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Structural Approach To Design Sensor Networks for Fault Diagnosis

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Supporting Information

ABSTRACT: Key faults significantly affect the normal operation of the process originating risk conditions. These failures should be identified even in the presence of missing measurements or outliers. In this work a new strategy to design sensor networks, which are able to resolve a set of key faults when sensors fail, is presented. The procedure deals with failure isolation using the Fault Resolution Degree concept. This is incorporated as a constraint of the minimum-cost design formulation, and the resulting optimization problem is solved using MILP codes. The strategy only uses low uncertainty data that are readily available at the process design stage. Application results of the methodology to case studies extracted from the literature are presented and compared with those provided by other existing techniques.

1. INTRODUCTION

The selection of the set of process variables to be measured, which is optimal with respect to some specified criteria and simultaneously satisfies certain information requirements of the system under analysis, is called the sensor network design problem (SNDP). Even though a great number of works in the literature devoted to the design of SNs have focused on monitoring the normal process operation, the optimal location of instruments to effectively diagnose plant faults has central importance for safety, environmental protection, and process economy.

For the sake of brevity, only the most relevant works related to the design of SNs for fault diagnosis purposes are briefly reviewed in this section, given that this work is devoted to presenting a contribution on that particular SNDP. Comprehensive reviews about the design of SNs for monitoring purposes can be found elsewhere (Bagajewicz[1] and Nguyen and Bagajewicz[2]).

The first work about the location of sensors for fault diagnosis was presented by Lambert[3] who used the probabilistic importance of events in causing a hazardous condition to optimally locate instruments in a chemical plant. Later on, Chuei-Tin et al.[4] proposed a monitoring system based on a parallel parameter estimation method designed to reduce the chance of bias. Fault observability (O) and resolution (R) criteria were selected to evaluate the performance of alternative designs.

Raghuraj et al.[5] developed a design strategy based on the use of directed graphs to find a set of instruments that ensures the O and the highest R of all failures under the assumption of the occurrence of one fault at a time. The methodology was extended to tackle sensor location problems in which some faults had high probability of occurring simultaneously. Later Bhushan and Rengaswamy[6] used signed directed graphs to represent the failure cause−effect model. Minimum-number SNs were designed subject to fault O and single/multiple resolution (S/MR) constraints.

A different approach was introduced by Bagajewicz et al.[7] to solve the SNDP from a fault diagnostic perspective. A MILP formulation was analyzed to obtain minimum-cost SNs subject to fault O and S/MR requirements, which were stated using matrix algebra concepts. The solution of the optimization problem was tackled using the procedure CPLEX of the GAMS program.

Later on Bhushan and Rengaswamy[8] proposed a design formulation based on quantitative information about the occurrence of faults and sensor failure probabilities. A comprehensive analysis of the maximum-reliability and minimum-cost SNDPs was later presented by the same authors (Bhushan and Rengaswamy[9]). A lexicographic optimization procedure that combined both objectives and heuristics to solve large scale problems were also examined.

The robustness of the network to uncertainties/errors in the underlying failure cause−effect model and probability data was considered next by Bhushan et al.[10] More distributed SNs were preferred to incorporate robustness to modeling errors. Regarding the treatment of inaccurate data, the main idea was to ensure that the constraints involving uncertain data were far from being active at the optimal solution. This robust design formulation was solved by Kotecha et al.[11] using Constraint Programming, and recently it was extended to consider robust upgrade and reallocation problems (Kolluri and Bhushan[12]).

Other contributions analyzed the design of SNs devoted to detecting and identifying a set of process faults by means of particular monitoring strategies. These works considered the effect of measurement uncertainties but took no account on the capability of the sensor structure to observe and resolve faults under the presence of sensor failures. In this sense, Musulin et al.[13] and Gerkens and Heyen[14] proposed methodologies to design SNs for processes monitored using Principal Component Analysis and model-equation residuals based techniques, respectively. The resulting optimization problems were solved using Genetic Algorithms.

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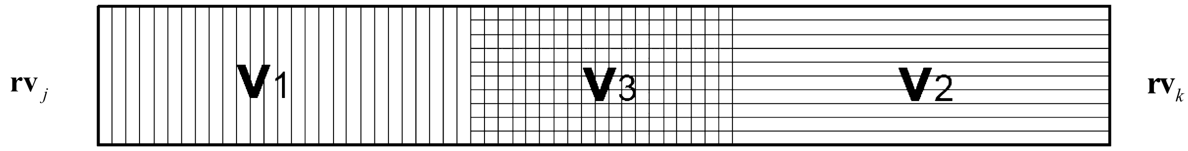

Figure 1. Conjunction between $\mathbf{rv}_j$ and $\mathbf{rv}_k$.

In chemical processes the occurrence of some particular abnormal events may originate important economic losses, hazardous working conditions, and huge environmental damage. Therefore it is essential to quickly detect and identify these key faults when they take place and perform adequate control actions to reduce their consequences. In this sense, the installed SN should be able to resolve the set of key failures under the presence of missing measurements or outliers. In this work a new strategy to deal with the optimal location of instruments for satisfying this purpose is presented. It is assumed that no selection of the technique used for monitoring the future process operation is performed at the plant design stage. The proposed approach tackles the key faults isolation from a structural point of view. The resolution degree (RD) of a key fault is defined with this aim. In contrast to previous works, the design is addressed using simpler formulations, which only use low uncertainty data that are readily available at the process design stage.

The rest of the paper is structured as follows. In Section 2, the definitions of fault O and S/MR are briefly revisited using Boolean algebra vectors, and the concept of fault RD is introduced. Different formulations for the design of SNs for fault diagnosis are presented in Section 3, and they are also put into the context of some previous works in the area. Next, application examples of the proposed formulations for two case studies extracted from the literature are provided. Results are analyzed and compared with those obtained by other existing techniques devoted to the design of robust SNs. A Conclusions section ends this article.

2. CONSTRAINTS FORMULATION

In this work the constraints of the proposed SNDPs are formulated in terms of the basic operators of Boolean algebra (conjunction $\wedge$, disjunction $\vee$, complement $\neg$) for binary variables. These are used to express SN configurations and failure cause−effect relationships. In this sense, a SN is represented by a vector $\mathbf{x}$ of dimension I, such that $x_i = 1$ if variable i is measured and $x_i = 0$ otherwise. Regarding failure cause−effect relationships, they are stated in matrix form using the $(I \times J)$ fault-sensor connectivity matrix $\mathbf{A}$, where J stands for the number of failures (Bagajewicz et al.[7]). This matrix is made up of binary elements such that $\mathbf{a}_{ij} = 1$ if the ith variable is affected by the occurrence of the jth fault, and $\mathbf{a}_{ij} = 0$ otherwise. Therefore the jth column of the connectivity matrix, $\mathbf{a}_j$, is a binary vector that represents all the variables which reveal the jth failure if they are measured. There exist different techniques to establish the cause−effect relationship between a fault and the variables that it affects. Comprehensive reviews about this issue can be found elsewhere (Raghuraj et al.;[5] Musulin et al.[13]).

2.1. Observability. A fault diagnostic system should observe fault symptoms and determine the root cause of the failure. A fault is categorized as observable if it is revealed by at least one sensor. The O of the jth fault for a given SN can be verified using the conjunction operation between the fault vector $\mathbf{a}_j$ and the sensor vector $\mathbf{x}$. That failure is observable if the sum of all the elements of $(\mathbf{a}_j \wedge \mathbf{x})$ is at least equal to or greater than 1. Therefore the O of all process faults is formulated as follows:

$$\sum_{i=1}^{I} (\mathbf{a}_j \wedge \mathbf{x})_i \geq 1 \quad (j = 1, ..., J) \tag{1}$$

where $\mathbf{rv}_j = (\mathbf{a}_j \wedge \mathbf{x})$ is defined as the jth fault resolution vector.

2.2. Simple Fault Resolution. If only one failure occurs at a given time, the instrumentation system must be able to observe and resolve it. This means that the correct fault source should be identified among all possible candidates. The R of faults strongly depends on the topology of the process. If two faults, j and k, affect the same process variables, that is,

$$\mathbf{a}_j = \mathbf{a}_k \tag{2}$$

these failures cannot be resolved, and they are considered as a single fault for the rest of the analysis.

Given the measurement vector $\mathbf{x}$, let us consider the conjunction operation between vectors $\mathbf{rv}_j$ and $\mathbf{rv}_k$, which are represented in Figure 1. Three different vectors are defined from that conjunction:

$$\mathbf{v}_1\text{: } \mathbf{rv}_j \wedge \neg\mathbf{rv}_k \tag{3}$$

$$\mathbf{v}_2\text{: } \neg\mathbf{rv}_j \wedge \mathbf{rv}_k \tag{4}$$

$$\mathbf{v}_3\text{: } \mathbf{rv}_j \wedge \mathbf{rv}_k \tag{5}$$

It can be seen that $\mathbf{v}_1$ contains all the instruments of $\mathbf{rv}_j$ not included in $\mathbf{rv}_k$; in contrast $\mathbf{v}_2$ involves the sensors of $\mathbf{rv}_k$ not contained in $\mathbf{rv}_j$, and $\mathbf{v}_3$ has the common elements. If the three vectors are nonzero, the occurrence of the jth process failure is resolved in terms of the measurements contained in $\mathbf{v}_1$ and $\mathbf{v}_3$, whose values deviate with respect to the steady state ones. Regarding the kth fault, it can be identified using the sensors included in $\mathbf{v}_2$ and $\mathbf{v}_3$.

If only two vectors are nonzero, the R of both faults is still possible. If $\mathbf{v}_2$ is null, the jth process fault is revealed using the observations included in $\mathbf{v}_1$ and $\mathbf{v}_3$, and the kth fault is identified by means of the sensors in $\mathbf{v}_3$. If $\mathbf{v}_1$ is null, the measurements contained in $\mathbf{v}_3$ allow solving the jth process failure, and those in $\mathbf{v}_2$ and $\mathbf{v}_3$ provide the R of the kth fault. Finally, if $\mathbf{v}_3$ is null, the R's of the jth and kth faults are achieved using the observations contained in $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively.

Three different cases arise when only one vector has elements. The first one corresponds to the nonobservability of the jth fault ($\mathbf{v}_1 = 0$ and $\mathbf{v}_3 = 0$). The second one is associated with the nonobservability of the kth fault ($\mathbf{v}_2 = 0$ and $\mathbf{v}_3 = 0$), and the last one is related to the lack of R of both failures ($\mathbf{v}_1 = 0$ and $\mathbf{v}_2 = 0$). Previous analysis indicates that the SR of faults j and k is achieved if at least two of the three vectors are non-null, that is,

$$\text{NNV}_{jk}(\mathbf{x}) \geq 2 \quad \forall\, j \neq k \tag{6}$$

where $\mathrm{NNV}_{jk}(\mathbf{x})$ is the number of non-null vectors that arise from the conjunction between $\mathbf{rv}_j$ and $\mathbf{rv}_k$. If only $\mathbf{v}_1$ and $\mathbf{v}_2$ are non-null, the SR of the faults is maximum as it has been stated by Raghuraj et al.[5]

Condition 6 implies that the resolution vectors (**rv**'s) of both faults should differ in at least one element. A difference of exactly one element arises if two of the three vectors are non-null, $\mathbf{v}_3 \neq 0$, and the other non-null vector has only one element. Therefore, the SR of the jth fault is attained if $\mathbf{rv}_j$ is different from the **rv**'s corresponding to all the other process failures, that is,

$$\mathbf{rv}_j \neq \mathbf{rv}_k \quad \forall\, j \neq k \tag{7}$$

It should be highlighted that if eq 6 is satisfied for a given SN, the O of faults j and k is fulfilled since there exists a variable affected by each failure in at least two of the three vectors. Therefore, the constraints of O and SR of these faults can be simultaneously tested using a single formulation.

2.3. Multiple Fault Resolution. If two or more faults have a high probability of occurring simultaneously, they are grouped and solved with respect to the others as a single fault (Raghuraj et al.[5]). Let us consider two failures represented by the fault vectors $\mathbf{a}_j$ and $\mathbf{a}_k$. The associated multiple failure vector $\mathbf{a}_{jk}$ is defined as

$$\mathbf{a}_{jk} = \mathbf{a}_j \vee \mathbf{a}_k \tag{8}$$

and it is incorporated into the connectivity matrix **A**.

The R of multiple faults is tackled considering the SR among all the multiple and simple fault vectors. For each multiple fault represented by the vector $\mathbf{a}_{jk}$, all the possible combinations between the **rv** of $\mathbf{a}_{jk}$ and the **rv**'s of the other faults denoted as $\mathbf{a}_\mathrm{p}$ are analyzed, where $\mathbf{a}_\mathrm{p}$ may be a multiple fault vector different from $\mathbf{a}_{jk}$ or a simple failure one. For each combination the following vectors should be examined:

$$\mathbf{v}_1\text{: } \mathbf{rv}_{jk} \wedge \neg\mathbf{rv}_\mathrm{p} \tag{9}$$

$$\mathbf{v}_2\text{: } \neg\mathbf{rv}_{jk} \wedge \mathbf{rv}_\mathrm{p} \tag{10}$$

$$\mathbf{v}_3\text{: } \mathbf{rv}_{jk} \wedge \mathbf{rv}_\mathrm{p} \tag{11}$$

to determine if $\mathrm{NNV}_{jk,\mathrm{p}}(\mathbf{x}) \geq 2$.

2.4. Resolution Degree. The risk associated with a fault depends on its probability of occurrence and its consequences. On the basis of the experience obtained from similar processes, a categorization of faults can be performed at the design stage. If hazard conditions for humans and the environment and important economic losses are associated with the occurrence of a process fault, it should be identified. Therefore the SN must be able to resolve this key failure even if some of the measurements that reveal the faulty state are not available.

The RD of the jth fault, RD_j, is defined as the amount of measurements belonging to $\mathbf{rv}_j$ that can be missed while the failure remains distinguishable. The RD strongly depends on the topology of the process. Some faults have RD = 0, which means no measurement can be missed at any time if it is required, keeping the isolation of the failures. Other faults have RD ≥ 1; therefore, they admit the loss of some observations.

2.4.1. Unitary Resolution Degree. A SN has a RD = 1 for the jth key fault if it remains identifiable when one measurement contained in $\mathbf{rv}_j$ is missed. If $\mathbf{rv}_j$ is constituted by R_j measurements, then R_j sensor configurations, represented by vectors $\mathbf{rv}_j^r$ ($r = 1...R_j$) of dimension ($R_j - 1$), exist which are able to identify the occurrence of the event. The following inequalities should be examined to verify $\mathrm{RD}_j = 1$

$$\mathrm{NNV}_{j^r k^r}(\mathbf{x}) \geq 2 \quad \forall\, j^r \neq k^r \quad r = 1...R_j \tag{12}$$

This is a special case of the general one that is stated next.

2.4.2. General Resolution Degree. A SN has a RD = g for the jth key fault if the failure remains identifiable when g observations affected by the fault are not available. In this case there exists $t_j = (R_j!)/(g_j!(R_j - g_j)!)$ sensor configurations of dimension $(R_j - g_j)$, which are able to identify the occurrence of the jth key failure. To verify $\mathrm{RD}_j = g_j$, the NNVs that arise from the conjunctions between each $\mathbf{rv}_j^r$and $\mathbf{rv}_k^r$ are inspected.

$$NNV_{j^r k^r}(\mathbf{x}) \geq 2 \quad \forall\, j^r \neq k^r \quad r = 1...t_j \tag{13}$$

Application examples about constraints evaluation are provided in Supporting Information.

3. SENSOR NETWORK DESIGN FORMULATIONS FOR FAULT DIAGNOSIS

In this section different formulations of the SNDP for failure diagnosis are presented. Single objective optimization problems are stated subject to constraints on faults O and S/MR and key faults RD. At first minimum-cost and minimum sensor-number designs are addressed, and then fixed-budget instrumentation problems are tackled. Furthermore the work is put into the context of previous publications in the area of SND.

In general let us assume that a set of J single/multiple faults are considered for the process under analysis, whose fault vectors are denoted as $\mathbf{a}_j$. If some of them are equal at first, a preprocessing step is required to merge the corresponding failures in a single one because they are indistinguishable.

The structure of the SN that allows the O of all process faults at minimum cost is determined by solving Problem 14, which comprises a set of J linear inequalities formulated in terms of the binary variables contained in vector **x**:

$$\begin{aligned} &\min_{\mathbf{x}}\ \mathbf{c}^\mathbf{T}\mathbf{x} \\ &s.t. \\ &\qquad \sum_{i=1}^{I} (\mathbf{a}_j \wedge \mathbf{x})_i \geq 1 \quad j = 1...J \end{aligned} \tag{14}$$

where c_i is the cost of the ith sensor.

A compact formulation of the SNDP that simultaneously takes into account faults O and R constraints is represented by means of eq 15:

$$\begin{aligned} &\min_{\mathbf{x}}\ \mathbf{c}^\mathbf{T}\mathbf{x} \\ &s.t. \\ &\qquad \mathrm{NNV}_{j,k}(\mathbf{x}) \geq 2 \quad j = 1...J,\ k = 1...J,\ j \neq k \end{aligned} \tag{15}$$

This problem comprises $[J(J - 1)/2]$ nonlinear inequalities restrictions. For a given pair of vectors $\mathbf{rv}_j(\mathbf{x})$ and $\mathbf{rv}_k(\mathbf{x})$, the restriction $\mathrm{NNV}_{j,k}(\mathbf{x}) \geq 2$ can be expressed as follows:

$$\begin{aligned} \mathrm{NNV}_{j,k} = {}& \min\Big[1, \sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i\Big] \\ &+ \min\Big[1, \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i\Big] + \min\Big[1, \sum_{i=1}^{I} (rv_j \wedge rv_k)_i\Big] \geq 2 \end{aligned} \tag{16}$$

To avoid the use of nonlinear constraints in the optimization problem, eq 16 can be replaced by the following set of three linear inequalities:

$$\sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_j \wedge rv_k)_i \geq 1 \quad \forall\, j \neq k \tag{17}$$

$$\sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i + \sum_{i=1}^{I} (rv_j \wedge rv_k)_i \geq 1 \quad \forall\, j \neq k \tag{18}$$

$$\sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i \geq 1 \quad \forall\, j \neq k \tag{19}$$

where eq 17 represents the sum of the elements of $\mathbf{rv}_j$, eq 18 is the sum of the elements of $\mathbf{rv}_k$, and eq 19 is the sum of the elements that belong to $(\mathbf{rv}_j \cup \mathbf{rv}_k)$ but not to their intersection. Therefore Problem 15 is transformed into Problem 20, which involves a set of $[3J(J-1)/2]$ linear inequalities restrictions. Identical restrictions come from many inequalities represented by eqs 17 and 18, which are eliminated. Therefore Problem 21 arises that contains $[J + (J(J-1))/2]$ constraints. This set of linear inequalities can be reduced further by eliminating redundant equations following the procedure outlined by Bhushan and Rengaswamy.[15] Problem 21 is solved using MILP codes, for example, the procedure CPLEX of the GAMS program.

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ &s.t. \\ &\left\{ \begin{aligned} &\sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_j \wedge rv_k)_i \geq 1 \quad \forall\, j \neq k \\ &\sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i + \sum_{i=1}^{I} (rv_j \wedge rv_k)_i \geq 1 \quad \forall\, j \neq k \\ &\sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i \geq 1 \quad \forall\, j \neq k \end{aligned} \right\} j = 1...J,\ k = 1...J,\ j \neq k \end{aligned} \tag{20}$$

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ &s.t. \\ &\sum_{i=1}^{I} (rv_j)_i \geq 1 \quad j = 1...J \\ &\sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i \geq 1 \quad j = 1...J,\ j \neq k \end{aligned} \tag{21}$$

The aim of this work is to address the design of SNs which are able to resolve a set of key failures under the presence of missing measurements or outliers. This is accomplished by incorporating RD restrictions for a set of S key faults to Problem 15. In compact form, the SNDP to be solved is represented by eq 22, where g is the lower bound selected for the key faults RD:

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ &s.t. \\ &\qquad \mathrm{NNV}_{j,k}(\mathbf{x}) \geq 2 \quad j = 1...J,\ k = 1...J,\ j \neq k \\ &\qquad \mathrm{RD}_s(\mathbf{x}) \geq g_s \quad s = 1...S \end{aligned} \tag{22}$$

Given that RD_s restrictions ($s = 1...S$) are evaluated in terms of the NNVs between the $\mathbf{rv}_{s^r}$ ($r = 1...t_s$) and $\mathbf{rv}_{k^r}$ ($k = 1...J$, $k \neq s$), Problems 22 and 15 have the same type of constraints. Using the proposed reformulation of the NNV restrictions into a set of linear inequalities, Problem 22 is transformed into the following one:

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ &s.t. \\ &\qquad \sum_{i=1}^{I} (rv_j)_i \geq 1 \quad j = 1...J \\ &\qquad \sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i \geq 1 \quad j = 1...J,\ j \neq k \\ &\qquad \sum_{i=1}^{I} (rv_{s^r})_i \geq 1 \quad s = 1...S;\ r = 1...t'_s \\ &\qquad \sum_{i=1}^{I} (rv_{s^r} \wedge \neg rv_k^r)_i + \sum_{i=1}^{I} (rv_k^r \wedge \neg rv_{s^r})_i \geq 1 \quad s = 1...S;\ r = 1...t'_s;\ s \neq k \end{aligned} \tag{23}$$

where $t_s' = (R_s'!)/(g_s!(R_s' - g_s)!)$ and R_s' is the cardinality of $\mathbf{a}_s$. The resulting optimization problem involves $[J + (J(J-1)/2) + J(\sum_{s=1}^{S} t_s')]$ constraints and, after the elimination of redundant equations, it can be solved, for instance, using the code CPLEX of the GAMS package.

It should be noticed that the incorporation of RD constraints for the key faults frequently enhances their R when no sensor fails. In general, the solution of Problem 23 involves more sensors associated with the key faults in comparison with that obtained when Problem 21 is solved. Therefore the R of the key variables can be done using more than one **rv** under the absence of sensor malfunctions, which means that alternative ways to verify the failure condition of these particular faults may be available.

If all the instruments have the same cost, the minimization of the instrumentation cost can be replaced by the minimization of the number of instruments; therefore, the previous problem is stated as follows:

$$\begin{aligned} &\min_{\mathbf{x}} \sum_{i=1}^{I} x_i \\ &s.t. \\ &\qquad \sum_{i=1}^{I} (rv_j)_i \geq 1 \quad j = 1...J \\ &\qquad \sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i \geq 1 \quad j = 1...J, \ j \neq k \\ &\qquad \sum_{i=1}^{I} (rv_{s^r})_i \geq 1 \quad s = 1...S; \ r = 1...t_s' \\ &\qquad \sum_{i=1}^{I} (rv_{s^r} \wedge \neg rv_k^r)_i + \sum_{i=1}^{I} (rv_k^r \wedge \neg rv_{s^r})_i \geq 1 \quad s = 1...S; \ r = 1...t_s'; \ s \neq k \end{aligned} \tag{24}$$

When the instrumentation project has a fixed budget, C_T, the maximization of the RD of the most critical failure s^* is proposed. The design should satisfy faults O and R, the amount of available resources, and the RD of all the key faults. It is a practical assumption that they remain solvable when at least one measurement is missed. The formulation of this problem is presented next.

$$\begin{aligned} &\max \mathrm{RD}_{s^*}(\mathbf{x}) \\ &s.t. \\ &\mathbf{c}^{\mathbf{T}}\mathbf{x} \leq C_T \\ &\qquad \sum_{i=1}^{I} (rv_j)_i \geq 1 \quad j = 1...J \\ &\qquad \sum_{i=1}^{I} (rv_j \wedge \neg rv_k)_i + \sum_{i=1}^{I} (rv_k \wedge \neg rv_j)_i \geq 1 \quad j = 1...J, \ j \neq k \\ &\qquad \sum_{i=1}^{I} (rv_{s^r})_i \geq 1 \quad s = 1...S; \ r = 1...t_s' \\ &\qquad \sum_{i=1}^{I} (rv_{s^r} \wedge \neg rv_k^r)_i + \sum_{i=1}^{I} (rv_k^r \wedge \neg rv_{s^r})_i \geq 1 \quad s = 1...S; \ r = 1...t_s'; \ s \neq k \end{aligned} \tag{25}$$

3.1. Relations between the Proposed Formulations and Other Existing Works. This work is devoted to presenting a new strategy to design an optimal SN, which is able to resolve a set of key faults under the presence of failed sensors or outliers. Only the following information is required: (a) the cause−effect relations among faults and variables, which come from the available process knowledge and simulation results, (b) the set of key failures selected from previous experience, and (c) sensor costs. All these low uncertainty data are readily available by plant designers. The categorization of faults allows taking into account both their occurrence probability and consequences at the design stage in a simple way.

The procedure deals with key failures R when sensor malfunctions occur from a structural point of view. With this purpose key faults RD constraints are incorporated into the optimization problem. It involves a linear objective function and a set of linear inequality restrictions; consequently, it is solved using MILP codes. It should be noticed that all the constraints are only stated in terms of the original I binary variables.

There exist some connections between this work and other contributions in the area of SND for fault diagnosis. In this sense, Bagajewicz et al.[7] used the concepts of matrix algebra to formulate a minimum-cost SNDP subject to faults O and R restrictions. The original optimization problem was reformulated as a MILP by increasing the number of constraints from $[J + ((J - 1)/2)]$ to $[J + (5J(J - 1)/2)]$ and the number of binary variables from I to $[I + (J(J - 1)/2)]$. The formulation presented in this work with the same purpose (Problem 21) involves $[J + ((J - 1)/2)]$ linear inequalities, and the number of binary variables remains equal to I. Furthermore the consequences of the presence of failed sensors or outliers on the fault detection capabilities of the SN are not addressed in their work.

Regarding reliabilities based methods, Bhushan and Rengaswamy[9] defined the Detection Unreliability of the jth fault, U_j, as the probability that it occurs and the sensors installed to observe it fail. Furthermore, a measure of the Detection Unreliability of all the faults and pseudofaults, U, is stated as the maximum U_j $\{j = 1...[J + ((J - 1)/2)]\}$. A pseudofault is defined by means of the fault set $B_{ij} = A_i \cup A_j - A_i \cap A_j$, where A_i and A_j are the sets of measurable nodes affected by faults i and j, respectively. For each pair (i,j) of faults, a pseudofault is generated and incorporated into the original set of S/M faults to analyze faults R. The evaluation of U requires knowing the fault cause−effect model, the probability of occurrence of all the faults, and the reliabilities of all the sensors that may participate in the network. It should be noticed that these data are uncertain. First, the aforementioned authors presented different formulations of the SNDP. They dealt with the minimization of the U subject to cost constraints, the minimization of the cost subject to U value restrictions, the lexicographic optimization with reliability as primary objective and cost as the secondary one, the minimization of the cost subject to the O and R of all the faults, etc. In their formulation of the last problem no failure probability data are used; therefore, it is equivalent to Problem 21. Among all the proposed formulations, those authors emphasized the use of the lexicographic procedure.

Later on Bhushan et al.[10] presented a rather complex lexicographic optimization framework for enhancing SN robustness to uncertain data. They proposed to reduce the effect of imprecise data on U by ensuring that the constraints involving inaccurate information are far from being active at the optimal solution. The methodology requires categorizing all process faults previous to its application, taking into account the inaccuracies associated with the evaluation of U_j. In their formulation, three objectives are appropriately combined in a single objective function. The primary objective is the minimization of U, the second one is the maximization of the minimum slack variable among all the slack ones contained in the unreliability detection constraints of uncertain faults, and the third one is to minimize the cost. The calculation of all the weighting constants used in the objective function is based on problem data. Three different scenarios of data inaccuracies are analyzed. The second one corresponds to inaccuracies in failure probabilities of some sensors with all fault occurrence probabilities accurately known. For this scenario, the optimization problem involves $[2 + J + (J - 1)/2 + 4J_u]$ linear constraints and $[I + J_u]$ binary variables, where J_u stands for the number of uncertain faults and pseudofaults and can be solved using MILP codes.

The strategy proposed in this work and the one presented by Bhushan et al.[10] have different purposes. The first one is devoted to satisfy both the R of all faults when no sensor fails and the R of the key faults under the presence of sensor malfunctions at minimum cost. That is, the optimal SN should satisfy two requisites, and one of them corresponds to the whole set of faults and the other one is associated with a specific set of important failures. The second methodology is focused on the whole set of faults and aims at optimizing system performance measures at minimum cost using reliabilities data. Even though no uncertainties are considered in our methodology, the second scenario of Bhushan at al.'s[10] procedure might be used for comparison purposes, taking into account that our categorization of faults is fixed.

Given that neither to guarantee the R of all the faults nor to satisfy the R of a specific set of faults are objectives of the aforementioned reliabilities based strategy, no indication is provided about how to verify if these conditions are achieved. In this article additional steps for this technique are devised to tackle those issues using the second scenario. To satisfy the R of all the faults, different runs of the optimization procedure should be performed increasing the upper bound of the cost restriction, and the $(\mathrm{PFP}_j - U_j)$ differences for each run should be inspected, where PFP_j is the jth process failure probability $(j = 1...[J + ((J - 1)/2)])$. For a given capital resource, the solution satisfies the R of all the faults when all the differences $(PFP_j - U_j)$ are greater than zero.

Regarding the R of key faults when sensors fail, different runs of the optimization procedure should also be performed increasing the upper bound of the cost restriction. For each capital resource, the original procedure gives a solution $\mathbf{x}$ which involves a set of $\mathbf{J_x}$ sensors. In this article it is proposed to define the set $(\mathbf{J_x} \cap \mathbf{J_s})$ for each key fault, where $\mathbf{J_s}$ represents the set of sensors affected by the key fault, and to recalculate the differences $(PFP_j - U_j)$, where j stands for the key fault and their associated pseudofaults. The new values of U_j are obtained taking into account that the probabilities of the failed sensors that participate in $(\mathbf{J_x} \cap \mathbf{J_s})$ are equal to 1. The total number of differences that should be evaluated for the sth key fault is $R_s!/(g_s!(R_s - g_s)!)$. The solution satisfies the R of all key faults, when the $R_s!/(g_s!(R_s - g_s)!)$ $(s = 1...S)$ differences $(PFP_j - U_j)$ are greater than zero for a given capital resource. It should be noticed that the uncertainties of sensor reliabilities affect U_j values; therefore, it cannot be accurately declared that faults are resolvable.

4. CASE STUDIES

A continuous stirred tank reactor (CSTR) and a fluid catalytic cracker unit (FCCU) are selected as case studies to show the application of the proposed strategy and compare its results with respect to those provided by a reliabilities-based method (Bhushan et al.[10]). Both processes are nonlinear and have been used in previous works related to the design of SNs for fault diagnosis.

4.1. Continuous Stirred Tank Reactor. The CSTR flowsheet, extracted from the work by Kramer and Palowitch,[16] is represented in Figure 2, and process variables notation is included in Table 1. In this reactor a positive order reaction of A to B takes place. Part of the reactor outlet stream is recycled to that unit through a heat exchanger to provide temperature control. Also the recycle flow rate is controlled, and the reactor residence time is controlled by maintaining a constant level in

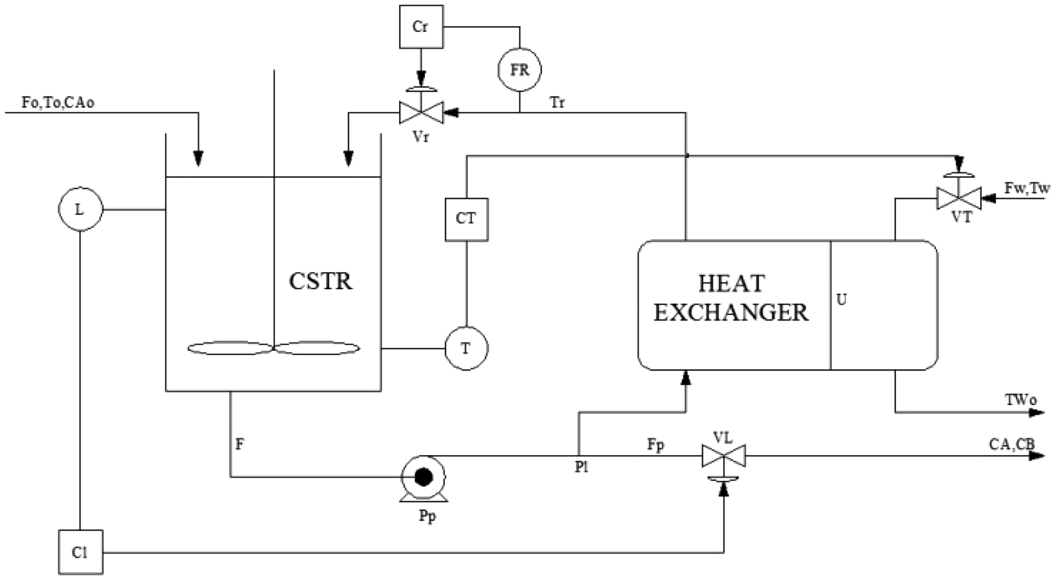

Figure 2. CSTR flowsheet.

Table 1. CSTR Variables Notation

notation	variable
CA CB	reactor concentrations of A and B
C_{A0}	feed concentration of A
C_l C_r C_T	controller output signals: level, recycle flow, temperature
F F_0 F_P F_R F_W	flow rates: reactor outlet, feed, product, recycle, cooling water
L	reactor liquid level
P_p	pump head
T T_0 T_R T_{W0} T_W	temperatures: reactor, feed, recycle, cooling water inlet and outlet
U	heat transfer coefficient
V_L V_r V_T	control valve stem positions: level, recycle flow, temperature
P_l	recycle takeoff pressure

this unit. Pressure and physical properties are assumed constant.

Raghuraj et al.[5] analyzed the fault cause−effect model for this system and determined seven identifiable faults, which are shown in Table 2.

Four minimum-cost SNDPs are formulated considering fault O, SR, and RD constraints. The selected key process fault corresponds to vector $\mathbf{a}_4$. Instrumentation costs are reported in Table 3. Because measurements of temperature and level are necessary for reactor control loops, their costs are set at zero. The results of the proposed design problems are shown in Table 4.

The amounts of inequalities constraints associated with each problem are 7 (for O constraint), 28 (for SR constraint), 49 (for SR, $RD_4 \geq 1$ constraints), and 49 (for SR, $RD_4 \geq 2$ constraints). But after the procedure of reduction of redundant equations they become 1, 7, 9, and 9, respectively.

When SR constraints are imposed, the problem solution is equal to that reported by Raghuraj et al.,[5] while the solution that satisfies the O of all the faults at minimum cost is different because no costs are considered in that work. Table 4 shows that the number of sensors of the solution set increases if RD restrictions are imposed. In comparison with the solution that satisfies only SR constraints, the measurement of T_{W0} should be incorporated if a $RD_4 \geq 1$ is required. Furthermore CB and T_{W0} participate in the solution vector when the lower bound of the RD increases to 2. An analysis of the RD for the rest of the faults indicates that none of them admit a RD greater than zero.

Table 5 presents the **rv**'s of each fault for the last three design problems contained in Table 4. For each design, the **rv**'s differ in at least one element. This fact demonstrates that all faults can be resolved by means of the optimal SN. Furthermore, it can be seen that an increment in the required RD originates an increase in the number of elements of the **rv**'s. It should be also noticed that the incorporation of RD restrictions enhances the R of key variables when no sensor malfunctions are present. In this sense Table 5 shows that the **rv** for the key fault $\mathbf{a}_4$ when $RD_4 \geq 1$ is made up of variables T and T_{W0}; therefore, there exist two alternative ways of resolving the key fault when no sensor fails.

Furthermore, the reliabilities-based methodology proposed by Bhushan et al.[10] is applied to design robust SNs for the system under analysis. The following values are selected for the process faults probabilities (PFPs) and sensor faults probabilities (SFPs): $PFP\mathbf{a}_1 = 10^{-1}$, $PFP\mathbf{a}_2 = 10^{-2}$, $PFP\mathbf{a}_3 = 10^{-2}$, $PFP\mathbf{a}_4 = 10^{-1}$, $PFP\mathbf{a}_5 = 10^{-2}$, $PFP\mathbf{a}_6 = 10^{-2}$, $PFP\mathbf{a}_7 = 10^{-2}$, $SFP_{CB} = 10^{-3}$, $SFP_{T} = 10^{-2}$, $SFP_{TW0} = 10^{-2}$, $SFP_{FW} = 10^{-3}$, $SFP_{FR} = 10^{-3}$, $SFP_{TR} = 10^{-2}$, $SFP_{CA} = 10^{-3}$, $SFP_{L} = 10^{-3}$, $SFP_{FP} = 10^{-3}$, which are similar to those provided by Bhushan and Rengaswamy.[9]

For comparative purposes, the aforementioned strategy is run assuming that only SFPs are uncertain, and among all the instruments only those affected by the occurrence of fault $\mathbf{a}_4$ ($\mathbf{Js}$ = [CB, T, T_{W0}]) are inaccurate. For different upper bounds on the capital resources (C^*), Table 6 presents the results of the methodology, that is, the solution vector $\mathbf{x}$, the values of U, $\phi_{s,4}$ (slack variable associated with the unreliability detection constraint of the uncertain fault), and Xs (slack variable for the cost restriction). To analyze if all the faults are resolvable when no sensor failure occurs, the ($PFP_j - U_j$) differences ($j = 1...[J + ((J - 1)/2)]$) are inspected, as it is explained in the previous section of this work, and the nonresolvable pairs of faults are determined. These are also included in Table 6. It can be seen that, for low budgets of the instrumentation project ($C^* < 2000$), some faults are indistinguishable. For $C^* = 2000$, the R of all faults is attained

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Table 2. CSTR Fault Cause−Effect Model

fault vector	fault source	affected variables	fault vector	fault source	affected variables
$\mathbf{a}_1$	$U\ T_W$	$CB\ T\ T_{W0}\ T_R$	$\mathbf{a}_5$	$C_l\ P_l\ P_p\ F_0$	$CB\ T\ T_{W0}\ CA\ L$
$\mathbf{a}_2$	V_T	$CB\ T\ T_{W0}\ F_W\ T_R$	$\mathbf{a}_6$	V_L	$CB\ T\ T_{W0}\ CA\ L\ F_P$
$\mathbf{a}_3$	$V_r\ C_r$	$CB\ T\ T_{W0}\ F_R\ T_R$	$\mathbf{a}_7$	C_{A0}	$CB\ T\ T_{W0}\ CA$
$\mathbf{a}_4$	$C_T\ T_0$	$CB\ T\ T_{W0}$			

Table 3. CSTR Sensor Costs

sensor	cost	sensor	cost	sensor	cost
CB	700	F_W	200	CA	700
T	0	F_R	300	L	0
T_{W0}	500	T_R	500	F_P	300

Table 4. CSTR Results

constraints	solution	cost
O	T	0
SR	$T\ F_W\ F_R\ T_R\ CA\ L\ F_P$	2000
$SR,\ RD_4 \geq 1$	$T\ T_{W0}\ F_W\ F_R\ T_R\ CA\ L\ F_P$	2500
$SR,\ RD_4 \geq 2$	$CB\ T\ T_{W0}\ F_W\ F_R\ T_R\ CA\ L\ F_P$	3200

Table 5. CSTR Resolution Vectors

fault vector	SR	SR, $RD_4 \geq 1$	SR, $RD_4 \geq 2$
$\mathbf{a}_1$	$T\ T_R$	$T\ T_{W0}\ T_R$	$CB\ T\ T_{W0}\ T_R$
$\mathbf{a}_2$	$T\ F_W\ T_R$	$T\ T_{W0}\ F_W\ T_R$	$CB\ T\ T_{W0}\ F_W\ T_R$
$\mathbf{a}_3$	$T\ F_R\ T_R$	$T\ T_{W0}\ F_R\ T_R$	$CB\ T\ T_{W0}\ F_R\ T_R$
$\mathbf{a}_4$	T	$T\ T_{W0}$	$CB\ T\ T_{W0}$
$\mathbf{a}_5$	$T\ CA\ L$	$T\ T_{W0}\ CA\ L$	$CB\ T\ T_{W0}\ CA\ L$
$\mathbf{a}_6$	$T\ CA\ L\ F_P$	$T\ T_{W0}\ CA\ L\ F_P$	$CB\ T\ T_{W0}\ CA\ L\ F_P$
$\mathbf{a}_7$	$T\ CA$	$T\ T_{W0}\ CA$	$CB\ T\ T_{W0}\ CA$

at minimum cost, and the solution is equal to that obtained solving Problem 21 or Problem 23 using the constraint $RD_4 \geq 0$. An increment of the capital resources ($C^* > 2000$) originates the increment of $\phi_{s,4}$ and/or Xs, but U remains at (−3) because the pseudofault (1, 4) imposes a limit on the U value.

Furthermore, it is simulated that the sensors included in $(\mathbf{J_x} \cap \mathbf{J_s})$ are out of service, one at a time, for different C^* values, and the R of the key fault $\mathbf{a}_4$ is analyzed. The procedure outlined in Subsection 3.1 of this work is applied, and the results are shown in Table 7. It point outs that for low budgets the key fault is nondistinguishable ($C^* < 2000$) or nonobservable ($2000 \leq C^* < 2500$). For $C^* = 2500$, the R of the key fault is achieved at minimum cost, and the solution is equal to that obtained solving Problem 23 using the constraint $RD_4 \geq 1$. For all the combinations of two instruments, a similar analysis is conducted by simulating that two of the sensors included in $(\mathbf{J_x} \cap \mathbf{J_s})$ are not available.

Simulation results are included in Table 8, which shows that the key fault is nondistinguishable for $C^* < 2000$ or nonobservable for $2000 \leq C^* < 3200$). For $C^* = 3200$, the R of the key fault is attained at minimum cost and the solution is equal to that achieved solving Problem 23 using the constraint $RD_4 \geq 2$.

Furthermore, worst-case simulations are carried out by increasing the SFPs, one at a time, for some capital resources. Increases of 1 from the SFPs nominal values on the $\log_{10}$ scale are performed. Results show that some solutions differ from the ones presented in Table 7. For instance, increasing the SFP of either T or Tr for $C^* = 2000$, the new solution is $\mathbf{x} = [\mathrm{CB}\ T\ T_{W0}\ T_R\ L]$, and by applying the modified procedure proposed in subsection 3.1 it is determined that the pairs of faults 4,7; 1,2; 1,3; 2,3; and 5,6 are nonresolvable. The same situation arises increasing the SFP of Tr for $C^* = 2500$.

Previous results highlight that the proposed modifications of the reliabilities-based methodology can be applied to analyze the R of all the faults when no sensor fails and the R of the key faults in the presence of missing measurements or outliers, but at the expense of an extra computational work. Furthermore it should be noticed that the categorization of faults in resolvable and nonresolvable is not unique, because it depends on the selected values of the SFPs.

4.2. Fluid Catalytic Cracker Unit.

The process flowsheet for the FCCU, extracted from the work by McFarlane et al.,[17] is represented in Figure 3, and variables notation is included in Table 9.

Preheated feed and hot slurry recycle are mixed and injected into the reactor riser, where they make contact with hot regenerated catalyst, and totally vaporize. As a result of the cracking reactions, coke is deposited on the surface of the catalyst, which should be regenerated because coke poisons it.

In the disengaging zone of the reactor, gas and catalyst are separated. Then catalyst is returned to the stripping section of the reactor where steam is injected to remove entrained hydrocarbons. Reactor product gas is passed to the main fractionator for heat recovery and separation into various product streams.

Spent catalyst is transported from the reactor to the regenerator where it is fluidized with air. Carbon and hydrogen on the catalyst react with oxygen to produce carbon monoxide and water. Gas travels up the regenerator into the cyclones where entrained catalyst is removed and returns to the bed. The regenerator is run at conditions of temperatures and excess oxygen to ensure that virtually all carbon monoxide produced in the bed is converted to carbon dioxide before entering the cyclones. Regenerated catalyst flows over a weir into the regenerator standpipe. The level of catalyst in the standpipe provides the driving force for catalyst flow through the regenerated catalyst U-bend to the reactor riser.

Raghuraj et al.[5] reported seven identifiable faults for this case study, which are included in Table 10. Furthermore the assumed sensor costs are shown in Table 11.

The minimum-cost sensor configurations obtained by solving Problems 14 and 21 are displayed in the first two rows of Table 12, and they are coincident with those reported in previous works. The following rows of this table correspond to the solution of Problem 23 for different RD constraints. Failures associated with fault vectors $\mathbf{a}_4$, $\mathbf{a}_5$, and $\mathbf{a}_7$ are selected as key process faults. For the process under analysis, it is not possible to require RDs greater than zero for the other faults. The amounts of inequalities constraints associated with each problem are:7 (for O constraint), 28 (for SR constraint), 70 (for SR, $RD_4 \geq 1$ constraints), 133 (for SR, $RD_4 \geq 2$ constraints), 84 (for SR, $RD_5 \geq 1$ constraints), 49 (for SR, $RD_7 \geq 1$ constraints), and 49 (for SR, $RD_7 \geq 2$ constraints). But after the procedure of reduction of redundant equations those quantities become 2, 5, 7, 7, 5, 7, and 7, respectively.

Table 6. CSTR—Resolution of All Faults for Different Capital Resources

C*	X	U	$\phi_{s,4}$	Xs	nonresolvable pairs of faults
1200	$CB\ T\ T_R$	−2	4	0	1,2; 1,3; 2,3; 4,5; 4,6; 4,7; 5,6; 5,7; 6,7
1500	$CB\ T\ T_R$	−2	4	300	1,2; 1,3; 2,3; 4,5; 4,6; 4,7; 5,6; 5,7; 6,7
1800	$CB\ T\ T_R$	−2	4	600	1,2; 1,3; 2,3; 4,5; 4,6; 4,7; 5,6; 5,7; 6,7
2000	$T\ F_W\ F_R\ T_R\ CA\ L\ F_P$	−3	0	0	-
2200	$T\ F_W\ F_R\ T_R\ CA\ L\ F_P$	−3	0	200	-
2500	$T\ T_{W0}\ F_W\ F_R\ T_R\ CA\ L\ F_P$	−3	2	0	-
2800	$CB\ T\ F_W\ F_R\ T_R\ CA\ L\ F_P$	−3	3	100	-
3000	$CB\ T\ F_W\ F_R\ T_R\ CA\ L\ F_P$	−3	3	300	-
3200	$CB\ T\ T_{W0}\ F_W\ F_R\ T_R\ CA\ L\ F_P$	−3	5	0	-

Table 7. CSTR—Resolution of key Faults for Different Capital Resources: Loss of One Measurement of $(J_x \cap J_s)$

C*	failed sensor	nonobservable and nonresolvable faults
1200	CB	4,5; 4,6; 4,7
	T	4,5; 4,6; 4,7
1500	CB	4,5; 4,6; 4,7
	T	4,5; 4,6; 4,7
1800	CB	4,5; 4,6; 4,7
	T	4,5; 4,6; 4,7
2000	T	4
2200	T	4
2500	T	-
	T_{W0}	-
2800	CB	-
	T	-
3000	CB	-
	T	-
3200	CB	-
	T	-
	T_{W0}	-

Table 8. CSTR—Resolution of Key Faults for Different Capital Resources: Loss of Two Measurements of $(J_x \cap J_s)$

C*	failed sensor	nonobservable and nonresolvable faults
1200	$CB\ T$	4; 4,5; 4,6; 4,7
1500	$CB\ T$	4; 4,5; 4,6; 4,7
1800	$CB\ T$	4; 4,5; 4,6; 4,7
2000	T	4
2200	T	4
2500	$T\ T_{W0}$	4
2800	$CB\ T$	4
3000	$CB\ T$	4
3200	$CB\ T$	-
	$T\ T_{W0}$	-
	$CB\ T_{W0}$	-

In Table 13 the **rv**'s of each fault for the design problems associated with rows 2 to 4 of Table 12 are displayed. For each design, it can be seen that the **rv**'s differ in at least one element. The same behavior is verified from the analysis of the **rv**'s obtained using the solutions of the last three optimization problems presented in Table 12. These **rv**'s are not included for the sake of space.

From Table 13 it can be seen that the **rv** associated with $\mathbf{a}_4$ is made up of measurements SN2 and F_{wg} if only SR restrictions are considered. When the constraint $RD_4 \geq 1$ is imposed to the optimization problem, that **rv** also contains the measurement SN1. Consequently there exist three alternative ways of resolving the key fault when no sensor fails. These are

Table 9. FCCU Variables Notation

notation	variable
Ψ_f	effective coking factor
H_{ris}	reactor riser height
$F_1\ F_2\ F_3\ F_4$	flow rates: wash oil, diesel fuel oil, fresh feed, slurry
F_c	coke production
$F_{sc}\ F_{wg}$	flow rates: spent catalyst, wet gas
F_{v11}	flow through wet gas compressor suction valve
$F_B\ F_{rgc}$	effect of feed type on: coke, regenerated catalyst flow
$P_{atm}\ P_{rb}\ P_4$	pressures: atmospheric, at riser bottom, in the reactor
C_{rgc}	carbon regenerated catalyst
V_{12}	wet gas flare valve position
K_{12}	wet gas flare valve flow rating
$T_2\ T_{sc}$	temperatures: fresh feed, spent catalyst
SN1: $Q_{in}\ dT_r\ T_r\ Q_{slurry}\ Q_{out}\ Q_{crack}\ Q_{catout}\ Q_{ff}\ \Delta H_{crack}$	Sensor Network 1
SN2: $F_{v12}\ dP_5\ P_5$	Sensor Network 2

Table 10. FCCU Fault Cause−Effect Model

fault vector	fault source	affected variables
$\mathbf{a}_1$	$C_{rgc}\ F_{sc}$	SN3
$\mathbf{a}_2$	$H_{ris}\ F_1$	SN3 F_c
$\mathbf{a}_3$	$\Psi_F\ F_2$	SN3 $F_c\ F_B$
$\mathbf{a}_4$	$T_2\ Q_{rgc}$	SN2 $P_4\ P_{rb}\ F_{wg}$ SN1 T_{sc}
$\mathbf{a}_5$	F_{rgc}	SN2 $P_4\ P_{rb}$ SN3 $F_c\ F_{wg}$ SN1 T_{sc}
$\mathbf{a}_6$	$F_4\ F_3$	SN2 $P_4\ P_{rb}$ SN3 $F_c\ F_B\ F_{wg}$ SN1 T_{sc}
$\mathbf{a}_7$	$F_{V11}\ P_{atm}\ V_{12}\ k_{12}$	SN2 $P_4\ P_{rb}$

Table 11. FCCU Sensor Costs

sensor	cost	sensor	cost	sensor	cost
SN2	100	SN3	700	F_{wg}	300
P_4	100	F_{coke}	300	SN1	500
P_{rb}	100	F_B	300	T_{sc}	500

Table 12. FCCU Results

constraints	solution	cost
O	P_{rb} SN3	800
SR	SN2 SN3 $F_c\ F_B\ F_{wg}$	1700
SR, $RD_4 \geq 1$	SN2 SN3 $F_c\ F_B\ F_{wg}$ SN1	2200
SR, $RD_4 \geq 2$	SN2 SN3 $F_c\ F_B\ F_{wg}$ SN1 T_{sc}	2700
SR, $RD_5 \geq 1$	SN2 SN3 $F_c\ F_B\ F_{wg}$	1700
SR, $RD_7 \geq 1$	SN2 P_4 SN3 $F_c\ F_B\ F_{wg}$	1800
SR, $RD_7 \geq 2$	SN2 $P_4\ P_{rb}$ SN3 $F_c\ F_B\ F_{wg}$	1900

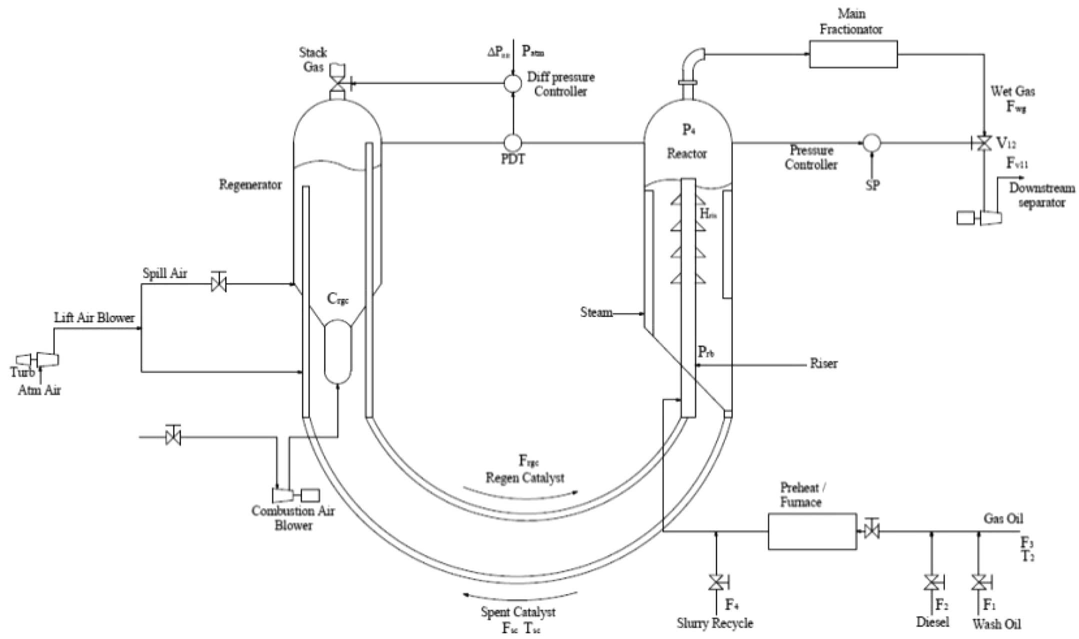

Figure 3. FCCU flowsheet.

Table 13. FCCU Resolution Vectors—SR and RD_4 Constraints

fault vector	SR	SR, $RD_4 \geq 1$	SR, $RD_4 \geq 2$
$\mathbf{a}_1$	*SN3*	*SN3*	*SN3*
$\mathbf{a}_2$	*SN3* F_c	*SN3* F_c	*SN3* F_c
$\mathbf{a}_3$	*SN3* F_c F_B	*SN3* F_c F_B	*SN3* F_c F_B
$\mathbf{a}_4$	*SN2* F_{wg}	*SN2* F_{wg} *SN1*	*SN2* F_{wg} *SN1* T_{sc}
$\mathbf{a}_5$	*SN2 SN3* F_c F_{wg}	*SN2 SN3* F_c F_{wg} *SN1*	*SN2 SN3* F_c F_{wg} *SN1* T_{sc}
$\mathbf{a}_6$	*SN2 SN3* F_c F_B F_{wg}	*SN2 SN3* F_c F_B F_{wg} *SN1*	*SN2 SN3* F_c F_B F_{wg} *SN1* T_{sc}
$\mathbf{a}_7$	*SN2*	*SN2*	*SN2*

constituted by the following pairs of measurements: (SN2, F_{wg}), (SN2, SN1), and (F_{wg}, SN1). A similar analysis of the **rv**'s for the last three optimization problems presented in Table 12 indicates that the incorporation of RD constraints for the key fault $\mathbf{a}_7$ enhances its R when no sensor fails, but this behavior is not verified for the fault vector $\mathbf{a}_5$.

Next, the reliabilities-based methodology presented by Bhushan et al.[10] is applied to design robust SNs for the FCCU system. The following values are selected for failure probabilities: $PFP\mathbf{a}_1 = 10^{-2}$, $PFP\mathbf{a}_2 = 10^{-2}$, $PFP\mathbf{a}_3 = 10^{-2}$, $PFP\mathbf{a}_4 = 10^{-1}$, $PFP\mathbf{a}_5 = 10^{-2}$, $PFP\mathbf{a}_6 = 10^{-2}$, $PFP\mathbf{a}_7 = 10^{-2}$, $SFP_{SN2} = 10^{-3}$, $SFP_{P4} = 10^{-3}$, $SFP_{Prb} = 10^{-3}$, $SFP_{SN3} = 10^{-3}$, $SFP_{Fcoke} = 10^{-3}$, $SFP_{FB} = 10^{-2}$, $SFP_{Fwg} = 10^{-3}$, $SFP_{SN1} = 10^{-2}$, $SFP_{Tsc} = 10^{-2}$, which are similar to those used by Bhushan and Rengaswamy.[9] The strategy is run considering that the uncertain data are the SFPs, and the inaccurate sensors are those which reveal the occurrence of the key faults. For reasons of space only the results obtained when the inaccurate sensors are those associated with the key fault vector $\mathbf{a}_4$ ($\mathbf{Js}$ = [SN2, P_4, P_{rb}, F_{wg}, SN1, T_{sc}]) are included in this work.

For different upper bounds on the capital resources (C^*), Table 14 presents the solution vectors and the values of U, $\phi_{s,4}$ and Xs. Furthermore, the procedure described in the previous section is applied to study the R of all the faults when no sensor fails. For each C^*, the set of nonresolvable faults is determined and also displayed in Table 14. It shows that, for $C^* < 1700$, some faults are nonresolvable. For $C^* = 1700$, the R of all the faults is achieved at minimum cost, and the solution is equal to that obtained solving Problem 21 or Problem 23 using the constraint $RD_4 \geq 0$. An increment of the capital resources ($C^* > 1700$) originates the increase of $\phi_{s,4}$ and/or Xs, but U remains at (−5) because the fault $\mathbf{a}_1$ and the pseudofaults (1,2), (2,3), and (5,6) restrict the U value.

For the same C^* values considered in Table 14, a simulation study is carried out to analyze the R of the key fault $\mathbf{a}_4$ when the instruments included in ($\mathbf{J}_\mathbf{x} \cap \mathbf{J}_\mathbf{s}$) are out of service one at a time. Table 15 contains some results of the analysis performed using the modified technique outlined in Subsection 3.1. From that table, it can be observed that for $C^* < 2300$ the key fault is nondistinguishable or nonobservable. For $C^* = 2300$, the R of the key fault is attained, but the optimal SN cost is greater than that corresponding to the sensor arrangement obtained solving Problem 23 using the constraint $RD_4 \geq 1$ (see the third row of Table 12 and the sixth row of Table 14).

A similar analysis is performed considering that two of the sensors included in ($\mathbf{J}_\mathbf{x} \cap \mathbf{J}_\mathbf{s}$) are out of service for the same C^* values reported in Table 14. For reasons of space the corresponding results table has been omitted from this article. Simulation results indicate that the R of the key fault is achieved when C^* is set at 2900. The solution is made up of all the sensors. Therefore its cost is greater than that corresponding to the SN obtained when Problem 23 is solved considering the restriction $RD_4 \geq 2$ (see the fourth row of Table 12 and the last row of Table 14).

These simulation studies demonstrate that if the modified reliabilities-based methodology is run to satisfy the R of the key

Table 14. FCCU—Resolution of All Faults for Different Capital Resources

C*	X	U	$\phi_{s,4}$	Xs	nonresolvable pair of faults
1000	SN2 P_4 P_{rb} F_{wg}	−2	11	400	1; 2; 3; 1,2; 1,3; 2,3; 4,5; 4,6; 5,7
1200	SN2 P_4 P_{rb} F_{wg} SN1	−2	13	100	1; 2; 3; 1,2; 1,3; 2,3; 4,5; 4,6; 5,7
1500	SN2 P_4 P_{rb} F_{wg} SN1	−2	13	100	1; 2; 3; 1,2; 1,3; 2,3; 4,5; 4,6; 5,7
1700	SN2 SN3 F_c F_B F_{wg}	−5	2	0	-
2200	SN2 SN3 F_c F_B F_{wg}	−5	2	500	-
2300	SN2 P_4 SN3 F_c F_B F_{wg} SN1	−5	7	0	-
2700	SN2 P_4 SN3 F_c F_B F_{wg} SN1	−5	7	400	-
2900	SN2 P_4 P_{rb} SN3 F F_B F_{wg} SN1 T_{sc}	−5	12	0	-

Table 15. FCCU—Resolution of Key Faults for Different Capital Resources: Loss of One Measurement of $(J_x \cap J_s)$

C*	failed sensor	nonobservable and nonresolvable faults
1200	SN2	4,5; 4,6
	P_4	4,5; 4,6
	P_r	4,5; 4,6
	F_w	4,5; 4,6
	SN1	4,5; 4,6
1700	SN2	-
	F_{wg}	4,7
2200	SN2	-
	F_{wg}	4,7
2300	SN2	-
	P_4	-
	F_w	-
	SN1	-
2900	SN2	-
	P_4	-
	P_r	-
	F_{wg}	-
	SN1	-
	T_{sc}	-

faults when sensors fail, even at the expense of extra computational work, the solution set may differ from that obtained using the strategy proposed in this work because the first procedure uses inaccurate information.

For some capital resources, worst-case simulations are also carried out by increasing the SFPs one by one. Increases of 1 from the SFPs nominal values on the $\log_{10}$ scale are performed. Results show that some solutions are different from the ones presented in Table 14. For instance, increasing the SFP of F_{wg} for $C^* = 1700$, the new solution is $\mathbf{x}$ = [SN2 P_4 P_{rb} F_{wg} SN1 T_{sc}]. By applying the modified procedure described in the previous section, it is determined that the faults $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are nonobservable and the pairs of faults 1,2; 1,3; 2,3; 4,5; 4,6; and 5,6 are nonresoluble. Therefore the categorization of faults in resolvable and nonresolvable is not unique, since it depends on the SFP values.

5. CONCLUSIONS

This work is devoted to presenting a new strategy to design an optimal SN, which is able to resolve a set of key faults under the presence of failed sensors or outliers.

The required data are the cause−effect relations among faults and variables, the set of key failures, and sensor costs. All these low uncertainty data are readily available by plant designers. The categorization of faults allows taking into account both faults occurrence probability and consequences at the design stage in a simple way.

The procedure deals with the key failures isolation when sensor malfunctions occur from a structural point of view. With this purpose key faults RD constraints are incorporated into the minimum-cost design problem. It involves a linear objective function and a set of linear inequality restrictions; consequently this problem is solved using MILP codes. Given that constraints are only expressed in terms of the original *I* binary variables and existing procedures for eliminating redundant equations are applied, the solution of the resulting optimization problem is rather simple.

It should be noticed that the incorporation of RD constraints for the key faults frequently enhances their R when no sensor fails. In general the R of these failures can be done using more than one **rv**; that is, alternative ways to verify their failure condition exist under the absence of sensor malfunctions.

Furthermore a modified reliabilities-based methodology is outlined in this work which can be applied to analyze the R of all the faults when no sensor fails and the R of the key faults under the presence of missing measurements or outliers, but at the expense of an extra computational work. It should be noticed that the categorization of faults in resolvable and nonresolvable obtained using the modified technique is not unique, because it depends on the selected values of the SFPs. For the same reason the solution set may differ from the one achieved using the strategy proposed in this work. These conclusions arise from the results attained by the application of the structural method to case studies extracted from the literature and those provided by the simulations conducted to put it into the context of other existing techniques.

In conclusion, the structural procedure presented in this article can be straightforwardly used by engineers to tackle the proposed SNDP due to the low uncertainty of the required information and the simplicity of the optimal design formulations.

In future works studies about the design of SNs devoted to detect and identify process faults using particular monitoring strategies will be carried out.

■ ASSOCIATED CONTENT

Supporting Information

Application examples of the evaluation of constraints. This material is available free of charge via the Internet at http://pubs.acs.org.

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Notes

The authors declare no competing financial interest.

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NOTATION

A fault-sensor connectivity matrix
$\mathbf{a}_j$ *j*th fault vector
C sensor cost vector
C_T Budget of the instrumentation project
g number of missing measurements
I number of measurable variables
J number of failures
NNV number of non-null vectors
PFP process failure probability
R_j number of measurements contained in $\mathbf{rv}_j$
RD_j *j*th fault resolution degree
$\mathbf{rv}_j$ *j*th fault resolution vector
$\mathbf{rv}_j^r$ *j*th fault residual resolution vector
S number of key faults
SFP sensor fault probability
t_j number of sensors configurations used to resolve the *j*th fault when sensors fail
U_j *j*th fault detection unreliability
x solution vector
Xs slack variable for the cost restriction
$\phi_{s,j}$ slack variable associated with the unreliability detection constraint of the uncertain *j*th fault

ACRONYMS

CSTR continuous stirred tank reactor
FCCU fluid catalytic cracker unit
MILP mixed-integer linear programming
O observability
R resolution
RD resolution degree
rv resolution vector
S/MR single/multiple resolution
SNDP sensor network design problem
U detection unreliability

REFERENCES

(1) Bagajewicz, M. J. A review of techniques for instrumentation design and upgrade in process plants. *Can. J. Chem. Eng.* **2002**, *80*, 3.
(2) Nguyen, D. Q.; Bagajewicz, M. J. New efficient breadth-first/level traversal tree search method for the design and upgrade of sensor networks. *AIChE J.* **2011**, *57*, 1302.
(3) Lambert, H. E. Fault trees for locating sensors in process systems. *Chem. Eng. Prog.* **1977**, *73*, 81.
(4) Chuei-Tin, C.; Kai-Nan, M.; Chii-Shiang, T. A simple design strategy for fault monitoring systems. *AIChE J.* **1993**, *39*, 1146.
(5) Raghuraj, R.; Bhushan, M.; Rengaswamy, R. Locating sensors in complex chemical plants based on fault diagnostic observability criteria. *AIChE J.* **1999**, *45*, 310.
(6) Bhushan, M.; Rengaswamy, R. Design of sensor network based on the signed directed graph of the process for efficient fault diagnosis. *Ind. Eng. Chem. Res.* **2000**, *39*, 999.
(7) Bagajewicz, M.; Fuxman, A.; Uribe, A. Instrumentation network design and upgrade for process monitoring and fault detection. *AIChE J.* **2004**, *50*, 1870.
(8) Bhushan, M.; Rengaswamy, R. Design of sensor location based on various fault diagnostic observability and reliability criteria. *Comput. Chem. Eng.* **2000**, *24*, 735.
(9) Bhushan, M.; Rengaswamy, R. Comprehensive design of a sensor network for chemical plants based on various diagnosability and reliability criteria. 1. Framework. *Ind. Eng. Chem. Res.* **2002**, *41*, 1826.
(10) Bhushan, M.; Narasimhan, S.; Rengaswamy, R. Robust sensor network design for fault diagnosis. *Comput. Chem. Eng.* **2008**, *32*, 1067.
(11) Kotecha, P. R.; Bhushan, M.; Gudi, R. D. Constraint programming based robust sensor network design. *Ind. Eng. Chem. Res.* **2007**, *46*, 5985.
(12) Kolluri, S.; Bhushan, M. Robust Reallocation and Upgrade of Sensor Networks for Fault Diagnosis. *Comput.-Aided Chem. Eng.* **2011**, *29*, 769.
(13) Musulin, E.; Bagajewicz, M.; Nougués, J. M.; Puigjaner, L. Instrumentation Design and Upgrade for Principal Components Analysis Monitoring. *Ind. Eng. Chem. Res.* **2004**, *43*, 2150.
(14) Gerkens, C.; Heyen, G. Sensor placement for fault detection and localization. *Comput.-Aided Chem. Eng.* **2008**, *25*, 355.
(15) Bhushan, M.; Rengaswamy, R. A framework for Sensor Network design for efficient and reliable fault diagnosis. *Proceedings of the 4th IFAC workshop on On-line Fault Detection and Supervision in the Chemical Process Industries*, Korea, June 7–8, 2001; Pergamon: London, 2001; p 33.
(16) Kramer, M. A; Palowitch, B. L. A rule-based approach to fault diagnosis algorithm using the signed directed graph. *AIChE J.* **1987**, *33*, 1067.
(17) McFarlane, R. C.; Reineman, R. C.; Bartee, J. F.; Gerogakis, C. Dynamic Simulator for a model IV in Fluid Catalytic Cracking Unit. *Comput. Chem. Eng.* **1993**, *17*, 275.