A Genetic Algorithm for Adaptive Tomography of Elliptical Objects

Marcelo Venere, Hstau Liao, and Alejandro Clausse

Abstract—**A probabilistic algorithm for on-line tomographic reconstruction of ellipse-like images is presented. The algorithm takes advantage of the characteristic preferential direction of the objects, constructing a guidance function to select the angles for subsequent radiographic projections. The simulation results confirm that the technique reduces the number of projections required to achieve a given quality limit.**

Index Terms—**Adaptive reconstruction, genetic algorithm, mutation filter, tomography.**

I. Introduction

Recent advances in radiation technology have given rise to new challenges in image processing and recognition. Radiation pulses emitted by dense plasmas provide a unique characteristic compared with other radiation devices, namely, very short flashes (10 ns) of high intense beams. This feature opens interesting possibilities in industry and medicine. Defectoscopy, failure identification, and hydride detection are some examples of the possible services that can be performed online, leading to substantial cost reductions and quality enhancement.

Images of the internal structure of key components or critical parts in production lines have an enormous potential from the point of view of quality control. One can imagine a step in a production line where the quality of certain critical components can be monitored automatically by means of tomographic identification of material defects. However, this step will introduce some time cost, which should be minimized to be applicable in real production lines.

The present work is oriented to develop an image processing system that takes advantage of the features of plasma radiation flashes, optimizing the emission-detection-reconstruction procedure. A genetic algorithm (GA) is used to design a strategy for an interactive on-line scanning of elliptical objects from limited projection data. The aim is the detection of cracks in materials, which generally grow developing ellipse-like shapes located in grain boundaries [1]. The present technique is in the line of other geometric-based reconstruction methods [2], [3], introducing the novelty of the adaptive setting.

II. Angle Spacing Distribution

A radiation beam passing through a body at a given angle θ produces a projection profile on a screen located behind the object. Given a set of M projection profiles, the original image can be reconstruoted solving the inverse problem, known as computed tomography. A popular reconstruction algorithm is the algebraic reconstruction technique (ART), which solves a set of algebraic equations representing the digital projections [4]. This is the procedure followed whenever a reconstruction is required along the present paper. ART is particularly suited for adaptive tomography for additional projections in that the transformation matrix is expanded, reducing computational costs when expanding the set of projections.

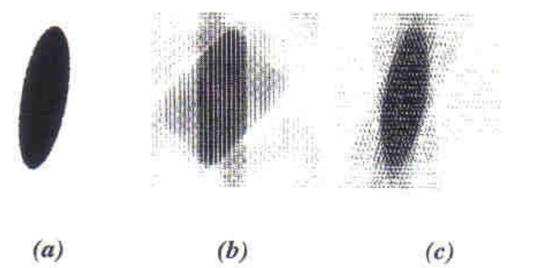

Fig. 1. Reconstruction of an ellipse using four projections. (a) Original image, (b) uniform spacing (0°, 45°, 90°, 135°), and (c) nonuniform spacing (0°, 63°, 80°, 98°).

Usually, the M projection angles are distributed uniformly in the interval $0, \pi$. However, this is not always the best strategy. Fig. 1 compares the reconstructions of an ellipse in (a), using four projections, equally spaced in (b), and with an appropriate selection of the perspectives in (c). It can be seen that substantial quality differences can be achieved with a careful selection of the view angles, and it is reasonable to take this into account for the design of an intelligent control of the angles the object will be rayed through.

The particular characteristic of the angle distribution of Fig. 1(c) is that most of the angles accumulate close to the direction of the main axis of the ellipse. The latter is valid for ART reconstructions. However, when using other methods, it is always possible to find a projection distribution that maximizes the information about the underlying object. An interesting study of this problem can be found in [5].

In this example, the ellipse is known prior to the reconstruction. In a real tomographic process, the position and orientation of the object is not known *a priori* and consequently, a suitable algorithm should be provided to estimate these parameters.

Let us consider a digital representation e_{ij} of an ellipse in a square domain

$$e_{ij} = \begin{cases} 1, & \text{inside the ellipse} \\ 0, & \text{otherwise} \end{cases}$$

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The position of the center of the ellipse can be calculated by determining the position of the center of mass x_c, y_c of the image

$$(\boldsymbol{x}_c, \boldsymbol{y}_c) = \frac{\sum\limits_{pixels} (\boldsymbol{x}_{ij}, \boldsymbol{y}_{ij}) \boldsymbol{e}_{ij}}{\sum\limits_{pixels} \boldsymbol{e}_{ij}}. \tag{1}$$

A useful indicator of the projection angles that provide information about the ellipse orientation is the standard deviation of the image with respect to a line passing through x_c, y_c, that is

$$\sigma(\theta) = \sqrt{\frac{\sum\limits_{pixels} e_{ij} d_{ij}^2(\theta)}{N_{pixels}}} \tag{2}$$

where $d_{ij}(\theta)$ represents the distance of the pixel i, j from a line with an orientation θ. The minimum σ corresponds to the direction of the main axis. Since better reconstructions can be achieved accumulating more views about the main axis, the function $\sigma(\theta)$ is useful to construct an estimate of the convenient angle distribution

$$p(\theta) = \frac{e^{-\sigma(\theta)}}{\int_0^\pi e^{-\sigma(\theta')}\, d\theta'}. \tag{3}$$

Equation (3) can be seen as the energy distribution of a collection of thermodynamic states, where $e^{-\sigma(\theta)}$ plays the role of a Boltzmann factor, and $\sigma(\theta)$ corresponds to an energy to be minimized [6]. The reconstruction of the ellipse shown in Fig. 1(c) corresponds to four angles distributed according to (3).

III. GA for Adaptive Tomography

GA's have been successfully applied to the detection of voids in geotomography [7]. GA is an iterative technique, in which the system is described by a population that randomly mutate following fitness criteria, which indicates the closeness to the optimum solution. Each individual of the population is a representation of the problem under consideration. The population size is usually kept constant over the entire run. In contrast, if the algorithm is applied *during* the radiographic session, each new angular projection introduces additional information. Consequently, the number of individuals (i.e., the angles) increases with each step. The "chromosome" of our GA is represented by an array whose elements are real numbers in the interval $(0, \pi)$, representing the directions of the projection views. The algorithm is genetic in the sense that each new angle is determined randomly, although it is biased by the information obtained in previous steps.

In real tomographies, the guidance angle distribution $p(\theta)$ cannot be used as the fitness function for the GA, because the actual object is unknown. However, it is still possible to construct successive approaches of $p(\theta)$ using partial reconstructions, which can be applied to bias the "birth" of each new angle.

The described GA should be complemented by a *filter mutation* to prevent the occurrence of redundant angles adding little information to the solution. For simplicity, the *"I've-already-got-that"* filter is incorporated into the fitness function as modulation factor, which creates dark zones around the angles previously visited. The fitness function for the n-angle is

$$p_n(\theta) = ke^{-\sigma_n(\theta)} \prod_{i=1}^{n-1} \frac{|\theta - \theta_i|}{5^\circ + |\theta - \theta_i|} \tag{4}$$

where

θ_i angle projection in the step-i;

$\sigma_n(\theta)$ standard deviation of the partial reconstruction in the n-step;

k appropriate normalization coefficient.

The filter can be any function that vanishes close to θ_i and is ineffective anywhere else. The range of influence, 5°, was chosen for practical convenience.

The complete algorithm is summarized in the following set of rules:

1) Start with two perpendicular projections.
2) Reconstruct a partial image, $e_{ij}(n)$.
3) Calculate an estimate of $\sigma_n(\theta)$ applying (2) to $e_{ij}(n)$.
4) Calculate the guidance function, $p_n(\theta)$ applying (4).
5) Randomly generate a new projection angle with probability distribution $p_n(\theta)$.
6) Repeat rules 2)–5) while the quality enhancement between $e_{ij}(n)$ and $e_{ij}(n-1)$ exceeds certain convergence criterion.

IV. Results and Discussion

The GA was implemented in $C++$ and applied to an ellipse with aspect ratio $\varepsilon = 0.25$. Fig. 2 shows the evolution of $p_n(\Theta)$ during the reconstruction sequence of an ellipse. In the first step [Fig. 2(a)], the shadow resulting from the partial reconstruction using two projections is used to produce a preliminary estimate of the orientation and to assess where the next view should be rayed (i.e., angles with larger $p_1(\theta)$. Fig. 2(b) shows the second step in the reconstruction. A new angle was taken (75°) and consequently, the filter algorithm reduced the probability in the corresponding neighborhood. Finally, in the forth step [Fig. 2(c)], the quality of the image was highly improved, and the maxima of the guidance function $p_7(\theta)$ indicated the views that would provide more information in future shots. Fig. 3 shows the evolution of the deviation $\sigma(\theta)$ during the adaptive process. It can be seen that the exact solution is quickly approached in a few steps.

The proposed interactive GA was compared to tomographies using equally spaced angles in order to assess its efficiency. The quality of the reconstruction can be measured using a distortion indicator defined by

$$d = \sqrt{\sum_{ij} (e_{ij} - e'_{ij})^2} \tag{5}$$

where e'_{ij} is the reconstruction of the original pixel e_{ij}. The performance is measured by counting the number of projections needed to reduce the distortion below a given value. Ultimately, this metric will represent savings in irradiation doses or monitoring costs, depending on the particular application of the tomography. Fig. 4 compares the performance of the present algorithm with uniform spacing. It can be seen that the same distortion limit can be achieved more efficiently using the proposed algorithm. Studies with different ellipses show good results even for eccentricities up to 0.9.

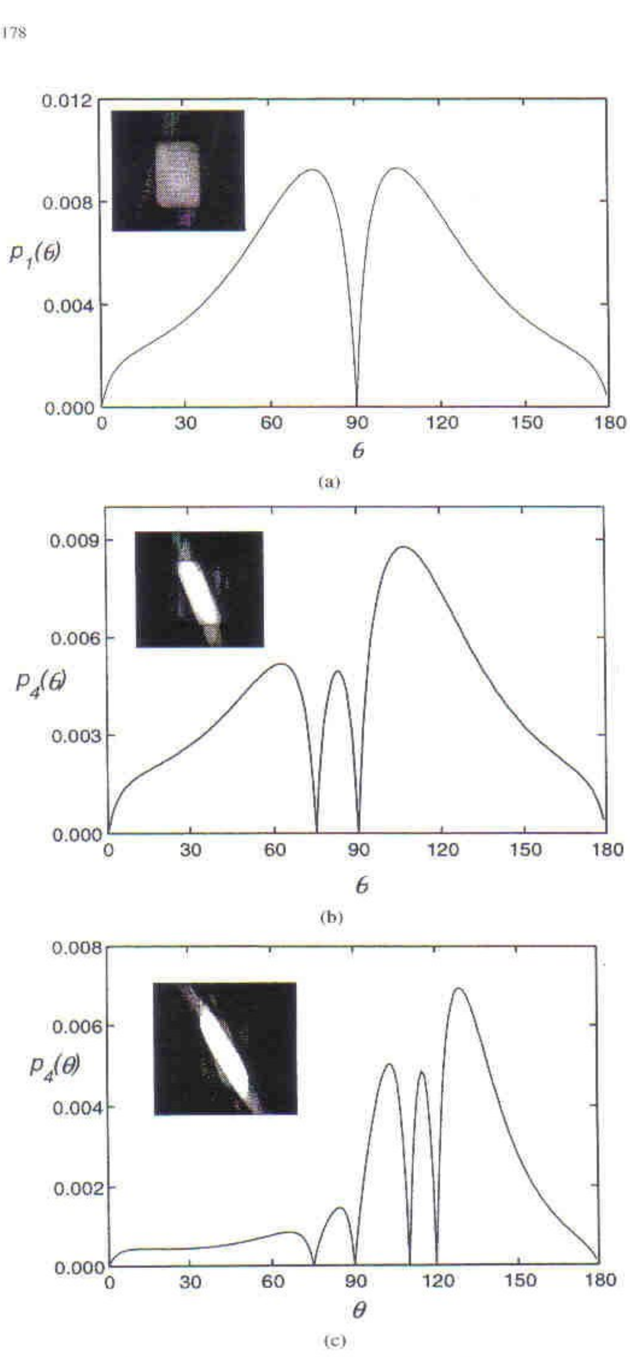

Fig. 2. (a) Partial reconstruction (step 4), (b) partial reconstruction (step 2), and (c) partial reconstruction (step 4).

V. Conclusions

A GA has been successfully applied to the design of an interactive procedure for on-line tomographic reconstructions of ellipse-like images, which are predominant in fracture cracks. The algorithm takes advantage of the preferential direction characteristic of the objects, constructing a fitness function to guide the selection of the angles for subsequent radiographic projections. The simulation results confirm that the technique allows the reduction of the number of projections required to achieve a given

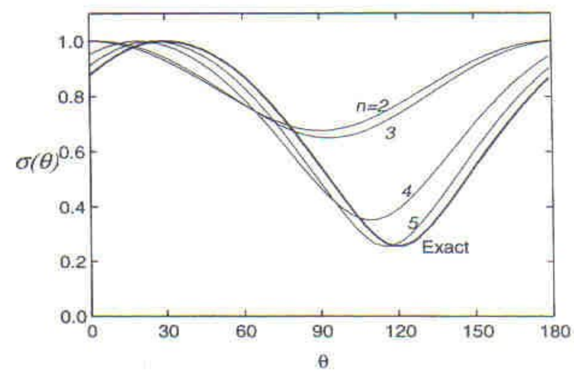

Fig. 3. Adaptive approach of the deviation σ to the exact solution.

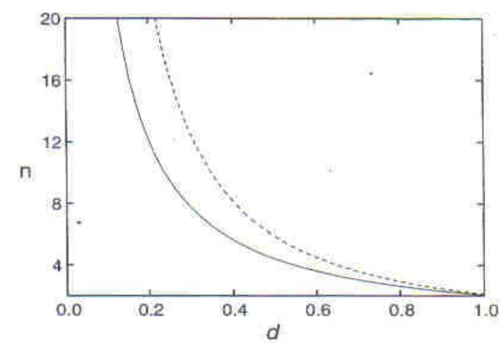

Fig. 4. Number of projections required to achieve a given distortion. Uniform spacing (dashed line), probabilistic algorithm (solid line).

quality reconstruction limit. Other optimization techniques can also be applied to the idea of adaptive tomography. Furthermore, the conceptual procedure can be extended to three dimensions and more general objects involving other appropriate fitness functions, covering families of objects sharing similar shapes. In general, the degree of improvement obtained and the definition of efficient indicators remain to be studied. The results of the present work suggest that symmetry measures can be further explored in dealing with other shapes.

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