of our open proposal evident. As we observed in a previous study carried out in the same context (Villarreal et al. 2015), the strong presence of pure mathematics with no applications to real world problems, and the scarce use of technologies in the teacher education program might act as barriers for our pedagogical proposal.

Despite the difficulties reported above, we argue that the implementation of modelling activities should be imperative during pre-service teacher education. In this sense, we agree with the opinion of many of the reviewed authors that there is still little room for modelling activities in teacher education programs.

7 Conclusions

The creation, and the implementation, of a *modelling scenario* in a mathematics education course for future teachers have revealed two aspects that deserve attention. On the one hand, the fact that future teachers learn about modelling and live the experience of a complete modelling process allows the teachers in charge of that course to focus on modelling as a teaching object. On the other hand, a deep exploration of the mathematical content that emerges in modelling projects has been limited due to some contextual as well as time constraints. That is to say, important advances were made in the *modelling-as-content* perspective but the *modelling-as-vehicle* perspective (Julie and Mudaly 2007) was not explored deeply enough. Related to this, it is important to note that, due to the characteristics of our *modelling scenario*, some requirements are needed. For example, for the modelling projects to progress significantly, it is essential that the teacher who guides them has not only mathematical knowledge, but also knowledge of other fields, including knowledge of the use of technologies. This fact raises the need for interdisciplinary work with specialists from other fields of knowledge. Specifically, in order to make advances in the *modelling-as-vehicle* perspective, coordinated actions between mathematics educators and mathematicians in charge of mathematics courses are vital. Such coordination could be achieved in different ways. On one hand, when students are engaged in modelling projects within the mathematics education course, mathematicians could participate by making contributions related to the mathematical ideas underlying the modelling projects. On the other hand, when students attend mathematics courses, they should have the opportunity to experience modelling activities enriched by digital technologies, and mathematics educators could support them in order for them to understand modelling from a pedagogical perspective as well. Perhaps in those interdisciplinary and dialogical scenarios, the students will be able to explore, apply, and create models with mathematical purposes as well as with didactical aims. The creation of such scenarios is a challenge not only for mathematics educators, but also for mathematicians. In this sense, it could be important to work in this direction, encouraging interactions between mathematics educators and mathematicians, at least at our university. Nevertheless, it is worth noting that these issues are not necessarily unique to pre-service teacher education in our university, for they occur elsewhere as well. Our attempt at addressing them, implementing alternative proposals, and researching the ways pre-service teachers respond, could be of interest to other mathematics teacher educators.

Regarding future research focused on experiences lived in modelling scenarios of an interdisciplinary nature and enriched with technologies, we could ask: What impact may such experiences have on future teachers' teaching practices? What new sense could they make of the modelling as a pedagogical approach? What role does technology play for advancing the mathematics embodied in a modelling project? These questions deserve our attention.

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