On the evolution of ultra-massive white dwarfs

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ABSTRACT

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ABST

Ultra-massive white dwarfs are powerful tools to study various phy pernova explosions and the theory of crystallization through white there are few evolutionary studies in the literature devoted to the sequences that constitute an improvement over previous ones. In process of phase separation expected during the crystallization sta diagram of dense oxygen/neon mixtures. Realistic chemical profile ing the semidegenerate carbon burning along the super-AGB phase for our evolving models are provided by detailed non-gray white We assessed the impact of all these improvements on the evolution evolutionary sequences for these stars. We conclude that crystallization is necessary to accurately determine the cooling age and to colors in the GAIA photometric bands for our H-rich white dwarf Finally, these new white dwarf sequences provide a new theoretic detected ultra-massive pulsating white dwarfs.

Key words. stars: evolution — stars: interiors — stars: white dwarf evolution. Indeed, more than 97% of all stars will eventually become white dwarfs. These old stellar remnants preserve information about the evolutionary history of their progenitors, providing a wealth of information about the physical evolutionary processes of stars, the star formation history, and about the characteristics of various stellar populations. Furthermore, their structure and evolutionary properties are well understood — see Fontaine & Brassard (2008); Winget & Kepler (2008), and Althaus et al. (2010b) for specific reviews — to the point that the white dwarf cooling times are currently considered one of the best age indicators for a wide variety of Galactic populations. Ultra-massive white dwarfs are powerful tools to study various physical processes in the Asymptotic Giant Branch (AGB), type Ia supernova explosions and the theory of crystallization through white dwarf asteroseismology. Despite the interest in these white dwarfs, there are few evolutionary studies in the literature devoted to them. Here, we present new ultra-massive white dwarf evolutionary sequences that constitute an improvement over previous ones. In these new sequences, we take into account for the first time the process of phase separation expected during the crystallization stage of these white dwarfs, by relying on the most up-to-date phase diagram of dense oxygen/neon mixtures. Realistic chemical profiles resulting from the full computation of progenitor evolution during the semidegenerate carbon burning along the super-AGB phase are also considered in our sequences. Outer boundary conditions for our evolving models are provided by detailed non-gray white dwarf model atmospheres for hydrogen and helium composition. We assessed the impact of all these improvements on the evolutionary properties of ultra-massive white dwarfs, providing up-dated evolutionary sequences for these stars. We conclude that crystallization is expected to affect the majority of the massive white dwarfs observed with effective temperatures below 40 000 K. Moreover, the calculation of the phase separation process induced by crystallization is necessary to accurately determine the cooling age and the mass-radius relation of massive white dwarfs. We also provide colors in the GAIA photometric bands for our H-rich white dwarf evolutionary sequences on the basis of new models atmospheres. Finally, these new white dwarf sequences provide a new theoretical frame to perform asteroseismological studies on the recently

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Althaus et al. (2010b) for specific reviews — to the point that the white dwarf cooling times are currently considered one of the best age indicators for a wide variety of Galactic populations, including open and globular clusters (see Winget et al. 2009; García-Berro et al. 2010; Jeffery et al. 2011; Bono et al. 2013; Hansen et al. 2013, for some applications).

The mass distribution of white dwarfs exhibits a main peak at $M_{\rm WD} \sim 0.6 M_{\odot}$, and a smaller peak at the tail of the distribution around $M_{\rm WD} \sim 0.82 M_{\odot}$ (Kleinman et al. 2013). The existence of massive white dwarfs ($M_{\rm WD}$ \gtrsim $0.8M_{\odot}$) and ultra-massive white dwarfs ($M_{\rm WD} \gtrsim 1.10M_{\odot}$) has been revealed in several studies (Castanheira et al. 2010, 2013; Hermes et al. 2013; Kepler et al. 2016; Curd et al. 2017). In-

deed, Rebassa-Mansergas et al. (2015) reports the existence of a distinctive high-mass excess in the mass function of hydrogenrich white dwarfs near $1M_{\odot}$.

An historic interest in the study of ultra-massive white dwarfs is related to our understanding of type Ia Supernova. In fact, it is thought that type Ia Supernova involve the explosion of an ultra-massive white dwarf or the merger of two white dwarfs. Also, massive white dwarfs can act as gravitational lenses. It has been proposed that massive faint white dwarfs can be responsible of "microlensing" events in the Large Magallanic Cloud.

The formation of an ultra-massive white dwarf is theoretically predicted as the end product of the isolated evolution of a massive intermediate-mass star —with a mass larger than 6-9 M_{\odot} , depending on metallicity and the treatment of convective boundaries. Once the helium in the core has been exhausted, these stars reach the Super Asymptotic Giant Branch (SAGB) with a partially degenerate carbon(C)-oxygen(O) core as their less massive siblings. However, in the case of SAGB stars their cores develop temperatures high enough to start carbon ignition under partially degenerate conditions. The violent carbonignition leads to the formation of a Oxygen-Neon core, which is not hot enough to burn Oxygen or Neon (Ne) (Siess 2006) and is supported by the degenerate pressure of the electron gas. If the hydrogen-rich envelope is removed by winds before electron captures begin in the O-Ne core, an electron-capture supernova is avoided and the star leaves the SAGB to form a white dwarf. As a result, ultra-massive white dwarfs are born with cores composed mainly of ¹⁶O and ²⁰Ne, with traces of ¹²C, ²³Na and ²⁴Mg (Siess 2007). In addition, massive white dwarfs with C-O cores can be formed through binary evolution channels; namely the single-degenerate channel in which a white dwarf gains mass from a nondegenerate companion, and double-degenerate channel involving the merger of two white dwarfs (Maoz et al. 2014). The study of the predicted surface properties and cooling times of ultra-massive CO- and ONe-core white dwarfs can help to assess the relevance of different channels in the formation of these stars.

During the last years, q(gravity)-mode pulsations have been detected in many massive and ultra-massive variable white dwarfs with hydrogen-rich atmospheres (DA), also called ZZ Ceti stars (Kanaan et al. 2005; Castanheira et al. 2010, 2013; Hermes et al. 2013; Curd et al. 2017). The ultra-massive ZZ Ceti star BPM 37093 (Kanaan et al. 1992, 2005) was the first object of this kind to be analyzed in detail. The existence of pulsating ultra-massive white dwarfs opens the possibility of carrying out asteroseismological analyses of heavy-weight ZZ Ceti stars, allowing to obtain information about their origin and internal structure through the comparison between the observed periods and the theoretical periods computed for appropriate theoretical models. In particular, one of the major interests in the study of pulsating ultra-massive DA white dwarfs lies in the fact that these stars are expected to have a well developed crystallized core. The occurrence of crystallization in the degenerate core of white dwarfs, resulting from Coulomb interactions in very dense plasmas, was first suggested by several authors about 60 yr ago, see Kirzhnits (1960); Abrikosov (1961); van Horn (1968) for details, and the more recent works by Montgomery & Winget (1999); Metcalfe et al. (2004); Córsico et al. (2005); Brassard & Fontaine (2005) for discussions. However, this theoretical prediction was not observationally demonstrated until the recent studies of Winget et al. (2009) and García-Berro et al. (2010), who inferred the existence of crystallized white dwarfs from the study of the white dwarf luminosity function of stellar clusters. Since ultra-massive ZZ Ceti stars are expected to have a core partially or totally crystallized, these stars constitute unique objects to detect the presence of crystallization. Thus, asteroseismology of ultra-massive DA white dwarfs is expected to contribute to our understanding of the Coulomb interactions in dense plasmas. The first attempt to infer the existence of crystallization in an ultramassive white dwarf star from the analysis of its pulsation pattern was carried out by Metcalfe et al. (2004) in the case of BPM 37093 (Kanaan et al. 2005), but the results were inconclusive (Brassard & Fontaine 2005).

Asteroseismological applications of ultra-massive DA white dwarfs require the development of detailed evolutionary models for these stars, taking into account all the physical processes responsible for interior abundance changes as evolution proceeds. The first attempts to model these stars by considering the evolutionary history of progenitor stars were the studies by Garcia-Berro et al. (1997) and Althaus et al. (2007). These studies, however, adopted several simplifications which should be assessed. To begin with, they consider a core chemical profile composed mainly of ¹⁶O and ²⁰Ne, implanted to white dwarf models with different stellar masses. A main assumption made in Althaus et al. (2007) (from here on, we refer to as A07) is that the same fixed chemical profile during the entire evolution is assumed for all of their models. Also, phase separation dur-

ing crystallization is an important missing physical ingredient in these studies. In fact, when crystallization occurs, energy is released in two different ways. First, as in any crystallization process, latent heat energy is released. And second, a phase separation of the elements occurs upon crystallization, releasing gravitational energy (Isern et al. 1997) and enlengthening the cooling times of white dwarfs. This process of phase separation has been neglected in all the studies of ultra-massive white dwarfs. Finally, progress in the treatment of conductive opacities and model atmospheres has been made in recent years, and should be taken into account in new attempts to improve our knowledge of these stars.

This paper is precisely aimed at upgrading these old white dwarf evolutionary models by taking into account the abovementioned considerations. We present new evolutionary sequences for ultra-massive white dwarfs, appropriate for accurate white dwarf cosmochronology of old stellar systems and for precise asteroseismology of these white dwarfs. We compute four hydrogen-rich and four hydrogen-deficient white dwarf evolutionary sequences. The initial chemical profile of each white dwarf model is consistent with predictions of the progenitor evolution with stellar masses in the range $9.0 \le M_{\rm ZAMS}/M_{\odot} \le 10.5$ calculated in Siess (2010). This chemical structure is the result of the full evolutionary calculations starting at the Zero Age Main Sequence (ZAMS), and evolved through the core hydrogen burning, core helium burning, the SAGB phase, including the entire thermally-pulsing phase. An accurate nuclear network has been used for each evolutionary phase. Thus, not only a realistic O-Ne inner profile is considered for each white dwarf mass, but also realistic chemical profiles and intershell masses built up during the SAGB are taken into account. In our study, the energy released during the crystallization process, as well as the ensuing core chemical redistribution were considered by following the phase diagram of Medin & Cumming (2010) suitable for ¹⁶O and ²⁰Ne plasmas¹. We also provide accurate magnitudes and colors for our hydrogen-rich models in the filters used by the spacial mission GAIA: G, G_{BP} and G_{RP}.

To the best of our knowledge, this is the first set of fully evolutionary calculations of ultra-massive white dwarfs including realistic initial chemical profiles for each white dwarf mass, an updated microphysics, and the effects of phase separation process duration crystallization². This paper is organized as follows. In Sect. 2 we briefly describe our numerical tools and the main ingredients of the evolutionary sequences, while in Sect. 3 we present in detail our evolutionary results and compare them with previous works. Finally, in Sect. 4 we summarize the main findings of the paper, and we elaborate on our conclusions.

2. Numerical setup and input physics

The white dwarf evolutionary sequences presented in this work have been calculated using the LPCODE stellar evolutionary code (see Althaus et al. 2005, 2012, for details). This code has been well tested and calibrated and has been amply used in the study of different aspects of low-mass star evolution (see García-Berro et al. 2010; Althaus et al. 2010a; Renedo et al. 2010, and references therein). More recently, the code has been used to generate a new grid of models for post-AGB stars (Miller Bertolami 2016) and also new evolutionary sequences for hydrogen-deficient white dwarfs (Camisassa et al. 2017). We

¹ A. Cumming, personal communication.

² These evolutionary sequences are available at http://evolgroup.fcaglp.unlp.edu.ar/TRACKS/ultramassive.html

mention that LPCODE has been tested against another white dwarf evolutionary code, and the uncertainties in the white dwarf cooling ages that result from the different numerical implementations of the stellar evolution equations were found to be below 2% (Salaris et al. 2013).

For the white dwarf regime, the main input physics of LPCODE includes the following ingredients. Convection is treated within the standard mixing length formulation, as given by the ML2 parameterization (Tassoul et al. 1990). Radiative and conductive opacities are from OPAL (Iglesias & Rogers 1996) and from Cassisi et al. (2007), respectively. For the low-temperature regime, molecular radiative opacities with varying carbon to oxygen ratios are used. To this end, the low temperature opacities computed by Ferguson et al. (2005) as presented by Weiss & Ferguson (2009) are adopted. The equation of state for the low-density regime is taken from Magni & Mazzitelli (1979), whereas for the high-density regime, we employ the equation of state of Segretain et al. (1994), which includes all the important contributions for both the solid and liquid phases. We considered neutrino emission for pair, photo, and bremsstrahlung processes using the rates of Itoh et al. (1996), while for plasma processes we follow the treatment presented in Haft et al. (1994). Outer boundary conditions for both H-rich and H-deficient evolving models are provided by non-gray model atmospheres, see Rohrmann et al. (2012), Camisassa et al. (2017), and Rohrmann (2018) for references. The impact of the atmosphere treatment on the cooling times becomes relevant for effective temperatures lower than 10 000 K. LPCODE considers a detailed treatment of element diffusion, including gravitational settling, chemical and thermal diffusion. As we will see, element diffusion is a key ingredient in shaping the chemical profile of evolving ultra-massive white dwarfs, even in layers near the core.

2.1. Treatment of crystallization

A main issue in the modelling of ultra-massive white dwarfs is the treatment of crystallization. As temperature decreases in the interior of white dwarfs, the Coulomb interaction energy becomes increasingly important, until at some point, they widely exceed the thermal motions and the ions begin to freeze into a regular lattice structure. Since the crystallization temperature of pure ²⁰Ne is larger than the crystallization temperature of ¹⁶O, this crystallization process induces a phase separation. In a mixture of ²⁰Ne and ¹⁶O, the crystallized plasma will be enriched in ²⁰Ne and, consequently, ²⁰Ne will decrease in the remaining liquid plasma. This process releases gravitational energy, thus constituting a new energy source that will impact the cooling times.

We used the most up-to-date phase diagram of dense O-Ne mixtures appropriate for massive white dwarf interiors (Medin & Cumming 2010). This phase diagram, shown in Fig. 1, yields the temperature at which crystallization occurs, as well as the abundance change at a given point in the solid phase during the phase transition. Γ is the coulomb coupling parameter, defined as $\Gamma = \frac{e^2}{k_B a_e T} Z^{5/3}$, where $a_e = \left(\frac{3}{4\pi n_e}\right)^{1/3}$ is the mean electron spacing. Γ_{crit} is set to 178.6, the crystallization value of a mono-component plasma. Γ_O is the value of Γ of $\Gamma^{16}O$ at which crystallization of the mixture occurs, and is related to the temperature and the density through the relation $\Gamma_O = \frac{e^2}{k_B a_e T} 8^{5/3}$. For a given mass fraction of $\Gamma^{20}O$, the solid red line in Fig. 1 gives us Γ_O , and, consequently, the temperature of crystallization is obtained. Once we obtain this temperature, it can be related to the

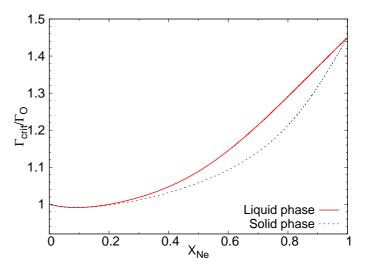


Fig. 1. Phase diagram of crystallization for a $^{16}\text{O}/^{20}\text{Ne}$ mixture (Medin & Cumming 2010). X_{Ne} is the ^{20}Ne abundance. Γ_{crit} is set to 178.6. Γ_{O} is given by $\Gamma_{\text{O}} = (e^2/k_B a_e T) 8^{5/3}$, see text for details.

 Γ of the mixture, by replacing T in the formula $\Gamma = \frac{e^2}{k_B a_c T} Z_{\text{mixture}}^{5/3}$, where Z_{mixture} is the mean ionic charge of the mixture. The Γ obtained using this procedure is larger than the value of Γ commonly used in the white dwarf evolutionary calculations, which is artificially set to 180. For a given abundance of 20 Ne in the liquid phase, the solid red line predicts $\Gamma_{\text{crit}}/\Gamma_{\text{O}}$, and the corresponding value of $\Gamma_{\text{crit}}/\Gamma_{\text{O}}$ at the dashed black line predicts the 20 Ne abundance in the solid phase, which is slightly larger than the initial 20 Ne abundance. The final result of the crystallization process is that the inner regions of the star are enriched in 20 Ne, and the outer regions are enriched in 16 O.

The energetics resulting from crystallization processes has been self-consistently and locally coupled to the full set of equations of stellar evolution (see Althaus et al. 2010c, for details of the implementation). The local change of chemical abundance resulting from the process of phase separation at crystallization leads to a release of energy (in addition to the latent heat). The inclusion of this energy in LPCODE is similar to that described in Althaus et al. (2010c), but adapted to the mixture of ¹⁶O and ²⁰Ne characterizing the core of our ultra-massive white dwarf models. At each evolutionary time step, we calculate the change in chemical composition resulting from phase separation using the phase diagram of Medin & Cumming (2010) for an oxygenneon mixture. Then, we evaluate the net energy released by this process during the time step. This energy is added to the latent heat contribution, which is considered as $0.77k_BT$ per ion. The total energy is distributed over a small mass range around the crystallization front. This local energy contribution is added to the luminosity equation (see Althaus et al. 2010c, for details).

The increase of ²⁰Ne abundance in the solid core as a result of crystallization leads to a Rayleigh-Taylor instability and an ensuing mixing process at the region above the crystallized core, inducing the oxygen enrichment in the overlying liquid mantle (Isern et al. 1997). Thus, those layers that are crystallizing contribute as an energy source, and the overlying unstable layers will be a sink of energy.

2.2. Initial models

As we have mentioned, an improvement of the present calculations over those published in A07 is the adoption of detailed chemical profiles which are based on the computation of all the

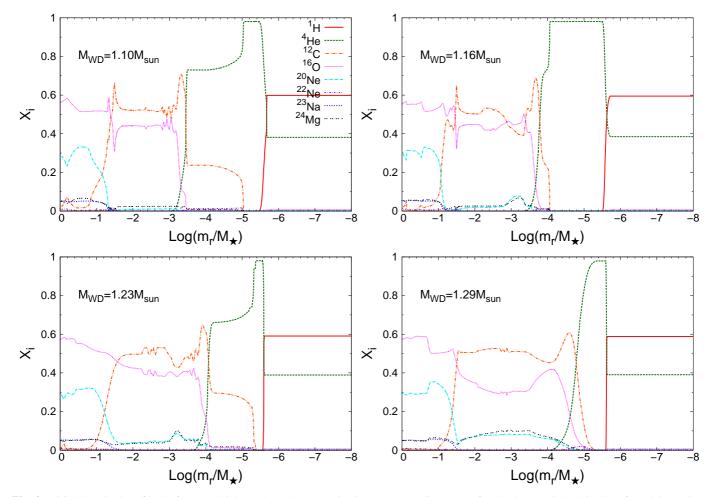


Fig. 2. Initial chemical profiles before Rayleigh-Taylor rehomogeneization corresponding to our four hydrogen-rich white dwarf models as given by nuclear history of progenitor stars.

previous evolutionary stages of their progenitor stars. This is true for both the O-Ne core and the surrounding envelope. In particular, the full computation of previous evolutionary stages allows us to assess the mass of the helium-rich mantle and the hydrogen-helium transition, which are of particular interest for the asteroseismology of ultra-massive white dwarfs. Specifically, the chemical composition of our models is the result of the entire progenitor evolution calculated in Siess (2007, 2010). These sequences correspond to the complete single evolution from the ZAMS to the thermally pulsating SAGB phase of initially $M_{\rm ZAMS} = 9, 9.5, 10$, and $10.5 M_{\odot}$ sequences with an initial metallicity of Z = 0.02. Particular care was taken by Siess (2007, 2010) to precisely follow the propagation of the carbon burning flame where most carbon is burnt (Siess 2006). This is of special interest for the final oxygen and neon abundances in the white dwarf core. In addition, Siess (2010) computed in detail the evolution during the thermally pulsing-SAGB phase where the outer chemical profiles and the total helium-content of the final stellar remnant are determined. No extra mixing was included at any convective boundary at any evolutionary stage. The absence of core overshooting during core hydrogen- and helium-burning stages implies that, for a given final remnant mass $(M_{\rm WD})$, initial masses $(M_{\rm ZAMS})$ represent an upper limit of the expected progenitor masses. Indeed, considering moderate overshooting during core helium burning lowers the mass range of SAGB stars in $2 M_{\odot}$ (Siess 2007; Gil-Pons et al. 2007). It is worth noting that the initial final mass relation is poorly constrained from observations (Salaris et al. 2009) and it is highly uncertain in stellar evolution models. On the other hand, considering overshooting during the thermally-pulsing SAGB, would induce third dredge-up episodes, altering the carbon and nitrogen abundances in the envelope. Finally, in this work we have not explored the impact on white dwarf cooling that could be expected from changes in the core chemical structure resulting from the consideration of extra- mixing episodes during the semi-degenerate carbon burning.

The stellar masses of our white dwarf sequences are $M_{\rm WD}$ = $1.10M_{\odot}$, $1.16M_{\odot}$, $1.23M_{\odot}$ and $1.29M_{\odot}$. Each evolutionary sequence was computed from the beginning of the cooling track at high luminosities down to the development of the full Debye cooling at very low surface luminosities, $\log(L_{\star}/L_{\odot}) = -5.5$. The progenitor evolution through the thermally-pulsing SAGB provides us with realistic values of the total helium content, which is relevant for accurate computation of cooling times at low luminosities. In particular, different helium masses lead to different cooling times. The helium mass of our $1.10M_{\odot}$, $1.16M_{\odot}$, $1.23M_{\odot}$ and $1.29M_{\odot}$ models are $3.24 \times 10^{-4}M_{\odot}$, $1.82 \times 10^{-4}M_{\odot}$ $10^{-4} M_{\odot}$, $0.78 \times 10^{-4} M_{\odot}$ and $0.21 \times 10^{-4} M_{\odot}$, respectively. By contrast, the total mass of the hydrogen envelope left by prior evolution is quite uncertain, since it depends on the occurrence of carbon enrichment on the thermally pulsing AGB phase (see Althaus et al. 2015), which in turn depends on the amount of overshooting and mass loss, as well as on the occurrence of late thermal pulses. For this paper, we have adopted the max-

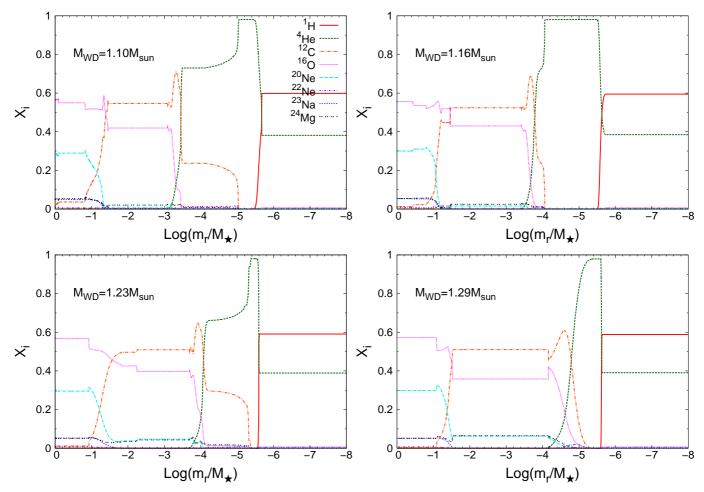


Fig. 3. Initial chemical profiles of our four hydrogen-rich white dwarf models once Rayleigh-Taylor rehomogeneization has occurred.

imum expected hydrogen envelope of about $\sim 10^{-6} M_{\odot}$ for ultra-massive white dwarfs. Larger values of the total hydrogen mass would lead to unstable nuclear burning and thermonuclear flashes on the white dwarf cooling track.

Fig. 2 illustrates the chemical profiles resulting from the progenitor evolution of our four hydrogen-rich white dwarf sequences³. The core composition is $\sim55\%^{-16}\mathrm{O}$, $\sim30\%^{-20}\mathrm{Ne}$, with minor traces of $^{22}\mathrm{Ne}$, $^{23}\mathrm{Na}$, $^{24}\mathrm{Mg}$. At some layers of the models, the mean molecular weight is higher than in the deeper layers, leading to Rayleigh-Taylor unstabilities. Consequently, these profiles are expected to undergo a rehomogeneization process in a timescale shorter than the evolutionary timescale. Thus, we have simulated the rehomogeneization process assuming to be instantaneous. The impact of this mixing process on the abundance distribution in the white dwarf core results apparent from inspecting Fig. 3. Clearly, rehomogeneization mixes the abundances of all elements at some layers of core, erasing preexisting peaks in the abundances.

3. Evolutionary results

We present in Fig. 4 a global view of the main phases of the evolution of an ultra-massive hydrogen-rich white dwarf model during the cooling phase. In this figure, the temporal evolution of the different luminosity contributions is displayed for our $1.16 M_{\odot}$

hydrogen-rich white dwarf sequence. The cooling time is defined as zero at the beginning of the white dwarf cooling phase, when the star reaches the maximum effective temperature. During the entire white dwarf evolution, the release of gravothermal energy is the dominant energy source of the star. At early stages, neutrino emission constitutes an important energy sink. In fact, during the first million yr of cooling, the energy lost by neutrino emission is of about the same order of magnitude as the gravothermal energy release, remaining larger than the star luminosity until the cooling time reaches about $\log(t) \sim 7$. As the white dwarf cools, the temperature of the degenerate core decreases, thus neutrino emission ceases and, consequently, the neutrino luminosity abruptly drops. It is during these stages that element diffusion strongly modifies the internal chemical profiles. The resulting chemical stratification will be discussed below. At $log(t) \sim 8.3$ crystallization sets in at the center of the white dwarf. This results in the release of latent heat and gravitational energy due to oxygen-neon phase separation. Note that, as a consequence of this energy release, during the crystallization phase the surface luminosity is larger than the gravothermal luminosity. This phase lasts for 2.5×10^9 years. Finally, at $\log(t) \sim 9$, the temperature of the crystallized core drops below the Debye temperature, and consequently, the heat capacity decreases. Thus, the white dwarf enters the so-called "Debye cooling phase", characterized by a rapid cooling.

The cooling times for all of our white dwarf sequences are displayed in Fig. 5. These cooling times are also listed in Table 1 at some selected stellar luminosities. Our hydrogen-deficient

³ The chemical profiles of our hydrogen-deficient white dwarf models are the same, except that no hydrogen is present in the envelope.

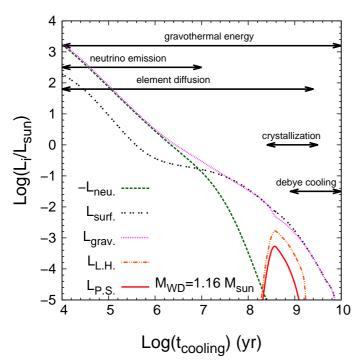


Fig. 4. Temporal evolution of surface luminosity (double dotted line) and different luminosity contributions: neutrino luminosity (dashed line), gravothermal luminosity (dotted line), latent heat (dot-dot-dashed line) and phase separation energy (solid line). The arrows indicate the main physical processes responsible for the evolution at different moments.

sequences have been calculated by considering recent advancement in the treatment of energy transfer in dense helium atmospheres, (see Camisassa et al. 2017; Rohrmann 2018, for details). As shown in Camisassa et al. (2017), detailed non-gray model atmospheres are needed to derive realistic cooling ages of cool, helium-rich white dwarfs. At intermediate luminosities, hydrogen-deficient white dwarfs evolve slightly slower than their hydrogen-rich counterparts. This result is in line with previous studies of hydrogen-deficient white dwarfs (Camisassa et al. 2017) and the reason for this is that convective coupling (and the associated release of internal energy) occurs at higher luminosities in hydrogen-deficient white dwarfs, with the consequent lengthening of cooling times at those luminosities. By contrast, at low-luminosities, hydrogen-deficient white dwarfs evolve markedly faster than hydrogen-rich white dwarfs. This is due to the fact that, at those stages, the thermal energy content of the hydrogen-deficient white dwarfs is smaller, and more importantly, because in these white dwarfs, the outer layers are more transparent to radiation. Note in this sense that the $1.10 \, M_{\odot}$ hydrogen-rich sequence needs 8.2 Gyr to reach the lowest luminosities, while the hydrogen-deficient sequence of the same mass evolves in only 4.6 Gyr to the same luminosities. Note also the different cooling behavior with the stellar mass, particularly the fast cooling of the $1.29 M_{\odot}$ hydrogen-rich sequence, our most massive sequence, which reaches $\log(L_{\star}/L_{\odot}) = -5$ in only 3.6 Gyr, which is even shorter (2.4 Gyr) in the case of the hydrogendeficient counterpart. These short cooling times that characterize the most massive sequences reflect that, at such stages, matter in most of the white dwarf star has entered the Debye regime, with the consequent strong reduction in the specific heat of ions (see Althaus et al. 2010b, for details).

All our hydrogen-deficient white dwarf sequences experience carbon enrichment in the outer layers as a result of convec-

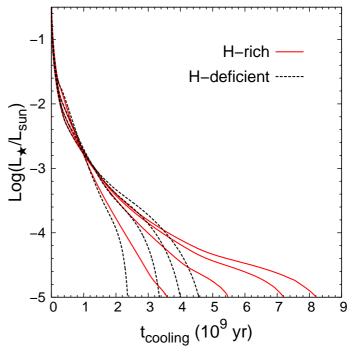


Fig. 5. Solid (dashed) lines display the cooling times for our hydrogen-rich (deficient) white dwarf sequences. At low luminosities and from left to right, stellar masses of both set of sequences are $1.29M_{\odot}, 1.23M_{\odot}, 1.16M_{\odot}$ and $1.10M_{\odot}$.

tive mixing. The outer convective zone grows inwards and when the luminosity of the star has decreased to $\log(L_{\star}/L_{\odot}) \sim -2.5$, it penetrates into deeper layers where heavy elements such as carbon and oxygen are abundant. Consequently, convective mixing dredges up these heavy elements, and the surface chemical composition changes. In particular, the surface layers are predominantly enriched in carbon. These results are in line with the predictions of Camisassa et al. (2017) for hydrogen-deficient white dwarfs of intermediate mass.

The evolution of our ultra-massive white dwarf sequences in the plane $log(g) - T_{eff}$ is depicted in Fig. 6 together with observational expectations taken from Mukadam et al. (2004); Nitta et al. (2016); Gianninas et al. (2011); Kleinman et al. (2013); Bours et al. (2015); Kepler et al. (2016); Curd et al. (2017). In addition, isochrones of 0.1, 0.5, 1, 2, and 5 Gyr connecting the curves are shown. For these white dwarfs, we estimate from our sequences the stellar mass and cooling age (we elect those for which their surface gravities are larger than 8.8). Results are shown in Table 2. Note that for most of the observed white dwarfs, the resulting cooling age is in the range 1 - 4 Gyr, and many of them have stellar masses above $1.25M_{\odot}$. Note also from Fig. 6 the change of slope of the isochrones, reflecting the well known dependence of cooling times on the mass of the white dwarf, i.e, at early stages, evolution proceeds slower in more massive white dwarfs, while the opposite trend is found at advanced stages.

In Fig. 7 we display our hydrogen-rich sequences in the plane $\log(g)-T_{\rm eff}$ together with observational expectations for pulsating massive white dwarfs taken from Mukadam et al. (2004); Hermes et al. (2013); Curd et al. (2017); Nitta et al. (2016). Also with solid lines we show the 0, 20, 40 ,60, 80, 90, 95 and 99 % of the crystallized mass of the star. Note that all of the observed pulsating white dwarfs with masses larger than $1.1 M_{\odot}$ fall in the region where more than 80% of their mass is expected to be crystallized. It is expected, as we will discuss in a forth-

Table 1. Cooling times of our hydrogen-rich (HR) and hydrogen-deficient (HD) white dwarf sequences at selected luminosities.

$\log(L_{\star}/L_{\odot})$	t (Gyr)							
	1.10(HR)	1.16(HR)	1.23(HR)	1.29 (HR)	1.10 (HD)	1.16 (HD)	1.23 (HD)	1.29 (HD)
-2.0	0.274	0.290	0.356	0.437	0.266	0.289	0.361	0.479
-3.0	1.318	1.310	1.320	1.185	1.367	1.354	1.325	1.173
-3.5	2.236	2.173	2.043	1.692	2.457	2.268	2.010	1.590
-4.0	3.625	3.427	2.999	2.265	3.547	3.217	2.793	2.048
-4.5	6.203	5.390	4.132	2.876	4.209	3.739	3.171	2.273
-5.0	8.225	7.213	5.467	3.594	4.580	3.996	3.346	2.362

Table 2. Stellar mass (solar mass) and cooling ages (Gyr) as predicted by our sequences under the assumption that they harbour O-Ne cores for selected ultra-massive white dwarfs in the literature. The letter "V" (variable) indicates that the star is a ZZ Ceti star. The last column gives the references from which the T_{eff} and $\log g$ values have been extracted.

SDSS 1909349,46+134507.87 DA	Star	Spectral Type	$\log(g)(\cos)$	$T_{\rm eff}({ m K})$	M_{\star}/M_{\odot}	t(Gyr)	Reference
SDSS J002113.16-192433.62 DA	SDSS J 090549.46+134507.87	DA	8.875	6774	1.110	3.966	Kepler et al. (2016)
DA 9,250 10,351 1,344 1,305 1,345	SDSS J 000901.20+202606.80	DA	8.857	11081	1.104	1.706	"
SDSS J 0030042-25-16-20208-66 DA 9.080 14593 1.197 1.244 " SDSS J 013853.194-283207.13 DA 9.402 9385 1.288 2.305 " SDSS J 013853.194-283207.13 DA 8.959 11768 1.153 1.652 " SDSS J 014859.15+253616.37 DA 8.812 10051 1.081 1.982 " SDSS J 001489.15+253616.37 DA 8.812 10051 1.081 1.982 " SDSS J 001489.15+253616.37 DA 8.885 9388 1.116 2.322 " SDSS J 004806.14+254703.56 DA 9.170 10976 1.226 2.069 " SDSS J 0015122.96+2241801.15 DA 9.170 10976 1.226 2.069 " SDSS J 224517.61+255043.70 DA 8.990 11570 1.165 1.734 " SDSS J 2224517.61+255043.70 DA 8.991 11570 1.165 1.734 " SDSS J 1222720.65+240601.31 DA 8.947 9921 1.146 2.190 " SDSS J 322257.27+252807.42 DA 8.882 6190 1.113 4.581 " SDSS J 1306401.31 DA 9.470 11694 1.307 1.828 " SDSS J 11544.64+2.94249.50 DA 9.970 11694 1.307 1.828 " SDSS J 111544.64+2.94249.50 DA 9.136 8837 1.214 2.770 " SDSS J 110927.047+285746.16 DA 9.053 8874 1.186 2.713 " SDSS J 110997.08+484805.90 DA 9.161 6639 1.222 3.893 " SDSS J 130846.79+424119.60 DA 9.731 12582 1.243 1.691 " SDSS J 13087.679+42419.60 DA 9.231 12582 1.243 1.691 " SDSS J 110970.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.287 1.282 1.243 1.691 " SDSS J 110970.88+484805.90 DA 9.280 1.1845 1.220 1.849 " SDSS J 110970.88+484805.90 DA 9.280 1.1845 1.220 1.849 " SDSS J 110970.88+484805.90 DA 9.280 1.1845 1.220 1.849 " SDSS J 110970.88+484805.90 DA 9.280 1.1845 1.220 1.849 " SDSS J 110970.88+484805.90 DA 9.280 1.1845 1.220 1.849 " SDSS J 1109820.88+484805.90 DA 9.380 1.276 2.359 " SDSS J 1109820.88+484805.90 DA 9.380 1.276 2.359 " SDSS J 1109820.88+484805.90 DA 9.380 1.28	SDSS J 002113.16+192433.62	DA	8.920	11555	1.134	1.655	"
DA	SDSS J 003608.73+180951.52	DA	9.250	10635	1.248	2.121	"
SDSS J 013853; 78324947.71 DA 8,959 11768 1.153 1.652 " SDSS J 001459; 15+253616.37 DA 8,812 10051 1.081 1.982 " SDSS J 010459; 15+253616.37 DA 8,812 10051 1.081 1.982 " SDSS J 010459; 15+253616.37 DA 8,812 10051 1.081 1.982 " SDSS J 003460; 14+254703.56 DA 8,885 9388 1.116 2.322 " SDSS J 005122; 96+241801.15 DA 9,170 10976 1.226 2.069 " SDSS J 005122; 96+241801.15 DA 9,170 11076 1.226 2.069 " SDSS J 224517.61+255043.70 DA 8,990 11570 1.165 1.734 " SDSS J 22257; 27+252807,42 DA 8,882 6190 1.113 4.581 " SDSS J 164642; 67+483207.96 DA 8,999 15324 1.169 1.042 " SDSS J 110054,91+230604.01 DA 9,470 11694 1.307 1.828 " SDSS J 110054,91+230604.01 DA 9,470 11694 1.307 1.828 " SDSS J 11054,042+2494.90 DA 9,136 8837 1.214 2.770 " SDSS J 1107204,7+285746.16 DA 9,053 8874 1.186 2.713 " SDSS J 1109404, 29+302102.03 DA 9,161 6639 1.222 3.893 " SDSS J 1109407,08+484805.90 DA 9,267 6369 1.222 3.893 " SDSS J 1109407,08+484805.90 DA 9,231 12582 1.243 1.691 " SDSS J 1109507,08+484805.90 DA 9,231 12582 1.243 1.691 " SDSS J 115010,11+474804.08 DA 9,240 16889 1.246 1.083 " SDSS J 115010,17+474804.08 DA 9,089 9538 1.198 2.460 " SDSS J 115009,87+510705.21 DA 9,180 11845 1.229 1.849 " SDSS J 115009,87+510705.21 DA 9,180 11845 1.229 1.849 " SDSS J 115208,52+551939.16 DAH 9,098 17136 1.204 0.939 " SDSS J 104092,03-087451 DA 9,180 11845 1.229 1.849 " SDSS J 105553,68+622929.41 DAH 9,086 17136 1.204 0.939 " SDSS J 1102553,68+622929.41 DAH 9,356 9380 1.276 2.359 " SDSS J 1103253,68+622929.41 DAH 9,356 9380 1.276 2.359 " SDSS J 110326,47+263952,68 DA 8,829 9680 1.090 2.134 " SDSS J 11036,4827,74+563952,68 DA 8,829 9680 1.090 2.134 " SDSS J 100363,665949,42 DA 9,383 22251 1.286 0.760 " SDSS J 100351,118-20012,197 DA 9,303 11648 1.229 1.849 " SDSS J 100369,74-602940,06 DA 8,845 1.3611 1.099 1.121 " SDSS J 100369,74-602940,06 DA 8,845 1.3611 1.099 1.121 " SDSS J 103600,74-75,75-75,75-75 DA 9,063 1.0088 1.257 " SDSS J 103600,74-75,75-75 DA 9,063 1.0088 1.257 " SDSS J 103600,75-765008.4 DQ 9,230 10112 1.236 2.299 (Kleinman et al. (2017) SDSS J 1	SDSS J 005142.50+200208.66	DA	9.080	14593	1.197	1.244	"
SDSS J 013423,76+26494,71 DA 8,939 11768 1.135 1.02 SDSS J 004806,14+254703,56 DA 8,885 9388 1.116 2.322 " SDSS J 004806,14+254703,56 DA 8,885 9388 1.116 2.322 " SDSS J 005122,96+241801.15 DA 9,170 10976 1.226 2.069 " SDSS J 222720,65+240601.31 DA 8,947 9921 1.146 2.190 " SDSS J 2225727,2552807.42 DA 8,882 6190 1.113 4.581 " SDSS J 1064642,67+483207.96 DA 8,999 15324 1.169 1.042 " SDSS J 110054,91+230604.01 DA 9,470 11694 1.307 1.828 " SDSS J 110054,91+230604.01 DA 9,136 8837 1.214 2.770 " SDSS J 1102720,47+285746.16 DA 9,053 8874 1.186 2.713 " SDSS J 110944,29+302102.03 DA 9,161 6639 1.222 3.893 " SDSS J 110944,29+302102.03 DA 9,161 6639 1.222 3.893 " SDSS J 110947,8484805,90 DA 9,231 12582 1.243 1.691 " SDSS J 1109107,084,484805,90 DA 9,231 12582 1.243 1.691 " SDSS J 110510,71+474804.08 DA 9,089 9538 1.198 2.460 " SDSS J 110510,71+474804.08 DA 9,089 9538 1.198 2.460 " SDSS J 132208,52+551939.16 DA 9,180 1.1845 1.229 " SDSS J 103208,52+551939.16 DA 9,180 1.1845 1.229 " SDSS J 1032503,03-042354.59 DA 9,180 1.1845 1.220 1.659 " SDSS J 1032503,03-042354.59 DA 9,180 1.1845 1.220 1.659 " SDSS J 112322,47+602940.06 DA 8,845 13611 1.099 1.121 " SDSS J 112322,47+602940.06 DA 8,845 13611 1.099 1.121 " SDSS J 103351,41,812,00121.97 DA 9,330 1.276 2.359 " SDSS J 103351,41,812,00121.97 DA 9,330 1.154 8.829 9.601 1.150 2.234 " SDSS J 103652,69+29122,48 DA 9,003 1.0088 1.276 2.359 " SDSS J 103652,69+29122,48 DA 9,003 1.0088 1.270 2.395 Kleinman et al. (2013) SDSS J 1003652,69+29122,48 DA 9,000 1.230 1.196 1.779 SDSS J 100515,11+661033.3)(V) DA 9,080 12030 1.196 1.779 Curd et al. (2017)	SDSS J013853.19+283207.13	DA	9.402	9385	1.288	2.305	"
SDSS 004806,14+254703,56	SDSS J 015425.78+284947.71	DA	8.959	11768	1.153	1.652	"
SDSS J005122,96+241801.15 DA 9,170 10976 1,226 2,069 " SDSS J22457.61+255043,70 DA 8,990 11570 1,165 1,734 " SDSS J222720,65+240601.31 DA 8,947 9921 1,146 2,190 " SDSS J232257,27+252807,42 DA 8,882 6190 1,113 4,581 " SDSS J164642,67+483207,96 DA 8,999 15324 1,169 1,042 " SDSS J110054,91+230604.01 DA 9,470 11694 1,307 1,828 " SDSS J110544,64+294249,50 DA 9,136 8837 1,214 2,770 " SDSS J101720,47+285746,16 DA 9,053 8837 1,186 2,713 " SDSS J10944,29+302102.03 DA 9,161 6639 1,222 3,893 " SDSS J109944,29+302102.03 DA 9,161 6639 1,222 3,893 " SDSS J1091097,08+484805,90 DA 8,970 7237 1,156 3,668 " SDSS J110970,17+474804,08 DA 9,240 16889 1,246 1,083 " SDSS J110510,71+474804,08 DA 9,089 9538 1,198 2,460 " SDSS J1150417,23+553900,45 DO 9,267 6360 1,244 2,929 " SDSS J132208,52+551939,16 DAH 9,098 17136 1,204 0,939 " SDSS J103208,52+551939,16 DAH 9,098 17136 1,204 0,939 " SDSS J103550,03-042354,59 DA 9,150 12651 1,220 1,659 " SDSS J104827,74+563952,68 DA 8,887 7516 1,116 3,367 " SDSS J103550,03-042354,59 DA 9,150 12651 1,220 1,659 " SDSS J103550,03-042354,59 DA 9,150 12651 1,220 1,659 " SDSS J103550,03-042354,59 DA 9,150 12651 1,220 1,659 " SDSS J103550,03-042354,59 DA 9,356 9380 1,276 2,359 " SDSS J103550,03-042354,59 DA 9,350 1,251 1,258 1,259 1,849 " SDSS J103550,03-042354,59 DA 9,350 1,251 1,200 1,659 " SDSS J103550,03-042354,59 DA 9,350 1,251 1,200 1,659 " SDSS J103550,04-0425,66 DA 8,845 13611 1,099 1,121 " SDSS J10350,03-042354,59 DA 9,350 1,251 1,220 1,659 " SDSS J103550,04-0425,66 DA 8,845 13611 1,099 1,121 " SDSS J103514,18+200121,97 DA 9,363 10,983 1,246 1,242 3,957 " SDSS J03514,400260,60 DA 8,969 703	SDSS J 001459.15+253616.37	DA	8.812	10051	1.081	1.982	"
SDSS 1224517,61+255043,70 DA	SDSS J 004806.14+254703.56	DA	8.885	9388	1.116	2.322	"
DA	SDSS J 005122.96+241801.15	DA	9.170	10976	1.226	2.069	"
SDSS J 222257.27+252807.42 DA 8.882 6190 1.113 4.581 " SDSS J 164642.67+483207.96 DA 8.999 15324 1.169 1.042 " SDSS J 110054.91+230604.01 DA 9.470 11694 1.307 1.828 " SDSS J 110054.91+230604.01 DA 9.470 11694 1.307 1.828 " SDSS J 110074.94+294249.50 DA 9.136 8837 1.214 2.770 " SDSS J 1102720.47+285746.16 DA 9.053 8874 1.186 2.713 " SDSS J 100944.29+302102.03 DA 9.161 6639 1.222 3.893 " SDSS J 100944.29+302102.03 DA 9.161 6639 1.222 3.893 " SDSS J 100944.29+302102.03 DA 9.161 6639 1.222 3.893 " SDSS J 1010907.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 112943.28+493451.45 DA 9.240 16889 1.246 1.083 " SDSS J 112943.28+493451.45 DA 9.240 16889 1.246 1.083 " SDSS J 110510.71+474804.08 DA 9.089 9538 1.198 2.460 " SDSS J 1150417.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 145009.87+510705.21 DA 9.180 11845 1.229 1.849 " SDSS J 132208.52+551939.16 DAH 9.098 17136 1.204 0.939 " SDSS J 1004825.11+350527.94 DA 8.887 7516 1.116 3.367 " SDSS J 100350.30-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 102553.68+6622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 102553.68+6622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 110320.3-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 110350.30-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 110350.30-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 110350.30-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 110350.30-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 103550.08-662949.42 DA 9.383 22251 1.286 0.760 " SDSS J 110350.93+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 110350.93+665949.42 DA 9.383 1.91 1.286 0.760 " SDSS J 112322.47+602940.06 DA 8.845 3611 1.099 1.121 " SDSS J 1035514.18+200121.97 DA 9.370 17134 1.281 1.130 " SDSS J 103554-890121.97 DA 9.370 17134 1.281 1.130 " SDSS J 103554-890121.97 DA 9.063 10088 1.190 2.234 " SDSS J 234044.83+091625.96 DA 9.230 10112 1.236 2.299 KIeinman et al. (2013) SDSS J 1000052.44-002610.5 DQ 9.320 10112 1.236 2.299 KIeinman et al. (2013) SDSS J 1000052.44-002610.5 DQ 9.320 10088 1.257 2.192 Gianninas et al. (2011) SDSS J 1000052.44-002610.5 DQ 9.	SDSS J 224517.61+255043.70	DA	8.990	11570	1.165	1.734	"
SDSS J 164642.67+483207.96 DA 8.999 15324 1.169 1.042 " SDSS J 110054.91+230604.01 DA 9.470 11694 1.307 1.828 " SDSS J 1101544.64+294249.50 DA 9.136 8837 1.214 2.770 " SDSS J 102720.47+285746.16 DA 9.053 8874 1.186 2.713 " SDSS J 100944.29+302102.03 DA 9.161 6639 1.222 3.893 " SDSS J 130846.79+424119.60 DA 8.970 7237 1.156 3.668 " SDSS J 101907.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 122943.28+493451.45 DA 9.240 16889 1.246 1.083 " SDSS J 110510.71+474804.08 DA 9.089 9538 1.198 2.460 " SDSS J 150417.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 145009.87+510705.21 DA 9.180 11845 1.229 1.849 " SDSS J 132208.52+55193.916 DAH 9.098 17136 1.204 0.939 " SDSS J 1004825.11+350527.94 DA 8.887 7516 1.116 3.367 " SDSS J 104827.74+563952.68 DA 9.150 12651 1.220 1.659 " SDSS J 102553.68+622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 104827.74+563952.68 DA 8.885 3611 1.099 1.121 " SDSS J 10036.93+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 10036.93+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 103514.18+200121.97 DA 9.403 17134 1.281 1.130 " SDSS J 1035251.28+511935.12 DA 8.969 7030 1.155 3.827 " SDSS J 1035251.28+511935.12 DA 8.969 7030 1.155 3.827 " SDSS J 1035251.08+154751.27 DA 9.063 10083 1.190 2.234 " SDSS J 234044.83+091625.96 DA 9.234 6166 1.242 3.957 " SDSS J 1030525.09+291229.48 DA 9.070 10284 1.192 2.182 " SDSS J 1030525.09+291229.48 DA 9.070 10284 1.192 2.182 " SDSS J 1030525.09+291229.48 DA 9.070 10284 1.192 2.182 " SDSS J 100052.44+002610.5 DQ 9.320 10088 1.257 2.192 " Gianninas et al. (2013) SDSS J 100052.44+002610.5 DQ 9.320 10088 1.257 2.192 Gianninas et	SDSS J 222720.65+240601.31	DA	8.947	9921	1.146	2.190	"
SDSS J 110054.91+230604.01 DA 9.470 11694 1.307 1.828 " SDSS J 110544.64+294249.50 DA 9.136 8837 1.214 2.770 " SDSS J 102720.47+285746.16 DA 9.053 8874 1.186 2.713 " SDSS J 100942.93-20102.03 DA 9.161 6639 1.222 3.893 " SDSS J 100944.29+302102.03 DA 9.161 6639 1.222 3.893 " SDSS J 100947.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 101907.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 1101907.08+484805.90 DA 9.240 16889 1.246 1.083 " SDSS J 110510.71+474804.08 DA 9.089 9538 1.198 2.460 " SDSS J 110510.71+474804.08 DA 9.180 11845 1.229 " SDSS J 150417.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 150417.23+55390.45 DA 9.180 11845 1.229 1.849 " SDSS J 1032208.52+551939.16 DAH 9.098 17136 1.204 0.939 " SDSS J 1032208.52+551939.16 DAH 9.098 17136 1.204 0.939 " SDSS J 103550.03-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 103555.08+622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 104827.74+563952.68 DA 8.829 9680 1.090 2.134 " SDSS J 110303.693+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 110303.93+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 104827.74+563952.68 DA 8.845 13611 1.099 1.121 " SDSS J 104827.74+563952.68 DA 9.403 11648 1.289 1.849 " SDSS J 104827.74+563952.68 DA 8.829 9680 1.090 2.134 " SDSS J 104827.74+563952.68 DA 8.829 9680 1.090 2.134 " SDSS J 104827.74+563952.69 DA 9.383 22251 1.286 0.760 " SDSS J 104827.74+563952.69 DA 9.383 22251 1.286 0.760 " SDSS J 104827.74-563952.69 DA 9.370 17134 1.281 1.130 " SDSS J 234942.80+154751.27 DA 9.063 10083 1.190 2.234 " SDSS J 234929.60+185119.52 DA 8.969 7030 1.155 3.827 " SDSS J 234929.60+185119.52 DA 8.963 10083 1.190 2.234 " SDSS J 232512.08+154751.27 DA 9.063 10083 1.190 2.234 " SDSS J 23000015.57-085008.4 DQ 9.230 10112 1.236 2.299 Kleinman et al. (2013) SDSS J 0000052.44-002610.5 DQ 9.320 10088 1.257 2.192 Gianninas et al. (2013) SDSS J 0000052.44-002610.5 DQ 9.320 10088 1.257 2.192 Gianninas et al. (2011) GDS18 (WD J 165915.11+661033.3) (V) DA 9.000 12030 1.196 1.719 " SDSS J 072724.66+403622.0 DA 9.010 1.2350 1.172 1.573 Curd et al. (2017)	SDSS J 232257.27+252807.42	DA	8.882	6190	1.113	4.581	"
SDSS J 11054.4.64-294249.50 DA 9.136 8837 1.214 2.770 " SDSS J 102720.47+285746.16 DA 9.053 8874 1.186 2.713 " SDSS J 10094.294302102.03 DA 9.161 6639 1.222 3.893 " SDSS J 130846.79+424119.60 DA 8.970 7237 1.156 3.668 " SDSS J 130846.79+424119.60 DA 9.231 12582 1.243 1.691 " SDSS J 130846.79+424119.60 DA 9.231 12582 1.243 1.691 " SDSS J 120943.28+493451.45 DA 9.240 16889 1.246 1.083 " SDSS J 150417.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 150417.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 145009.87+510705.21 DA 9.180 11845 1.229 1.849 " SDSS J 132208.52+551939.16 DAH 9.088 17136 1.204 0.939 " SDSS J 104825.11+350527.94 DA 8.887 7516 1.116 3.367 " SDSS J 104825.11+350527.94 DA 9.150 12651 1.220 1.659 " SDSS J 104825.368+622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 104827.74+563952.68 DA 8.829 9680 1.090 2.134 " SDSS J 112322.47+602940.06 DA 8.845 13611 1.099 1.121 " SDSS J 110036.93+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 1004920.03-080141.71 DA 9.403 11648 1.288 1.289 1.849 " SDSS J 103514.18+200121.97 DA 9.370 17134 1.281 1.130 " SDSS J 003710.25+511935.12 DA 8.969 7030 1.155 3.827 " SDSS J 03710.25+511935.12 DA 8.969 7030 1.155 3.827 " SDSS J 037312.41-8119.52 DA 8.969 7030 1.155 3.827 " SDSS J 037312.41-8119.52 DA 8.969 7030 1.155 3.827 " SDSS J 03652.69+291229.48 DA 9.063 10088 1.257 2.192 Kleinman et al. (2013) SDSS J 000052.44-002610.5 DQ 9.320 10088 1.257 2.192 Kleinman et al. (2013) SDSS J 000052.44-002610.5 DQ 9.320 10088 1.257 2.192 Kleinman et al. (2013) SDSS J 000052.44-002610.5 DQ 9.320 10088 1.257 2.192 Kleinman et al. (2011) GDSS J 072724.66+403622.0 DA 9.000 1.2535 1.172 1.573 Curd et al. (2017)	SDSS J 164642.67+483207.96	DA	8.999	15324	1.169	1.042	"
SDSS J 102720,47+285746.16 DA	SDSS J 110054.91+230604.01	DA	9.470	11694	1.307	1.828	"
SDSS J 100944.29+30/2102.03 DA 9.161 6639 1.222 3.893 " SDSS J 101907.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 101907.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 1122943.28+493451.45 DA 9.240 16889 1.246 1.083 " SDSS J 11251.114474804.08 DA 9.089 9538 1.198 2.460 " SDSS J 11251.1147.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 145009.87+510705.21 DA 9.180 11845 1.229 1.849 " SDSS J 132208.52+551939.16 DAH 9.098 17136 1.204 0.939 " SDSS J 104825.11+350527.94 DA 8.887 7516 1.116 3.367 " SDSS J 103550.03-042354.59 DA 9.150 12651 1.220 1.659 " SDSS J 102533.68+622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 112322.47+602940.06 DA 8.845 13611 1.099 1.121 " SDSS J 11036.93+665949.42 DA 9.383 22251 1.286 0.760 " SDSS J 004920.03-080141.71 DA 9.403 11648 1.289 1.849 " SDSS J 003551.41.8+200121.97 DA 9.370 17134 1.281 1.130 " SDSS J 093710.25+511935.12 DA 8.969 7030 1.155 3.827 " SDSS J 2349429.60+185119.52 DA 8.935 6966 1.139 3.848 " SDSS J 234944.83+091625.96 DA 9.234 6166 1.242 3.957 " SDSS J 234944.83+091625.96 DA 9.234 6166 1.242 3.957 " SDSS J 2000052.44-002610.5 DQ 9.230 10112 1.236 2.299 Kleinman et al. (2013) SDSS J 000052.44-002610.5 DQ 9.320 10088 1.257 2.192 " SDSS J 000551.11+661033.3) (V) DA 9.080 12030 1.196 1.719 " SDSS J 072724.664-403622.0 DA 9.080 12030 1.196 1.719 " SDSS J 072724.664-403622.0 DA 9.080 12030 1.196 1.719 " SDSS J 072724.664-403622.0 DA 9.010 12350 1.172 1.573 Curd et al. (2017)	SDSS J 111544.64+294249.50	DA	9.136	8837	1.214	2.770	"
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SDSS J 130846.79+424119.60 DA 8.970 7237 1.156 3.668 " SDSS J 101907.08+484805.90 DA 9.231 12582 1.243 1.691 " SDSS J 122943.28+493451.45 DA 9.240 16889 1.246 1.083 " SDSS J 110510.71+474804.08 DA 9.089 9538 1.198 2.460 " SDSS J 150417.23+553900.45 DO 9.267 6360 1.244 2.929 " SDSS J 145009.87+510705.21 DA 9.180 11845 1.229 1.849 " SDSS J 1004825.11+350527.94 DA 8.887 7516 1.116 3.367 " SDSS J 102553.68+622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 102553.68+622929.41 DAH 9.356 9380 1.276 2.359 " SDSS J 102362.68-bds DA 8.829 9680 1.090 2.134 " SDSS J 102362.2.47+602940.06 DA 8.845 1361	SDSS J 100944.29+302102.03						"
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							Curd et al. (2017)
SDSS J 084021, 23+522217, 4 (V) DA 8.930 12160 1.139 1.523 "	SDSS J 084021.23+522217.4 (V)	DA	8.930	12160	1.139	1.523	"
SDSS J 165538.93+253346.0 DA 9.200 11060 1.234 2.035 "							"
SDSS J 005047.61–002517.1 DA 8.980 11490 1.162 1.744 Mukadam et al. (2004)							Mukadam et al. (2004)
BPM 37093 (LTT 4816) (V) DA 8.843 11370 1.097 1.608 Nitta et al. (2016)							

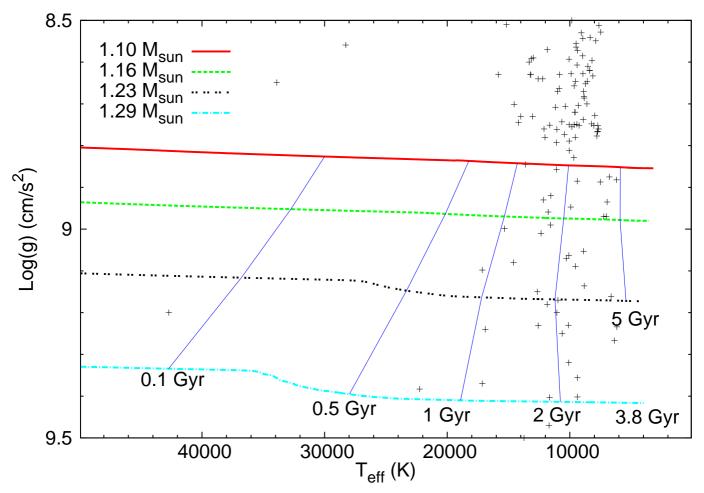


Fig. 6. The hydrogen-rich sequences in the plane $\log(g) - T_{\text{eff}}$. Blue solid lines display isochrones of 0.1, 0.5, 1, 2, and 5 Gyr. Crosses indicate the location of ultra-massive white dwarfs from Mukadam et al. (2004); Nitta et al. (2016); Gianninas et al. (2011); Kleinman et al. (2013); Bours et al. (2015); Kepler et al. (2016); Curd et al. (2017).

Table 3. Percentages of crystallized mass of our hydrogen-rich sequences and effective temperature at which they occur.

Crystallized mass	$\log(T_{\mathrm{eff}})$ (K)				
	$1.10M_{\odot}$	1.16 <i>M</i> _⊙	$1.23M_{\odot}$	$1.29M_{\odot}$	
0%	4.31	4.38	4.46	4.58	
20%	4.26	4.32	4.41	4.54	
40%	4.22	4.29	4.38	4.51	
60%	4.17	4.23	4.34	4.46	
80%	4.09	4.16	4.26	4.39	
90%	4.03	4.10	4.20	4.33	
95%	3.91	3.95	4.10	4.25	
99%	3.77	3.83	4.02	4.10	

coming paper, that crystallization process affects the pulsation properties of massive ZZ Ceti stars, as it has also been shown by Montgomery & Winget (1999); Córsico et al. (2004, 2005); Brassard & Fontaine (2005).

The effective temperature at various percentage of crystallized mass is also listed in Table 3. Note that, at the onset of crystallization, the highest mass sequences exhibit a marked increase in their surface gravities. This behavior is a consequence of the change in the chemical abundances of ¹⁶O and ²⁰Ne during the crystallization. As the abundance of ²⁰Ne grows in the inner regions of the white dwarf, its radius decreases, and consequently its surface gravity increases. Crystallization sets in at similar luminosities and effective temperatures in a hydrogen-deficient as in a hydrogen-rich white dwarf with the same mass. Hydrogen-deficient cooling sequences are not shown in this Figure since they exhibit a similar behavior but their surface gravities are slightly larger, since their radius are relatively smaller.

Element diffusion profoundly alters the inner abundance distribution from the early cooling stages of our massive white dwarf models. This is borne out by Figs. 8 and 9, which display the abundance distribution in the whole star at three selected effective temperatures for the 1.10 and $1.29M_{\odot}$ hydrogen-rich white dwarf models, respectively. Note that, as a result of gravitational settling, all heavy elements are depleted from the outer layers. Note also that that initial chemical discontinuities are strongly smoothed out. But more importantly, the initial helium and carbon distribution in the deep envelope result markedly changed, particularly in the most massive models, where the initial pure helium buffer has almost vanished when evolution has reached low effective temperatures. This is quite different from the situation encountered in white dwarfs of intermediate mass. These changes in the helium and carbon profiles affects the radiative opacity in the envelope and thus the cooling times at late stages.

The other physical process that changes the core chemical distribution during white dwarf evolution is, as we mentioned, phase separation during crystallization. The imprints of phase

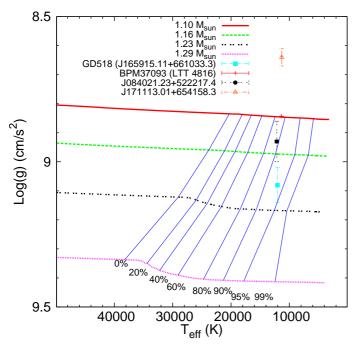


Fig. 7. The hydrogen-rich sequences in the plane $\log(g) - T_{\rm eff}$. Blue solid lines indicate 0, 20, 40, 60, 80, 90, 95 and 99% of crystallized mass. The symbols with error bars indicate the location of the known pulsating ultra-massive DA white dwarfs (Mukadam et al. 2004; Hermes et al. 2013; Curd et al. 2017; Nitta et al. 2016).

separation on the core chemical composition can be appreciated in the bottom panels of Fig. 8 and 9, and more clearly in Fig. 10, which illustrates the change in the abundances of ²⁰Ne and $^{16}\mathrm{O}$ in a $1.16M_{\odot}$ model shortly after the occurrence of crystallization (top panel) and by the time a large portion of the star has crystallized (bottom panel). The chemical abundances of ²⁰Ne and ¹⁶O right before the crystallization sets in, are plotted with thick dashed lines. For this stellar mass, crystallization starts at the center of the star at $\log(L_{\star}/L_{\odot}) \sim -1.8$. Note that, in the top panel, (the crystallization front is at $\log(M_r/M_{\star}) \sim -0.4$) the initial ²⁰Ne and ¹⁶O abundances have strongly been changed by the process of phase separation and the induced mixing in the fluid layers above the core, which extends upwards to $\log(M_r/M_{\star}) \sim$ -1. Other elements apart from ¹⁶O and ²²Ne are not taken into account in the phase separation process, and the slight change shown in their abundances is due only to element diffusion.

To properly assess the phase separation process during crystallization, it should be necessary to consider a 5-component crystallizing plasma composed in our case by ¹²C , ¹⁶O, ²⁰Ne, ²³Na and ²⁴Mg, which are the most abundant elements in the white dwarf core (see Figure 3). Such 5-component phase diagram is not available in the literature. However, Prof. A. Cumming has provided us with the final abundances in the solid phase in the center of the $1.10\,\mathrm{M}_\odot$ white dwarf model, considering a given 5-component composition ⁴. The abundances of ¹²C, ¹⁶O, ²⁰Ne, ²³Na and ²⁴Mg at the center of this model right before crystallization occurs are listed in Table 4, together with the final abundances in the solid phase predicted by the 5-component calculations, and those predicted by the phase diagram for a ¹⁶O-²⁰Ne mixture shown in Figure 1. The abundances of ¹⁶O and ²⁰Ne are noticeably altered by crystallization regardless of the treatment considered. However, considering a 2-component phase diagram results in a stronger phase separation of ¹⁶O and

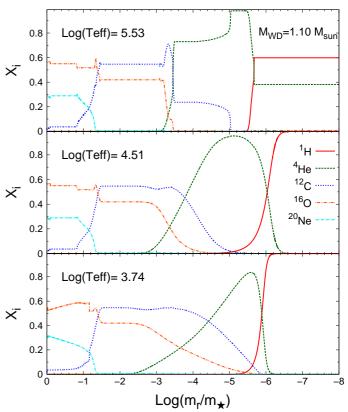


Fig. 8. Inner abundance distribution for $1.10M_{\odot}$ hydrogen-rich models at three selected effective temperatures, as indicated.

Table 4. Abundances at the center of the $1.10 \, M_{\odot}$ white dwarf model before crystallization, and the final abundances in the solid phase resulting of considering a 5-component mixture of 12 C , 16 O, 20 Ne, 23 Na and 24 Mg, and the 2-component 16 O- 20 Ne phase diagram shown in Figure 1.

	Initial	Solid 5-component	Solid 2-component
¹² C	0.0167	0.0082	0.0167
^{16}O	0.5624	0.5561	0.5450
20 Ne	0.2921	0.3289	0.3311
²³ Na	0.0538	0.0579	0.0538
24 Mg	0.0513	0.0489	0.0513

²⁰Ne. Nevertheless, in this treatment the abundances of trace elements ¹²C, ²³Na and ²⁴Mg are not altered by the crystallization process. The sum of the abundances of these trace elements is lower than 15% in the core of all our ultra-massive white dwarf models and we do not expect this to alter substantially the evolutionary timescales. To properly assess the effects of considering a 5-component phase diagram on the cooling times of white dwarfs it should be necessary calculate the evolution of the white dwarf model through the entire crystallization process, for which we would require the full phase diagram, not available at the moment of this study.

The phase separation process of 20 Ne and 16 O releases appreciable energy, see Fig. 4, so as to impact the white dwarf cooling times. This can be seen in Fig. 11, which shows the cooling times for our $1.22M_{\odot}$ hydrogen-rich sequence (upper panel) when crystallization is neglected (double-dotted line), when only latent heat is considered during crystallization (dotted line), and when both latent heat and energy from phase separation are considered during crystallization (solid line). Clearly, the energy resulting from crystallization, in particular the release

⁴ A. Cumming, personal communication.

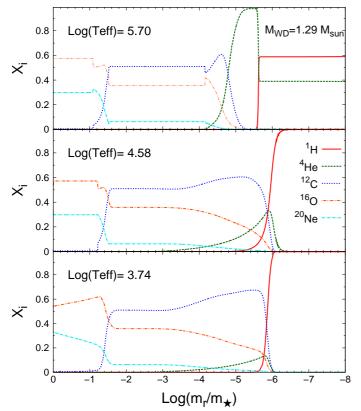


Fig. 9. Same as Fig. 8 but for $1.29M_{\odot}$ models.

of latent heat, increases substantially the cooling times of the ultra-massive white dwarfs. The inclusion of energy from phase separation leads to an additional delay on the cooling times (admittedly less than the delay caused by latent heat) at intermediate luminosities. But below $\log(L_{\star}/L_{\odot}) \sim -3.6$, when most of the star has crystallized, phase separation accelerates the cooling times. At these stages, no more energy is delivered by phase separation, but the changes in the chemical profile induced by phase separation have strongly altered both the structure and thermal properties of the cool white dwarfs, impacting their rate of cooling. Note in this sense, the change in the radius of the white dwarf that results from the inclusion of phase separation (bottom panel of Fig. 11). In fact, the star radius becomes smaller due to the increase of neon in the core during crystallization. As we mentioned, this explains the increase of the surface gravity of our sequences in the case of phase separation is considered, see Figs. 6 and 7

The present evolutionary sequences of ultra-massive white dwarfs constitute an improvement over those presented in A07. The comparison between the evolutionary sequences of both studies is presented in Fig. 12 for the $1.10 M_{\odot}$, $1.16 M_{\odot}$ and $1.23 M_{\odot}$ hydrogen-rich sequences. Note that appreciable differences in the cooling times exist between both set of sequences. In particular, the present calculations predict shorter ages at intermediate luminosities, but this trend is reversed at very low surface luminosities, where our new sequences evolve markedly slower than in A07.

To close the paper, we attempt to trace back the origin of such differences. We begin by examining the impact of the new chemical profiles, as compared with that used in A07 (as illustrated in Figure 4 of Córsico et al. (2004)) which is the same used for all white dwarf sequences in A07. To this end, we have computed two artificial white dwarf sequences by neglecting phase sep-

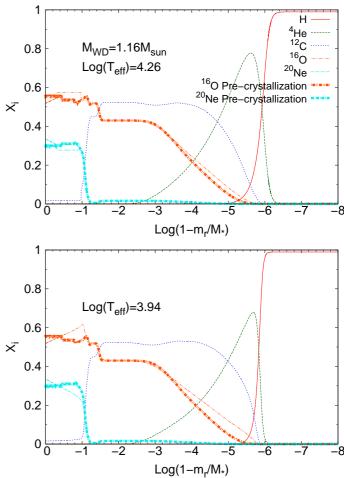


Fig. 10. Change in the chemical profiles of our $1.16M_{\odot}$ hydrogenrich white dwarf model induced by the phase separation process during crystallization. Top (bottom) panel depicts the chemical profile at $\log(T_{\rm eff}) = 4.26(3.94)$. For comparison, the abundances of $^{16}{\rm O}$ and $^{20}{\rm Ne}$ right before phase separation are also plotted with thick lines in both panels.

aration during crystallization. Comparison is made in Fig. 13, which shows the cooling times of a $1.16M_{\odot}$ hydrogen-rich white dwarf model resulting from the use of the chemical profile considered in A07 (solid line) and the chemical profile employed in the current study (dotted line). Note that the use of new chemical profiles employed in the present study (Siess 2010) predicts larger cooling times than the use of the chemical profiles of García-Berro et al. (1997) considered in A07. This is due to not only to the different core chemical stratification in both cases but also to the different predictions for the helium buffer mass expected in the white dwarf envelopes, which affects the cooling rate of cool white dwarfs. In this sense, the full computation of evolution of progenitor stars along the thermally pulsing SAGB constitutes an essential aspect that cannot be overlooked in any study of the cooling of massive white dwarfs.

Improvements in the microphysics considered in the computation of our new sequences also impact markedly the cooling times; this is particularly true regarding the treatment of conductive opacities and the release of latent heat during crystallization. Specifically, in the present sequences we make use of the conductive opacity as given in Cassisi et al. (2007), in contrast to A07 where the older conductive opacities of Itoh et al. (1994) were employed. The resulting impact on the cooling time becomes apparent from Fig. 14. Here we compare the cooling

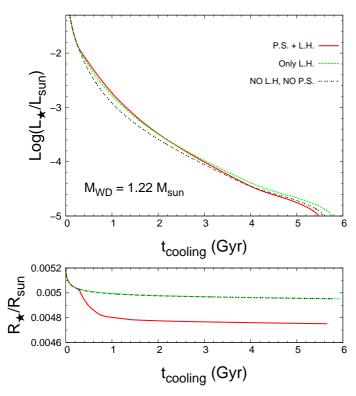


Fig. 11. Top panel: Cooling times of our $1.22M_{\odot}$ hydrogen-rich sequence when crystallization is neglected (double-dotted line), when only latent heat is considered during crystallization (dotted line), and when both latent heat and energy from phase separation are considered during crystallization (solid line). Bottom panel: White dwarf radius in terms of the cooling time for these evolutionary tracks.

times for $1.16M_{\odot}$ white dwarf white dwarf models having the same chemical composition as in A07 but adopting different microphysics. A close inspection of this figure reveals that the improvement in the microphysics considered in our current version of LPCODE as compared with that used in A07, particularly the conductive opacity at intermediate luminosity and the treatment of latent heat during the crystallization phase at lower luminosities, lead to shorter cooling times. Note that when we use the old microphysics (and the same chemical profile) we recover the results of A07.

We conclude from Figs. 13 and 14 that the inclusion of detailed chemical profiles appropriate for massive white dwarfs resulting from SAGB progenitors and improvements in the microphysics results in evolutionary sequences for these white dwarfs much more realistic than those presented in A07. These improvements together with the consideration of the effects of phase separation of ²⁰Ne and ¹⁶O during crystallization yield accurate cooling times for ultra-massive white dwarfs.

Finally, we present our ultra-massive white dwarf cooling tracks in GAIA photometry bands: G, G_{BP} and G_{RP} . These magnitudes have been obtained using detailed model atmospheres for H-composition described in Rohrmann et al. (2012). The cooling tracks are plotted in the color-magnitude diagram in Figure 15, together with the local sample of white dwarfs within 100 pc from our sun of Jiménez-Esteban et al. (2018), in the color range: $-0.52 < (G_{BP} - G_{RP}) < 0.80$. The onset of crystallization in our cooling sequences is indicated with filled squares. Note that crystallization occurs at approximately the same magnitude, $G + 5 + 5log(\pi) \sim 12$. The moment when convective coupling is occurring in each white dwarf sequence is also indicated using filled triangles. Clearly, our ultra-massive white

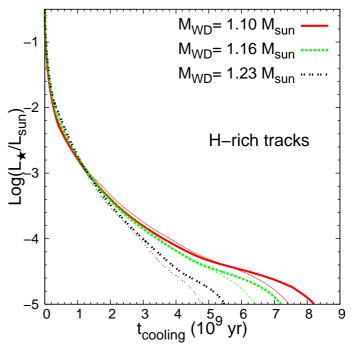


Fig. 12. Cooling times of our hydrogen-rich white dwarf sequences with $1.10M_{\odot}$, $1.16M_{\odot}$ and $1.23M_{\odot}$ (thick lines), as compared with the cooling sequences of A07 of similar masses (thin lines).

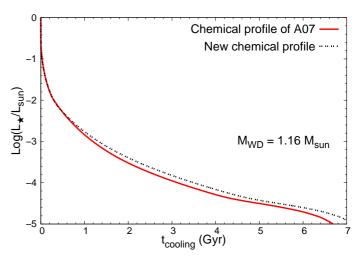


Fig. 13. Cooling times of $1.16M_{\odot}$ hydrogen-rich white dwarf models without phase separation resulting from the use of different chemical profiles. Solid red line corresponds to the cooling sequence using our current stellar evolutionary code but implanting the chemical profile considered in A07. Black dotted line corresponds to the cooling sequence calculated using our new chemical profile (plotted in the top right panel of Fig. 3).

dwarf cooling tracks fall below the vast majority of the white dwarf sample. The reason for this relies on the mass distribution of the white dwarf sample, that exhibits a sharp peak around $0.6\,M_\odot$ (Tremblay et al. 2019). Thus, the vast majority of white dwarfs will be characterized by larger luminosities than the ones present in our ultra-massive white dwarfs. However, a detailed analysis of this color-magnitude diagram is beyond the scope of the present paper and we simply present white dwarf colors for our ultra-massive white dwarfs, which are available for downloading.

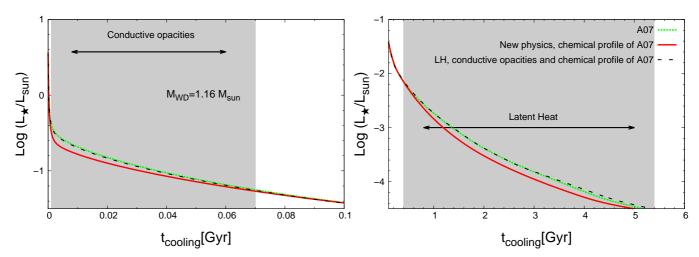


Fig. 14. Cooling times of $1.16M_{\odot}$ hydrogen-rich white dwarf models without phase separation and considering the same chemical profiles of A07, but adopting different microphysics. The cooling sequence of A07 is displayed using a green dotted line, which is based on an old treatment of conductive opacity and latent heat. The red solid line shows the cooling times calculated using our current numerical code. The dashed line shows prediction given by our current numerical code but with old microphysics, this is, the same conductive opacities and treatment of latent heat as considered in A07. The left panel amplifies the early stages of the white dwarf stage. The right panel shows the rest of the white dwarf cooling track.

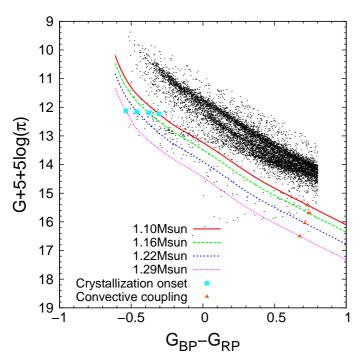


Fig. 15. H-rich white dwarf cooling sequences in the color-magnitude diagram in GAIA bands, together with the sample of white dwarfs within 100 pc, obtained by Jiménez-Esteban et al. (2018). The filled squares indicate the moment when crystallization begins in each white dwarf cooling sequence and the filled triangles indicate the moment when convective coupling occurs.

4. Summary and conclusions

In this paper we have studied the evolutionary properties of ultramassive white dwarfs with ^{16}O and ^{20}Ne cores. For this purpose, we have calculated hydrogen-rich and hydrogen-deficient white dwarf cooling sequences of 1.10, 1.16, 1.23 and $1.29M_{\odot}$, resulting from solar metallicity progenitors with the help of LPCODE evolutionary code. These cooling sequences are appropriate for the study of the massive white dwarf population in the solar neighborhood resulting from single evolution of pro-

genitor stars. In our study we have considered initial chemical profiles for each white dwarf model consistent with predictions of the progenitor evolution with stellar masses in the range $9.0 \le M_{\rm ZAMS}/M_{\odot} \le 10.5$, as calculated in Siess (2010). These chemical profiles are the result of the computation of full evolutionary sequences from the ZAMS, through the core hydrogen burning, core helium burning, and the semidegenerate carbon burning during the thermally-pulsing SAGB phase. Hence, not only a realistic O-Ne inner profile is considered for each white dwarf mass, but also realistic chemical profiles and intershell masses built up during the SAGB are taken into account. In particular, the evolution through the entire SAGB phase provides us with realistic values of the total helium content necessary to compute realistic cooling times at low luminosities. We have calculated both hydrogen-rich and hydrogen-deficient white dwarf evolutionary sequences. In particular our hydrogendeficient sequences have been calculated by considering recent advancement in the treatment of energy transfer in dense helium atmospheres. Each evolutionary sequence was computed from the beginning of the cooling track at high luminosities down to the development of the full Debye cooling at very low surface luminosities, $\log(L_{\star}/L_{\odot}) = -5.5$. We also provide colors in the GAIA photometric bands for these white dwarf evolutionary sequences on the basis of models atmospheres of Rohrmann et al. (2012).

A relevant aspect of our sequences is that we have included the release of energy and the ensuing core chemical redistribution resulting from the phase separation of ¹⁶O and ²⁰Ne induced by the crystallization. This constitutes a major improvement as compared with previous studies on the subject, like those of A07 and Córsico et al. (2004). To this end, we incorporate the phase diagram of Medin & Cumming (2010) suitable for ¹⁶O and ²⁰Ne plasma, which provides us also with the correct temperature of crystallization. In addition, our white dwarf models include element diffusion consistently with evolutionary processes.

The calculations presented here constitute the first set of fully evolutionary calculations of ultra-massive white dwarfs including realistic initial chemical profiles for each white dwarf mass, an updated microphysics, and the effects of phase separation process duration crystallization. All these processes impact to a dif-

ferent extent the cooling times of ultra-massive white dwarfs. We find a marked dependence of the cooling times with the stellar mass at low luminosity and a fast cooling in our most massive sequences. In particular, our 1.29 M_{\odot} hydrogen-rich sequence reaches $\log(L_{\star}/L_{\odot}) = -5$ in only 3.6 Gyr, which is even shorter (2.4 Gyr) in the case of the hydrogen-deficient counterpart. Our results also show an enrichment of carbon in the outer layers of the hydrogen-deficient sequences at intermediate luminosities. We have also investigated the effect of element diffusion, and found that these processes profoundly change the inner abundance distribution from the very early stages of white dwarf evolution. In particular, the initial helium and carbon distributions below the hydrogen-rich envelope result substantially changed when evolution reaches low effective temperature, thus impacting the cooling times at such advanced stages of evolution.

Our new cooling sequences indicate that all pulsating white dwarfs existing in the literature with masses higher than $1.10 M_{\odot}$ should have more than 80% of their mass crystallized if they harbour O-Ne cores. This is a relevant issue since crystallization has important consequences on the pulsational properties of massive ZZ Ceti stars. This aspect has recently been thoroughly explored in De Gerónimo et al. (2018) on the basis of these new sequences, with relevant implications for the pulsational properties characterizing ultra-massive white dwarfs.

In summary, we find that the use of detailed chemical profiles as given by progenitor evolution and their time evolution resulting from element diffusion processes and from phase separation during crystallization constitute important improvements as compared with existing calculations that has to be considered at assessing the cooling times and pulsational properties of ultramassive white dwarfs. We hope that asteroseismological inferences of ultramassive white dwarfs benefit from these new evolutionary sequences, helping to shed light on the crystallization in the interior of white dwarfs.

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