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The multi-echelon vehicle routing problem with cross docking in supply chain management

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ABSTRACT

Multi-echelon distribution networks are quite common in supply chain and logistics. Deliveries of multiple items from factories to customers are managed by routing and consolidating shipments in warehouses carrying on long-term inventories. On the other hand, cross-docking is a logistics technique that differs from warehousing because products are no longer stored at intermediate depots. Instead, cross-dock facilities consolidate incoming shipments based on customer demands and immediately deliver them to their destinations. Hybrid strategies combining direct shipping, warehousing and cross-docking are usually applied in real-world distribution systems. This work deals with the operational management of hybrid multi-echelon multi-item distribution networks. The goal of the N-echelon vehicle routing problem with cross-docking in supply chain management (the VRPCD-SCM problem) consists of satisfying customer demands at minimum total transportation cost. A monolithic optimization framework for the VRPCD-SCM based on a mixed-integer linear mathematical formulation is presented. Computational results for several problem instances are reported.

1. Introduction

Industrial companies usually accomplish a series of activities such as purchasing raw materials from suppliers, manufacturing and storing end-products at intermediate facilities, and delivering them to final customers. Suppliers, manufacturers, warehouses and customers are the major components of the so-called *supply chain* (SC) carrying goods from the upstream to the downstream side of the SC. Four major business functions are performed in a supply chain: purchasing, manufacturing, inventory and distribution. The latter one is concerned with both the transportation of raw materials or parts from suppliers to factories, and the shipping of finished products from factories to demand locations. Since the major supply chain functions are strongly interrelated by materials and information flows, they cannot be individually managed (Cohen & Lee, 1989; Vidal & Goetschalckx, 1997). A good coordination of them is a critical issue in most manufacturing companies. *Supply chain management* (SCM) aims to efficiently control the material flow through the supply chain so as to improve its performance as a system. An effective SCM helps to substantially reduce operational costs and increase the customer service level. On the downstream side of the supply chain, *distribution* involves the transfer of multiple final items from factories to demand points directly or via transshipment facilities. These additional components of a transportation network are usually distribution centers (DCs) or warehouses. They act as intermediate locations between factories and end customers to both facilitate the consolidations of shipments from different suppliers and meet customer demands on peak periods through the accumulation of product inventories. In this way, lower transportation costs and faster response times are achieved at the expense of increasing terminal and inventory costs. The difficulties of managing inventories rise substantially for a distribution network with multiple tiers of locations. Distribution from many origins to many destinations is the essence of logistics (Langevin, Mbaraga, & Campbell, 1996). Savings in transportation costs by using N-echelon networks ($N \geq 2$) are also partly due to economies of scale because vehicles of different sizes are used at different levels. Line haul trucks are assigned to inbound transportation moving end products from factories to intermediate facilities where they are stored. Loads are later transferred to delivery vehicles having lower capacity and serving between such facilities and the final destinations. In addition to storing products for some period of time, two further tasks are usually performed at distribution networks involving DCs and warehouses, namely consolidation and break-bulk operations. *Consolidation* consists of combining shipments of similar or different products from several origins at the distribution center. *Break-bulk* is the opposite function through which a large load from a given origin is split into multiple, smaller shipments that are delivered to customers.

Another type of intermediate stage is the *cross-dock* facility where break-bulk operations over ingoing, consolidated shipments are carried out right after they arrive at the depot. Such loads are

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Nomenclature

Sets

- A minimum-cost arcs
- I nodes (factories, warehouses, distribution centers, customers)
- N events
- P products
- V vehicles
- I_v nodes that can be serviced by vehicle v
- ID destination nodes ($ID \subset I$)
- IM cross-dock facilities ($IM \subset I$)
- IS factories ($IS \subset I$)
- IB_v candidate base nodes for vehicle v
- ID_p set of destinations requiring product p
- IS_p set of factories producing for product p
- IM_p set of warehouses delivering product p
- N_i set of events for node i
- V_i set of vehicles that can visit node i

Parameters

- D_{ip} amount of product p demanded by node i
- II_{ip} initial inventory of product p at source i
- $FINV_{ip}$ end inventory of product p specified for cross-dock facility i
- a_i earliest service time at node i
- b_i latest service time at node i
- c_{ij} routing cost between nodes i and j
- co_v penalty cost per unit overtime for vehicle v
- fc_v fixed cost of using vehicle v
- ft_i fixed stop time at node i
- qw_v maximum weight capacity of vehicle v
- qv_v maximum volume capacity of vehicle v
- t_v^{max} maximum allowed routing time for vehicle v
- t_{ij} travel time between nodes i and j
- uv_p unit-volume of product p
- uw_p unit-weight of product p
- vt_{ip} unit load/unload time for product p at node i
- M_C,M_T,M_L big-M values for travel cost, travel time and load constraints

Binary variables

- $X_{ni,n'i'}$ variable sequencing events n and n' taking place at nodes i and i'
- Y_{niv} variable denoting that vehicle v visits node i at event $n \in N_i$

Continuous variables

- AI_{nip} additional amount of product p received from other sources at node i after the event n
- $L_{ni,pv}$ amount of product p loaded on vehicle v during stop (n, i) at source i
- $U_{ni,pv}$ amount of product p delivered by vehicle v to node i during stop (n, i)
- $AL_{ni,pv}$ total amount of product p loaded on vehicle v from the start to stop (n, i)
- $AU_{ni,pv}$ total amount of product p delivered by vehicle v from the start to stop (n, i)
- C_{ni} travel cost for the vehicle visiting node i from the start to stop (n, i)
- CV_v overall travelling cost for vehicle v
- T_{ni} travel time for the vehicle visiting node i from the start to stop (n, i)
- OT_v total travel time for vehicle v

properly sorted and dispatched to customers by outgoing vehicles without delay. In other words, cross-docking implies the rapid movement of products from the receiving dock to the shipping dock at the cross-dock facility, where they stay for a short time before delivery to customers. Residence time of shipments at cross-dock facilities, also called satellite platforms or simply *satellites*, is typically less than 24 h. In addition to providing a good customer service, cross-docking strategy has some important advantages over the traditional warehousing because it reduces inventory costs, storage space needs and order-cycle time, and accelerates cash flow (Cook, Gibson, & MacCurdy, 2005). Success stories on cross docking that resulted in considerable competitive advantages have been reported by several industries having significant proportions of distribution costs like food and beverage producers, pharmaceutical companies, automobile manufacturers and retail chains. A real world setting from the food industry has recently been presented by Boysen (2010). The peculiarity of frozen foods and other refrigerated products, e.g. pharmaceuticals, is that the cooling chain must be intact. Once a shipment is unloaded at the intermediate facility, it must be instantaneously loaded into a cooled outbound trailer. No intermediate storage inside the uncooled terminal is allowed. Cross docking systems also work well for perishable products that need to reach the marketplace faster to preserve quality and freshness.

Different kinds of distribution networks are implemented by industrial companies. In *manufacturer storage with direct shipping*, the supply points are factories and the demand points are customers, i.e. a single-echelon strategy. When *distributor storage* is adopted, the distributors are intermediate facilities like DCs or warehouses and the demand points include customers and retailers. In this case, product stocks are exclusively located at the distribution center and all shipments are dispatched from the DC to customers. Moreover, there are no transshipment points and a single-echelon distribution network is still used. Manufacturer storage is usually planned for high-value products whose demands are difficult to forecast, while inventories of fast-moving items are stored at the DC to get a better responsiveness. Another type of distribution system includes factories and warehouses as supply points with product stocks located at both kinds of facilities and shipments going directly from manufacturers or via warehouses to customers, i.e. a two-echelon distribution network. If the stock on-hand at some warehouse is positive but lower than the demand size, it is used to partially meet the demand and the remaining portion is fulfilled through either transshipment of products from another source or direct shipping from the manufacturer storage. Complex N-echelon distribution systems may include more than a single layer of intermediate warehouses. On the other hand, the *cross-docking* strategy implies that product inventories are consolidated at the manufacturer storage with all shipments going from the factory to a central DC, where the receiving loads move from the receiving to the shipping dock in 24–48 h before dispatching them to consumer zones. It is also a two-echelon transportation network. Cross-docking retains the advantages of a centralized inventory at the manufacturing site and the consolidation of shipments at cross-dock facilities, i.e. manufacturing storage with in-transit merge. In any case, the selected network design should be tailored to the types of items to distribute and the needs of customers to service. A tailored distribution policy requires to operate hybrid networks combining manufacturer storage with warehousing and cross-docking. Companies in the same industrial segment often choose different network designs, mainly because their operational strategies are focused on different performance measures such as response time, product availability or customer satisfaction.

To effectively design and manage large-scale distribution networks, long-run strategic planning, medium-term tactical planning and short-term operational planning should be periodically developed (Simchi-Levi, Kaminsky, & Simchi-Levi, 2004). Distribution at the operational level is concerned with short-term inventory management and transportation planning. Transportation represents a substantial fraction of the total logistics cost. During the operational planning, vehicle routes and schedules are generated based on available resources, supplier and customer locations, and product demands. The problem objective is to minimize transportation costs while meeting customer service-level requirements like on-time deliveries. Although considerable research on the distribution problem has been carried out, the attention was mainly focused on strategic and tactical planning. Most optimization approaches for operational planning of N-echelon distribution networks are extensions of methods for the classical vehicle routing problem (VRP). To do so, they are usually decomposed into a number of single-echelon distribution problems. Moreover, there are very few papers dealing with N-echelon transportation networks involving cross-docking. This paper introduces a new monolithic optimization framework for the short-term operational planning of N-echelon multi-item distribution networks using warehousing and cross-docking strategies. Deliveries of products from manufacturers to clients through direct shipping and/or via warehouses and cross-dock points are simultaneously considered. Customer requirements at demand points that may include several types of products, and initial stocks at factories and warehouses are all known at the start of the planning horizon. Besides, the number and locations of suppliers, warehouses and cross-dock points are problem data.

2. Literature review

Extensive work has been done on N-echelon distribution systems but mainly focused on facility location and flow assignment issues (Amaro & Barboa-Povoa, 2008; Bonfill, Espuña, & Puigjaner, 2008; Jayaraman & Ross, 2003; Tsiakis, Shah, & Pantelides, 2001; Verderame & Floudas, 2009; You & Grossmann, 2008). Instead, vehicle routing has been treated in a simplified way or not explicitly considered. Two well-known distribution problems at the tactical level are the N-echelon location routing problem (NE-LRP) and the inventory routing problem (IRP). Most of the studies are related to two-echelon systems ($N=2$). The aim of the NE-LRP is to define the structure of the distribution system by optimizing the number and location of facilities in both echelons, the vehicle fleet size for each level and the material flow distribution on each echelon. On the other hand, the inventory routing problem is a long-term planning problem that provides a good starting point for studying the integration of two important functions in the supply chain, namely inventory management and transportation. It considers customer usage rate rather than customer orders to establish when to serve and how much is delivered to a customer. However, less attention is paid on the detailed routes to be followed to reach customer locations. The objective of the IRP is to minimize the average distribution costs over the planning horizon, while avoiding stockouts at customer sites. Complete surveys on NE-LRP and IRP problems can be found in Salhi and Nagy (2007) and Moin and Salhi (2007), respectively.

Only recently, the N-echelon vehicle routing and scheduling problem (NE-VRP) has received some attention. The most common instance is the two-echelon vehicle routing problem (2E-VRPCD). It was introduced by Perboli, Tadei, and Vigo (2011) as an extension of the classical VRP, where the freight delivery from a single depot to customers is managed by routing and consolidating the load at intermediate depots called satellites. Afterwards, the freight is sent from satellites to customers. Therefore, the 2E-VRPCD deals with the vehicle routing and scheduling for a cross-docking system. The problem assumes a single depot or origin, and a fixed number of capacitated satellites. Direct shipping from the depot to customers is not allowed and only one type of freight is considered. Vehicles belonging to the same level have the same fixed capacity. Moreover, all customer demands are fixed and known in advance and must be satisfied within the scheduling horizon. The time domain does not arise in the problem formulation, and consequently no time windows are defined for deliveries and satellite operations. To solve the 2E-VRPCD problem, the transportation network is usually decomposed into two levels, with the upper one connecting the depot to satellite platforms and the lower level linking satellites to customers. The objective is the minimization of the total transportation cost in both levels. Several versions of the 2E-VRPCD have been studied. In the most general case, each satellite can be served by more of than one 1st-level vehicle and, therefore, the related satellite demand can be split into two or more trucks. In the 2nd level, however, each customer should be served by a single vehicle. Each transportation level has its own fleet, and vehicles for some level cannot be reassigned to another one. Since it was recently introduced, the literature on the 2E-VRPCD problem is rather limited. Perboli et al. (2011) proposed a mixed-integer linear programming formulation together with valid cuts to get better lower bounds by strengthening linear relaxations. Transportation costs from the depot to each satellite, and from a satellite to every customer location are given. Instead, temporal aspects like travel times, duration of loading/unloading operations and time windows are not considered. A set of benchmark problems involving one depot, 2 satellites and up to 32 customers was mostly solved to optimality. When the number of satellites rises and around 50 customers are served, the average optimality gap was above 30% after a CPU time of 5000 s. To decrease the computational cost, a pair of math-based heuristics based on a linear relaxation of the model was applied. By doing so, non-optimal solutions featuring an average gap of 21% with regards to the best lower bound were found in a short CPU time.

Crainic, Mancini, Perboli, and Tadei (2010) applied a separation strategy that splits the 2E-VRP problem into two major routing subproblems, one at each level. The second-level subproblem is further decomposed into as many VRPs as the number of satellites, assuming that the set of customers assigned to each satellite is known. The customer-to-satellite assignment problem is solved through a clustering-based heuristic procedure allocating customers to closer satellites. In the same way, the VRP for the first level involves a single depot and a set of satellites with each one featuring a demand equal to the sum of the demands of customers assigned to it. The resulting VRPs at the two levels are iteratively solved, while adjusting satellite demands through customer-to-satellite reassignments. Temporal aspects are still ignored. Compared with Perboli et al. (2011), the so-called multi-start heuristics for the 2E-VRPCD presented a much better computational performance. Good solutions for problems involving up to 5 satellites and 50 customers were found at low CPU times.

A closely related problem is the so-called vehicle routing problem with cross-docking (VRPCD). The VRPCD is the problem of transporting products from a set of suppliers (pickup nodes) to a set of customers (delivery nodes) via a single cross-dock. Products from the suppliers are picked up by a fleet of homogeneous vehicles, consolidated at the cross-dock, and immediately delivered to customers by the same set of vehicles, without intermediate storage. Then, the problem involves vehicle route design and consolidation at the cross-dock. The major features of the VRPCD are the following: (*i*) a single type of product is handled; (*ii*) each node must be visited by a single vehicle only once; (*iii*) vehicles can pick up or deliver more than one supplier or customer; (*iv*) pickup and delivery routes start and end at the cross-dock; (*v*) amounts to load/unload at pickup/delivery nodes are known data; (*vi*) the total quantity unloaded at the receiving dock and the total one loaded in the shipping dock should be equal, i.e. there is no end inventory at the cross-dock. The problem goal is to minimize the total transportation cost while satisfying all node requests within the planning horizon. Service time windows for the nodes are usually specified. There are some major differences between the VRPCD and the 2E-VRPCD problems: (*a*) a single cross-dock vs. several satellite platforms; (*b*) a single vehicle fleet based on the cross-dock facility vs. several vehicle fleets (a different one for each depot); (*c*) multiple sources vs. single source; (*d*) time windows for node services vs. temporal aspects ignored; (*e*) pickup and delivery requests vs. customer demands. Lee, Jung, and Lee (2006) were the first authors to study the VRPCD problem. They developed an MILP integrated model that considers cross-docking operations and vehicle routing scheduling, assuming that all vehicles coming from suppliers arrive at the cross-dock *simultaneously*. Such temporal constraints tend to avoid vehicle waiting times at the cross-dock. Time windows were not specified and customer needs must be satisfied within the planning horizon. Since the problem is NP-hard, a heuristic algorithm based on tabu search was applied. The linear relaxation of the model provides a lower bound with which to compare the objective value for the solution found. Recently, Liao, Lin, and Shih (2010) proposed a new tabu search algorithm for the VRPCD and solved again the set of benchmark problems introduced by Lee et al. (2006). Good feasible solutions were obtained at much less computational time.

A similar problem was studied by Wen, Larsen, Clausen, Cordeau, and Laporte (2009) but, in this case, pickup and delivery tasks have predetermined time windows and vehicles coming from suppliers not necessarily arrive at the cross dock simultaneously. Besides, customer requests are defined in terms of two nodes, namely the pickup node where the freight is loaded and the delivery node to which is destined. Since pickup and delivery operations are carried out at the cross-dock (CD), the CD is represented by four nodes with the first two standing for the starting and ending points of pickup routes, and the last two for the extreme points of delivery routes. A mixed integer programming formulation was developed. By ignoring the set of constraints linking pickup and delivery activities, the resulting model corresponds to a problem with two independent VRPTW, i.e. a 2-VRPTW problem. The optimal solution to 2-VRPTW provides a lower bound for the VRPCD. To solve the problem, a tabu search heuristic embedded within an adaptive memory procedure was developed. Examples involving up to 200 pairs of nodes were tackled. Non-optimal solutions with objective values less than 5% away from the 2-VRPTW lower bound were found in a short computational time. The VRPCD as defined by Wen et al. (2009) can be regarded as a pickup and delivery problem with time windows and transshipment (PDPTWT). The PDPTWT was introduced by Mitrovic-Minic and Laporte (2006) to investigate the usefulness of operating systems in which two vehicles can handle the same request through the use of transshipment points. In the PDPTWT, each request may be split into two sub-requests, namely a pickup and a delivery sub-request, that can be handled by two different vehicles. The incorporation of transshipment points may yield solutions with shorter travel distances or fewer vehicles.

Dondo, Méndez, and Cerdá (2009) introduced the so-called vehicle routing problem in supply chain management (VRP-SCM). The VRP-SCM problem is a generalization of the N-echelon vehicle routing problem because it handles multiple items and also allows direct shipping of products from manufacturer storages to customers. Moreover, it better resembles the logistics activities at multi-site manufacturing firms by allowing multiple events at every location. As a result, two or more vehicles can visit a given site to perform pickup and/or delivery operations, and vehicle routes may include several stops at the same site, i.e. multiple tours for a vehicle. More important, the allocation of customers to suppliers and the quantities of products shipped from each source to a particular client are additional model decisions. Dondo et al. (2009) proposed an MILP model that relies on a continuous-time representation and applies the global precedence concept to model the sequencing constraints controlling the ordering of vehicle stops on every route. The approach provides a very detailed set of optimal vehicle routes and schedules to meet all product demands at minimum total transportation cost. However, the approach has two major limitations. On one hand, it cannot handle cross-docking operations and lots of products received at the distribution center from the manufacturer cannot be delivered to customers during the same planning horizon. Moreover, the intermediate DCs are regarded as suppliers of products to customer locations and simultaneously as demand points for manufacturer sites with specific product needs.

This work introduces a mixed-integer linear programming (MILP) formulation for the N-echelon multi-item vehicle routing and scheduling problem with cross-docking and time windows (NE-VRPCD). It can be regarded as a generalization of the mathematical model proposed by Dondo et al. (2009). In this newly defined NE-VRPCD problem, that can be called the VRPCD problem in supply chain management (VRPCD-SCM), multiple types of products are handled and customer demands involving more than one item can be satisfied through either direct shipping or via intermediate facilities. The final decisions are left to the model. Moreover, intermediate depots may keep finite stocks of fast-moving products (warehousing) and/or act as cross-dock platforms for slow-moving and high-value items. Besides, some customers can be pre-assigned to a given depot. Transshipment operations are automatically triggered when the initial stock of some product in a warehouse is insufficient to meet both the overall demand of the assigned customers and the target inventory at the end of the planning horizon. Supplies may come from factories or other warehouses, and the related model decisions will aim to minimize fixed and variable transportation costs. In contrast to previous approaches on 2E-VRPCD and VRPCD, the best distribution strategy for the new VRPCD-SCM problem is found by solving the proposed MILP formulation through a branch-and-cut algorithm instead of using heuristic procedures.

3. Problem description

Similarly to Dondo et al. (2009), a multi-echelon distribution network is described by a graph G (*I*, *A*). The node set *I* includes factories, warehouses, distribution centers and customer locations, and the arc set *A* represents minimum-cost routes linking nodes in the network (see Fig. 1). Those routes in the set *A* connect manufacturers to warehouses, and manufacturers and warehouses to customer zones. A customer order may include several products often available at different production sites. Then, the consolidation of shipments from multiple suppliers to intermediate DCs should be made before transporting the products on another truck to a single destination. In this work, it is considered a transportation infrastructure that allows: (i) direct shipping; (ii) shipping via DC or regional warehouses, including cross-docking; and (iii) a combination of both types of shipments, i.e. a hybrid strategy. Besides, some routes can interconnect manufacturing sites or warehouses among themselves to also account for *milk runs*, i.e. a sequence of pickup/delivery operations carried out by the same vehicle. Three types of nodes are considered: (1) "Pure" source nodes (*IS*), usually manufacturer storages, delivering products to DCs, warehouses and customer locations. Trucks stopping at a pure source node just carry out pickup operations. (2) Intermediate nodes (*IM*), like distribution centers or regional warehouses that receive and store products from manufacturers, and deliver them to customers. Vehicles stopping at intermediate nodes can accomplish pickup and/or

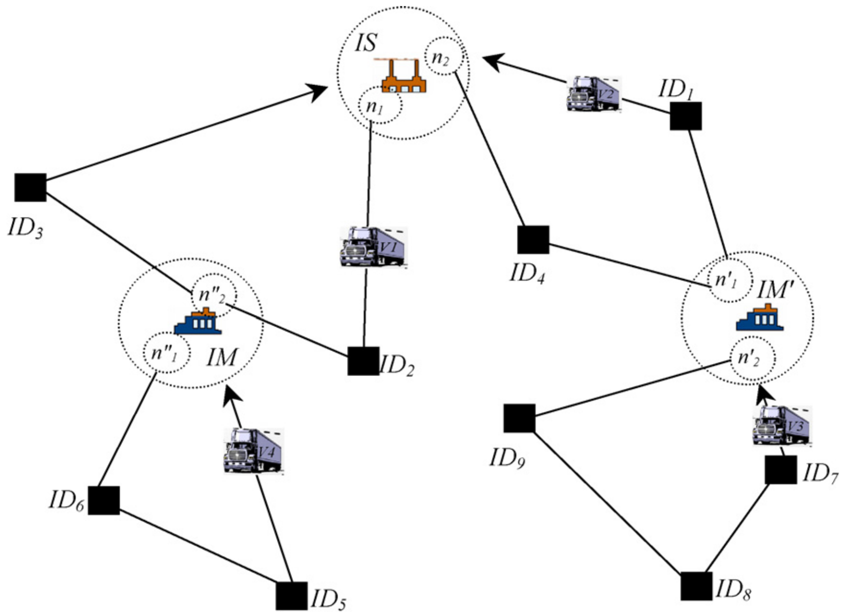

Fig. 1. A two-echelon distribution network.

delivery services. (3) Destination nodes (*ID*), like consumer locations, receive products from manufacturers and DCs and the visiting trucks just accomplish delivery operations. The elements of IS and IM are supplier sites (i.e. SS = IS ∪ IM) providing products to downstream locations in the supply chain, while the elements of ID are destination nodes for product shipments. Initial product inventories II_{ip} ($i \in SS$, $p \in P$) are usually available at source nodes, and product demands d_{ip} ($i \in$ ID, $p \in P$) are only specified for customer locations. Intermediate nodes like DCs or warehouses have a special treatment because cross-docking is now allowed. They may need to receive lots of some products to meet the assigned customer demands and/or to reach the final target inventory levels. Therefore, their product needs are not known before solving the problem. In contrast to Dondo et al. (2009), product demands at DCs are no longer problem data. Instead, target inventory levels to reach at the horizon end are specified for each distribution center.

Every arc $(i, j) \in A$ between nodes (i, j) is characterized by a distance-based transportation cost c_{ij} and a travel-time t_{ij}. It is assumed that the travel cost c_{ij} satisfies the triangle inequality, i.e. $c_{ij} + c_{jk} \le c_{ik}$, where $(i, j, k) \in I$. In addition, the problem definition includes the set P comprising the range of products to move from factories and warehouses to customers, and the vehicle set V standing for the available trucks carrying products to the assigned destinations. Since the total shipment size must never exceed the volume/weight truck capacity, the weight (uw_p) and the volume (uv_p) of a single unit of product p as well as the weight capacity (qw_v) and the volume capacity (qv_v) of each truck are important problem data. Furthermore, each vehicle has a base from which it starts and finishes the journey. A vehicle base can be located at manufacturing sites or warehouses. Let $B \subset SS$ be the set of candidate bases for the available trucks and B_v ($\subset B$) the subset of alternative bases for a particular vehicle v. Moreover, a customer zone i should usually be serviced from some pre-defined sources (factories or warehouses). Then, it should be visited by vehicles V_i ($\subset V$) that start their journeys from such pre-assigned supply points.

In real-life distribution problems, several vehicles can stop at the same manufacturing site or warehouse to accomplish pickup or delivery operations. Moreover, a vehicle may be visiting a source node several times during the same tour, and product requirements at some destination may be satisfied through various partial shipments using more than one vehicle. Therefore, a sequence of operations may be performed at every location and a vehicle stop is no longer characterized by just the visited node. To overcome this problem, the proposed mathematical formulation assumes that an ordered set of events $n \in N_i$ may happen at every location i and the vehicle stop (n, i) is characterized by the visited node i and the time event n at which it occurs. The nth-event at site i, if accomplished, will occur before the vehicle stop $(n+1, i)$. The maximum number of events at node i given by $|N_i|$ should be at least as large as the optimal number of vehicle stops at location i. During a stop, a truck performs loading and/or unloading operations. The tasks carried out by a vehicle during stop (n, i) are defined by some model variables to be described in the next Section.

By considering multiple events at every location, the formulation of the VRP-SCM problem with cross-docking better describes the operations in real-world N-echelon distribution networks. Similarly to Dondo et al. (2009), the proposed model for the VRPCD-SCM problem is able to consider (i) allocation of suppliers to customers, (ii) load splitting, (iii) milk runs, (iv) selection of types and amounts of products to pick up at source nodes and their destinations, (v) construction of vehicle routes featuring multiple tours with intermediate stops at the base to load further lots of products, provided that the maximum service time is not exceeded, (vi) customer time windows and maximum service time, (vii) initial inventories at manufacturer and distributor storages and (viii) cross-docking operations at intermediate facilities.

4. Model assumptions

The problem formulation presented in Section 6 is based on the following assumptions:

1. Problem data are known with certainty and remain unchanged with time.
2. Every vehicle can transport lots of different products but its weight/volume capacity must never be exceeded.
3. A customer location may demand several products provided by either the same or different sources.

4. There are no pre-defined suppliers for some customers, and the amounts of products to pick up at source nodes are not problem data but model variables. The product flow pattern through the distribution network is then a model decision.
5. Each vehicle can accomplish loading and unloading tasks, but pickup operations do not necessarily precede delivery operations. Intermediate stops at source nodes to load further amounts of products are permitted.
6. Partial deliveries are allowed, and several vehicles can stop at the same source/destination node during the planning horizon.
7. Each location can be visited by the same vehicle more than once. Consequently, a vehicle route may include a series of tours with intermediate stops at the vehicle base for further pickup operations.
8. During the stop at mixed nodes (i.e. warehouses), a vehicle can accomplish pick-up and delivery tasks. Certainly, such loading and unloading operations will involve different products.
9. The total amount of a particular product picked up by a given vehicle at source nodes should be equal to the total quantity of that product that it delivers to demanding locations.
10. Each vehicle route should start and end at the vehicle base selected by the model among the alternative choices.
11. The length of a vehicle stop has a fixed and a variable component. The fixed-contribution may depend on the site, while the variable component is proportional to the amount of products to pick-up or deliver by the vehicle.
12. Cross-docking operations at intermediate facilities (i.e. warehouses) are allowed.
13. If lots of products received at DCs should not be immediately loaded into outbound trucks, they can be temporarily stored until the time of shipping them to the assigned destinations.
14. Inbound and outbound vehicles must stay in receiving/shipping docks of cross-dock facilities until they complete their delivery/pickup tasks.
15. Target product inventories at distribution centers, given as problem data, must be available at the end of the planning horizon.
16. There is a maximum service time for each vehicle that cannot be exceeded.
17. Time-window and service-time constraints can be relaxed by including penalty cost terms in the objective function that linearly increases with the violation size.

5. Problem variables

Most of the model variables were already presented in Dondo et al. (2009). However, new ones are necessary to handle cross-docking operations at intermediate depots. Those facilities are no longer regarded as demand points with specific product needs. In this work, such requirements depend on the requests of the assigned customers and their prescribed end target inventory levels $FINV_{ip}$ ($i \in IM$, $p \in P$). Among the 0–1 variables included in the model, the most important ones are:

(a) Assignment variables Y_{niv} denoting that the event $n \in N_i$ at node $i \in I$ has been allocated to vehicle $v \in V_i$. When $Y_{niv} = 1$, vehicle v will visit node i at time event n, i.e. the stop (n, i) for vehicle v. A set of preordered events is assigned to every node i with the event n taking place before $(n+1)$. If $Y_{niv} = 0$ for any vehicle $v \in V_i$, the events $\{n, n+1, \ldots |N_i|\}$ never occur at node i. They will be fictitious events.

(b) Sequencing variables $X_{ni,n'i'}$ denoting that the vehicle stop (n, i) at node i will occur before the event n' at location i', whenever $X_{ni,n'i'} = 1$ ($n \in N_i$, $n' \in N_{i'}$) and $Y_{niv} = Y_{n'i'v} = 1$. Assuming that node i is visited by vehicle $v \in V_i$ at event n ($Y_{niv} = 1$) and the route for $v' \in V_{i'}$ includes a stop (n', i') at node i' ($Y_{n'i'v'} = 1$), then the vehicle stop (n, i) will occur earlier than (n', i') whenever $X_{ni,n'i'} = 1$. In contrast to Dondo et al. (2009), vehicles v and v' might be different to account for cross-dock operations. A single variable $X_{ni,n'i'}$ is enough to sequence a pair of stops (n, i) and (n', i'). Then, the variable $X_{ni,n'i'}$ with $i < i'$ (or $n < n'$ if $i = i'$) is just included in the model. The separate handling of allocation and sequencing decisions permits to get a substantial saving in binary variables.

Continuous variables C_{ni} and T_{ni} (with $n \in N_i$, $i \in I$) introduced by Dondo et al. (2009) are still considered to establish the distance-based transportation cost and the travel time from the assigned base to stop (n, i) for vehicle v whenever $Y_{niv} = 1$. When the route includes multiple tours, the travel time and transportation cost up to stop (n, i) are referred to the start of the journey. Besides, CV_v and OT_v stand for the overall transportation cost and travel time incurred by vehicle v to complete the assigned tasks and returns to its base. On the other hand, the continuous variables $L_{ni,pv}$ and $U_{ni,pv}$ indicate the nature and extent of the tasks carried out by vehicle v during the stop (n, i) at site i, in case $Y_{niv} = 1$. If $L_{ni,pv} > 0$, then $L_{ni,pv}$ units of product $p \in P$ are picked-up by vehicle v during stop (n, i). If instead $U_{ni,p'v} > 0$, then $U_{ni,p'v}$ units of product $p' \in P$ are delivered to location i by vehicle v at event $n \in N_i$. Values of $L_{ni,pv}$ and $U_{ni,p'v}$ are set to 0 if $Y_{niv} = 0$. To determine the current load transported by vehicle v after stop (n, i) to avoid overcapacity or product shortage, variables $AL_{ni,pv}$ and $AU_{ni,pv}$ representing the accumulated amount of product p picked up and delivered by vehicle v, respectively, from the start to stop (n, i) are defined. The difference $(AL_{ni,pv} - AU_{ni,pv})$ provides the amount of product p transported by vehicle v after stop (n, i). Furthermore, the handling of cross-dock operations requires to introduce a new set of continuous variables AI_{nip} to represent the additional inventory of product p received at the intermediate warehouse $i \in IM$ from other sources up to the event $n \in N_i$.

The proposed MILP formulation for the VRP-SCM with cross-docking includes seven constraint categories: (a) *Route building constraints* assigning a particular stop (n, i) at node $i \in I$ to at most a single truck, and ordering vehicle stops (n, i) on the same route. (b) *Product inventory constraints* restraining the overall amount of products loaded by visiting vehicles at source nodes. To account for cross-docking, two sub-categories of inventory constraints are defined, one for pure sources IS and the other for intermediate facilities IM. The later one is introduced in this work to also consider cross-docking. Product stocks at pure sources are those available at the start. Shipments to pure sources are not expected during the planning horizon. On the contrary, DCs and warehouses can receive additional lots of products from other sources. Then, tracking the variation of product inventories with time at DCs to avoid product shortages and backorders becomes necessary. (*c*) *Product demand constraints* ensuring that customer requests are satisfied. (*d*) *Null in-transit inventory constraints* requiring that every product unit picked up by a vehicle must be delivered to a demanding location before the end of the vehicle trip. (*e*) *Loading/unloading constraints* monitoring the total amount of products transported by each vehicle to prevent from overcapacity or product shortages. *(f) Time window and maximum service time constraints* ensuring that the customer service begins within the specified time window and the vehicle returns to its base within the allowed working period. All of these restrictions have been grouped into the vehicle-related constraint set. *(g) Additional inventory constraints* monitoring the amount of every product received at each warehouse from different sources over the planning horizon.

6. The MILP mathematical model

6.1. Route building constraints

(i) Allocating base nodes to vehicles. Eq. (1) states that every vehicle v, if used in the distribution schedule, must start and end its trip at the assigned base node $l \in IB_v$. The node set $IB_v \subset I$ includes all the possible operational bases for vehicle v. Since multiple vehicles v can depart from the same base node l, usually a factory or warehouse, every vehicle must be allocated to a different event n predefined for node l:

$$\sum_{l \in IB_v} \sum_{n \in N_l} Y_{nlv} \le 1 \quad v \in V \tag{1}$$

(ii) Allocating events at every node to vehicles. Eq. (2) states that every predefined event n at node i, i.e. the vehicle stop (n, i), can at most be allocated to a single visiting truck v. Consequently, multiple vehicle stops at the same node will always be assigned to different events:

$$\sum_{v \in V_i} Y_{niv} \le 1 \quad n \in N_i, \quad i \in I \tag{2}$$

(iii) Pre-ordering events occurring at the same node. Eq. (3) enforces the condition that the stop (n', i) can only be allocated to vehicle v if all the previous stops (n, i) at node i, with $n < n'$, have already been assigned to some visiting vehicles:

$$\sum_{v \in V_i} Y_{niv} \ge \sum_{v \in V_i} Y_{n'iv} \quad (n, n') \in N_i : n' > n, i \in I \tag{3}$$

(iv) Used vehicle condition. This constraint relates the decision variables Y_{niv} introduced in Eqs. (2) and (3) between themselves. It states that a given vehicle v can be allocated to multiple stops (n, i) at the same or different nodes whenever it has been previously assigned to a base node l. The parameter M_v defines the maximum number of stops (n, i) that can be allocated to vehicle v:

$$\sum_{i \in I_v} \sum_{n \in N_i} Y_{niv} \le M_v \left(\sum_{l \in IB_v} \sum_{n \in N_l} Y_{nlv} \right) \quad v \in V \tag{4}$$

6.2. Travelling cost constraints

(v) Travelling cost from the base node l to the first serviced node i for vehicle v. Constraint (5) states that the minimum cost to reach any node i must be equal or greater than the travelling cost to go directly from the base node l to node i, given by the parameter c_{li}:

$$C_{ni} \ge \sum_{l \in IB_v} \sum_{n' \in N_l} c_{li} Y_{n'lv} - M_C(1 - Y_{niv}) \quad n \in N_i, \quad i \in I, \quad v \in V_i \tag{5}$$

(vi) Accumulated travelling cost for vehicle v up to the stop (n, i). The travelling cost along the route of vehicle v from the start to the stop (n, i) at node i is computed through the pair of Eq. (6). To calculate such an accumulated travelling cost from the base node up to every stop (n, i), sequencing variables $X_{ni,n'i'}$ are defined to determine the order in which such a pair of nodes is visited. The parameter $c_{ii'}$ defines the distance-based cost for travelling from i to i':

$$\left\{ \begin{array}{l} C_{n'i'} \ge C_{ni} + c_{ii'} - M_C(1 - X_{ni,n'i'}) - M_C(2 - Y_{niv} - Y_{n'i'v}) \\ C_{ni} \ge C_{n'i'} + c_{i'i} - M_C X_{ni,n'i'} - M_C(2 - Y_{niv} - Y_{n'i'v}) \end{array} \right\} \quad n \in N_i, n' \in N_{i'}, (i, i') \in I, v \in V_{ii'} : i < i' \tag{6}$$

Considering the fact that the proposed model is able to consider situations where the same node i is visited several times by vehicle v, Eq. (6) can be rewritten to ordering multiple stops of vehicle v at node i as given by Eq. (6.1). Note that event n always occurs before n' whenever $n < n'$:

$$C_{n'i} \ge C_{ni} - M_C(2 - Y_{niv} - Y_{n'iv}) \quad (n, n') \in N_i, \quad i \in I, \quad v \in V_i : n < n' \tag{6.1}$$

(vii) Overall travelling cost for vehicle v. To compute the total cost for the route assigned to vehicle v, Eq. (7) incorporates the travelling cost from the last visited node i on the v-trip to the base depot l:

$$CV_v \ge C_{ni} + \sum_{l \in B_v} \sum_{n' \in N_l} c_{li} Y_{n'lv} - M_C(1 - Y_{niv}) \quad n \in N_i, \quad i \in I, \quad v \in V \tag{7}$$

6.3. Travelling time constraints

(viii) Travelling cost from the assigned base node $l \in IB_v$ to the first serviced node for vehicle v. Eq. (8) computes the minimum time needed to arrive at the first visited node. Then, it includes the travelling time for the arc (l, i) defined by t_{li} as well as the fixed and variable time required for pick-up operations at the base node l, usually a factory or warehouse:

$$T_{ni} \ge \sum_{l \in IB_v} \sum_{n' \in N_l} t_{li} Y_{n'lv} + ft_l + vt_l \sum_{p \in P_l} L_{nlpv} - M_C(1 - Y_{niv}) \quad n \in N_i, \quad i \in I, \quad v \in V \tag{8}$$

(ix) Travelling time for vehicle v from the assigned base node to the stop (n, i). The pair of Eq. (9) computes the time required to go from the assigned base to any node visited by vehicle v. Travelling times for the edges between nodes on the route of vehicle v, as well as a fixed and variable times for pickup and delivery operations at visited locations are considered by Eq. (9) to compute vehicle arrival times:

$$\left\{ \begin{array}{l} T_{n'i'} \ge T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) + t_{ii'} - M_C(1 - S_{ni,n'i'}) - M_C(2 - Y_{niv} - Y_{n'i'v}) \\ T_{n'i'} \ge T_{ni} + ft_{i'} + vt_{i'} \left(\sum_{p \in P_i} L_{n'i',pv} + U_{n'i',pv} \right) + t_{i'i} - M_C S_{ni,n'i'} - M_C(2 - Y_{niv} - Y_{n'i'v}) \end{array} \right\} \quad n \in N_i, n' \in N_{i'}, (i, i') \in I, v \in V_{ii'} : i < i' \tag{9}$$

In case of multiple stops of vehicle v at the same node i (usually, a source node) taking place at different time events, the pair of equations (9) can be replaced by a single one by considering that event n occurs before n' if $n < n'$ (see Eq. (9.1)):

$$T_{n'i'} \ge T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) - M_C(2 - Y_{niv} - Y_{n'i'v}) \quad (n, n') \in N_i, i \in I, v \in V_i : n < n' \tag{9.1}$$

Usually, different vehicles v and v' stop at the same warehouse i at time events n and n' (with $n < n'$) to perform delivery and pickup activities, respectively, if the initial stocks available at node i are not large enough to meet the assigned customer demands. In such cases, pickup operations by vehicle v' must start after unloading the cargo from vehicle v, and the pair of Eq. (9) reduces to Eq. (9.2):

$$T_{n'i} \geq T_{ni} + ft_i + vt_i\left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv}\right) - M_C(2 - Y_{niv} - Y_{n'iv'})$$
$$(n, n') \in N_i, i \in I, v, v' \in V_i : n < n' \quad (9.2)$$

(x) Overall travelling time for vehicle v. The duration of the trip assigned to vehicle v is computed by Eq. (10). It adds both the duration of unloading/loading activities carried out at node i and the travelling time to return to the base node to the time required for reaching the last visited node i:

$$OT_v \geq T_{ni} + ft_i + vt_i\left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv}\right) + \sum_{l \in B_v}\sum_{n' \in N} t_{il} Y_{n'lv} - M_C(1 - Y_{niv}) n \in N_i,\ i \in I, v \in V \quad (10)$$

(xi) Time window and maximum service time constraints. Product delivery at the customer location $i \in ID$ should start within the specified time window (a_i, b_i) and vehicle v must complete the assigned tasks before time $t_v^{\max}$:

$$a_i \leq T_{ni} \leq b_i \quad n \in N_i, i \in I \quad (11)$$

$$OT_v \leq t_v^{\max} \quad v \in V \quad (12)$$

6.4. Product availability and demand constraints

(xi) For factories (pure sources). The total amount of every product p supplies by a pure source i (a factory) to the assigned destinations can never exceed the initial inventory available on node i:

$$\sum_{v \in V_i}\sum_{n \in N_i} L_{ni,pv} \leq II_{ip} \quad i \in IS, p \in P_i \quad (13)$$

(xii) For warehousing and cross-docking facilities (mixed nodes). The total amount of every product p taken from a cross-docking facility i up to the event n can never exceed the initial stock plus the additional quantity of product p received from other sources up to the event n:

$$\sum_{v \in V_i}\sum_{\substack{n' \in N_i \\ n' \leq n}} L_{n'i,pv} \leq II_{ip} + AINV_{nip} \quad n \in N_i, \quad i \in IM, \quad p \in P_i \quad (14)$$

(xiii) Overall product balance at each intermediate facility. This constraint enforces an overall balance between the total amount of product p available in the warehousing or cross-docking facility i, including the one received from other sources during the planning horizon, and the overall quantity of p supplied by node i to customer locations. However, Eq. (15) allows that a positive inventory of product p, given by $FINV_{ip}$, remains at node i at the end of the horizon. In such a case, the total stock of product p available for delivery is reduced by the amount $FINV_{ip}$:

$$\sum_{v \in V_i}\sum_{n \in N_i} L_{ni,pv} \leq II_{ip} + \sum_{v \in V_i}\sum_{n \in N_i} U_{ni,pv} - FINV_{ip} \quad p \in P_i, i \in IM \quad (15)$$

(xiv) Products demands at customer nodes. The total amount of product p delivered to each customer node i must always satisfy its demand:

$$\sum_{v \in V_i}\sum_{n \in N_i} U_{ni,pv} \geq D_{ip} \quad i \in ID, p \in P_i \quad (16)$$

(xv) Relationship between variables $L_{ni,pv}$ and Y_{niv}. Eq. (17) enforces the condition that a pickup activity by vehicle v during stop (n, i) at source node i can take place, i.e. $L_{ni,pv} > 0$, only if vehicle v has been assigned to stop (n, i), i.e. $Y_{niv} = 1$:

$$L_{ni,pv} \leq M_L Y_{niv} \quad n \in N_i, \quad i \in (IS \cup IM), \quad p \in P_i, \quad v \in V_i \quad (17)$$

(xvi) Relationships between variables $U_{ni,pv}$ and Y_{niv}. The pair of Eqs. (18.1) and (18.2) enforce the condition that a delivery operation by vehicle v at stop (n, i) can only take place if such a stop has been allocated to v, i.e. $Y_{niv} = 1$:

$$\text{For suppliers:} \quad U_{ni,pv} \leq M_L Y_{niv} \quad n \in N_i, \quad i \in IM, \quad p \in P_i, \quad v \in V_i \quad (18.1)$$

$$\text{For customers:} \quad U_{ni,pv} \leq D_{ip} Y_{niv} \quad n \in N_i, \quad i \in ID, \quad p \in P_i, \quad v \in V_i \quad (18.2)$$

6.5. Vehicle-related constraints

(xvii) Overall product balance for every vehicle. Eq. (19) states that the total amount of product loaded on vehicle v must always be equal to the total amount of product delivered by v along its entire route:

$$\sum_{i \in IS \cup IM}\sum_{n \in N_i} L_{ni,pv} = \sum_{i \in (IM \cup ID)}\sum_{n \in N_i} U_{ni,pv} \quad p \in P, \quad v \in V \quad (19)$$

(xviii) Accumulated amount of product p picked up by vehicle v up to the stop (n, i). The pair of Eq. (20) computes the total amount of product p loaded on vehicle v from the start up to stop (n, i):

$$\left\{\begin{array}{l} AL_{n'i',pv} \geq AL_{ni,pv} + L_{n'i',pv} - M_L(1 - S_{ni,n'i'}) - M_L(2 - Y_{niv} - Y_{n'i'v}) \\ AL_{ni,pv} \geq AL_{n'i',pv} + L_{ni,pv} - M_L S_{ni,n'i'} - M_L(2 - Y_{niv} - Y_{n'i'v}) \end{array}\right\}$$

$$n \in N_i, n' \in N'_{i'}, (i, i') \in I, \quad p \in P, \quad v \in V_{ii'} : n < n', \quad i \neq i' \quad (20)$$

In case of multiple visits to node i by the same vehicle v, Eq. (20) takes the following form:

$$AL_{n'i,pv} \geq AL_{ni,pv} + L_{n'i,pv} - M_L(2 - Y_{niv} - Y_{n'iv}) \quad (n, n') \in N_i, \quad i \in I,$$
$$v \in V_i : n < n' \quad (20.1)$$

(xix) Accumulated amount of product p delivered by vehicle v up to the stop (n, i). The pair of Eq. (21) computes the total amount of product p unloaded from vehicle v from the start of the journey up to stop (n, i):

$$\left\{\begin{array}{l} UL_{n'i',pv} \geq UL_{ni,pv} + U_{n'i',pv} - M_L(1 - S_{ni,n'i'}) - M_L(2 - Y_{niv} - Y_{n'i'v}) \\ UL_{ni,pv} \geq UL_{n'i',pv} + U_{ni,pv} - M_L S_{ni,n'i'} - M_L(2 - Y_{niv} - Y_{n'i'v}) \end{array}\right\}$$

$$n \in N_i, n' \in N'_i, (i, i') \in I, \quad p \in P, \quad v \in V_{ii'} : n < n', i \neq i' \quad (21)$$

In case of multiple visits to node i by the same vehicle v, equation (21) takes the following form:

$$UL_{n'i,pv} \geq UL_{ni,pv} + U_{n'i,pv} - M_L(2 - Y_{niv} - Y_{n'iv}) \quad (n, n') \in N_i, \quad i \in I, \quad v \in V_i : n < n' \qquad (21.1)$$

(xx) Vehicle capacity constraints. Maximum weight and volume capacities are enforced on the total cargo transported by every vehicle v. The difference $(AL_{ni,pv} - AU_{ni,pv})$ provides the number of units of product p transported by vehicle v after the stop (n, i) at node i. Such a quantity can never be negative and the summation of $(AL_{ni,pv} - AU_{ni,pv})$ for all products should never exceed the maximum capacity of vehicle v:

$$\left\{ \begin{array}{l} \sum_{p \in P} uw_p(AL_{ni,pv} - AU_{ni,pv}) \leq qw_v \\ \sum_{p \in P} uv_p(AL_{ni,pv} - AU_{ni,pv}) \leq qv_v \\ AL_{ni,pv} - AU_{ni,pv} \geq 0 \end{array} \right\} \quad n \in N_i, \quad i \in I_v, \quad v \in V \qquad (22)$$

(xxi) Bounds on variables $AU_{ni,pv}$ and $AL_{ni,pv}$. The accumulated amount of product picked up/delivered by vehicle v from the start of the journey up to stop (n, i) is always bounded by both the quantity of product picked up/delivered at node i (the lower bound), and the total amount loaded/unloaded along the entire route (the upper bound):

$$\left\{ \begin{array}{l} L_{ni,pv} \leq AL_{ni,pv} \leq \sum_{i' \in IS \cup IM} \sum_{n' \in N'_i} L_{n'i',pv} \\ U_{ni,pv} \leq AU_{ni,pv} \leq \sum_{i' \in IS \cup IM} \sum_{n' \in N'_i} U_{n'i',pv} \end{array} \right\} \quad n \in N_i, \quad i \in I_v, \quad v \in V, \quad p \in P \qquad (23)$$

6.6. Additional inventory received at cross-docking facilities from other sources

(xxii) Additional inventory of product p received at the mixed node i up to stop (n', i). The amount of product p available at the cross-dock facility i up to the event n' depends on both the additional inventory at the previous event n and the amount of product received at event n':

$$AI_{n'ip} \geq AI_{nip} + \sum_{v \in V_i} U_{n'i,pv} \quad (n, n') \in N_i, \quad i \in IM, \quad p \in P : n < n' \qquad (24)$$

(xxiii) Bounds for the value of AI_{nip}. The accumulated amount of product p received at the cross-dock facility i from the beginning of the planning horizon up to the event n is never lower than the quantity unloaded at event n, and is never greater than the total amount of p supplied to node i from other sources during the entire horizon.

$$\sum_{v \in V_i} U_{ni,pv} \leq AI_{nip} \leq \sum_{n' \in N_i} \sum_{v \in V_i} U_{n'i,pv} \quad n \in N_i, \quad i \in IM, \quad p \in P \qquad (25)$$

6.7. The objective function

The selected objective function aims to minimizing the total transportation cost, including fixed and variable costs, over the whole planning horizon.

$$\text{Min} \left[\sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in IB_v} \sum_{n \in N_l} fc_v Y_{nlv} \right] \qquad (26)$$

The minimum total travel time has been adopted as a secondary target. In other words, the minimum-cost vehicle tours must be completed at the least possible travel times. After solving the MILP model involving Eqs. (1)–(26), the assignment variables Y_{niv} and the sequencing variables $S_{ni,n'i'}$ are fixed at their optimal values and the resulting LP model is solved again but now using the expression (27) as the problem objective.

$$\text{Min} \left[\sum_{v \in V} OT_v \right] \qquad (27)$$

7. Computational results and discussion

In this section, the performance of the proposed MILP formulation is evaluated by solving five examples all dealing with the operational planning of two-echelon multi-item supply chain networks. Such examples are modified instances of case studies previously tackled by Bonfill et al. (2008) and Dondo et al. (2009). They involve a single manufacturing site, one-to-three distribution centers and up to 29 customer locations. Distribution of four-to-six products from the factory to warehouses, and from these facilities to demand points is made through a fleet of two-to-six vehicles. With regards to Dondo et al. (2009), initial inventories at DCs have been substantially reduced to force the execution of cross-docking operations in intermediate facilities so as to meet product demands at the assigned destinations. In the examples, customers located in the neighborhood of a supplier (manufacturing plants or warehouses) have been preassigned to that source. If the demanding point is on the border line of the neighborhoods of two sources, the choice of the supplier is left to the model. Available stocks in the factory and warehouses at the start of the planning horizon, and the demands of products P1–P6 at customer sites for the five examples are reported in Tables 1 and 2, respectively. Data related to the vehicle fleet are given in Tables 3 and 4 provides the weight and volume per unit of each product. At every vehicle stop, lots of several products can be sequentially picked up and/or delivered at the visited node. As shown in Table 3, the stop time at each site for performing pickup and/or delivery operations comprises a fixed time of 1 h and a variable time period that directly increases with the total cargo at a rate of 250 units/h. However, the proposed formulation can easily handle non-equal load/unload rates. Furthermore, Tables 5A and 5B present the distances between locations in km. A maximum service time of 70 h is considered at all examples, except for Example 1 ($t_{\max}$ = 90 h) and Example 5 ($t_{\max}$ = 80 h). To avoid equivalent solutions from the vehicle routing viewpoint, vehicles are not allowed to perform pickup operations at source locations different from the assigned base node. All the examples were solved to global optimality by using a HP Z600 Workstation with six-core Intel Xeon Processor (2.93 GHz), the modelling language GAMS and GUROBI 3.0 as the MILP solver. A relative optimality tolerance of 0.001 has been adopted.

7.1. Example 1

Example 1 considers a two-echelon distribution network with storage facilities at both the Madrid-based factory (node MAD) and the distribution center (DC) at Barcelona (node BAR). Shipments from these two sites to other seventeen cities should be performed to meet their specified demands of four products P1–P4. Two vehicles V1–V2 are available, with V1 based at BAR and V2 housed at MAD. Most cities located within a radius of 200 km from Barcelona have the DC at BAR as the pre-assigned supplier. Such a choice is based on the fact that the average distance of such cities from the other source at MAD is over 650 km. Instead, Zaragoza (ZAR), Lerida (LER), Valencia (VAL) and Teruel (TER) located in the

Table 1
Product inventories at source nodes for all examples.

	P1	P2	P3	P4	P5	P6
Example 1						
Barcelona	1100	425	425	200		
Madrid	1500	1500	1500	1500		
Example 2						
Barcelona	1100	0	425	0		
Madrid	1500	1500	1500	1500		
Example 3						
Barcelona	1100	0	425	0		
Madrid	1500	1500	1500	1500		
Bilbao	200	50	200	100		
Example 4						
Barcelona	800	0	325	0		
Madrid	2500	2500	2500	2500		
Bilbao	200	50	200	100		
Malaga	300	300	300	300		
Example 5						
Barcelona	800	0	325	0	0	300
Madrid	2600	2600	2600	2600	800	900
Bilbao	200	50	200	100	300	0
Malaga	300	300	300	300	0	0

sphere of influence of both sources (MAD and BAR) can be visited by either V1 or V2. In other words, the supplier may be the factory (i.e. direct shipment) or the DC (i.e. via warehousing), with the assignment decisions left to the model. Initial inventories at node BAR are not enough to meet product demands from the group of cities pre-assigned exclusively to the DC (see Table 1). Then, some lots of products transported by vehicle V2 from node MAD should be received at the distribution center. As a result, delivery and pickup operations are sequentially performed by vehicles V2 and V1, respectively, at BAR. In contrast, loading tasks will be only accomplished by V2 at MAD. At each node, there will be as many events as the number of vehicle stops taking place. Therefore, at least two events are to be predefined for BAR ($|N_{BAR}| = 2$) and just one for MAD and the other locations. If instead $|N_{BAR}|$ is set to 1, the problem has no feasible solution. Time windows within which the service should be started at demanding locations have been omitted. Moreover, transfer times of products between receiving and shipping docks at BAR are neglected.

Two instances of Example 1, called Examples 1A and 1B, were considered. End inventories at BAR are forced to be zero at Exam-

Table 2
Product demands at destination nodes for all examples.

Demands (for all examples)						
	P1	P2	P3	P4	P5	P6
Girona	120		150		50	50
Lerida		75	75			70
Tarragona	50	200		100		
Vic	100		100		150	
Valencia	120	120			50	50
Zaragoza	200		250	150		
Perpignan	150	150			100	50
Andorra	800		200		100	
Valladolid	50	150		200		
S.Sebastián	100	50				
Bilbao[a]	120		120	120		
Teruel	200	100			100	
Soria		200	50	100		150
Santander		150	100	50	200	
Burgos		100	150		100	50
Lugo		100		100		
La Coruña	100		100		100	150
Badajoz		220	430			
Granada	300	250		370		
Murcia		380	200			
Sevilla			200	450		
Cadiz			340			
Córdoba		420		430		
Huesca		50	150			
Castellon	100			50		
Pamplona		100			50	
Zamora	100		100			
Alicante	50			50		
Almeria				100		

[a] Demands at Bilbao only for Examples 1 and 2.

Table 3
Vehicle parameters.

	V1	V2	V3	V4	V5	V6
Example 1						
Weight capacity (kg)	20000	20000		–	–	–
Volume capacity (m^3)	25	32		–	–	–
Example 2a						
Weight capacity (kg)	15000	12000	12000	–	–	–
Volume capacity (m^3)	25	18	23	–	–	–
Example 2b						
Weight capacity (kg)	5000	10000	12000	–	–	–
Volume capacity (m^3)	10	18	23	–	–	–
Example 3						
Weight capacity (kg)	10000	14000	10000	10000	–	–
Volume capacity (m^3)	18	22	20	20	–	–
Example 4						
Weight capacity (kg)	12000	14000	10000	8000	20000	15000
Volume capacity (m^3)	20	22	18	18	28	20
Example 5						
Weight capacity (kg)	12000	18000	12000	10000	18000	12000
Volume capacity (m^3)	20	28	22	15	29	20
Fixed cost					$ 5000 (V1–V4)	$ 4000 (V5–V6)
Variable cost					3 $/km (V1–V4)	2.5 $/km (V5–V6)
Loading/unloading times						
Fixed						1 h
Variable						250 units/h
Average speed	250 units/h					70 km/h

Table 4
Product specific weights and volumes.

	P1	P2	P3	P4	P5	P6
Weight (kg/unit)	3	6	5	5	5	5
Volume (m^3/unit)	0.005	0.015	0.010	0.005	0.005	0.005

ple 1A by writing Eq. (15) as a strict equality. In contrast, Eq. (15) is expressed as an inequality constraint at Example 1B allowing BAR to have finite stocks at the horizon end. Example 1A was solved to optimality in 18.0 s of CPU time. The optimal routes and schedules for vehicles V1 and V2 are depicted in Fig. 2. More details are given in Table 6, including the times at which vehicles leave their bases, together with arrival times and pickup/delivery operations performed by vehicles V1 and V2 at each visited node. Deliveries to the DC and customer nodes are reported with negative figures, while pickups at source nodes (MAD, BAR) are represented by positive numbers. Furthermore, the total distance and time travelled by the vehicles, the used weight/volume vehicle capacity, and the optimal fixed and variable transportation costs are also given in Table 6. As shown in Fig. 2, nodes ZAR and LER are supplied from source BAR and served by vehicle V1, while VAL and TER have been assigned to MAD and visited by V2. In this way, the volumetric capacities of both vehicles are almost fully employed, i.e. 89.9% for V1 and 98.1% for V2. The required CPU time, the amount of linear constraints and the number of binary and continuous variables are given in Table 7. From this table, it follows that vehicle V1 waits for the arrival of V2 at node BAR that occurs at time 30.8 h and the completion of delivery operations at time 34.6 h to start the trip from the DC. The amounts of products P1 (320 units), P3 (350 units) and P4 (50 units) unloaded from V2 at BAR exactly close the gap between the initial inventories of such items and the total requirements of the cities to be serviced by V1. Therefore, such quantities are subsequently loaded into vehicle V1 together with the initial stocks and sent to the assigned destinations. As a result, the inventories of products P1–P4 at the DC are null at the end of the planning horizon.

Example 1B allows finite end product inventories at the warehouse if by so doing the total transportation cost diminishes. Compared with Example 1A, the optimal solution for Example 1B features a lower transportation cost and a final stock of P4 as large as 100 units at the distribution center (see Fig. 3 and Table 8). Such cost savings were obtained by choosing MAD instead of BAR as the supplier of ZAR that is now visited by vehicle V2. As a result, the service time of V2 rises to 85.9 h still lower than the maximum service time of 90 h, and the total travel distance decreases from 4160 to 3893 km. Moreover, the optimal vehicle routes and schedules were found in a CPU time of 11.8 s.

7.2. Example 2

Compared with Example 1, two major changes have been introduced in Example 2. On one hand, initial stocks of P2 and P4 at Barcelona-based DC are no longer available and “pure” cross-docking operations for such items should be performed at the warehouse to service the assigned cities. On the other hand, two vehicles V2–V3 rather than a single one start their trips from MAD in order to reduce the maximum service time from 90 h to 70 h. Vehicle V2 replenishes product inventories at the DC and visits some locations on its route to/from BAR while vehicle V3 serves other cities in the sphere of influence of MAD. The overall capacity of vehicles V2 and V3 is lower than the one exhibited by the MAD-based vehicle in Example 1 (see Table 3). Two instances of Example 2 have been considered. The capacity of vehicle V1 is reduced from 15,000(w)/25(v) for Example 2A to 5000(w)/10(v) for Example 2B, thus forcing the BAR-based vehicle to make a pair of tours to service all the assigned cities. The other problem data are similar to those specified for Example 1. Pickup operations are carried out by vehicles V2 and V3 at MAD, while pickup and delivery tasks are performed by V1 and V2 at the distribution center respectively. Therefore, a pair of events is predefined for both sources MAD and BAR, and a single one for the other cities.

Table 5A
Distances between locations for Examples 1–4 (in km).

	Barcelona	Girona	Lerida	Tarragona	Vic	Valencia	Zaragoza	Perpignan	Andorra	Madrid	Bilbao	Valladolid	S.Sebastian	Teruel	Soria	Burgos	Coruña	Lugo	Santander
Barcelona		103	178	101	70	351	311	192	198	640	606	663	620	409	453	583	1043	1020	693
Girona	103		226	194	68	444	375	96	215	705	678	703	618	505	523	581	1091	1018	737
Lerida	178	226		107	158	348	149	316	183	479	454	507	464	319	297	427	865	864	537
Tarragona	101	194	107		162	260	240	283	260	560	535	598	555	311	388	518	972	955	628
Vic	70	68	158	162		411	307	158	151	637	610	635	550	473	455	513	1023	950	669
Valencia	351	444	348	260	411		328	535	501	370	607	580	605	167	376	517	961	863	673
Zaragoza	311	375	149	240	307	328		465	322	330	305	367	324	185	157	287	822	724	397
Perpignan	192	96	316	283	158	535	465		163	788	645	793	668	594	613	671	1181	1108	753
Andorra	198	215	183	260	151	501	322	163		625	545	660	505	472	450	580	1018	1017	653
Madrid	640	705	479	560	637	370	330	788	625		395	215	395	302	231	237	609	511	393
Bilbao	606	678	454	535	610	607	305	645	545	395		280	119	462	231	158	556	546	108
Valladolid	663	703	507	598	635	580	367	793	660	215	280		354	441	210	122	455	357	248
S.Sebastian	620	618	464	555	550	605	324	668	505	395	119	354		449	268	232	735	637	227
Teruel	409	505	319	311	473	167	185	594	472	302	462	441	449		231	372	896	798	528
Soria	453	523	297	388	455	376	157	613	450	231	231	210	268	231		141	665	567	297
Burgos	583	581	427	518	513	517	287	671	580	237	158	122	232	372	141		535	437	156
Coruña	1043	1091	865	972	1023	961	822	1181	1018	609	556	455	735	896	665	535		98	547
Lugo	1020	1018	864	955	950	863	724	1108	1017	511	546	357	637	798	567	437	98		449
Santander	693	737	537	628	669	673	397	753	653	393	108	248	227	528	297	156	547	449	
Huesca	274	374	119	210	314	399	73	466	472	397	322	440	256	254	431	230	359	694	597
Castellon	284	385	260	187	354	66	284	476	482	417	607	564	551	153	681	384	524	906	808
Pamplona	437	537	281	373	377	502	176	507	630	407	159	326	93	357	268	177	203	538	440
Zamora	759	859	603	694	709	600	464	951	957	248	759	859	603	694	709	600	464	951	957
Alicante	515	616	491	418	585	167	493	705	713	422	817	615	776	317	548	659	1031	933	815
Almeria	809	910	785	712	879	461	759	1001	1009	563	967	757	1013	602	794	795	1138	1045	973

Table 5B
Distances between locations for Example 5 (in km).

	Madrid	Valencia	Badajoz	Granada	Malaga	Murcia	Sevilla	Cadiz	Cordoba	Alicante	Almeria
Madrid		370	401	434	544	401	538	663	400	422	563
Valencia	370		716	519	648	241	697	808	545	167	461
Badajoz	401	716		438	436	675	217	342	272	697	605
Granada	434	519	438		129	278	256	335	166	354	167
Malaga	544	648	436	129		407	219	265	187	482	219
Murcia	401	241	675	278	407		534	613	444	76	220
Sevilla	538	697	217	256	219	534		125	138	610	423
Cadiz	663	808	342	335	265	613	125		263	689	485
Cordoba	400	545	272	166	187	444	138	263		520	333
Huesca	397	399	783	831	941	612	927	1051	788	564	830
Castellon	417	66	781	585	713	307	749	873	611	232	526
Pamplona	407	502	739	841	951	714	915	1049	798	667	933
Zamora	248	600	361	682	755	649	536	641	591	670	811
Alicante	422	167	697	354	482	76	610	689	520		295

Table 6
Optimal vehicle routes and schedules for Example 1A.

Allowed source-demand site allocations	
Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Tarragona, Zaragoza, Valencia, Lerida, Andorra, Perpignan, Girona, Vic, Teruel)
Madrid	V2 and (Madrid, Barcelona, Zaragoza, Valencia, Lerida, Teruel, Soria, Burgos, S.Sebastian, Bilbao, La Coruña, Lugo, Valladolid)

Vehicle	Detailed schedule of vehicle activities							
	Site	Arrival time	P1	P2	P3	P4	Used capacity	
							%w	%v
V1 (—)	Barcelona	34.6	+1420	+425	+775	+250	59.7	89.9
	Tarragona	48.6	−50	−200		−100		
	Zaragoza	54.4	−200		−250	−150		
	Lerida	59.9		−75	−75			
	Andorra	64.1	−800		−200			
	Perpignan	71.4	−150	−150				
	Gerona	75.0	−120		−150			
	Vic	78.0	−100		−100			
	Barcelona	80.9						
V2 (—)	Madrid	0.0	+1010	+970	+870	+620	81.5	98.1
	Teruel	19.2	−200	−100				
	Valencia	23.8	−120	−120				
	Barcelona	30.8	−320		−350	−50		
	Soria	41.1		−200	−50	−100		
	Burgos	45.5		−100	−150			
	S.Sebastian	50.8	−100	−50				
	Bilbao	54.1	−120		−120	−120		
	Santander	58.1		−150	−100	−50		
	Lugo	66.7		−100		−100		
	La Coruña	69.9	−100		−100			
	Valladolid	78.2	−50	−150		−200		
	Madrid	83.9						
Travelled distance								4160 km
Routing cost								$12,480
Fixed cost								$10,000
Total cost								$22,480

Table 7
Computational results for all examples.

Example	CPU time (s)	Binary variables	Continuous variables	Linear constraints
1A	18.0	170	439	4,007
1B	11.8	170	439	4,007
2A	12.4	172	599	4,513
2B	62.9	189	641	5,024
3	16.1	158	751	4,223
4A	37.3	232	1355	6,579
4B	1.2	232	1355	6,579
5	218.3	317	2339	12,322

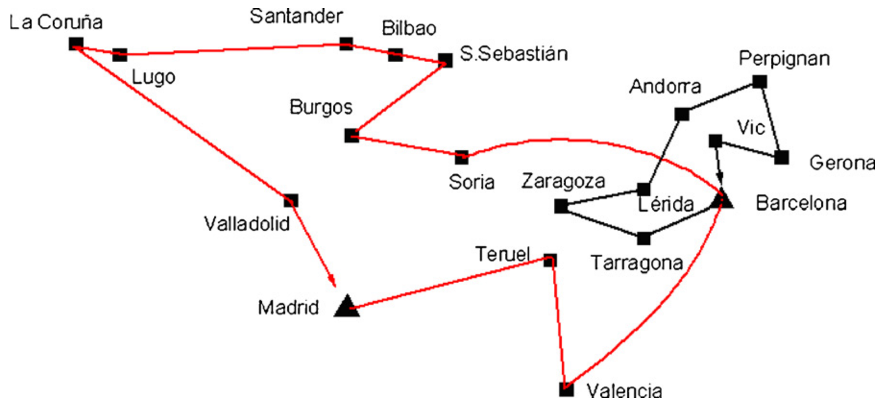

Fig. 2. The best vehicle routes for Example 1A.

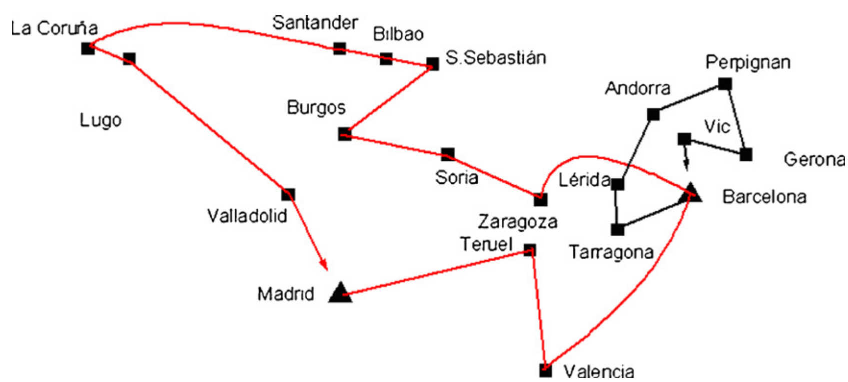

Fig. 3. The optimal solution for Example 1B.

Table 8
Optimal vehicle routes and schedules for Example 1B.

Allowed source-demand site allocations

Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Tarragona, Zaragoza, Valencia, Lerida, Andorra, Perpignan, Girona, Vic, Teruel)
Madrid	V2 and (Madrid, Barcelona, Zaragoza, Valencia, Lerida, Teruel, Soria, Burgos, S.Sebastian, Bilbao, La Coruña, Lugo, Valladolid)

Detailed schedule of vehicle activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	Used capacity	
							%w	%v
V1 (—)	Barcelona	31.2	+1220	+425	+575	+100	46.7	72.9
	Tarragona	44.6	−50	−200		−100		
	Lerida	48.5		−75	−75			
	Andorra	52.7	−800		−200			
	Perpignan	60.0	−150	−150				
	Gerona	63.6	−120		−150			
	Vic	66.7	−100		−100			
	Barcelona	69.5						
V2 (▬)	Madrid	0.0	+1010	+970	+870	+720	84.0	99.7
	Teruel	19.6	−200	−100				
	Valencia	24.2	−120	−120				
	Barcelona	33.0	−120		−100			
	Zaragoza	37.5	−200		−250	−150		
	Soria	43.1		−200	−50	−100		
	Burgos	47.5		−100	−150			
	S.Sebastian	52.9	−100	−50				
	Bilbao	56.2	−120		−120	−120		
	Santander	60.1		−150	−100	−50		
	La Coruña	70.1	−100		−100			
	Lugo	73.3		−100		−100		
	Valladolid	80.2	−50	−150		−200		
	Madrid	85.9						
Travelled distance								3893 km
Routing cost								$11,679
Fixed cost								$10,000
Total cost								$21,679

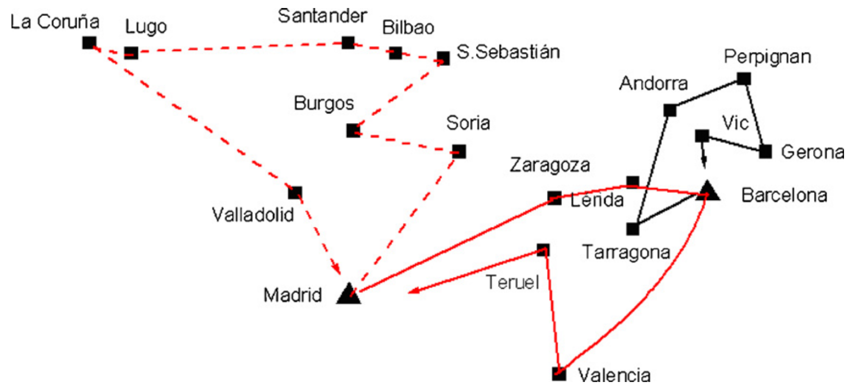

Fig. 4. The optimal vehicle routes for Example 2A.

The best vehicle routes and schedules for Example 2A are depicted in Fig. 4. They were found in 12.4 s of CPU time (see Table 7). More details on the optimal solution for Example 2A are given in Table 9. All vehicles have completed their assigned tasks at time 60.1 h, thus satisfying the maximum service time of 70 h. It was assumed that at least two shipping docks are available at MAD to allow vehicles V2-V3 to start their pickup operations at time $t = 0$. From Table 9, it is observed that pickup operations by vehicle V1 at the distribution center begins immediately after deliveries of products P1–P4 by V2 to BAR have been completed. Otherwise, vehicle V1 would have to perform a pair of tours and travel a longer distance. Note that the four cities with alternative sources (ZAR, LER, VAL, TER) are all visited by V2 coming from source MAD. In fact, routing costs become lower if such nodes are directly serviced from MAD instead of performing cross-dock operations at BAR and assigning vehicle V1 to visit them. The presence of an additional vehicle increases the number of variables and the number of constraints. In particular, the number of binary variables rises from 170 to 172. Though the vehicles complete their tasks much earlier, the total routing cost shows a 23.9% increase with regards to Example 1 because fixed and variable transportation costs both rise.

Table 9
Optimal vehicle routes and schedules for Example 2A.

Allowed source-demand site allocations

Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Gerona, Lerida, Tarragona, Vic, Valencia, Zaragoza, Teruel, Perpignan, Andorra)
Madrid	V2 and (Madrid, Barcelona, Zaragoza, Valencia, Teruel, Lerida, Valladolid, Soria, Burgos, Santander)
Madrid	V3 and (Madrid, Teruel, Valladolid, Soria, Burgos, Santander, Bilbao, La Coruña, Lugo, S. Sebastian)

Detailed schedule of vehicle-activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	Used capacity	
							%w	%v
V1 (—)	Barcelona	26.3	+1220	+350	+450	+100	56.7	65.4
	Tarragona	37.2	−50	−200		−100		
	Andorra	43.3	−800		−200			
	Perpignan	50.7	−150	−150				
	Gerona	54.2	−120		−150			
	Vic	57.3	−100		−100			
	Barcelona	60.1						
V2 (▬)	Madrid	0.0	+640	+645	+350	+250	73.2	97.9
	Zaragoza	13.3	−200		−250	−150		
	Lerida	18.8		−75	−75			
	Barcelona	22.9	−120	−350	−25	−100		
	Valencia	31.3	−120	−120				
	Teruel	35.7	−200	−100				
	Madrid	42.2						
V3 (–·–)	Madrid	0.0	+370	+750	+520	+570	92.2	92.0
	Soria	13.1		−200	−50	−100		
	Burgos	17.6		−100	−150			
	S.Sebastian	22.9	−100	−50				
	Bilbao	26.2	−120		−120	−120		
	Santander	30.2		−150	−100	−50		
	Lugo	38.8		−100		−100		
	La Coruña	42.0	−100		−100			
	Valladolid	50.3	−50	−150		−200		
	Madrid	55.9						
Travelled distance								4283 km
Routing cost								$12,849
Fixed cost								$15,000
Total cost								$27,849

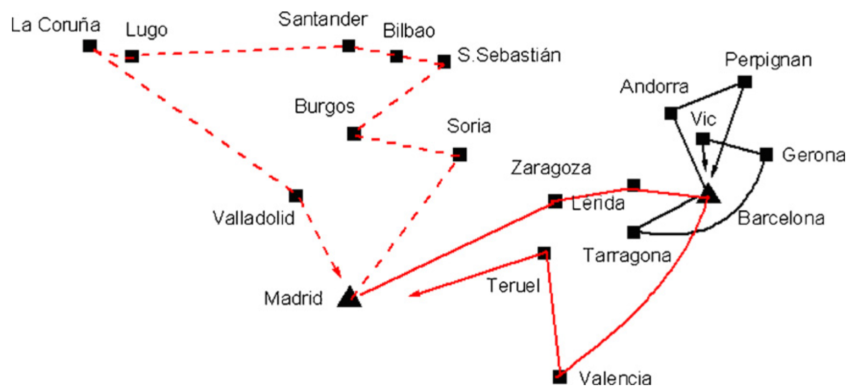

Fig. 5. The optimal solution for Example 2B.

If Example 2B is solved through using the same number of events adopted for Example 2A (i.e. two for nodes BAR and MAD and only one for the remaining locations), the resulting mathematical model has no feasible solution. This is so because the new vehicle capacity for V1 is not large enough to service all the demanding cities by making just a single tour. To overcome this problem, there are two remedial actions consisting of (a) using another vehicle based at BAR or MAD, or (b) allowing vehicle V1 housed at BAR, to make a pair of tours instead of a single one. Both alternatives require to assigning an additional event to either MAD or BAR. The second option was chosen and a further event was assigned to BAR so that V1 can make a second stop at the DC to pick up further amounts of products. Then, three events for BAR, two for MAD and only one for the other cities were predefined to solve Example 2B. Consequently, the model size becomes larger and the number of binary variables rises from 172 to 189. The optimal solution is shown in Fig. 5 and Table 10. It was determined in 62.9 s. It is observed that vehicle V1 waits for the arrival of V2 and the completion of the related delivery operations at BAR before leaving the base to service Andorra (node AND) and Perpignan (node PER). After that, it

Table 10
Optimal vehicle routes and schedules for Example 2B.

Allowed source-demand site allocations	
Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Gerona, Lérida, Tarragona, Vic, Valencia, Zaragoza, Teruel, Perpignan, Andorra)
Madrid	V2 and (Madrid, Barcelona, Zaragoza, Valencia, Teruel, Lérida, Valladolid, Soria, Burgos, Santander)
Madrid	V3 and (Madrid, Teruel, Valladolid, Soria, Burgos, Santander, Bilbao, La Coruña, Lugo, S. Sebastian)

Detailed schedule of vehicle-activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	Used capacity	
							%w	%v
V1 (—)	Barcelona	26.3	+950	+150	+200		95.0	90.0
	Andorra	35.3	−800		−200			
	Perpignan	42.7	−150	−150				
	Barcelona	47.6	+270	+200	+250	+100	75.2	73.5
	Tarragona	53.3	−50	−200		−100		
	Gerona	58.5	−120		−150			
	Vic	61.6	−100		−100			
	Barcelona	64.4						
V2 (▬)	Madrid	0.0	+640	+645	+350	+250	73.2	97.9
	Zaragoza	13.3	−200		−250	−150		
	Lerida	18.8		−75	−75			
	Barcelona	22.9	−120	−350	−25	−100		
	Valencia	31.3	−120	−120				
	Teruel	35.7	−200	−100				
	Madrid	42.2						
V3 (–·–)	Madrid	0.0	+370	+750	+520	+570	92.2	92.0
	Soria	13.1		−200	−50	−100		
	Burgos	17.6		−100	−150			
	S.Sebastian	22.9	−100	−50				
	Bilbao	26.2	−120		−120	−120		
	Santander	30.2		−150	−100	−50		
	Lugo	38.8		−100		−100		
	La Coruña	42.0	−100		−100			
	Valladolid	50.3	−50	−150		−200		
	Madrid	55.9						
Travelled distance								4511 km
Routing cost								$13,533
Fixed cost								$15,000
Total cost								$28,533

Table 11
Optimal vehicle routes and schedules for Example 3.

Allowed supplying-site and demanding-sites allocations	
Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Girona, Lérida, Tarragona, Vic, Perpignan, Andorra, Zaragoza)
Madrid	V2 and (Barcelona, Madrid, Zaragoza, Valencia, Teruel, Lérida, Valladolid, Soria, Burgos)
Bilbao	V3 and (Madrid, Santander, Bilbao, Valladolid, Soria, Burgos)
	V4 and (Bilbao, Santander, S.Sebastian, Lugo, Soria, Burgos, La Coruña, Valladolid)

Detailed schedule of vehicle-activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	Used Capacity	
							%w	%v
V1 (—)	Barcelona	29.2	+1220	+350	+450	+100	85.1	90.8
	Vic	39.7	−100		−100			
	Gerona	42.4	−120		−150			
	Perpignan	45.9	−150	−150				
	Andorra	50.4	−800		−200			
	Tarragona	59.1	−50	−200		−100		
	Barcelona	63.0						
V2 (▬)	Madrid	0.0	+640	+845	+400	+350	76.7	98.3
	Teruel	14.3	−200	−100				
	Valencia	18.8	−120	−120				
	Barcelona	25.8	−120	−350	−25	−100		
	Lerida	31.7		−75	−75			
	Zaragoza	35.5	−200		−250	−150		
	Soria	41.1		−200	−50	−100		
	Madrid	46.8						
V3 (–·–)	Madrid	0.0	+50	+500	+150	+250	51.5	52.5
	Burgos	8.2		−100	−150			
	Bilbao	12.4	−50	−400		−250		
	Madrid	26.6						
V4 (▬)	Bilbao	16.2	+250	+450	+200	+350	62.0	58.8
	S.Sebastian	23.9	−100	−50				
	Valladolid	30.6	−50	−150		−200		
	La Coruña	39.7	−100		−100			
	Lugo	42.9		−100		−100		
	Santander	51.1		−150	−100	−50		
	Bilbao	54.9						
Travelled distance								4666 km
Routing cost								$13,998
Fixed cost								$20,000
Total cost								$33,998

returns to BAR to pickup further amounts of products and starts another tour to satisfy the demand of the remaining cities to be serviced. As a result, the optimal travel distance increases from 4283 to 4511 km. In contrast, the optimal tours for vehicles V2 and V3 remain similar to those found for Example 2A. As before, there are no end product inventories at the DC. Example 2B shows another important feature of the proposed problem formulation. Some lots of products (i.e. 150 units of P2) are immediately moved from the receiving dock to the shipping dock at BAR and sent to AND and PER. The other lots received from MAD stay for more than 20 h before they are shipped to their destinations.

7.3. Example 3

Another warehouse placed at Bilbao (node BIL) to service the cities located within its sphere of influence is considered at Example 3. Distribution of products from BIL is performed by an additional vehicle V4 whose features are given in Table 3. The cities of Lugo

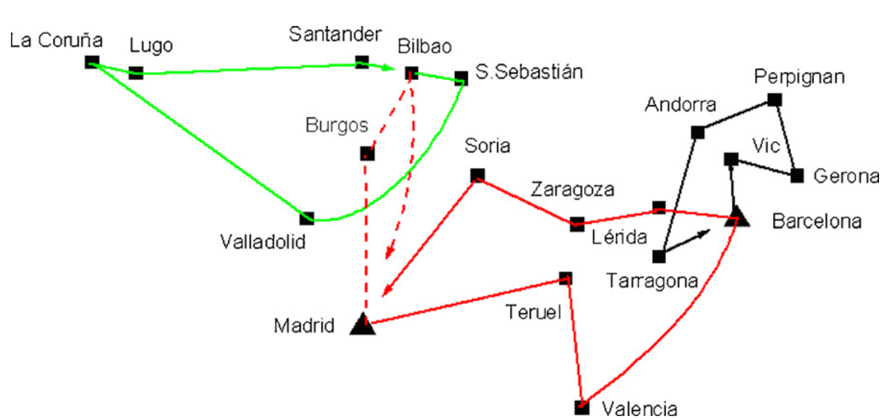

Fig. 6. The optimal vehicle routes for Example 3.

Table 12
Time windows for starting the customer service at Example 4B.

	A (h)	B (h)
Girona	30	50
Lérida	0	25
Tarragona	45	60
Vic	20	40
Valencia	10	35
Zaragoza	0	25
Perpignan	30	55
Andorra	30	55
Valladolid	10	30
S.Sebastián	30	55
Bilbao	0	15
Teruel	20	40
Soria	0	20
Santander	10	30
Burgos	0	15
Lugo	20	40
La Coruña	20	50
Badajoz	0	20
Granada	30	45
Murcia	20	45
Sevilla	10	50
Cadiz	30	60
Córdoba	30	55

(LUG), San Sebastian (SSEB) and La Coruña (LACO) located inside the service area of the warehouse at BIL have this facility as the pre-assigned supplier. Some other locations such as Santander (SAN) and Burgos (BUR) can be serviced from either MAD or BIL. Besides, nodes (BAR, MAD, BIL) are the alternative sources for Soria (SOR) and Valladolid (VALL). Initial stocks available at BIL are not enough to meet demands at customer nodes exclusively serviced by vehicle V4. Then, further amounts of products from MAD transported by vehicle V3 should be received and cross-docked at BIL. To allow delivery and pickup operations by vehicles V3 and V4, respectively, two events are predefined for node BIL. Customer demands and initial stocks at BAR are similar to the ones proposed for Example 2, and weight/volume capacities for V1–V4 are presented in Table 3. No time windows are specified and a maximum service time of 70 h cannot be exceeded. The best vehicle routes and schedules are shown in Fig. 6 and Table 11. Despite considering an additional warehouse, the optimal solution was found in a CPU time of 16.1 s. At the optimum, SOR and BUR are supplied from MAD through vehicles V2 and V3, respectively, while SAN and VALL are serviced by V4 based at Bilbao. Vehicle V4 should wait for the arrival of V3 and the completion of the related delivery activities (i.e. the first event at BIL) before it begins loading lots of products into V4 to meet the assigned demands (i.e. the second event at BIL). When the pickup operations have ended, vehicle V4 starts moving to San Sebastian (node SSEB). The amounts of products received from MAD at warehouses BAR and BIL are fully cross-docked and sent to their destinations. As a result, no product inventories remain at the two DCs when the planning horizon ends.

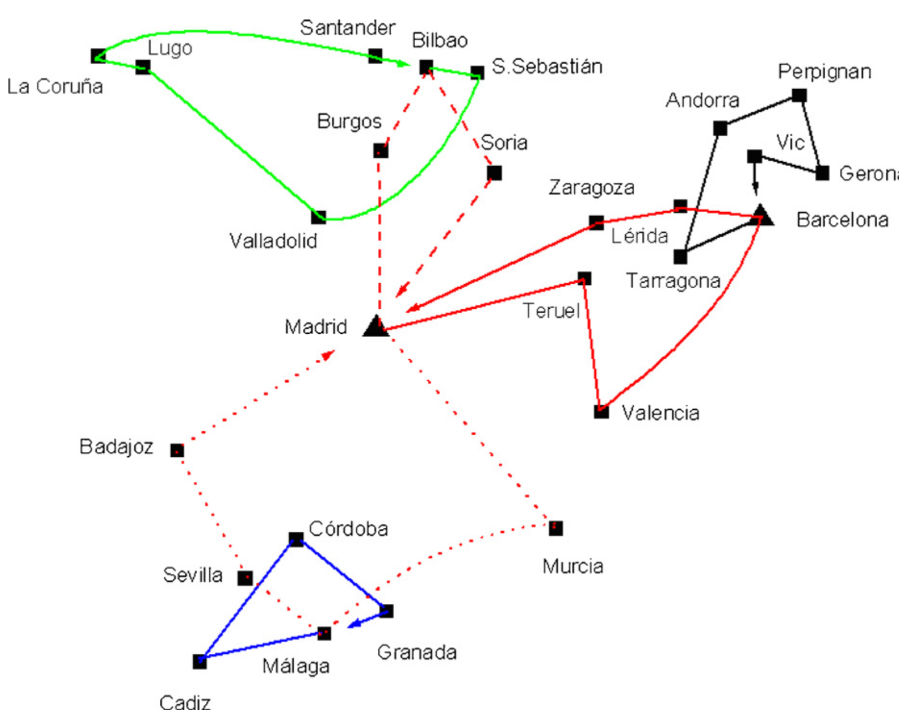

Fig. 7. The optimal vehicle routes for Example 4A.

Table 13
Optimal vehicle routes and schedules for Example 4A.

Allowed supplying-site and demanding-sites allocations

Supplying site	Vehicles and Demanding sites that can visit
Barcelona	V1 and (Barcelona, Girona, Lerida, Tarragona, Vic, Zaragoza, Perpignan, Andorra)
Madrid	V2 and (Madrid, Barcelona, Valencia, Zaragoza, Teruel, Lérida, Valladolid, Soria, Burgos)
Bilbao	V3 and (Madrid, Bilbao, Santander, Valladolid, Soria, Burgos)
Málaga	V5 and (Madrid, Malaga, Badajoz, Murcia, Granada, Sevilla, Córdoba)
	V4 and (Bilbao, Valladolid, Santander, S.Sebastian, Lugo, Soria, Burgos, La Coruña)
	V6 and (Málaga, Badajoz, Murcia, Granada, Sevilla, Cordoba, Cadiz)

Detailed schedule of vehicle-activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	Used capacity	
							%w	%v
V1 (—)	Barcelona	31.0	+1220	+350	+450	+100	70.9	81.8
	Tarragona	41.9	−50	−200		−100		
	Andorra	48.0	−800		−200			
	Perpignan	55.4	−150	−150				
	Gerona	58.9	−120		−150			
	Vic	62.0	−100		−100			
	Barcelona	64.8						
V2 (▬)	Madrid	0.0	+940	+645	+450	+250	72.8	91.5
	Teruel	14.5	−200	−100				
	Valencia	19.0	−120	−120				
	Barcelona	26.0	−420	−350	−125	−100		
	Lerida	33.5		−75	−75			
	Zaragoza	37.3	−200		−250	−150		
	Madrid	45.4						
V3 (‒ . ‒)	Madrid	0.0	+50	+700	+200	+350	71.0	72.5
	Burgos	9.6		−100	−150			
	Bilbao	13.8	−50	−400		−250		
	Soria	20.9		−200	−50	−100		
	Madrid	26.6						
V4 (▬)	Bilbao	17.6	+250	+450	+200	+350	77.5	65.3
	S.Sebastian	25.3	−100	−50				
	Valladolid	32.0	−50	−150		−200		
	Lugo	39.7		−100		−100		
	La Coruña	42.9	−100		−100			
	Santander	52.5		−150	−100	−50		
	Bilbao	56.3						
V5 (▪ ▬ ▪)	Madrid	0.0		+970	+870	+950	74.6	100.0
	Murcia	17.9		−380	−200			
	Málaga	27.0		−370	−40	−500		
	Sevilla	34.8			−200	−450		
	Badajoz	41.5		−220	−430			
	Madrid	50.8						
V6 (▬)	Malaga	31.7	+300	+670	+340	+800	33.6	100.0
	Cadiz	44.9			−340			
	Cordoba	51.0		−420		−430		
	Granada	57.8	−300	−250		−370		
	Malaga	64.3						
Travelled distance								7143 km
Routing cost								$20195.00
Fixed cost								$28000.00
Total cost								$48195.00

7.4. Example 4

Example 4 considers a two-echelon distribution network involving a manufacturer storage at Madrid, three warehouses located at Barcelona, Bilbao and Malaga, six further demanding locations and two additional vehicles V5 and V6. Vehicle V5 is housed in MAD and replenishes inventories at Malaga (MAL), while V6 is based at MAL and distribute lots of products to neighboring cities. Only the city of Cadiz (CAD) much closer to Malaga than to the other sources has been pre-assigned to the new warehouse at MAL. Therefore, a fleet of three heterogeneous vehicles (V2, V3, V5) is available at MAD-facility having a similar number of shipping docks. As a result, pickup operations by the three vehicles can be performed at the same time. The other three trucks (V1, V4, V6) start their trips from BAR, BIL and MAL, respectively. A total of 27 nodes are now considered. Since the initial stocks available in the three distribution centers are lower than the product requirements at the assigned locations, cross-dock operations must be performed. To allow the visit of two different vehicles, a pair of events was pre-assigned to cross-dock facilities at BAR, BIL and MAL, while three were predefined for MAD and only one for the demanding cities.

Two instances of Example 4, called Examples 4A and 4B, were considered. Example 4A specifies neither time windows nor finite end inventories at some warehouses. Such additional problem features are taking into account in Example 4B with time windows given by Table 12 and end inventories at BIL-based warehouse fixed at 20 units for all products. The best solution for Example 4A found in 37.3 s is depicted in Fig. 7 and Table 13. Distribution of products from the three distribution centers does not start until the vehicles coming from the manufacturer storage at MAD with additional

Table 14
Optimal vehicle routes and schedules for Example 4B (with time windows).

Allowed supplying-site and demanding-sites allocations

Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Girona, Lerida, Tarragona, Vic, Zaragoza, Perpignan, Andorra)
Madrid	V2 and (Madrid, Barcelona, Valencia, Zaragoza, Teruel, Lérida, Valladolid, Soria, Burgos)
Bilbao	V3 and (Madrid, Bilbao, Santander, Valladolid, Soria, Burgos)
Málaga	V5 and (Madrid, Malaga, Badajoz, Murcia, Granada, Sevilla, Córdoba)
	V4 and (Bilbao, Valladolid, Santander, S.Sebastian, Lugo, Soria, Burgos, La Coruña)
	V6 and (Málaga, Badajoz, Murcia, Granada, Sevilla, Cordoba, Cadiz)

Detailed schedule of vehicle-activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	Used capacity	
							%w	%v
V1 (—)	Barcelona	29.5	+1220	+350	+450	+100	70.9	81.8
	Vic	40.0	−100		−100			
	Gerona	42.8	−120		−150			
	Perpignan	46.2	−150	−150				
	Andorra	50.7	−800		−200			
	Tarragona	59.5	−50	−200		−100		
	Barcelona	63.3						
V2 (▬)	Madrid	0.0	+940	+645	+450	+250	72.8	91.5
	Zaragoza	14.9	−200		−250	−150		
	Lerida	20.4		−75	−75			
	Barcelona	24.5	−420	−350	−125	−100		
	Valencia	34.5	−120	−120				
	Teruel	38.9	−200	−100				
	Madrid	45.4						
V3 (–·–)	Madrid	0.0	+70	+720	+220	+370	74.8	76.0
	Burgos	9.9		−100	−150			
	Bilbao	14.2	−20	−270	−20	−70		
	Soria	20.0		−200	−50	−100		
	Valladolid	25.4	−50	−150		−200		
	Madrid	31.1						
V4 (▬)	Bilbao	16.7	+200	+300	+200	+150	51.9	45.8
	Santander	22.6		−150	−100	−50		
	La Coruña	32.6	−100		−100			
	Lugo	35.8		−100		−100		
	S.Sebastian	46.7	−100	−50				
	Bilbao	50.0						
V5 (- ▬ -)	Madrid	0.0		+970	+870	+950	74.6	100.0
	Badajoz	17.4		−220	−430			
	Málaga	27.7		−370	−40	−500		
	Murcia	38.2		−380	−200			
	Sevilla	49.1			−200	−450		
	Madrid	60.4						
V6 (▬)	Malaga	32.4	+300	+670	+340	+800	33.6	100.0
	Granada	43.6	−300	−250		−370		
	Cordoba	50.7		−420		−430		
	Cadiz	58.8			−340			
	Malaga	65.0						
Travelled distance								7934 km
Routing cost								$22232.50
Fixed cost								$28000.00
Total cost								$50232.50

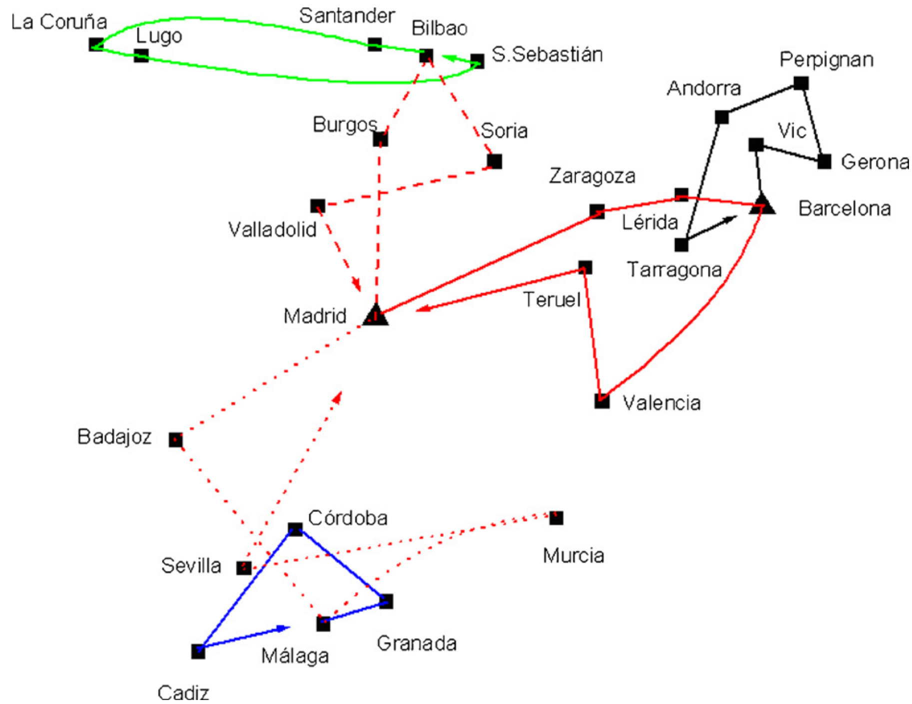

Fig. 8. The optimal solution for Example 4B.

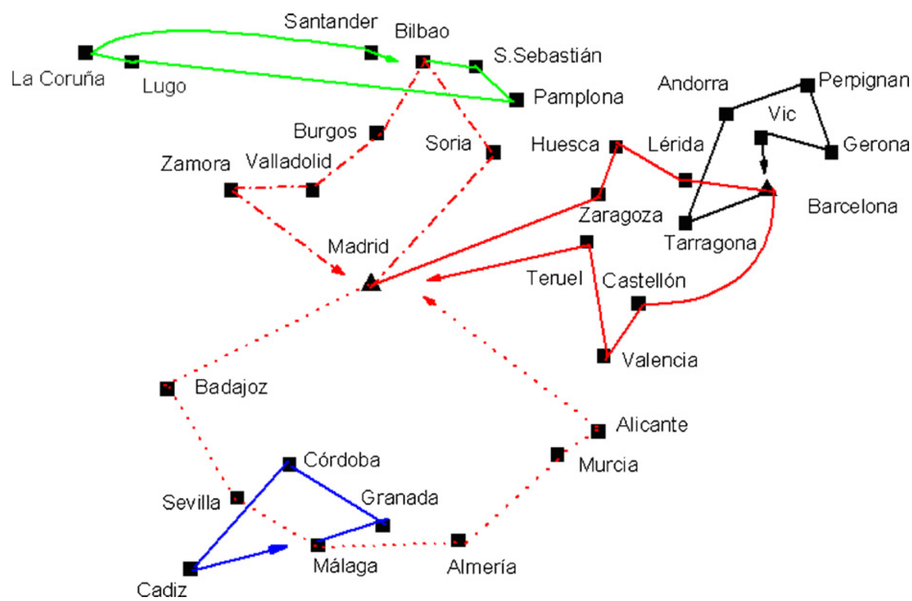

Fig. 9. The optimal solution to Example 5.

amounts of products arrive. Otherwise, vehicles (V1, V4, V6) would have to make an intermediate stop at their bases for loading further lots of products, and consequently the resulting routes will present higher transportation costs and travel longer distances.

Compared with Example 4A, the optimal solution to Example 4B features a longer total travel distance and a higher transportation cost (see Fig. 8 and Table 14). It was found in 1.2 s of CPU time. Some routes no longer have tear-drop shapes and some crossing points appear, i.e. distorted routes. This is the case for the tours travelled by vehicles V3 and V5. The overall travel distance rises from 7143 to 7934 km and the transportation cost grows by 4.22%.

7.5. Example 5

Example 4A is revisited but this time six products P1–P6 are to be distributed and six additional customer locations were considered. Moreover, the maximum service time has been increased to 80 h and time windows for four locations are given: Zaragoza (0-20 h), Soria (0-20 h), San Sebastian (20–35 h), and Badajoz (0–20 h). Other problem data for Example 5 involving 33 locations are given in Tables 1–5. The new products P5-P6 are demanded at cities serviced from MAD, BAR and BIL. From Table 1, it follows that no initial stocks of products (P2, P4, P5) are available at BAR, and the amount

Table 15
Optimal vehicle routes and schedules for Example 5.

Allowed supplying-site and demanding-sites allocations

Supplying site	Vehicles and demanding sites that can visit
Barcelona	V1 and (Barcelona, Girona, Lerida, Tarragona, Vic, Zaragoza, Perpignan, Andorra, Huesca, Castellon)
Madrid	V2 and (Madrid, Barcelona, Valencia, Zaragoza, Teruel, Lerida, Valladolid, Soria, Castellon, Huesca, Alicante)
Bilbao	V3 and (Madrid, Bilbao, Santander, Valladolid, Soria, Burgos, Pamplona, Zamora)
Málaga	V5 and (Madrid, Malaga, Badajoz, Murcia, Granada, Sevilla, Cordoba, Almeria, Alicante)
	V4 and (Bilbao, Valladolid, Santander, S.Sebastian, Lugo, Soria, Burgos, La Coruña, Pamplona, Zamora)
	V6 and (Malaga, Badajoz, Murcia, Granada, Sevilla, Cordoba, Cadiz, Almeria, Alicante)

Detailed schedule of vehicle-activities

Vehicle	Site	Arrival time	P1	P2	P3	P4	P5	P6	Used capacity	
									%w	%v
V1 (—)	Barcelona	37.9	+1220	+350	+450	+100	+400	+100	91.8	94.3
	Tarragona	50.8	−50	−200		−100				
	Andorra	56.9	−800		−200		−100			
	Perpignan	64.6	−150	−150			−100	−50		
	Gerona	68.8	−120		−150		−50	−50		
	Vic	72.3	−100		−100		−150			
	Barcelona	75.7								
V2 (▬)	Madrid	0.0	+1040	+695	+600	+300	+550	+120	84.1	94.6
	Zaragoza	18.9	−200		−250	−150	−100			
	Huesca	23.4		−50	−150					
	Lerida	26.9		−75	−75			−70		
	Barcelona	31.3	−420	−350	−125	−100	−400			
	Castellon	41.9	−100			−50				
	Valencia	44.5	−120	−120			−50	−50		
	Teruel	49.2	−200	−100						
	Madrid	56.1								
V3 (–·–)	Madrid	0.0	+150	+800	+300	+350	+150	+350	91.7	90.9
	Soria	12.7		−200	−50	−100		−150		
	Bilbao	19.0		−350		− 50	−50	−150		
	Burgos	26.3		−100	−150		−100	−50		
	Valladolid	30.6	−50	−150		−200				
	Zamora	34.6	−100		−100					
	Madrid	39.9								
V4 (▬)	Bilbao	22.4	+200	+400	+200	+150	+350	+150	72.5	81.7
	S.Sebastian	30.9	−100	−50						
	Pamplona	33.8		−100			−50			
	Lugo	41.7		−100		−100				
	La Coruña	44.9	−100		−100		−100	−150		
	Santander	55.5		−150	−100	−50	−200			
	Bilbao	60.1								
V5 (▪▬▪)	Madrid	0.0	+50	+970	+870	+1100			87.9	100.0
	Badajoz	18.7		−220	−430					
	Sevilla	25.4			−200	−450				
	Málaga	32.1		−370	−40	−500				
	Almeria	39.9				−100				
	Murcia	44.4		−380	−200					
	Alicante	48.8	−50			−50				
	Madrid	56.3								
V6 (▬)	Malaga	36.8	+300	+670	+340	+800			88.5	94.8
	Granada	48.0	−300	−250		−370				
	Cordoba	55.1		−420		−430				
	Cadiz	63.2			−340					
	Malaga	69.4								
Travelled distance										7470 km
Routing cost										$21111.50
Fixed cost										$28000.00
Total cost										$49111.50

of P6 on hand at BIL in the beginning of the planning horizon is zero. Each of the additional cities [Huesca (HUE), Castellon (CAS), Pamplona (PAM), Zamora (ZAM), Alicante (ALI), Almeria (ALM)] can be serviced from two alternative sources and by two different vehicles. Despite the larger number of products and cities, the optimal solution was found in 218.3 s. It is shown in Fig. 9 and Table 15. As the pickup/delivery operations at sources (MAD, BAR, BIL) takes more time and more cities are serviced, vehicle routes become longer with regards to Example 4A. In particular, V1 ends at time 75.7 h still below the maximum service time of 80 h. No change is observed in the optimal value of the objective function if two events instead of a single one are associated to customer locations with two alternative suppliers.

8. Conclusions

A MILP mathematical formulation for the vehicle routing problem with cross-docking in supply chain management (VRPCD-SCM) has been developed. The VRPCD-SCM addresses the problem of managing hybrid multi-echelon distribution networks transporting multiple products from factories to customers through direct shipping and/or via intermediate depots using warehousing and cross docking strategies. The approach is a generalization of the mathematical model introduced by Dondo et al. (2009) for the VRP-SCM problem without cross-dockings. Factories, warehouses and customer locations are the problem nodes where a number of events can take place. During an event, a vehicle stop occurs and pickup and/or delivery operations are accomplished. The nature and extent of such operations are established by solving the proposed formulation. The number of events predefined for each location is a model parameter. It should be at least equal to the related number of vehicle stops at the optimum solution. The selected problem goal is to minimize the total routing cost, including fixed and distance-based transportation costs. The lowest travel time has been adopted as a 2nd-level objective. Several features of the proposed approach were illustrated by tackling a wide variety of examples. All of them were solved to optimality in a reasonable amount of CPU time. Among the model features highlighted by the examples, it should be especially mentioned: (a) the automatic execution of cross-dock operations when initial stocks at warehouses are scarce to meet demands at the assigned destinations; (b) combined warehousing and cross-docking strategies at intermediate facilities when finite end inventories are specified; (c) the use of both direct shipping and distribution via intermediate facilities to satisfy customer requirements at the optimum; (d) the visit of two or more different vehicles to the same location; (e) the generation of vehicle routes involving more than a single tour with multiple stops at the base for reloading operations, if the vehicle capacity is lower than the total demand of the assigned locations; (f) the distribution of multiple products (up to six) via several intermediate facilities (up to three), (g) the straightforward handling of customer requests including more than one item; and (h) the effective management of heterogeneous vehicle fleets housed at different bases.

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