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# Dynamical behaviour of beams and plates

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## ABSTRACT

This article presents a review of the research results obtained in the ICMASA (Institute of Civil Engineering and Environment of Salta). Reference information sources are primarily papers in the open literature generated in the last years. The results reviewed correspond to the area of analysis of the statical and dynamical behaviour of beams and plates with complicating effects. Also some relevant results obtained by other investigators are included.

## 1. INTRODUCTION

The basis of modern vibration analysis can be found in the classical textbooks: A Treatise on the Mathematical Theory of Elasticity by A. Love and The Theory of Sound by Lord Rayleigh. The advent of the digital computer made it possible to generate semi-closed form solutions of a great number of vibrating systems. On the other hand the development of large digital computers allow the construction of algorithms which simulate the systems directly using the famous Finite Element Method. Today the natural frequencies, mode shapes and other responses of almost all linear system can be obtained with the use of the mentioned method. Nevertheless the closed form solutions and approximate analytical solutions which can be obtained, for example, with the classical methods of Galerkin and Ritz are particularly interesting. The physical insight into the nature of the solutions is a great advantage of this type of solutions.

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The determination of natural frequencies in transverse vibration of beams and plates with complicating effects, are problems that have been extensively studied by several researches in the last decades.

This article presents a review of the research results obtained in the ICMASA (Institute of Civil Engineering and Environment of Salta). Information sources referenced are primarily papers in the open literature generated in the mentioned institute in the last years.

The major part of the articles present solutions obtained with the classical Rayleigh and Rayleigh-Ritz methods and some of the extensions, such as the optimized Rayleigh method.

## 2. VIBRATING BEAMS.

Several works dealing with transverse vibrations of tapered beams have been published. Mabie and Rogers [1,3] have presented analyses for tapered beams with different end conditions. Goel [4] analysed the case of transverse vibrations of linearly tapered beams elastically restrained against rotation at either end. Laura et. al. [5,6] treated several cases of non-uniform beams.

Grossi and Bhat [7] presented the analysis of the approximate determination of frequency coefficients of linearly tapered beams with ends elastically restrained against rotation. Two approaches were used: the modified Rayleigh-Schmidt method and the characteristic orthogonal polynomials method developed by Bhat. The approximate determination of frequency coefficients of linearly tapered beams elastically restrained against rotation at one end and with a concentrated mass at the other, was considered in [8].

Grossi and Aranda [9] presented the construction and application of a functional that yields the mathematical model which describes the dynamical behaviour of Timoshenko Beams with general boundary conditions.

Auciello [10] showed that a discrepancy of values obtained by Grossi and Bhat in [7] and the values obtained by Goel in [4], was due to a trivial error in the interesting Goel theory.

Grossi and Aranda [11] considered the application of an extension of the Rayleigh method to generate results for a vibrating beam which supports a concentrated mass. A discussion about the analytical and numerical performances is included.

Grossi and Arenas [12] presented a work concerned with the use of the Rayleigh-Schmidt method in the determination of frequencies corresponding to the first two modes of vibration of a linearly tapered beam with both ends elastically restrained against rotation and translation.

Grossi et. al. [13] considered the problem of weak solutions in beams. The existence and uniqueness of the weak solutions of a boundary value problem and an eigenvalue problem, which correspond, respectively, to the statical and dynamical behaviour of a tapered beam with ends generally restrained, has been demonstrated.

Arenas and Grossi [14] analysed the exact and approximate determination of frequency coefficients of a uniform beam, with one end spring-hinged and a rotational restraint in a variable position.

Nallim and Grossi [15] presented a simple, accurate and flexible general algorithm for the study of a great number of beam vibration problems. The approach has been developed based on the Rayleigh-Ritz method with characteristic orthogonal polynomial shape functions.

## 3. VIBRATING PLATES.

### 3.1 ISOTROPIC PLATES.

The determination of natural frequencies in transverse vibration of isotropic plates with complicating effects, such as elastically restrained edges, presence of holes with free edges, variable thickness, etc, is a problem that has been extensively studied by several researches.

Leissa's works [16,17] constitute excellent compilations of the literature concerning isotropic plates.

Grossi and Bhat [18] studied the problem of natural frequencies of tapered plates with edges elastically restrained against rotation and translation by using boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method and applying the Rayleigh-Schmidt method.

Grossi and Laura [19] used the optimized Rayleigh-Ritz method to generate values of the fundamental frequency and the one corresponding to the first fully antisymmetric mode for rectangular plates elastically restrained against rotation and with located circular holes.

Arenas and Grossi [20] dealt with the development and application of a general algorithm for the determination of values of frequency coefficients for a rectangular plate with a central free hole.

### 3.2 ORTHOTROPIC AND ANISOTROPIC PLATES.

Natural frequencies of orthotropic and anisotropic plates with any combination of classical boundary conditions, (i.e.: free, clamped and simply supported) have been studied extensively. Reference [21] is an important survey of the literature concerning dynamics of plate-type structural elements of composite material. Laura and co-workers have supplied much of the information regarding the use of polynomial expressions as approximating functions and the treatment of elastically restrained boundaries [22,28].

In the articles [29] and [30], Ashton considered the analysis of anisotropic plates using the Ritz method in conjunction with beam mode shape functions to approximate the deflected shape. These papers considered clamped, simply-supported and free edges. In reference [31] this author extended the analysis to include elastically restrained edges.

Grossi [32] considered the application of the Rayleigh-Ritz method to generate results for a great number of flexural vibrations problems for rectangular orthotropic and anisotropic plates.

Laura, Bambill and Grossi [33] presented a discussion about relevant references which have been omitted by the authors of the article [34] in which they analysed the free vibration of anisotropic rectangular plates with various boundary conditions.

Nallim, Grossi and Laura [35] proposed a simple approach for determining the fundamental frequency of transverse vibration of a circular plate of rectangular orthotropy carrying a central concentrated mass.

Nallim and Grossi [36] studied the problem of natural frequencies of tapered orthotropic rectangular plates with a central free hole and edges restrained against rotation and translation by using orthogonal polynomials in the Rayleigh-Ritz method and applying a generalization of the Rayleigh method.

Grossi and Lebedev [37] dealt with the application of the Ritz method to the determination of the natural frequencies of a rectangular anisotropic plate with a free corner formed by the intersection of two free edges.

Grossi [38] demonstrated the existence and uniqueness of the weak solutions of boundary value problems and eigenvalue problems, which correspond, respectively, to the statical and dynamical behaviour of rectangular anisotropic plates with edges elastically restrained against rotation.

## 4. CONSIDERATION ABOUT VARIATIONAL METHODS.

After Walter Ritz presented in 1908 his now famous variational method, several mathematician became interested in his method and gave it lengthy treatment, [39, 43]. On the other hand, investigators in the field of applied sciences generated an immense quantity of papers in which approximate solutions of various problems of mathematical physics were constructed with the aid of the mentioned method. Particularly, the Ritz method has been used extensively over the years to study the problem of flexural vibration of rectangular isotropic, orthotropic and anisotropic plates.

Grossi [44] used a problem to illustrate the following relevant property: when using the Ritz method we choose a sequence of functions $v_i$ which constitute a base in the space $V$, where only the homogeneous stable boundary conditions are included, so there is no need to subject the functions $v_i$ to the natural boundary conditions. The fact that the natural boundary conditions of a system need not be satisfied by the chosen co-ordinate functions is a very important characteristic of the Ritz method, especially when dealing with problems for which such satisfaction is very difficult to achieve. For instance, this is the case of a rectangular anisotropic, orthotropic or isotropic plate with edges elastically restrained against rotation.

Grossi [45] presented an informative review of several applications of the Rayleigh-Schmidt method and also he has shown that this technique yields accurate results in several rather difficult elastodynamic problems. In [46] a discussion about certain general boundary conditions is presented. Rigorous considerations by means of functional analysis are stated. In [47] it is shown that the use of a two-term approximating function with several undetermined exponents in the Rayleigh- Schmidt method leads to a simple and accurate algorithm for the determination of the fundamental frequency of a vibrating beam.

Grossi and Mac Gaul [48] demonstrated that for certain assumed functions, the use of more than one adjustable exponent in the optimised Rayleigh method, does not increase the labour and difficulties in the analytical developments, but instead it leads to an improvement of the accuracy in the numerical results.

Grossi [49] presented a brief review of asssumed-mode methods and the approximate solutions of several vibrating problems.

Grossi and Albarracin [50] presented a variant of Bhat's method, based on the use of the Rayleigh-Schmidt method of undetermined powers. This new procedure allows the use of a lower number of orthogonal polynomials than the classical Bhat's method, avoiding cases of numerical instability.

Grossi and Albarracin [51] dealt with the applicability of the Rayleigh-Ritz method for the determination of frequency coefficients of beams and plates. It has been shown that the approximate satisfaction of boundary conditions introduces additional constraints into the formulation that bring unexpected results. The adequate procedure for constructing the co-ordinate functions to avoid numerical errors has also been included.

Lebedev and Grossi [52] outlined the connection between the traditional ideas of mechanics and the newer mathematical concepts of generalised solutions and distributions.

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