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Uncertainty in the estimation of the postmortem interval based on rectal temperature measurements: a Bayesian approach

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- Uncertainties of model parameters must be included in the estimation of PMI.
- Bayesian inference increases the reliability of PMI estimation.
- The posterior probability distribution of PMI is more useful than a time interval.
- The Bayesian framework allows combining different methods of PMI estimation.

Abstract

In this work, the postmortem interval is estimated by means of Bayesian inference using rectal temperature data from a published database. First, a systematic analysis of the uncertainties in each of the model parameters is carried out in order to assess their relative influences on the postmortem interval uncertainty. Then, the method is applied to the whole database and proves to be more reliable than the well-established nomogram method. Moreover, the result of the Bayesian inference process is the full posterior probability distribution of the postmortem interval, which provides more information than a simple point estimate or a time interval. This distribution can be used to assign probabilities to specific time intervals that may arise in a criminal investigation. The application of this statistical analysis can be extended to any method of estimating the postmortem interval.

1 Introduction

In the context of finding a human corpse, the postmortem interval (PMI), or time since death, is defined as the time elapsed between the death and the examination of the body. Its estimation is a common forensic practice and is essential, among other things, to validate or refute a suspect's alibi in criminal investigations. Although there is a increasing variety of techniques to estimate the PMI, the method developed mainly by Henssge [1, 2], known as the "nomogram method" (although nowadays the calculations are done by computer rather than using a nomogram), is the most widely used in practice. This method is based on the model of body cooling by Marshall and Hoare [3, 4], which describes the changes in the standardized temperature Q as a function of time since death and has two parameters A and B. This method can be applied directly at the crime scene and in a wide variety of settings. Although there are several studies questioning its precision [5, 6, 7], it is still the current standard in determining the PMI [8].

In order to obtain an estimate of the PMI using the nomogram method, the following data must be acquired at the scene: the rectal temperature of the body, the mean ambient temperature, the body mass and a corrective factor c wich depends on the cooling conditions. The method assumes that the probability distribution of the time since death is Gaussian, with mean value $\hat{t}$ and standard deviation σ_t, and the result is commonly expressed as $\hat{t} \pm 2\sigma_t$, where the minimum and maximum times define the 95.45 % confidence interval for the PMI [9] according to a frequentist statistical approach. However, there are a number of drawbacks associated with this procedure, namely:

1. The mean value $\hat{t}$ is obtained by performing the calculations considering all variables as being deterministic (i.e. with a fixed value), and the error assigned to the estimation arises from a histogram of residuals which was obtained using the same method in a series of experiments [9]. The only relationship between the empirical error assigned to the estimation of the time since death in a specific case and the evidence obtained at the scene is the standardized temperature value Q (related to the degree of cooling progress). Therefore, measurement errors and uncertainty of the model parameters are not directly taken into account by this method.

2. Even assuming that sufficient data are available to estimate the mean ambient temperature over an adequate time interval (i.e. an interval large enough to contain the time since death, starting for example at the time when the deceased has been last seen alive), the uncertainty of that estimate is not taken into account. In cases where the ambient temperature varies considerably throughout the cooling process, the solution proposed in the bibliography is to consider the extreme temperatures and make two estimates of $\hat{t}$ that finally define an interval [10]. The recommendation is to report this interval only if it is greater than that obtained using the empirical error. However, in this case only the uncertainty in the ambient temperature would be considered, ignoring the other sources of uncertainty.

3. While many examples have been published showing how the value of the corrective factor c is determined, there is no systematic way to "translate" the set of cooling conditions to a specific value of that factor. The tabulated values of c were determined by fitting experimental data, but the fitting errors are neither reported nor included in the calculations [10].

4. The functional dependence of the parameter B with the body mass and the corrective factor c was empirically determined by fitting a data set, but the fitting error is neither reported nor included in the calculations. A similar situation is observed for the parameter A and its functional dependence on the mean ambient temperature.

5. The human core temperature is known to fluctuate naturally throughout the day (circadian cycles) and depending on many factors such as health status, menstrual cycle, intense physical activity, etc. [11, 12]. However, this method assumes a fixed initial temperature without uncertainty.

6. Rectal temperature is not constant throughout the entire rectum, but depends on the depth and anatomy of the individual [13]. Although it is possible to standardize the insertion depth of the thermometer, the interindividual variability in the dimensions of the rectum makes the location of a representative point always be subject to error.

7. The uncertainty in the estimated time since death is taken as the standard deviation of a set of experimental errors obtained in different ranges of standardized temperature Q. These empirical errors informally encompass all the uncertainties previously mentioned, but are specific to the cases included in their determination. The following question arises: are the samples used to obtain these errors representative of the entire population of possible cases? In other words: do these empirical errors have a universal quality?

A couple of works were recently published analyzing the error in PMI estimation caused by "input errors" (i.e. measurement errors) in the nomogram method. The authors performed error propagation calculations and Monte Carlo simulations to quantify the effect of errors on rectal temperature, ambient temperature, initial temperature, and time measurement [14], as well as the errors on body mass and corrective factor [15]. They then used an experimental database of postmortem rectal cooling to determine the absolute error between the estimated mean time since death according to the nomogram method and the actual time since death [7], and their results show that the empirical errors used according to the nomogram method are in many cases less than the real ones, and therefore the confidence interval of 95.45% obtained with this method is not always reliable. The conclusion of this last work was that it is necessary to perform a recalibration of the nomogram method, recalculating the corrective factors in a large number of samples (according to its recommendations, at least 400 samples are required).

Another important flaw, inherent to the analysis of the data according to the nomogram method, consists in the interpretation of the result. Formally speaking, a frequentist confidence interval of 95.45% in the reported PMI is not an interval where the probability of finding the true PMI is equal to 95.45%. Indeed, the only thing that can be assured from the frequentist perspective is that, if confidence intervals of 95.45% are built from infinite samples[1], 95.45% of these intervals will contain the true value of the PMI. However, given a specific sample and the associated confidence interval, an important question arises: what is the probability that the true value of the time since death is contained in this specific interval? Unfortunately, it is not possible to answer this question from a frequentist perspective. In fact, following this approach, the time since death is a fixed quantity and therefore the probability that it is contained in the interval will be 0 or 1, and this conclusion is not particularly useful.

In this work, we analyze the effect of the different variables and parameters involved in the nomogram method from a different point of view, namely from the Bayesian perspective, using a published database of postmortem rectal cooling [16]. The result of this approach is not a mean estimate of the PMI with an associated empirical error, but the full posterior probability distribution of the time since death. This distribution is obtained, given a statistical model of the data, through an inference process in which all the uncertainties associated with the model parameters are introduced. The use of conditional probability distributions has been previously proposed to compute confidence intervals for the reported time since death by means of the nomogram method combined with non-temperature-based methods [17, 18], but the uncertainties of the model parameters were not taken into account. Bayesian methods have also been applied to the estimation of PMI based on multiple temperature measurements and using mechanistic models [19]. Although these complex models are very interesting from a theoretical point of view, they are not yet applicable in daily forensic casework. The method we propose in this work is simple to implement and requires the same effort at the crime scene as the nomogram method.

One of the advantages of this approach is to avoid the error associated with the hypothesis that the PMI is normally distributed around a mean value, since, as will be seen in Section 3, in some cases its distribution is rather asymmetric. However, the greatest benefit of having the posterior distribution of the PMI is to be able to make concrete claims about probabilities. In this sense, a credibility region of 95.45% can be calculated, and its interpretation, in contrast to the frequentist approach, is precisely the probability that the true time since death is contained in that region, given the prior information and the data obtained at the scene. On the other hand, it is also possible to answer specific questions of the type: what is the probability that death occurred in a specific time interval? This question may arise, for example, when analyzing a suspect's alibi, and it has a potentially enormous value as scientific evidence in court. In addition, this method allows new information to be incorporated (e.g. the time at which the person was last seen alive, knowledge of a history of fever, changes in cooling conditions prior to finding the body, etc.) in a systematic manner, and therefore makes it possible to obtain posterior distributions of PMI specific to each case (instead of using an empirical error that does not depend on the case, as in the nomogram method). Furthermore, Bayesian inference allows combining distributions of PMI obtained with different techniques in order to achieve a final distribution that reflects all the available knowledge about a case.

2 Materials and methods

2.1 PMI estimation using the nomogram method

The steps to estimate the postmortem interval using the nomogram method are summarized below:

1. The rectal temperature u is measured at the scene, usually employing the same procedure proposed by Marshall and Hoare [3, 10, 20, 21].

2. The mean ambient temperature, u_A, is calculated from data collected at the scene (or near it) in the last hours previous to finding the body. If such a record of ambient temperature is not available, the measured value at the time of discovery is used.

3. The body mass, m, is indirectly estimated at the scene. If possible, this value is more accurately measured with a scale during the autopsy and the estimation of the time since death is corrected accordingly.

4. The set of environmental conditions K in which the body was found is registered. This corresponds to a verbal description of the clothing that the body had at the time of the discovery, the kind of surface on which it was laying, its position (face up, sideways, etc.), the presence of moisture on the body and/or clothing, the air flow, etc.

[1]In this case, this would be equivalent to repeating the death "experiment" on the same person infinite times, something that defies common sense.

5. Given K, the corrective factor c is determined by means of the recommendations given by Henssge, which are based on the results of different analyzes carried out on experimental data. The values of c are tabulated [2, 10, 22] and correspond to a standard mass of 70 kg.

6. Based on m and c, the value of the parameter B is calculated by means of an empirical equation proposed by Henssge, which is based on an adjustment he made on an experimental data set [9], and whose most complete version includes the corrections given in equation 6.10 of the book [10]:

$$B = b_1 . z^{-5/8} + b_2, \tag{1}$$

where:

$$\begin{aligned} z &= \left[\frac{b_1}{\left(m^{-5/8} - b_2\right) . \left(b_3 \exp(b_4 c)\right) + b_5} \right]^{8/5}, \\ b_1 &= -1.2815, \\ b_2 &= 0.0284, \\ b_3 &= -3.24596, \\ b_4 &= -0.89959, \\ b_5 &= -0.0354. \end{aligned}$$

7. A value of the parameter A is chosen depending on the mean ambient temperature [9]:

$$A = \begin{cases} 1.25, & \text{for } u_A \leq 23 \\ 1.11, & \text{for } u_A > 23 \end{cases} \tag{2}$$

8. An initial temperature value $u_0 = 37.2$ °C is assumed.

9. The standardized temperature value Q is calculated:

$$Q = \frac{u - u_A}{u_0 - u_A}. \tag{3}$$

10. The time since death is calculated by solving the Marshall and Hoare equation for variable t:

$$Q = A . \exp(Bt) + (1 - A) . \exp\left(\frac{AB}{A-1} . t\right). \tag{4}$$

    This equation cannot be explicitly solved for t, and therefore a nomogram or one of the existing computer programs is used (e.g. the freeware developed by E. Friedlander, the online page developed by W. Schweitzer or C. Henssge's commercial software) [9, 10].

11. Depending on the value of Q, an uncertainty σ_t is assigned to the estimated time since death. This uncertainty is taken from a histogram of absolute errors experimentally determined by Henssge [9, 10]:

$$\sigma_t = \begin{cases} 1.3, & \text{for } Q \geq 0.5 \\ 2.2, & \text{for } 0.3 \leq Q < 0.5 \\ 3.4, & \text{for } 0.2 \leq Q < 0.3 \end{cases}. \tag{5}$$

12. If $Q < 0.2$, this method is not used to estimate the PMI.

The result of the estimation is expressed as $\hat{t} \pm 2\sigma_t$, where $\hat{t}$ is the time value calculated with equation (4) and σ_t is obtained from equation (5).

2.2 PMI estimation by means of Bayesian inference

In the context of Bayesian inference, each of the variables and parameters involved in the body cooling model is considered to have an associated probability distribution. This means that none of the measured or estimated quantities have an exact value, but all of them have an associated uncertainty that depends, among other things, on the details of the procedure, the characteristics of the instrument and the inter-sample variability. From this perspective, the resulting PMI will also have an associated probability distribution, which can be obtained through an inference process based on the experimental data and prior information available on the particular case. In this article we present the result of an estimation as the posterior probability density of the time since death, $p(t|D, I)$, which should be read as "the probability that the time since death is t, given the data D and the prior information I" (this function is a density because t is a continuous variable). Here, D is the set of all data (evidence) obtained at the scene and I is a proposition representing all the information that is known and assumed about the case (the cooling model, the statistical model of the data, the prior distributions of the parameters, etc.). In this way, Bayesian inference consists of an update of the available information, and therefore the probability density function that is obtained is called posterior [23].

To correctly perform a Bayesian inference, it is important to clearly establish the relationships between the different variables and model parameters. Figure 1 shows a diagram representing these links, based on the description given in Section 2.1:

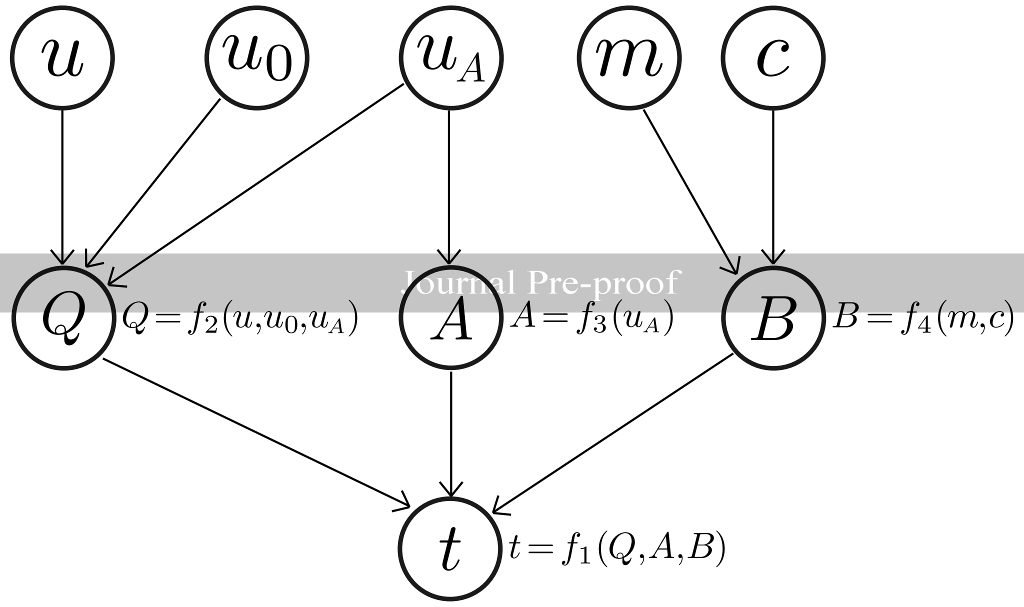

Figure 1: PMI estimation model using Bayesian inference. The result is the posterior distribution of the time since death given the data.

The data set obtained in a specific case for which the time since death is to be determined is $D = \{\tilde{u}, \tilde{u}_0, \tilde{u}_A, \tilde{m}, \tilde{c}\}$, where the tilde ($\sim$) over a variable represents its observed value. Here, u, u_0 and u_A are measured in [°C], m is measured in [kg] and c is dimensionless. According to this scheme, the functions $f_2(u, u_0, u_A)$, $f_3(u_A)$ and $f_4(m, c)$ allow us to obtain Q, A and B according to equations (3), (2) and (1), respectively, which are deterministic relationships. Finally, the time since death t is calculated from Q, A and B also in a deterministic fashion by means of $f_1(Q, A, B)$, which is obtained by numerically inverting function (4).

The first step in this Bayesian inference process is to obtain the posterior densities of each of the parameters u, u_0, u_A, m and c, and then make a series of variable transformations until finally obtaining the posterior density of the PMI. Given this density, the probability that death occurred in the time interval (a, b) (with $b > a$) is:

$$\Pr\left(t \in (a, b) \,|D, I\right) = \int_a^b p\left(t|D, I\right) dt. \tag{6}$$

Therefore, the Bayesian inference process allows obtaining time intervals that have a certain probability value or, conversely, the probability that death occurred in a specific time interval. Finally, we define the *credibility region of 95.45%* as the interval (a, b) with minimum amplitude[2] that satisfies equation (6) for $\Pr\left(t \in (a, b) \,|D, I\right) = 0.9545$. Throughout this article, we will call this interval R_{95}.

2.3 Determination of the posterior density of the time since death

To obtain the expression that allows calculating the posterior density of the time since death given the data, we start from the functional relationship between t and the variables Q, A and B, given by $f_1(Q, A, B)$ (see Figure 1). This relationship is deterministic and, therefore, following the guidelines given in [23], the sought expression is:

$$p\left(t|D, I\right) = \iiint_{V_1} \delta\left(t - f_1\left(Q, A, B\right)\right) p\left(Q, A|D, I\right) p\left(B|D, I\right) dQ\, dA\, dB, \tag{7}$$

where $\delta(t)$ is Dirac's delta function, $p\left(Q, A|D, I\right)$ is the joint posterior density of Q and A, $p\left(B|D, I\right)$ is the marginal posterior density of B, and V_1 is the integration region. The distributions appearing in the integrand of equation (7) can be calculated by means of the following expressions:

$$p\left(Q, A|D, I\right) = \iiint_{V_2} \delta\left(Q - f_2\left(u, u_0, u_A\right)\right) \delta\left(A - f_3\left(u_A\right)\right) p\left(u, u_0, u_A|D, I\right) du\, du_0\, du_A, \tag{8}$$

$$p\left(B|D, I\right) = \iint_{S} \delta\left(B - f_4\left(m, c\right)\right) p\left(m, c|D, I\right) dm\, dc, \tag{9}$$

where $p\left(u, u_0, u_A|D, I\right)$ is the joint posterior density of u, u_0 and u_A, $p\left(m, c|D, I\right)$ is the joint posterior density of m and c, and the corresponding integration regions are V_2 and S. Details of the derivation of these expressions can be found in Appendix A. Using this calculation scheme, the uncertainty in the time since death is completely determined by the uncertainties in the observed variables, which are contained in the distributions $p\left(u, u_0, u_A|D, I\right)$ and $p\left(m, c|D, I\right)$.

[2]Formally, the region calculated in this way is called *highest posterior density region*, and it satisfies the condition that $p\left(t|D, I\right)$ is greater at all points inside the interval (a, b) than outside it. However, for unimodal distributions this is also the interval with minimum amplitude containing the required probability [24].

In practice, these equations are not solved analytically, but are used to obtain samples of the corresponding distributions. The procedure for extracting a sample from the posterior distribution of the PMI is summarized below:

1. Extract a sample of u, u_0 and u_A from their posterior distributions, which depend on the observed values $\tilde{u}$, $\tilde{u}_0$ and $\tilde{u}_A$, as well as their uncertainties. This is detailed in Section 2.4.
2. Use the samples obtained in 1 to compute $Q = f_2(u, u_0, u_A)$ and $A = f_3(u_A)$, thus obtaining a sample of the joint distribution $p(Q, A|D, I)$.
3. Repeat step 1 with m and c and step 2 with B, to obtain a sample of the distribution $p(B|D, I)$.
4. Extract a sample of the posterior distribution of the PMI, $p(t|D, I)$, by computing $t = f_1(Q, A, B)$ using the samples of Q, A and B obtained in steps 1 - 3.

2.4 Posterior distributions of the parameters

Given a data point $\tilde{x}$ that represents the measured value of parameter x, Bayes' theorem [23] states that:

$$p(x|\tilde{x}, I) = \frac{p(\tilde{x}|x, I)\, p(x|I)}{\int_{D_x} p(\tilde{x}|x, I)\, p(x|I)\, dx}, \tag{10}$$

where $p(x|\tilde{x}, I)$ is the posterior density of x, $p(\tilde{x}|x, I)$ is the likelihood function and $p(x|I)$ is the prior density of x. In this work, for each of the data $\tilde{x} \in D$ we propose a normal likelihood of the form $\tilde{x}|x, I \sim \mathcal{N}(x, \sigma_x^2)$, with known uncertainty σ_x. Following the guidelines given in [25], to obtain the posterior density of x we assume an improper normal prior density that reflects a state of total ignorance: $p(x|I) \propto 1$. In this way, the posterior distribution of parameter x given the unique data value $\tilde{x}$ is also normal: $x|\tilde{x}, I \sim \mathcal{N}(\tilde{x}, \sigma_x^2)$, and the corresponding density function is:

$$p(x|\tilde{x}, I) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(x - \tilde{x})^2}{2\sigma_x^2}\right). \tag{11}$$

Although in this work we assume a prior information equivalent to a state of total ignorance about each of the parameters involved, it is possible to incorporate here relevant information by modifying the distribution $p(x|I)$ accordingly. For example, if the deceased person was known to have a fever at the time of death, it is possible to assign a prior density to u_0 that reflects this knowledge.

2.5 Incorporation of non-temperature-based information

Given the availability of information about the distribution of PMI which is independent of that used in the estimation by means of the body cooling model, it is straightforward to incorporate it using Bayes' theorem in a second inference process. Such information may come, for example, from other estimations carried out with different methods or from witnesses that identify the time at which the person was last seen alive.

Let $I_1 = D, I$ be the statement that represents all the information, prior and posterior, corresponding to the estimation of PMI by using the body cooling model. This proposition represents the prior information in this second inference process. On the other hand, let D_1 be the statement that represents the non-temperature-based data. So, based on Bayes' theorem, we have:

$$p(t|D_1, I_1) \propto p(D_1|t, I_1)\, p(t|I_1), \tag{12}$$

where $p(t|I_1)$ is the posterior density of the time since death obtained in the first inference process according to equation (7) and $p(D_1|t, I_1)$ is the new likelihood function, which represents the probability that the statement D_1 is true given the time since death t and the prior information I_1. The nature of this latter probability function depends on the format of the new information.

Consider the information given by the proposition $D_1 \equiv$ *"the maximum and minimum possible values for the postmortem interval are t_{max} and t_{min}, respectively"*. In this case, t_{max} and t_{min} represent the times elapsed since the last time a witness saw the person alive and since the body was found[3], respectively, with respect to the instant of the rectal temperature measurement, and therefore D_1 represents valuable information worth to be incorporated into the analysis. Given the nature of proposition D_1, we have a uniform likelihood, namely:

$$p(D_1|t, I_1) = \begin{cases} 1, & \text{for } t_{min} \le t \le t_{max} \\ 0, & \text{elsewhere} \end{cases},$$

and therefore:

$$p(t|D_1, I_1) = \begin{cases} k\, p(t|I_1), & \text{for } t_{min} \le t \le t_{max} \\ 0, & \text{elsewhere} \end{cases}, \tag{13}$$

where k is a normalization constant. Thus, this new posterior density contains both the information provided by the analysis of body cooling and the non-temperature-based information from witnesses. Of course, this distribution can be used again as a prior in a new inference process that involves data associated with another PMI estimation technique. In this way, different types of data associated with the estimation of the PMI can be systematically combined, updating the degree of belief and obtaining a more reliable distribution.

[3] Although the PMI is defined as the time elapsed from death to the examination of the corpse, there is generally a delay between discovery and examination.

2.6 Parameter uncertainties

In this section we analyze each of the model's parameters separately, in order to establish criteria to select adequate values for their uncertainties.

2.6.1 Uncertainty in u

There are mainly two sources of uncertainty for the measured rectal temperature u. First of all, each thermometer has a certain accuracy, which is given in °C, and this value generally varies between 0.1 and 0.5 °C depending on the manufacturer. Second, the body cooling model describes the temporal variation of temperature at the same position (depth) in the rectum. Therefore, a temperature measurement made at a different depth would imply a difference with respect to the value predicted by the model, introducing a source of uncertainty. Lee and others [13] carried out a series of experiments in which they measured the rectal temperature of living humans at rest at different insertion depths between 4 and 19 cm (measured from the sphincter), and the mean value of the difference between the minimum and maximum recorded temperatures was on the order of 0.5 °C. Although a standardized procedure for the insertion of the thermometer is used, the measurement site will always be subject to inevitable human error and anatomical differences between individuals, so these deviations must be taken into account.

2.6.2 Uncertainty in u_0

It is known that the core temperature of a healthy person is not constant, but varies throughout the day (circadian cycles) depending on metabolism, physical activity, age and many other factors, both endogenous and exogenous [11, 12]. In addition to this physiological variation, cases of postmortem hyperthermia have been reported [6] and there is also the possibility of hypo- or hyperthermia prior to death (e.g. caused by disease). Furthermore, it is postulated that long periods of agony can strongly modify the central temperature before death, both increasing and decreasing it [10]. All these factors contribute to the uncertainty in the initial temperature. The typical mean core temperature value used in the estimation of PMI is $\hat{u}_0 = 37.2$ °C. In cases where hypo- or hyperthermia can be ruled out, an uncertainty $\sigma_{u_0} = 0.5$ °C reflects the normal fluctuations of a healthy body around this value [11]. On the other hand, if no information is available in this regard, such ignorance should be reflected in a greater value of uncertainty. If the deceased person had a history of fever or hypothermia, or if a long period of agony is suspected, a modification of the mean value and standard deviation according to these situations should be considered.

2.6.3 Uncertainty in u_A

The uncertainty in the mean ambient temperature value is mainly due to its variation throughout the cooling process, and to a much lesser extent to the accuracy of the thermometer. The average ambient temperature is calculated with the following formula:

$$u_A = \frac{1}{n}\sum_{i=1}^{n} T_A(t_i), \tag{14}$$

where $T_A(t_i)$ is the ambient temperature recorded at time t_i, the subscript 1 corresponds to the first recorded value of ambient temperature and n is the number of measurements between the first record and the examination of the body (when the rectal temperature measurement is performed). The value of u_A thus depends on the length $\Delta t = t_n - t_1$ of the period considered. This is shown in Figure 2, using as an example data from a meteorological station located in the city of San Carlos de Bariloche (Argentina) between 3:00 AM on September 10, 2017 and 3:00 AM on September 12 of the same year (48 hours in total).

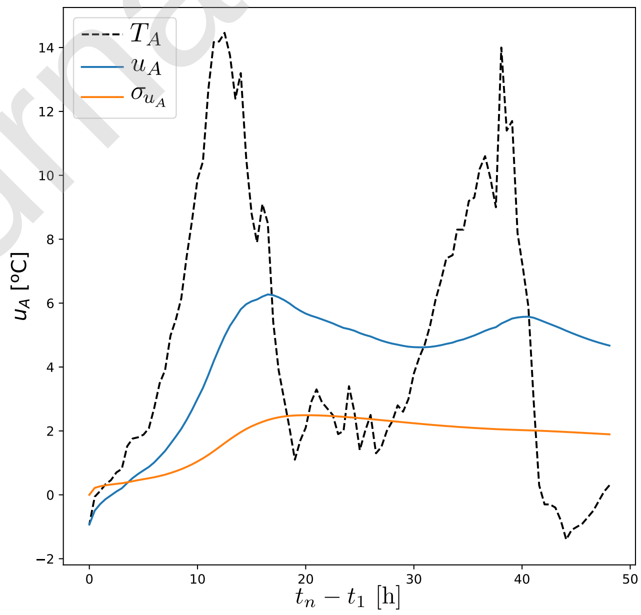

Figure 2: Temporal variation of ambient temperature (dotted black line) during two days of spring in 2017, in the city of San Carlos de Bariloche, Argentina. The curves corresponding to the mean value of the ambient temperature and the standard deviation of said mean value are also shown.

As can be seen, during the last 16 hours prior to the discovery, the fluctuation in the ambient temperature is such that u_A varies considerably depending on how many values of $T_A(t)$ are used for its calculation. The results of the PMI estimation depend in turn on the value adopted for u_A, so it is important to assign an adequate value to its uncertainty. In this example, we see that σ_{u_A} reaches values of up to 2 °C.

In cases where the ambient temperature record is not available throughout the considered interval, we propose to use a value of σ_{u_A} that reflects the usual seasonal climatic conditions at the site of the discovery.

2.6.4 Uncertainty in m

Although the body mass is first indirectly estimated at the scene, its value is usually re-determined using a scale at the autopsy, in which case its uncertainty is expected to depend solely on the instrument accuracy. However, there are different reasons that could generate a variation in the mass from the moment of death to its estimation at the scene to its measurement in the autopsy room (e.g. loss of blood and other fluids). Therefore, in each particular case, a value of σ_m must be selected that contemplates this type of situation. On the other hand, if a measurement of m is not performed with a scale before reporting the results of the PMI estimation, a rather conservative value of σ_m should be considered that reflects the observer's training level.

2.6.5 Uncertainty in c

The corrective factor c is the most problematic parameter of the nomogram method, since its estimation is made from a verbal description of the conditions at the scene (K) and using only a partially systematized procedure. This means that the same "value" of K can be translated into different values of c depending on the investigator's interpretation. Furthermore, the tabulated values of c correspond to mean values obtained from empirical adjustments, and therefore have an associated uncertainty that is not reported in the bibliography. For instance, by using tables 6.18 and 6.19 of [10] the value $c = 0.9$ corresponds to a body with the following description: $K_0 \equiv$ *"one to two layers of fine clothing, dry, in still air and supported on a thermally indifferent base, standard mass (70 kg)"*. However, this value was obtained from a series of experiments carried out under a set of conditions K_i similar to K_0 but not identical, and it is only an average value. Therefore, it is absolutely necessary to consider the uncertainty of parameter c in the estimation of the PMI, especially considering its great influence on the shape of the cooling curve, as will be shown in Section 3.1.6.

In a reference work on the nomogram method [26], the authors recommend using two values of c that represent the limits of an interval where the "true" value of said parameter is found. With these values, two different estimates of time since death are obtained, and the authors recommend selecting the widest interval among that obtained using the mean value of c and the one obtained using the two extreme values. However, it is not clear how to objectively select that range of c.

In two other recent works that analyze the errors of the nomogram method, the authors make the following statements regarding the corrective factor: (1) *"The corrective factor c neither represents a continuous variable nor can it be measured [...]"* [14] and (2) *"Actually, the variable c is defined on the class* $\mathbb{N} \times \mathbb{R}$ *of ordered pairs* (N, m) *with N being any possible non-standard cooling condition and m being any possible body mass, where N is an infinite space without a clear set definition and with no quantifiable mathematical structure or even a topology [...]"* [15]. We believe that these statements are imprecise and could lead to confusion. In practice, each of the experimental corrective factors is obtained, for a certain set K of non-standard conditions, according to the relationship $c = m_v/m$, where m_v is the "virtual mass" that a body should have in order to cool in the same way under standard conditions, and m is the actual body mass. Therefore, since m and m_v are continuous variables, c is also continuous. While the tabulated values of c correspond to a finite set of cooling conditions (represented by discrete variables), it is expected that two slightly different conditions (e.g. clothes a little more or less fine, slightly different body positions, slightly different air velocities [5], etc.) give two different values of c. It is impractical to define the cooling conditions with continuous variables (e.g. clothing thickness, water mass, wind speed, thermal conductivity of the supporting base, etc.); instead, the tabulated values of c are average values obtained from a series of similar experiments, and this means that there is only a finite (discrete) set of corrective factors to select from in the tables. In our opinion, the correct way to present the values of c would be by giving their probability distributions: for example, instead of giving the value $c = 0.9$ as described in the example at the beginning of this section, it would be much more significant to state $c \sim \mathcal{N}(\hat{c}, \sigma_c^2)$, where $\hat{c} = 0.9$ and σ_c is the standard deviation of the values obtained in the corresponding experiments.

2.7 Analysis of experimental data

To evaluate the Bayesian inference method, this study analyzes cases from the database published by Muggenthaler et al. [16], which consists of 84 postmortem cooling curves recorded under strictly controlled environmental conditions. Because the information was given in form of cooling diagrams, we used the free software Graph Grabber 2.0 to extract the data points from the figures. All calculations were performed with software programmed in Python 3, and the number of samples in all simulations was 10000. In general, the larger the number of samples, the better the description of the corresponding probability distribution. With N = 10000, the uncertainties introduced by the sampling process are negligible compared to the uncertainties in the model parameters.

In the first part of this study, the influence of the uncertainties in the different parameters on the estimation of the PMI is analyzed. Firstly, a specific case of interest is considered, namely case number 54 of the mentioned database, which was classified by the authors as one of the "representative cases" and generated discussions due to the great inaccuracy with which the nomogram method represents its cooling curve [27, 28]. From this experiment, the discovery of the body is simulated by choosing an arbitrary point on the cooling curve and the estimation of the PMI is carried out using Bayesian inference. The influence of parameter uncertainties is analyzed first separately and then the combined uncertainty is studied. Different values are assigned to σ_x in each case, using ranges according to the analysis of Section 2.6. Subsequently, similar simulations of body finding are performed for all cases in the database and in different parts of the cooling curves corresponding to 8 ranges of Q, respectively: [0.95,0.90), [0.90,0.80), [0.80,0.70), [0.70,0.60), [0.60,0.50), [0.50,0.40), [0.40,0.30) and [0.30,0.20]. The purpose of these simulations is to analyze the influence of the parameter uncertainties as a function of the cooling progress. In all cases, the value of Q corresponds to the experimental data (measurements).

In the second part of this work, a fixed value of uncertainty is selected for each parameter, according to the experimental conditions described in [16], and the PMI is estimated in all the cooling curves of the database and in three ranges of Q, namely: (1) $Q \geq 0.5$, (2) $0.3 \leq Q < 0.5$ and (3) $0.2 \leq Q < 0.3$. These ranges correspond to those used by Henssge in his analysis of error under non-standard conditions [10]. In parallel with the estimation of the PMI using Bayesian inference, the estimation is also performed using the nomogram method and both results are compared.

3 Results and discussion

3.1 Influence of parameter uncertainties on the estimation of the PMI

Characteristics of case number 54

The relevant information available about case number 54 in [16] is summarized below:

1. Delay between death and the first measurement: 1.83 h.
2. $\tilde{u}_A = 19.4$ °C.
3. $\tilde{m} = 158.4$ kg.
4. Clothing: "briefs, undershirt, shirt, jeans, shoes, abdomen/thorax uncovered".
5. Supporting base: steel trolley.

According to the description of the cooling conditions, the body had two layers of light clothing (only the lower trunk is considered), it was dry and in an environment with negligible air flow, therefore a corrective factor $\tilde{c} = 1.2$ would be adequate according to table 6.18 of [10]. However, since the supporting base is conductive and the clothing layer is thin, according to table 6.19 of the same reference it is appropriate to subtract 0.2 from the corrective factor, finally obtaining a value $\tilde{c} = 1.0$.

Simulation of discovery of the body

In the following, each PMI estimation is carried out using a single rectal temperature measurement, corresponding to an arbitrarily selected point of the cooling curve. This discovery simulation allows estimating the PMI in a specific case by performing different uncertainty analyzes. We will call the true time since death τ.

For the analysis of case number 54, the selected point was $(\tau, \tilde{u}) = (10.7, 34.0)$. The data set is then:

$$D = \{\tilde{u}, \tilde{u}_0, \tilde{u}_A, \tilde{m}, \tilde{c}\} = \{34.0, 37.2, 19.4, 158.4, 1.0\}\,.$$

In each of Sections 3.1.2 - 3.1.6, five different values of uncertainty in the corresponding parameter are analyzed, while all other parameters remain fixed (i.e. without uncertainty). The values of uncertainty were selected taking into account the analysis in section 2.6.

3.1.1 Empirical error

First, we analyze the estimation of PMI using the nomogram method with the empirical error suggested in the bibliography. According to the data, the estimated time since death with this method is $\hat{t} = 14.9$ h. Taking into account that $Q = 0.82$, then according to the equation (5) corresponds $\sigma_t = 1.3$ h. Therefore, the confidence interval of 95.45% for the PMI estimated by the nomogram method is $[12.3, 17.5]$ h (remember the difference between this interval and the region R_{95} described in Section 2.2). Figure 3 shows the results of this estimation, together with the experimental cooling curve and the simulated curve according to the body cooling model with the parameters obtained from the data. Note that the experimental curve begins at $t = 1.83$ h, corresponding to the delay between the discovery and the start of the measurements.

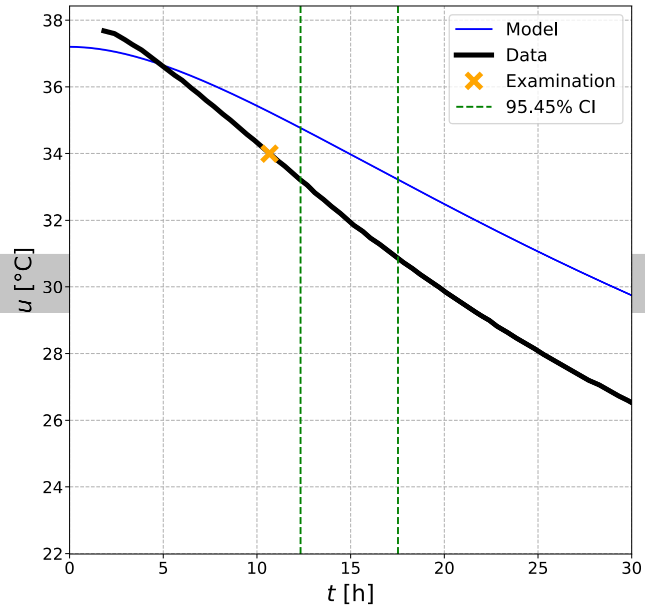

Figure 3: Estimation of the time since death using the nomogram method. The experimental (black) and simulated (blue) cooling curves are shown. The orange cross represents the point considered for the simulation of the discovery. Green dashed lines indicate the 95.45% confidence interval for t.

As can be seen, there is a large difference between the experimental and the simulated curve. At first glance, it is observed that the initial body temperature was above 37.2 °C, which is not taken into account by the nomogram method. On the other hand, the cooling rate according to the model is clearly lower than the actual cooling rate, so there is a large error in estimating parameter B. In turn, the possible sources of error in estimating B are mainly the corrective factor c and, to a lesser extent, the mass m, as will be shown in the next sections. As can be seen, the 95.45% confidence interval estimated with the nomogram method does not contain the true PMI in this case.

3.1.2 Uncertainty in the rectal temperature

The selected range of σ_u was [0.1,0.5] °C. Figure 4 shows the results for the two extreme values of σ_u in the range. Each upper panel shows the histogram created from the samples of the posterior distribution of t, obtained according to the method described in Sections 2.2 - 2.3. Each lower panel shows the experimental cooling curve (data), the simulated cooling curve according to the body cooling model (identical for all the samples of u), the 95.45% confidence interval for the PMI computed with the nomogram method and the region R_{95} determined by Bayesian inference, respectively. As can be seen, the dispersion in the estimated PMI increases with the uncertainty in the rectal temperature, and thus the inclusion of τ in R_{95} becomes more and more probable. It becomes evident that the consideration of the uncertainty in u has a noticeable effect on the result of the estimation and, taking into account that the body was in an initial stage of cooling, this result agrees with those obtained in another study [14].

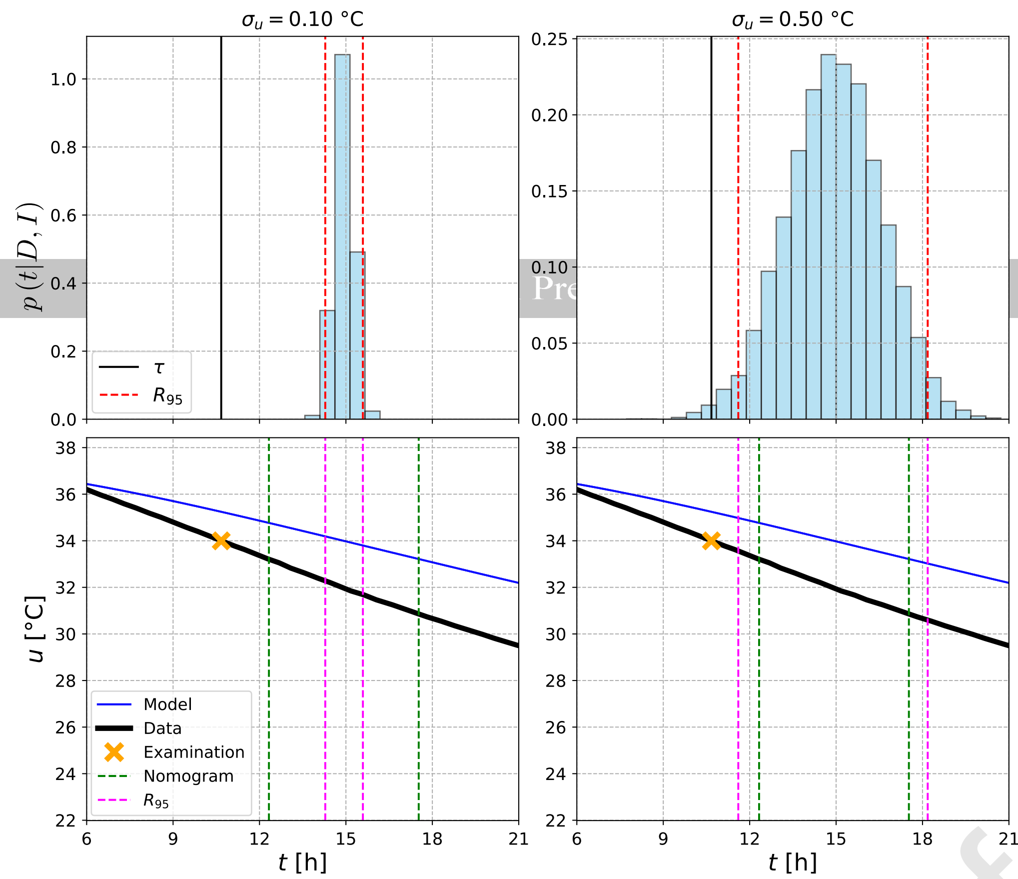

Figure 4: (Upper panels) Posterior distribution of PMI obtained by means of Bayesian inference for different magnitudes of the uncertainty in the rectal temperature. The point under analysis corresponds to a simulation of discovery based on the cooling data of case number 54 in the database. (Lower panels) Body cooling data and simulations. The vertical dotted lines show the 95.45% confidence interval (nomogram method, green) and the R_{95} (Bayesian inference, magenta).

The previous results correspond to a single case with $Q = 0.82$ (experimental value). Now considering all the cases in the database, Figure 9(a) shows the amplitude of the R_{95} obtained as a function of the degree of cooling progress (represented by the value of $1 - Q$), using the fixed value $\sigma_u = 0.2$ °C. For each range of Q, both the actual value $1 - Q_i$ and the amplitude of the resulting R_{95i} were computed for each sample i, obtaining a distribution of $1 - Q$ values and R_{95} amplitudes whose median represent the horizontal and vertical coordinate of each point in the figure, respectively. On the other hand, the error bars represent the region of credibility of 95.45% for both variables, computed in the same fashion as the R_{95}. As can be seen, the amplitude of R_{95} tends to increase for extreme values of Q. This can be explained by taking into account that the cooling curve of the model has a small slope $|du/dt|$ at the beginning ($Q \approx 1$), due to the relatively slow establishment of a temperature gradient between the interior of the body and the surrounding air; on the other hand, as the cooling progresses the curve tends asymptotically to u_A and the slope decreases again when $Q \to 0$. Given a small value of $|du/dt|$, even a small variation in the vertical coordinate u generates a large variation in the horizontal coordinate t. Therefore, the uncertainty in determining rectal temperature plays a more important role in situations where Q is very high or very low, its effect being more pronounced in the second case. The minimum uncertainty was found in the range $0.7 \leq Q < 0.9$, with 95.45% of the amplitudes of R_{95} between 0.2 and 2.3 h and a median equal to 0.9 h; on the other hand, the maximum uncertainty was found in the range $0.2 \leq Q < 0.3$, with 95.45% of the amplitudes of R_{95} between 0.7 and 4.8 h and a median equal to 2.2 h.

3.1.3 Uncertainty in the initial temperature

Here we consider $\sigma_{u_0} \in [0.1, 1.0]$ °C. Analogously to Section 3.1.2, Figure 5 shows the results for the extreme values of uncertainty in the range. In this case, however, different samples of u_0 generate different cooling curves, as can be seen in the lower panels of the figure, where 200 superimposed curves have been plotted. This effect is noticeable: as the dispersion of u_0 increases, there is a greater number of simulated curves whose points are vertically displaced with respect to the simulation performed using the nomogram method (Fig. 3), and the amplitude of this displacement decreases as time increases. This variability somewhat compensates the error caused by assuming a fixed initial temperature, in the sense that it increases the amplitude of the R_{95} and thus the probability that it contains the value of τ. Again, the dispersion in the estimated time since death increases with the uncertainty in u_0, and also a slight asymmetry is observed in the posterior distribution of the PMI, with a greater tendency towards higher times. It is important to note that the magnitude and type of asymmetry are specific to the case under study, since they depend essentially on the shape of the cooling curve (which, in turn, depends on other parameters besides u_0). The results show a slightly lower impact of the uncertainty in u_0 on the posterior distribution of t in comparison with that observed when considering the uncertainty in u, for comparable values of uncertainty in both quantities. However, it should be noted that in practice typically $\sigma_{u_0} > \sigma_u$, so that the uncertainty in the initial temperature also plays an important role in the posterior distribution of the PMI, in agreement with the results obtained in another study [14].

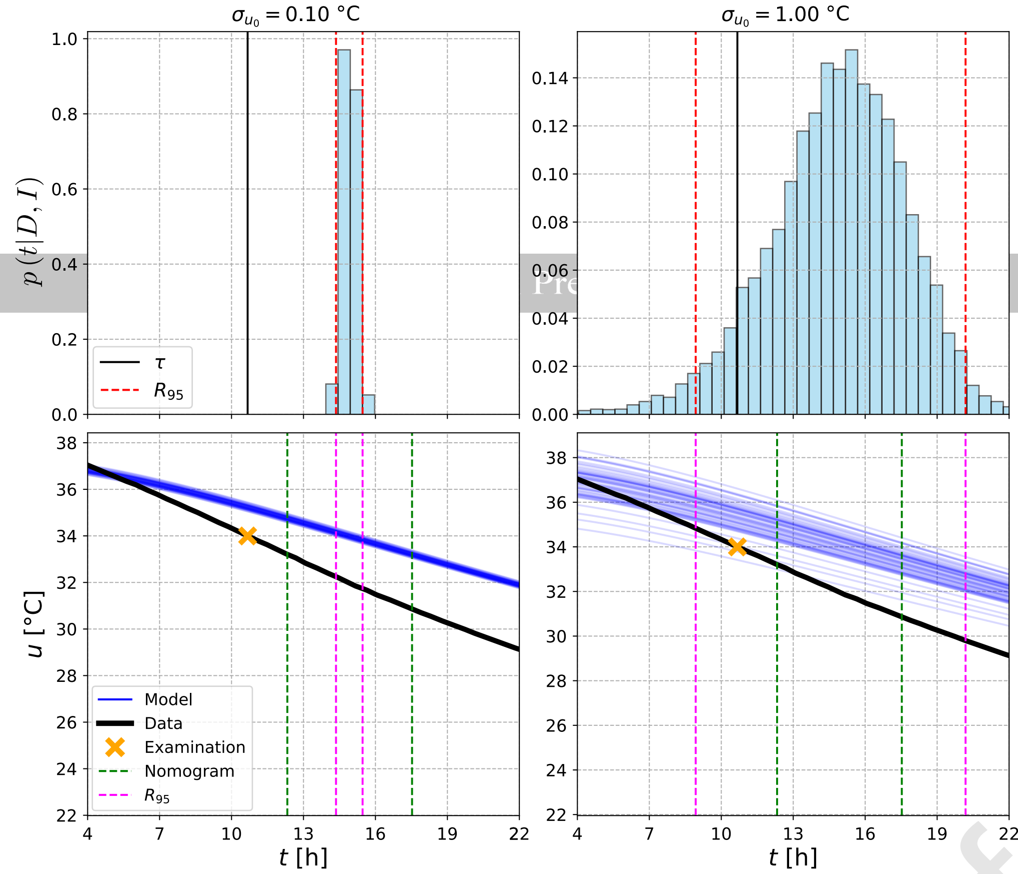

Figure 5: Analysis of uncertainty in the initial temperature. See caption of Figure 4 for details.

The variation of the amplitude of the R_{95} with respect to the degree of cooling progress for all cases in the database is shown in Figure 9(b), using the fixed value $\sigma_{u_0} = 0.75$ °C. In this case, a maximum amplitude value is observed in the range $0.9 \leq Q < 0.95$, with 95.45% of the values between 0.9 and 9.5 h (median equal to 3.8 h), followed by a monotonous decrease in the amplitude as the cooling progresses. This can be explained considering that the effect of a variation in u_0 is mainly reflected in the first part of the cooling curve of the model, and therefore its uncertainty will have a greater impact on the estimate for high values of Q. However, the amplitude of the R_{95} is not negligible for low values of Q, tending to a median of 2 h (95.45% of the values between 0.6 and 4.5 h).

3.1.4 Uncertainty in the mean ambient temperature

Now we analyze values of σ_{u_A} in the range [0.1,2.0] °C, and Figure 6 shows the results for the extreme values in this interval. As expected, the dispersion in the estimated time since death increases as the uncertainty in the mean ambient temperature increases. As demonstrated in another study through an error propagation analysis [14], the effect of varying this parameter on the cooling curve is more important for higher degrees of cooling progress, because u_A represents the asymptotic value to which the rectal temperature tends as time increases. For this reason, it is expected that the effect of an uncertainty in the estimate of u_A will be greater for lower values of Q. In this particular case, the influence of the uncertainty in this parameter is less than the influence of the uncertainties in u and u_0, but it is important to clarify that this is mainly because the simulated discovery corresponds to $Q = 0.82$. In other words, the body was in an initial stage of cooling, where uncertainties in u and u_0 exert a greater influence.

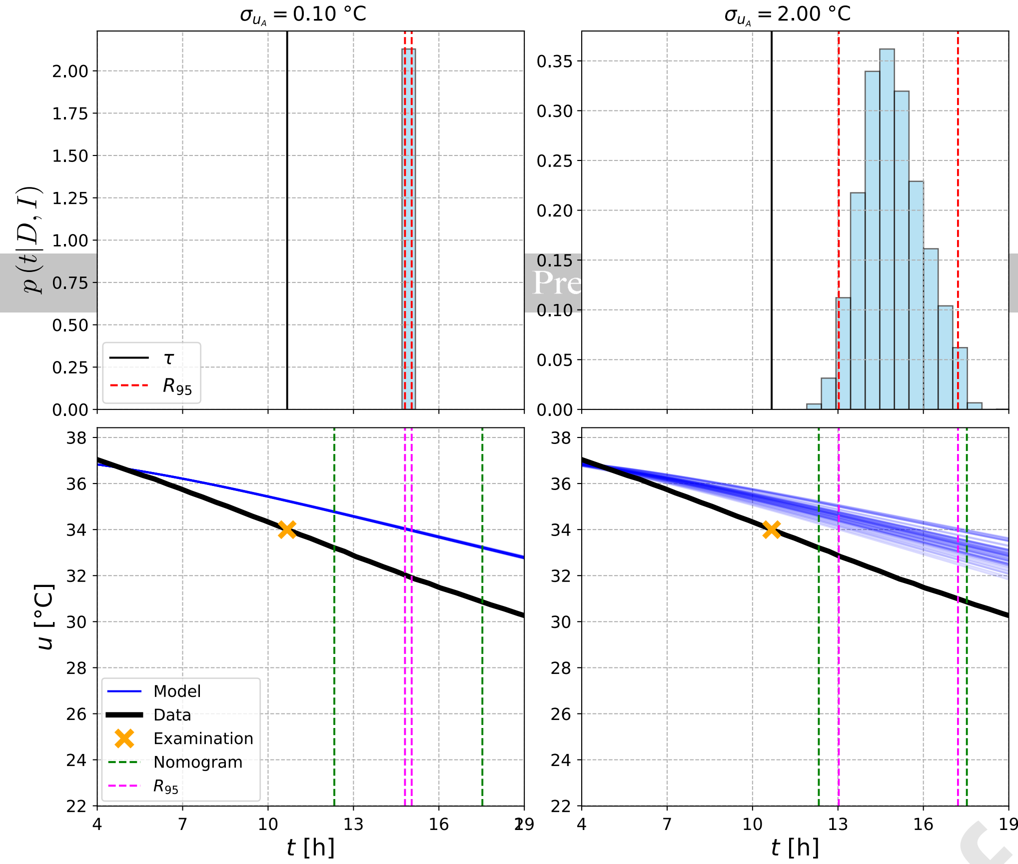

Figure 6: Analysis of uncertainty in the mean ambient temperature. See caption of Figure 4 for details.

Figure 9(c) presents the results obtained for all cases in the database as a function of the degree of cooling progress, using the fixed value $\sigma_{u_A} = 0.5$ °C. The behavior in this case is reciprocal to that corresponding to the uncertainty in u_0, showing a monotonous increase in the amplitude of R_{95} as the cooling progresses. As explained in Section 3.1.2, for low values of Q the cooling curve becomes almost horizontal, and small displacements thereof generate large deviations in the estimated PMI. This, added to the fact that a variation of u_A generates changes in the cooling curve that are more noticeable for low values of Q, explains that the uncertainty in the estimation of the PMI becomes arbitrarily large as the body approaches thermal equilibrium with the environment. The greatest amplitudes were obtained in the range $0.2 \leq Q < 0.3$, with a median of 4.3 h and 95.45% of the values between 1.4 and 9.4 h.

3.1.5 Uncertainty in the body mass

Here we study values of σ_m between 0.1 and 4 kg. As shown in Figures 7 and 9(d), the effect of uncertainty on the mass increases with the degree of cooling progress, in accordance with results obtained in other studies [15]. However, the effect of uncertainty in this parameter seems to be much lower than the others. The value of m affects the cooling rate through its influence on parameter B, according to equation (1) and, given the relative magnitudes of m and c, the contribution of the uncertainty in m to the total uncertainty is much lower than the contribution of the uncertainty in the correction factor[4].

[4]A linear error propagation for parameter B results in $\sigma_B^2 \approx E_m^2 + E_c^2$, where $E_x = (\partial B/\partial x)\,\sigma_x$. Because both the corrective factor and body mass influence only the computation of B, the quotient $(E_c/E_m)^2$ roughly indicates the relationship between the contributions of these parameters to the total uncertainty. The quotient of the derivatives is $(\partial B/\partial c)\,/(\partial B/\partial m) = \frac{m}{c}$. The standard values of these variables are $m_0 = 70$ kg and $c_0 = 1$, so that the standard value of the quotient of derivatives is equal to 70 kg. Therefore, $(E_c/E_m)^2 \approx 4900\,(\sigma_c/\sigma_m)^2$. Considering reasonable uncertainties such as $\sigma_c = 0.25$ and $\sigma_m = 0.4$ kg, we obtain $(E_c/E_m)^2 \approx 2000$, and thus the contribution of the uncertainty in the corrective factor on the total uncertainty is three orders of magnitude greater than the contribution of the uncertainty in the body mass.

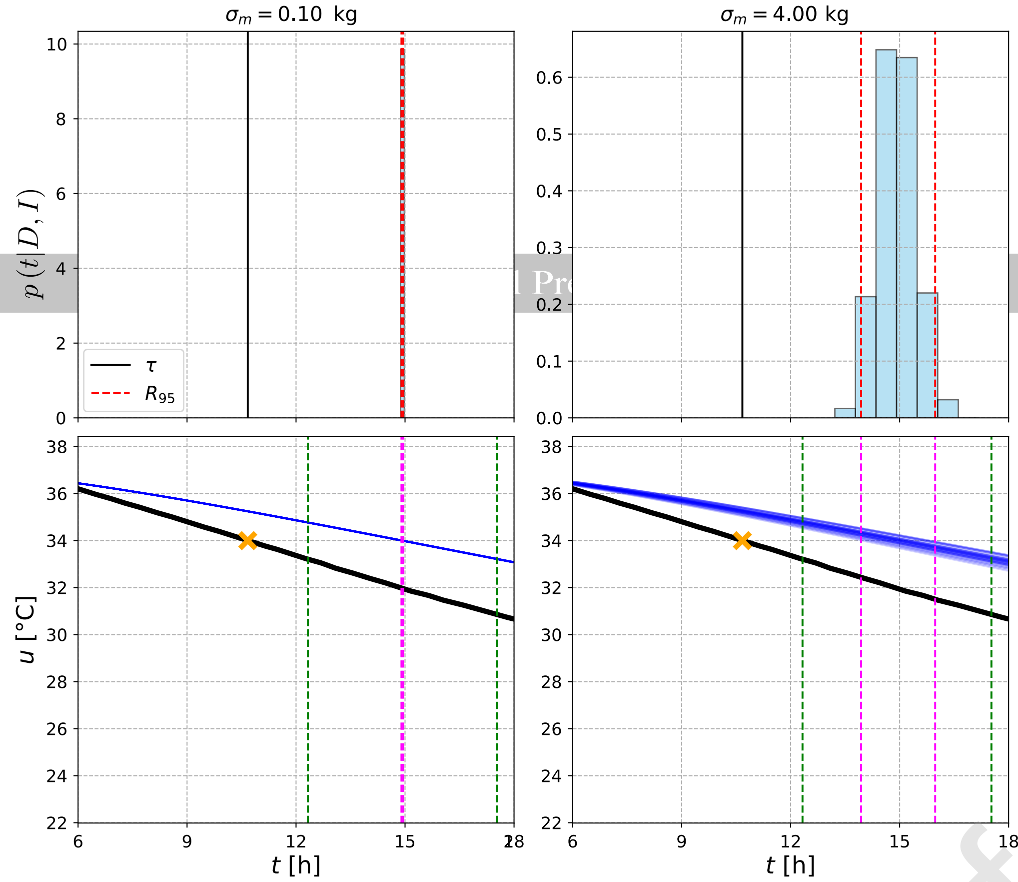

Figure 7: Analysis of uncertainty in the body mass. See caption of Figure 4 for details.

3.1.6 Uncertainty in the corrective factor

This section analyzes values of σ_c between 0.1 and 0.5. As can be seen in Figure 8, the magnitude of this uncertainty has a very significant impact on the shape of the cooling curve. This is reflected in the posterior distribution of t, the width of which increases dramatically as σ_c grows. In addition, the presence of asymmetries is observed for high values of uncertainty. This result indicates that the determination of the corrective factor represents one of the main sources of uncertainty in the PMI estimate. In fact, considering $\sigma_c \geq 0.3$ and the other parameters without uncertainty, the resulting R_{95} contains the value of τ, in contrast to the result obtained using the nomogram method. It is also worth noting that many of the simulated curves, obtained by considering the uncertainty in c, model the actual body cooling very accurately in contrast to the curve obtained through the nomogram method.

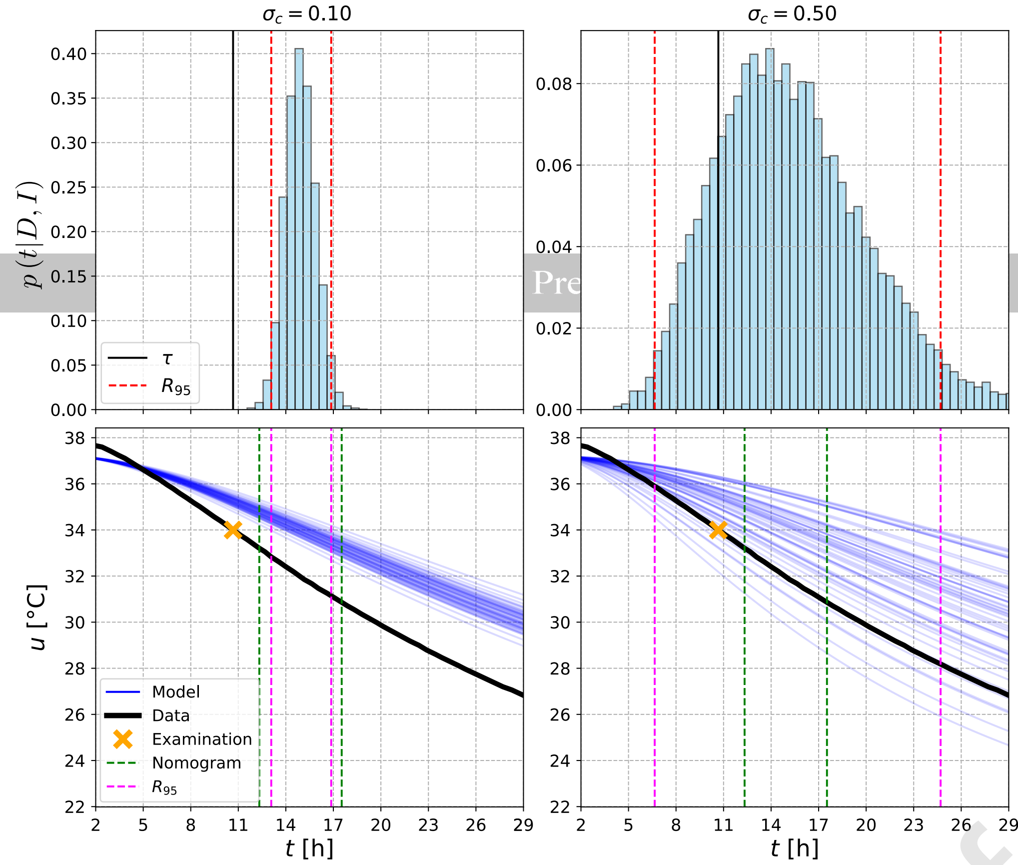

Figure 8: Analysis of uncertainty in the corrective factor. See caption of Figure 4 for details.

Figure 9(e) presents the results obtained for all cases in the database as a function of the degree of cooling progress, using the fixed value $\sigma_c = 0.25$. As can be seen, the amplitude of R_{95} increases monotonically as Q decreases, and this was to be expected considering the role of the corrective factor in computing parameter B (eq. (1)), which is analogous to that of the body mass. However, the posterior distributions of t obtained considering the uncertainty in c are much wider than the corresponding distributions obtained when considering the uncertainty in m. Indeed, in this case the distribution of the lowest amplitudes of R_{95}, corresponding to the range $0.90 \leq Q < 0.95$, presented a median of 2.7 h, with 95.45% of the values between 1.0 and 4.9 h. On the other hand, the maximum uncertainties (corresponding to the range $0.2 \leq Q < 0.3$) presented a median of 18.3 h, with 95.45% of the values between 8.9 and 30.2 h. These results indicate that the most influential parameter in the uncertainty associated with the determination of the PMI by the nomogram method is the corrective factor, since a reasonable uncertainty in its determination induces amplitudes of the R_{95} much higher in comparison with those obtained considering the uncertainties of the other parameters separately. On the other hand, the uncertainty in the corrective factor cannot be avoided due to the imprecision of the method developed for its determination.

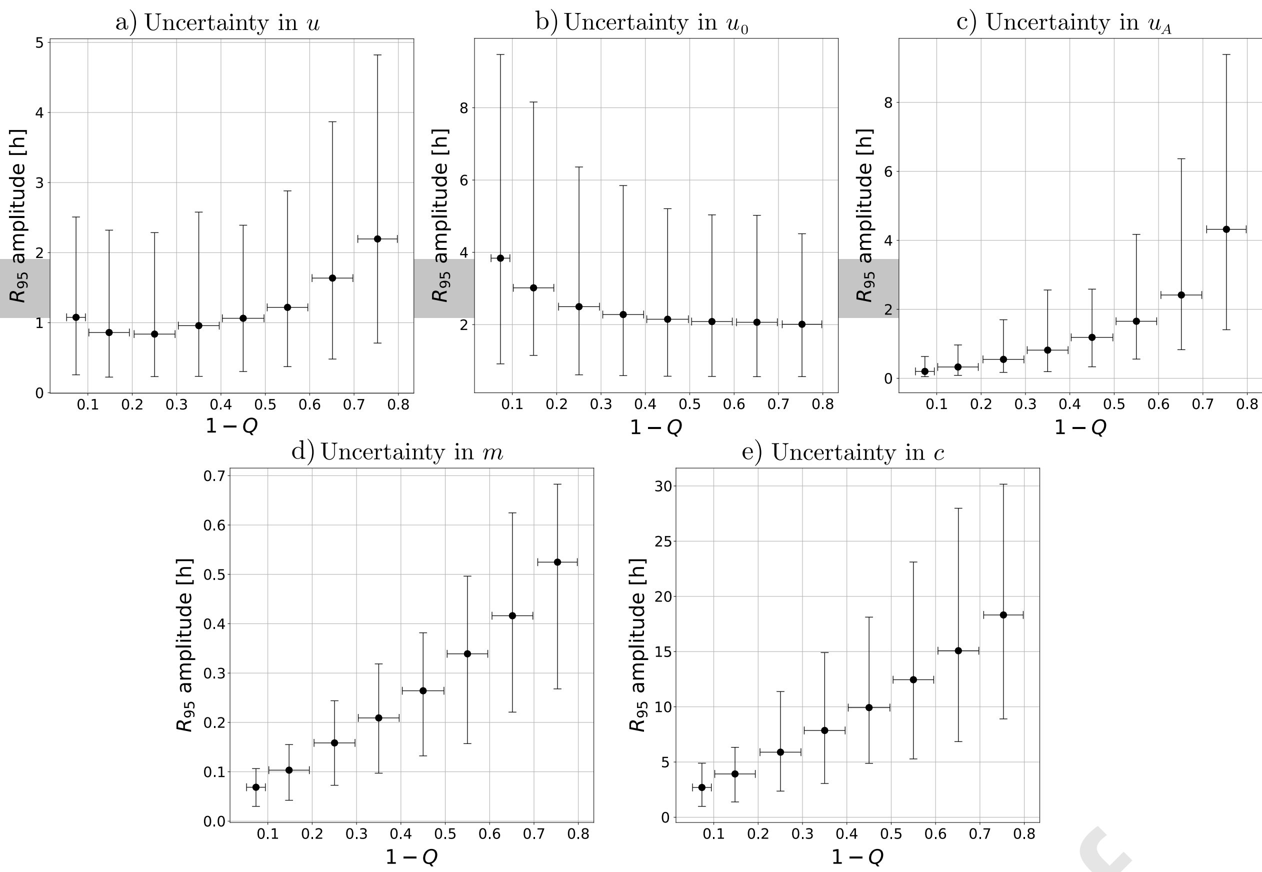

Figure 9: Amplitude of R_{95} in all cases of the database for different ranges of standardized temperature, considering only the uncertainty in a single parameter. (a) Rectal temperature: $\sigma_u = 0.2$ °C; (b) Initial temperature: $\sigma_{u_0} = 0.75$ °C; (c) Mean ambient temperature: $\sigma_{u_A} = 0.5$ °C; (e) Body mass: $\sigma_m = 0.4$ kg; (e) Corrective factor: $\sigma_c = 0.25$.

3.1.7 Uncertainties in all parameters

In this section, the joint effect of the uncertainties in all parameters on the estimation of the PMI for the simulated discovery based on case 54 is analyzed. Here we assign values to each uncertainty taking into account that the experiment was carried out under controlled conditions [16]. First, although the accuracy of the thermometer used for the rectal temperature measurement was 0.1 °C, considering the analysis of Section 2.6.1 we propose $\sigma_u = 0.2$ °C as an adequate value. On the other hand, the ambient temperature and the cooling conditions remained fixed throughout the experiment, but the delay between death and the start of the measurements was 1.8 h (which is not negligible) and no accurate information is available regarding these variables during that interval. Furthermore, although the conditions in the laboratory sought to replicate the conditions in the scene, they are at least slightly different. In particular, the corrective factor c must be different (different air flow, different humidity, different supporting base). Taking both these considerations and the analysis of Section 2.6 into account, we chose a fixed value $\sigma_{u_A} = 0.5$ °C for the uncertainty in the mean ambient temperature and a range of variation $0.05 \leq \sigma_c \leq 0.3$ for the uncertainty in the corrective factor. Regarding body mass, following the considerations of Section 2.6.4 we selected a fixed value $\sigma_m = 0.4$ kg. Finally, we considered a range of variation $0.1 \leq \sigma_{u_0} \leq 1.0$ °C for the uncertainty in the initial temperature. We then selected a total of 5 sets of values, which are labeled with an index j and are detailed below:

j	σ_u [°C]	σ_{u_0} [°C]	σ_{u_A} [°C]	σ_m [kg]	σ_c
0	0.20	0.10	0.50	0.40	0.05
1	0.20	0.33	0.50	0.40	0.11
2	0.20	0.55	0.50	0.40	0.17
3	0.20	0.78	0.50	0.40	0.24
4	0.20	1.00	0.50	0.40	0.30

Table 1: Sets of parameter uncertainties for the estimation of PMI in the simulated discovery of case number 54 by means of Bayesian inference.

The results are presented in Figure 10. As can be seen, the combined effect of uncertainties in all parameters strongly affects the shape of the cooling curves, even for relatively low values of individual uncertainties. Indeed, for $j = 2$ the resulting R_{95} contains the value of τ, something that does not occur in any of the results obtained considering each of the respective uncertainties individually. This again demonstrates one of the advantages of Bayesian inference, namely, that it allows different sources of uncertainty to be systematically incorporated into the analysis.

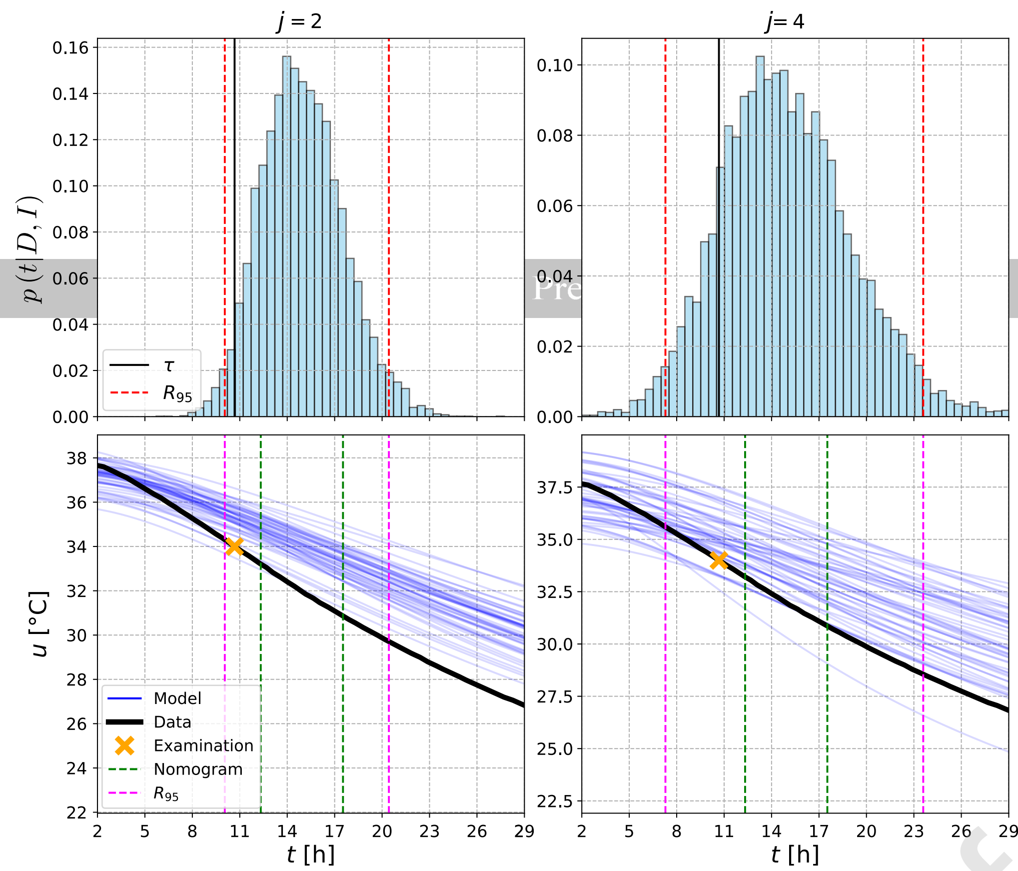

Figure 10: Analysis of the combined uncertainty of all parameters. The index j corresponds to the first column of Table 1. See caption of Figure 4 for details.

The results become more clear when looking at Figure 11. For each value of j, this figure shows the distribution of the PMI (shaded area) superimposed with the R_{95} (thick vertical line). It can be seen that the amplitude of the R_{95} increases very rapidly as the individual uncertainties of the parameters increase, and therefore there is a compromise relationship: on the one hand, if the individual uncertainties are underestimated, the resulting R_{95} will likely not contain the true PMI and, furthermore, excessively low probabilities will be assigned to intervals that are slightly away from the median (which could be misinterpreted in court as an impossibility, leading, for example, to the decision of dismissing an important suspect); on the other hand, if very conservative individual uncertainties are assigned, the resulting R_{95} will have an amplitude so large that it may not be useful for the cause. However, the latter situation more consistently reflects the low reliability of the method, and allows more realistic values to be assigned to the probabilities of time intervals far from the median of the distribution, reducing the probability of taking incorrect decisions based on an estimate of the PMI with an overestimated accuracy.

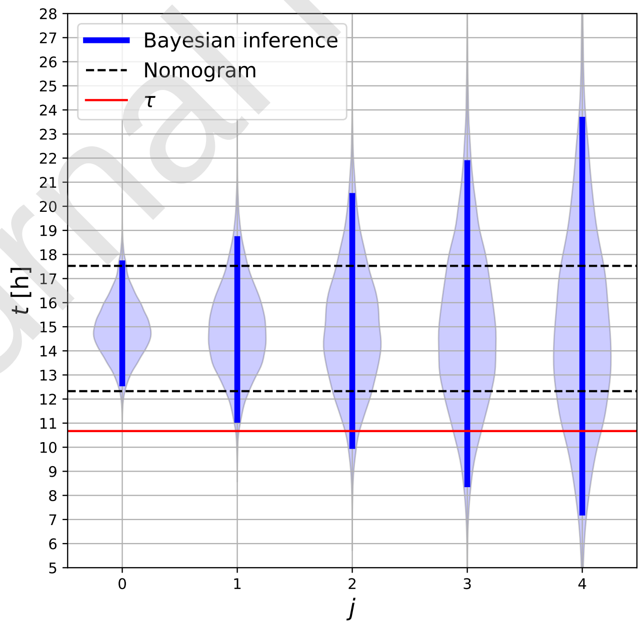

Figure 11: Postmortem interval estimated for different values of uncertainty in all parameters. Each blue shaded region represents the posterior distribution of the PMI obtained by Bayesian inference, and each thick blue vertical line represents the corresponding R_{95}. On the other hand, the black dotted horizontal lines represent the limits of the estimated PMI according to the nomogram method. The index j corresponds to the first column of Table 1.

3.2 Reliability of PMI estimation using Bayesian inference

The results of the previous sections illustrate the implementation of Bayesian inference in determining the PMI and its associated uncertainty for different values of uncertainty in the model parameters. In this section, we implement Bayesian inference in the complete database using a fixed set of parameter uncertainties, namely $\sigma_u = 0.2$ °C, $\sigma_{u_A} = 0.5$ °C, $\sigma_{u_0} = 0.75$ °C, $\sigma_m = 0.4$ kg and $\sigma_c = 0.25$. These values were selected on the basis of the discussion in Section 2.6 and the specific experimental conditions described in [16]. The objective of this analysis is to determine the reliability of this method as a function of cooling progress, as was done by Henssge in his study of empirical errors [9].

Table 2 summarizes the results obtained by the nomogram method and by Bayesian inference for the ranges $0.5 \leq Q < 1.0$, $0.3 \leq Q < 0.5$ and $0.2 \leq Q < 0.3$, respectively, for all cases in the database. The results are presented in terms of "correct estimations", i.e. estimations in which the true time since death was contained in the R_{95} (Bayesian inference) or in the 95.45% confidence interval (nomogram method). Since not all of the cooling curves in the database have temperature records in all ranges of Q, the number of samples N varies in each case.

Q range	N	Correct estimations, NM	Correct estimations, BI
[0.5,1.0)	84	47 (56%)	58 (69%)
[0.3,0.5)	75	41 (55%)	52 (69%)
[0.2,0.3)	53	29 (55%)	39 (74%)

Table 2: Proportion of correct estimations according to the nomogram method (NM) and the Bayesian inference method (BI), for different ranges of Q.

The results indicate that, for this database, the R_{95} obtained through Bayesian inference by assigning adequate uncertainties to the parameters is more reliable than the interval reported by using the empirical errors of the nomogram method in all ranges of Q. Although the proportion of correct estimations is relatively low, it could be significantly improved through a recalibration of the method (as proposed by the authors in [7]), by using a more complex statistical model (which takes into account, for instance, the uncertainties that arise when fitting equation (1) to experimental data) and by combining this technique with other methods of estimating PMI, always in the context of Bayesian inference, as explained in Section 2.5.

4 Practical application in casework

In this section we present a step-by-step application example to estimate the postmortem interval using the method of Bayesian inference based on rectal temperature measurements. For this purpose, we will use the data corresponding to a point on the cooling curve of case number 72 in [16].

Step 1: Collect the data and select the corrective factor

- Rectal temperature: $\tilde{u} = 34.8$ °C.
- Ambient temperature: $\tilde{u}_A = 21.9$ °C.
- Body mass: $\tilde{m} = 73.3$ kg.
- Clothing: "T-shirt, trousers, front of abdomen/thorax uncovered".
- Supporting base: steel trolley.

According to this data, the body had one layer of light clothing on the lower trunk, it was dry and in an environment with negligible air flow, therefore a corrective factor $\tilde{c} = 1.1$ is first selected from table 6.18 of [10]. Then, since the supporting base is conductive and the clothing layer is very thin, according to table 6.19 of the same reference we subtract 0.2 from the corrective factor, finally obtaining a value $\tilde{c} = 0.9$. The complete data set is then:

$$D = \{\tilde{u}, \tilde{u}_0, \tilde{u}_A, \tilde{m}, \tilde{c}\} = \{34.8, 37.2, 21.9, 73.3, 0.9\}.$$

Step 2: Define the uncertainties

Taking into account the considerations detailed in Section 3.1.7, we choose the following values for the different uncertainties involved in the computation: $\sigma_u = 0.2$ °C (rectal temperature), $\sigma_{u_A} = 0.5$ °C (ambient temperature), $\sigma_{u_0} = 0.75$ °C (initial temperature), $\sigma_m = 0.4$ kg (body mass) and $\sigma_c = 0.25$ (corrective factor).

Step 3: Sample the posterior distribution of the PMI

Given the data and the uncertainties, we use a script[5] programmed in Python 3 to extract $N = 10000$ samples from the posterior distribution of the PMI, following the inference scheme described in Figure 1. The steps involved are listed below:

1. We extract N samples of variables u, u_A, u_0, m and c from their posterior distributions given by Equation (11).

[5] A copy of the script can be obtained upon email request from the authors.

2. From the samples obtained in the previous step, we compute the N corresponding values of standardized temperature Q and the parameters A and B using Equations (3), (2) and (1), respectively.

3. For each of the N triplets (Q, A, B) calculated in the previous step, we determine the value of t that satisfies Equation (4). These are the samples of the posterior distribution $p(t|D, I)$ given by Equation (7). Since non-temperature-based information is not available, these samples represent the best probability distribution that we can obtain for the time since death given the evidence in this case.

In this example, the computation took approximately 36 s on a standard office laptop.

Step 4: Compute the 95.45% credibility region

In this final step, we calculate the interval R_{95} defined in Section 2.2:

$$R_{95} = (6.6, 16.0)\,\text{hpm}.$$

Although this interval is the traditional way of presenting the results, we want to emphasize the importance of preserving the posterior distribution $p(t|D, I)$ computed in Step 3, since it can be used to perform other inferences (eg, compute the probability of death in a specific time interval) and can also be combined with other PMI estimation techniques to obtain a more precise distribution based on all the available evidence.

5 Conclusions

Using the body cooling data published by Muggenthaler et al., we applied the Bayesian inference method to assess the influence of uncertainties in the different parameters involved in the estimation of PMI. An adequate assignment of these uncertainties results in a posterior distribution of the PMI that assigns greater probabilities to intervals containing the true time since death, compared to the use of empirical errors proposed by the nomogram method. Moreover, this approach facilitates an in-depth understanding of the main sources of uncertainty in a formal and systematic fashion.

The result of the inference process is the posterior probability density of the time since death, which provides more information than a simple point estimate or a time interval. This distribution can be used to assign probabilities to specific time intervals that may arise in a criminal investigation.

Despite the improvements brought about by using Bayesian inference in this context, some of the hypotheses considered are simplistic, and it is necessary to modify them to obtain more representative results of the underlying physical phenomenon. First, it is necessary to introduce a statistical model for parameter B that includes the uncertainties in the parameters b_1, b_2, b_3 and b_4 (equation (1)). Second, the discrete model of parameter A as a function of the mean ambient temperature (equation (2)) is very limited, considering its role in the body cooling model (equation (4)), and in this sense a simple continuous function would give more versatility to the method. Finally, the corrective factor should be very carefully analyzed. On the one hand, in this work we assigned a normal distribution to c, centered on the tabulated value $\tilde{c}$ and with fixed uncertainty σ_c. However, each tabulated value $\tilde{c}$ comes from a curve fit that has a specific uncertainty. This implies that the uncertainty σ_c must be a function of $\tilde{c}$ (each cooling condition has its own associated curve fit, and therefore its own uncertainty). On the other hand, the set of conditions K determined at the scene should uniquely define the value $\tilde{c}$. However, in many cases there may be an ambiguity in the interpretation of the tabulated conditions, leading to an error in the determination of $\tilde{c}$ and consequently to an incorrect estimation of the posterior distribution of c. It is therefore of immense importance to systematize the determination of the corrective factor $\tilde{c}$ to reduce (and, if possible, eliminate) these ambiguities.

The application of this statistical analysis can be extended to any method of estimating the postmortem interval. Furthermore, Bayesian inference allows different sources of information to be systematically combined to obtain a more reliable probability distribution of PMI. To achieve this, a posterior density of the PMI must be determined by one method and then used as the prior density in the inference process carried out with data from another method.

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Appendices

A Derivation of the expression for $p\,(t|D,I)$

Given two continuous variables x and y, a data set D and a prior information I, the marginal posterior distribution of x is obtained by integrating the joint posterior distribution $p\,(x,y|D,I)$ with respect to y:

$$p(x|D,I) = \int\limits_{D_y} p(x,y|D,I)\, dy,$$

where D_y represents the region of variation of y. Applying this concept, we can express the marginal posterior distribution of the time since death from the joint posterior distribution of t, Q, A and B:

$$p(t|D,I) = \iiint\limits_{V_1} p(t,Q,A,B|D,I)\, dQ\, dA\, dB, \tag{15}$$

where the integration region is $V_1 = D_Q \times D_A \times D_B$. Applying the product rule, we can write the integrand of equation (15) as:

$$p(t,Q,A,B|D,I) = p(t|Q,A,B,D,I)\, p(Q,A,B|D,I) \tag{16}$$

Because t is completely determined by the values of Q, A, and B through a deterministic relationship (function $f_1(Q,A,B)$), we have [23]:

$$p(t|Q,A,B,D,I) = \delta(t - f_1(Q,A,B)). \tag{17}$$

where $\delta(t)$ is Dirac's delta function. The other factor in equation (16), which corresponds to the joint posterior density of Q, A and B, can be expanded by applying the product rule again, and taking into account that Q and A are not independent (since both depend functionally on variable u_A):

$$p(Q,A,B|D,I) = p(Q,A|D,I)\, p(B|D,I). \tag{18}$$

The posterior density of the time since death according to equation (9) is then obtained by combining equations (15), (16), (17) and (18). On the other hand, following the scheme given in Figure 1 and again applying the marginalization process and the product rule, we have:

$$\begin{aligned} p(Q,A|D,I) &= \iiint\limits_{V_2} p(Q,A,u,u_0,u_A|D,I)\, du\, du_0\, du_A \\ &= \iiint\limits_{V_2} p(Q,A|u,u_0,u_A,D,I)\, p(u,u_0,u_A|D,I)\, du\, du_0\, du_A \\ &= \iiint\limits_{V_2} \delta(Q - f_2(u,u_0,u_A))\, \delta(A - f_3(u_A))\, p(u,u_0,u_A|D,I)\, du\, du_0\, du_A, \end{aligned} \tag{19}$$

$$\begin{aligned} p(B|D,I) &= \iint\limits_{S} p(B,m,c|D,I)\, dm\, dc \\ &= \iint\limits_{S} p(B|m,c,D,I)\, p(m,c|D,I)\, dm\, dc \\ &= \iint\limits_{S} \delta(B - f_4(m,c))\, p(m,c|D,I)\, dm\, dc, \end{aligned} \tag{20}$$

where the integration regions are $V_2 = D_u \times D_{u_0} \times D_{u_A}$ and $S = D_m \times D_c$.

Taking into account that the uncertainties in all the observed variables are independent of each other, the joint distributions that appear in the integrands of equations (19) and (20) can be written as the product of the marginal distributions of the variables involved. In this way, we obtain $p(u,u_0,u_A|D,I) = p(u|D,I)\, p(u_0|D,I)\, p(u_A|D,I)$, and similarly $p(m,c|D,I) = p(m|D,I)\, p(c|D,I)$. These latter distributions are described in Section 2.4 and are the starting point for calculating the posterior distribution of the PMI.

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