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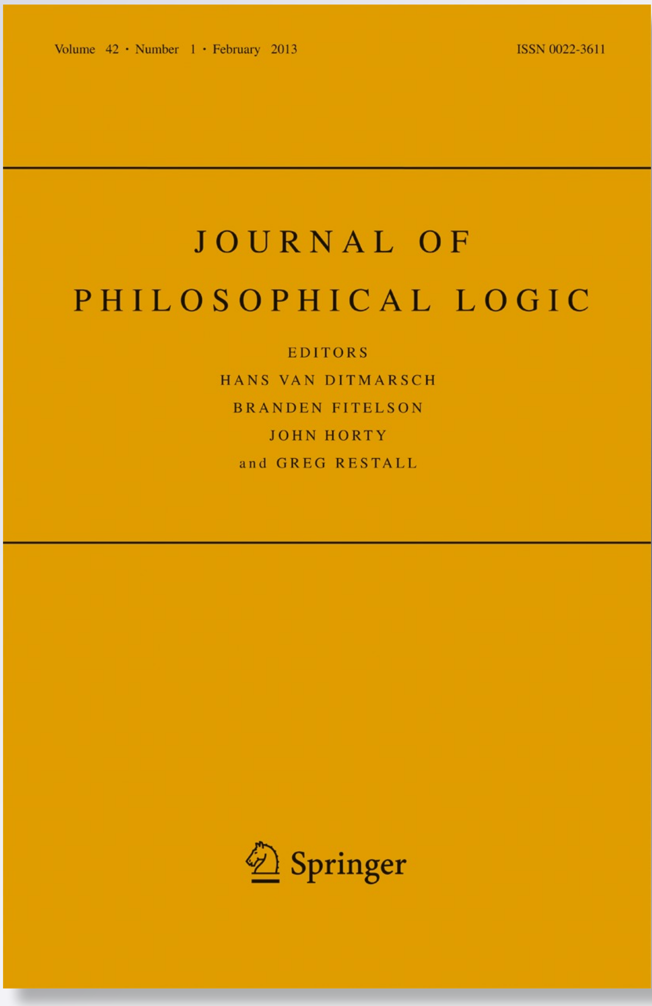

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Stratified Belief Bases Revision with Argumentative Inference

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Abstract We propose a revision operator on a stratified belief base, i.e., a belief base that stores beliefs in different strata corresponding to the value an agent assigns to these beliefs. Furthermore, the operator will be defined as to perform the revision in such a way that information is never lost upon revision but stored in a stratum or layer containing information perceived as having a lower value. In this manner, if the revision of one layer leads to the rejection of some information to maintain consistency, instead of being withdrawn it will be kept and introduced in a different layer with lower value. Throughout this development we will follow the principle of minimal change, being one of the important principles proposed in belief change theory, particularly emphasized in the AGM model. Regarding the reasoning part from the stratified belief base, the agent will obtain the inferences using an argumentative formalism.

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Thus, the argumentation framework will decide which information prevails when sentences of different layers are used for entailing conflicting beliefs. We will also illustrate how inferences are changed and how the status of arguments can be modified after a revision process.

Keywords Belief revision · AGM model · Argumentation systems

1 Introduction

Belief Revision is the process of changing beliefs by taking into account a new piece of information. The formalization of Belief Revision has been the subject of investigation in Philosophy and Logic, spreading to Computer Science where the research takes place in the areas of Artificial Intelligence and Databases. *Argumentation* is concerned primarily with accepting claims supported by arguments that are obtained by reasoning from accepted premises; acceptance proceeds in the context of disagreement where the support and interference of other arguments is analyzed. In settings where autonomous agents are involved, Belief Revision describes the way in which an agent is supposed to change its beliefs when new information arrives or when changes in the world are observed restoring the consistency of the knowledge base; Argumentation is concerned with establishing the agent's current beliefs from a potentially incomplete and/or inconsistent knowledge base maintaining the consistency of these beliefs.

It is tempting to take the standard AGM approach [1] for a direct comparison of Argumentation and Belief Revision. This approach, based on *partial meet contractions*, and all its variants: *safe contractions* [2], *epistemic entrenchment* [35], *sphere systems* [38], and *kernel contractions* [40], have been the most influential works in belief change and the dynamics of knowledge; no other work has influenced the development of belief revision theory in a similar way. The AGM postulates [1, 33] offer a clear framework that makes fundamental views explicit. The use of beliefs sets, i.e., deductively closed sets of formulas, is one of the basic elements of the standard AGM theory. Beliefs sets, however, are not useful as a formal device for developing an argumentation theory since the basic reasoning steps are abstracted away; this eliminates the fundamental distinction between explicit and implicit beliefs. Precisely, establishing which are the implicit beliefs represents the main focus of an argumentation theory. Thus, the elements contained on the formalisms used in belief base revision [27, 41], or in iterated epistemic change [18, 19, 42, 43] offer a richer and more appropriate base for our research, allowing us to obtain deeper insights into the change processes at all levels.

The connection between belief revision and argumentation was analyzed by Doyle in his seminal work on reason maintenance [22]. Later, his analysis was expanded in [23] where their similarities and differences were thoroughly discussed. In this work, Doyle contrasted the foundations approach and the coherence approach. The first one exemplified in the paper by his own *reason* *maintenance system* (also known as *Truth Maintenance System* or TMS), and the latter represented by the AGM theory. He concludes listing these similarities and remarking that: *"Perhaps the most fruitful way of viewing the issue is to focus on the great and fundamental similarities of the approaches rather than on the apparently minor differences."* While the philosophical discussion about these two forms of considering the representation and revision of beliefs is outside the scope of this paper, it is interesting to note that both perspectives have positively influenced each other [34].

In another line of work of the effort seeking to show connections between systems, the interrelation between belief revision and nonmonotonic inference was made clear in [47], and was further developed for iterative change operations in [44]; this is the BRDI[1] approach to belief revision [24]. As to argumentation, those approaches that are based on rules or use some sort of inference mechanism provide logical chains that establish links between what is accepted as premises and what is concluded. Regarding this aspect, belief revision theories remain on an abstract level, describing by postulates *what* good inferences are, while argumentation is more concerned with *how* conclusions are drawn, making explicit the reasons for belief.

One of the first approaches to connect belief revision and argumentation was presented in [29]. The knowledge representation language introduced recognizes *undefeasible* beliefs, represented as K, and *defeasible* beliefs, represented as Δ. In that framework, the reuse of rejected undefeasible conditionals from K is proposed by changing the type of relation between its components. Thus, an undefeasible conditional $\alpha \rightarrow \beta$ representing a strict relation between its antecedent and its consequent, instead of being eliminated from K during a change was transformed in a *defeasible conditional* $\alpha \rightsquigarrow \beta$ and stored in the set Δ of defeasible beliefs.

A stratified belief base is a belief base in which all beliefs are assigned a certain value and each stratum contains all the beliefs that share the same appraisal. The concept of value will remain as an abstract notion in this work providing a natural way to label each stratum; the set of labels will be assumed to have a total order. Furthermore, and characterizing our approach, each stratum is required to be internally consistent.[2] Notice that direct inconsistency between information stored in different strata could remain. Inconsistency among potential beliefs will be handled by the reasoning mechanism we will introduce leading to a consistent set of actual beliefs.

Such a stratified belief base provides a suitable foundation for the epistemic state of an intelligent agent. This epistemic state will contain explicit beliefs, i.e., the ones stored in the stratified belief base, and implicit beliefs, i.e., the ones that can be obtained by inference from the explicit beliefs. For the derivation of the implicit beliefs from such bases, we will resort to an

[1] BRDI: Belief Revision as Defeasible Inference.

[2] For a different, paraconsistent approach see [7–9], see Section 6 for brief datails.

argumentation system approach under one of Dung's style semantics [25]. Since each stratum is assumed to be consistent, it is necessary to devise a mechanism for revising stratified belief bases for handling the case when conflicting information attempts to be added to the same stratum. The decision to keep each stratum consistent, makes it necessary for a rational agent to resolve these conflicts before obtaining inferences from its stratified epistemic base. Our intention is to generalize the one-level reuse of beliefs shown in [29] by extending it to a multi-level formalism, i.e., beliefs that are discarded from one stratum during revision will provide the input for the revision of the next, less valued stratum. In this manner, beliefs are not given up completely, but can be preserved in a weakened form. We will exemplify how this reuse is relevant for the derivation of beliefs, and how argumentative inferences are modified after changes.

This paper joins the research effort aimed to understand the connections between Belief Revision and Argumentation by offering the following contributions: (i) the definition of a revision operator on stratified belief bases and several postulates characterizing it, (ii) the multi-level reuse of discarded beliefs from the revision of each stratum and, as a result of this, (iii) the ability to dynamically change the level of beliefs, (iv) the integration of an argumentation formalism that allows for warranted inferences taking into consideration pieces of information with different values.

The presentation is organized as follows. Section 2 provides opening material and basic notation on stratified belief bases. Section 3 shows the argumentation framework that will be used for reasoning. Section 4 presents a revision subsystem that will be used to preserve consistency on every stratum of a stratified belief base. Section 5 introduces a revision operator to be applied to a stratified belief base and presents some postulates for this operator addressing connections between belief revision and argumentation, Section 6 discusses pertinent related work, and Section 7 offers the conclusions of this paper.

2 Stratified Belief Bases: Preliminaries and Notation

In classical belief revision models, some sentences that are inconsistent with the epistemic input are fully discarded. This decision seems to be going against the idea that is good to preserve as much old information as possible. This criterion is known as the *principle of minimal change* [33] in the related literature. The multi-level reuse of beliefs offers the opportunity of taking those sentences that are eliminated by the revision process in one stratum and using them to revise the next less valued stratum. This reuse mechanism avoids the complete loss of that information, offering the advantage of having a *dynamic classification of beliefs*, that is, beliefs are dynamically classified using their value. The process of dynamic classification has been frequently used in the evolution of human knowledge. For instance, the belief establishing that all metals are solid under normal conditions of pressure and temperature was maximally reliable, i.e., having the highest value, for centuries. However, at some point in history, it was discovered that mercury is a metal that is in liquid state under afore said conditions, and the discovery effected a change in the belief describing that property of metals. Many other beliefs have changed in similar manner throughout the evolution of Science and undoubtedly will continue happening.

In this paper we will adopt a propositional language $\mathcal{L}$ with a complete set of boolean connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$. Formulæ in $\mathcal{L}$ will be denoted by lowercase Greek letters: $\alpha, \beta, \delta, \ldots, \omega$. Sets of sentences in $\mathcal{L}$ will be denoted by uppercase Latin letters: $A, B, C, \ldots, Z$. The symbol $\top$ represents a tautology or *truth*, and the symbol $\perp$ represents a contradiction or *falsum*. The characters γ and σ will be reserved to represent selection and incision functions for change operators respectively. We use a consequence operator $Cn : 2^{\mathcal{L}} \longrightarrow 2^{\mathcal{L}}$ that takes sets of sentences in $\mathcal{L}$ and produces new sets of sentences in $\mathcal{L}$. This operator satisfies *inclusion* ($A \subseteq Cn(A)$), *idempotence* ($Cn(A) = Cn(Cn(A))$), and *monotony* (if $A \subseteq B$ then $Cn(A) \subseteq Cn(B)$). It is assumed that the consequence operator includes classical consequences and verifies the standard properties of *supraclassicality* (if α can be derived from A by deduction in classical logic, then $\alpha \in Cn(A)$), *deduction* ($\beta \in Cn(A \cup \{\alpha\})$ if and only if $(\alpha \rightarrow \beta) \in Cn(A)$) and *compactness* (if $\alpha \in Cn(A)$ then $\alpha \in Cn(A')$ for some finite subset A' of A). In general, we will write $\alpha \in Cn(A)$ as $A \vdash \alpha$.

In our approach, an agent's epistemic state will be represented as a stratified belief base. In such base, each stratum will contain a set of sentences with the same value and each stratum will be assumed consistent. This decision contrast with other approaches, such as [7, 8] where a form of argumentation is used as a general inference mechanism from a (potentially) inconsistent stratified belief base rendering a paraconsistent approach.

Definition 1 $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$ is a *stratified belief base* (SBB) if and only if $\Sigma_i \subseteq \mathcal{L}$ and Σ_i is finite and consistent for all $0 \leq i \leq n$, and all the sentences in Σ_i have the same value. The strata are considered to be ordered in the following way: the beliefs of Σ_j have a higher assigned value than the ones in Σ_i when $j < i$. Sometimes, slightly abusing the notation, $x \in \Sigma$ will mean that $x \in \Sigma_i$ for some $\Sigma_i \in \Sigma, 0 \leq i \leq n$.

It seems a good design decision for a rational agent, faced with contradictory beliefs of different levels, should opt to believe only the stronger of them disregarding the weaker. Such agent will reserve the first stratum Σ_0 for proper beliefs, while the rest of the layers refer to information that: (i) it has assigned a lower level than the information in Σ_0, (ii) it is believed only when it does not contradict beliefs that are in Σ_0 or that are contained in a strata of higher level, and (iii) even when it is believed, it will remain more provisional than any belief in Σ_0. Therefore, in our approach, stratified belief bases are largely composed of information that is not fully believed, and the stratification is produced by the perceived value of that information. This characterization differs from other approaches that consider that beliefs have a certain strength or entrenchment [35, 46, 57].

It is important to note that our approach differs from the research presented in [7, 8]. In these works, our abstract notion of value is grounded as reliability and each stratum can be inconsistent. Moreover, the reliability assigned to the elements in the belief base cannot be modified. Our approach considers consistent strata and will allow to change the level of beliefs as a consequence of a revision.

3 Warranted Inference Through Argumentation

Stratified belief bases (SBB) are collections of sentences with different values attached, possibly containing multiple copies of sentences in different strata, each stratum being consistent, and the union of the set of strata could be inconsistent, i.e., sentences in one stratum can contradict sentences in a different stratum. To make good use of this information in selecting the implicit beliefs, a mechanism with the ability of prioritizing the beliefs with higher value and with the capability of solving conflicts is needed; argumentation frameworks were developed for that purpose. In this work, we introduce a formalism that defines an argumentation framework that allows to entail warranted conclusions from a stratified belief base.

The use of argumentation will introduce the possibility of pondering arguments for and against a given conclusion and extending the analysis to interference occurring in the intermediate steps of the reasoning process, i.e., at the level of sub-arguments. After presenting the argumentation framework, in Section 5 a revision operator for a stratified belief base will be introduced. This operator will allow the addition of incoming information to the SBB, maintaining the local consistency of each SBB's stratum and handling the change in the value assigned of each sentence.

3.1 Arguments from Stratified Belief Bases

Arguments are pieces of reasoning that support conclusions from evidence. They can have intermediate reasoning steps, and hence, can be structured with sub-arguments. Each sub-argument provides support for an intermediate conclusion. In the literature, structured arguments are assumed to be consistent, in the sense that an argument does not contain two sub-arguments that support contradictory conclusions. In other words, the set of formulas used in the reasoning to infer the conclusion is assumed to be consistent [10, 11, 52, 58]. Thus, an argument cannot be in conflict with itself, i.e., cannot be self-defeating, providing a consistent explanation for the conclusion it supports.

Our definition of an argument is adapted from the notion of argument structure introduced in [58] and widely accepted in many formalizations that consider how arguments are built [6, 55]. The intuitions behind these definitions are simple. An argument for a conclusion α is a minimal set of consistent formulas from the stratified belief base Σ that entails the conclusion. Furthermore, pondering the entity that builds the argument, lets consider an agent whose beliefs are represented as Σ. For this agent, Σ_0 contains its more valued beliefs, therefore it is assumed that this stratum represents a set of indisputable beliefs; they provide the foundation for the other accepted beliefs. Following the same conceptualization, no argument should contradict the sentences in Σ_0. The definitions and remarks below formalize the above mentioned intuitions.

Definition 2 Let $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$ be a stratified belief base and α a sentence. A set of sentences A is an *argument* for α from Σ, denoted $\langle A, \alpha\rangle_\Sigma$ or simply $\langle A, \alpha\rangle$, if:

1. $A \subseteq (\Sigma_0 \cup \ldots \cup \Sigma_n)$.
2. $A \cup \Sigma_0$ is consistent.
3. $\alpha \in Cn(A \cup \Sigma_0)$.
4. There is no $X \subset A$ that satisfies conditions 2 and 3.

Given the argument $\langle A, \alpha\rangle_\Sigma$, the sentence α is called the *conclusion* of the argument, and the set A is called the *support* of α. Sometimes, when no confusion could arise, we will simplify the notation for arguments as $\langle A, \alpha\rangle$.

Definition 3 Let Σ be a stratified belief base. An argument $\langle B, \beta\rangle_\Sigma$ is a *sub-argument* of $\langle A, \alpha\rangle_\Sigma$, if $B \subseteq A$.

Observation 1 Given an argument $\langle A, \alpha\rangle_\Sigma$ built from a stratified belief base $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$, the condition 2 of Definition 2 determines that $\neg\alpha \notin Cn(\Sigma_0)$.

Observation 2 Let Σ be a stratified belief base, and let $\langle A, \alpha\rangle_\Sigma$ be an argument from Σ with sub-arguments $\langle B, \beta\rangle_\Sigma$, and $\langle C, \gamma\rangle_\Sigma$. Since A is consistent (Definition 2), $B \cup C$ is a consistent set of formulas.

Example 1 Consider the stratified belief base $\Sigma = (\Sigma_0, \Sigma_1, \Sigma_2, \Sigma_3)$, where:

$$\Sigma_0 = \begin{Bmatrix} k \\ h \\ h \rightarrow j \end{Bmatrix} \quad \Sigma_1 = \begin{Bmatrix} b \rightarrow c \\ e \rightarrow \neg b \\ h \rightarrow b \\ k \rightarrow \neg f \end{Bmatrix} \quad \Sigma_2 = \begin{Bmatrix} e \\ a \rightarrow b \\ f \rightarrow \neg c \\ h \rightarrow f \end{Bmatrix} \quad \Sigma_3 = \begin{Bmatrix} a \\ j \end{Bmatrix}$$

The following arguments are only some of the arguments that can be obtained from Σ:

$\langle \{k\}, k\rangle$
$\langle \{h\}, h\rangle$
$\langle \{e\}, e\rangle$
$\langle A_0, \neg e\rangle \quad A_0 = \{a, a \rightarrow b, e \rightarrow \neg b\}$
$\langle A_1, c\rangle \quad A_1 = \{a, a \rightarrow b, b \rightarrow c\}$
$\langle A_2, b\rangle \quad A_2 = \{a, a \rightarrow b\}$

$\langle A_3, \neg b\rangle \quad A_3 = \{e, e \rightarrow \neg b\}$
$\langle A_4, \neg c\rangle \quad A_4 = \{h, h \rightarrow f, f \rightarrow \neg c\}$
$\langle A_5, f\rangle \quad A_5 = \{h, h \rightarrow f\}$
$\langle A_6, b\rangle \quad A_6 = \{h, h \rightarrow b\}$
$\langle A_7, \neg f\rangle \quad A_7 = \{k, k \rightarrow \neg f\}$

As an example, note that $\langle A_2, b\rangle$ is a sub-argument of both $\langle A_1, c\rangle$ and $\langle A_0, \neg e\rangle$, and that $\langle A_5, f\rangle$ is a sub-argument of $\langle A_4, \neg c\rangle$.

Since arguments can contain sentences of different strata, the comparison of arguments will take account of such information. The value of a set of sentences will depend on the strata the sentences belong to. Arguments will have a higher value if they are built with sentences of a stratum with a lower index. Since a sentence can belong to more of one stratum, the stratum with lower index (greatest value) should be chosen. Hence, the *index* of a sentence β in Σ, denoted $index_{\Sigma}(\beta)$, will be the index corresponding to the lowest indexed stratum – and hence most valued – in Σ to which β belongs to.

Definition 4 Let $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$ be a stratified belief base such that $\beta \in (\Sigma_0 \cup \ldots \cup \Sigma_n)$. Then, $index_{\Sigma}(\beta) = i$ if and only if $\beta \in \Sigma_i$ and there is no Σ_j $(j < i)$ such that $\beta \in \Sigma_j$. We say that i is the *stratum* of β.

The stratum of an argument is therefore defined by the index of its weakest sentence.

Definition 5 Let Σ be a stratified belief base and $A = \{x_1, x_2, \ldots, x_m\}$ be a set of sentences in Σ. The *stratum of the set* A is

$$str_{\Sigma}(A) = \max\{index_{\Sigma}(x_1), index_{\Sigma}(x_2), \ldots, index_{\Sigma}(x_m)\}$$

Example 2 Consider the Σ in Example 1, then:

$$\begin{array}{lll}
index_{\Sigma}(k) = 0, & index_{\Sigma}(j) = 3, & index_{\Sigma}(h \to b) = 1, \\
str_{\Sigma}(\langle\{k\}, k\rangle) = 0, & str_{\Sigma}(\langle\{h\}, h\rangle) = 0, & str_{\Sigma}(\langle\{e\}, e\rangle) = 2, \\
str_{\Sigma}(\langle\{j\}, j\rangle) = 3, & str_{\Sigma}(\langle\{h, h \to j\}, j\rangle) = 0, & str_{\Sigma}(\langle A_1, c\rangle) = 3, \\
str_{\Sigma}(\langle A_2, b\rangle) = 3, & str_{\Sigma}(\langle A_3, \neg b\rangle) = 2, & str_{\Sigma}(\langle A_4, \neg c\rangle) = 2, \\
str_{\Sigma}(\langle A_5, f\rangle) = 2, & str_{\Sigma}(\langle A_6, b\rangle) = 1, & str_{\Sigma}(\langle A_7, \neg f\rangle) = 1
\end{array}$$

Next, based on $str_{\Sigma}(\cdot)$, we introduce the order "$\succeq_{\Sigma}$" in the set of arguments that can be built from Σ. Recall that a lower number represents more value.

Definition 6 Let $\langle B, \beta\rangle_{\Sigma}$ and $\langle A, \alpha\rangle_{\Sigma}$ be two arguments from a stratified belief base Σ. We will say that $\langle B, \beta\rangle_{\Sigma}$ is *as least as good* than $\langle A, \alpha\rangle_{\Sigma}$, denoted as $\langle B, \beta\rangle_{\Sigma} \succeq_{\Sigma} \langle A, \alpha\rangle_{\Sigma}$, if and only if $str_{\Sigma}(B) \leq str_{\Sigma}(A)$.

The following proposition establishes that a sub-argument cannot be weaker than any of its super-arguments.

Proposition 1 *Let Σ be a stratified belief base, if $\langle B, \beta\rangle_{\Sigma}$ is a sub-argument of $\langle A, \alpha\rangle_{\Sigma}$ then $\langle B, \beta\rangle_{\Sigma} \succeq_{\Sigma} \langle A, \alpha\rangle_{\Sigma}$.*

Proof Suppose that $\langle B, \beta\rangle_{\Sigma} \not\succeq_{\Sigma} \langle A, \alpha\rangle_{\Sigma}$, then it should be the case that $str_{\Sigma}(B) > str_{\Sigma}(A)$. However, if $str_{\Sigma}(B) > str_{\Sigma}(A)$, then there should be some sentence in B that is not in A, but that is not possible because $B \subseteq A$. □

3.2 Conflict and Defeat Relations

Two arguments that support contradictory conclusions clearly interfere with each other; this will be considered as a relation of *conflict*. For instance, the arguments $\langle A_2, b\rangle$ and $\langle A_3, \neg b\rangle$ from Example 1 are in conflict. Furthermore, given two arguments A and B that are in conflict, such as $B \succeq_\Sigma A$, it is natural to establish that the interference of B over A is successful; this introduces the notion of *defeat*.

Defeat can be effected in different ways. In presenting the idea, we have introduced a particular type of defeat where the attack is aimed at the conclusion, this form of defeat is called *rebuttal*; other kinds of defeat exists leading to a richer relation. When the attack is indirect, i.e., being aimed at an intermediate step in the reasoning, the defeat is called *undercut*. In this work, undercuts will be accomplished by the attack of one argument to a sub-argument of another argument, that is, it is a rebuttal over a sub-argument of the attacked argument. There is another form of attack, called *assumption attack*. Given that we will not use assumptions in our formalism, this latter form of attack will not appear in our presentation. The formal definitions of conflict and defeat are introduced next.

Definition 7 Let Σ be a stratified belief base. Two arguments $\langle A, \alpha\rangle_\Sigma$ and $\langle B, \beta\rangle_\Sigma$ are *in conflict*, if $\Sigma_0 \cup \{\alpha, \beta\}$ is an inconsistent set.

We would like to discuss the particular use of the stratum Σ_0 as context to analyze the inconsistency of the conclusions of two arguments. The Σ_0 stratum is a set of consistent beliefs with maximum value and any argument, by its construction, should be consistent with it. Since each conclusion by itself is consistent with Σ_0, the conclusions must be contradictory with each other, possibly through the use of an appropriated subset of the beliefs in Σ_0. The example below shows the general case.

Example 3 Consider the stratified belief base $\Sigma = (\Sigma_0, \Sigma_1)$, where:

$$\Sigma_0 = \begin{Bmatrix} x \\ y \\ a \rightarrow c \\ b \rightarrow \neg c \end{Bmatrix} \qquad \Sigma_1 = \begin{Bmatrix} x \rightarrow a \\ y \rightarrow b \end{Bmatrix}$$

The arguments $\langle A, a\rangle$ with $A = \{x, x \rightarrow a\}$ and $\langle B, b\rangle$ with $B = \{y, y \rightarrow b\}$ are in conflict even though their conclusions are not in direct contradiction.

Definition 8 Let Σ be a stratified belief base. An argument $\langle B, \beta\rangle_\Sigma$ *defeats* $\langle A, \alpha\rangle_\Sigma$, if there exists a sub-argument $\langle C, \gamma\rangle_\Sigma$ of $\langle A, \alpha\rangle_\Sigma$, such that the following two conditions hold:

- $\langle B, \beta\rangle_\Sigma$ and $\langle C, \gamma\rangle_\Sigma$ are in conflict, and
- $\langle B, \beta\rangle_\Sigma \succeq_\Sigma \langle C, \gamma\rangle_\Sigma$.

We will say that $\langle B, \beta\rangle_\Sigma$ defeats $\langle A, \alpha\rangle_\Sigma$ at sub-argument $\langle C, \gamma\rangle_\Sigma$.

Example 4 Consider the stratified belief base of Example 1. Argument $\langle A_3, \neg b\rangle$ defeats $\langle A_1, c\rangle$ (at the sub-argument $\langle A_2, b\rangle$), and $\langle A_6, b\rangle$ defeats $\langle A_3, \neg b\rangle$. Argument $\langle A_4, \neg c\rangle$ defeats $\langle A_1, c\rangle$ and $\langle A_7, \neg f\rangle$ defeats $\langle A_4, \neg c\rangle$ (at the sub-argument $\langle A_5, f\rangle$).

Consider two arguments $\langle C, \gamma\rangle_{\Sigma}$ and $\langle A, \alpha\rangle_{\Sigma}$ that are in conflict. Suppose that $str_{\Sigma}(C) = str_{\Sigma}(A)$. Then it holds that $\langle C, \gamma\rangle_{\Sigma} \succeq_{\Sigma} \langle A, \alpha\rangle_{\Sigma}$ and also that $\langle A, \alpha\rangle_{\Sigma} \succeq_{\Sigma} \langle C, \gamma\rangle_{\Sigma}$. Hence, in this particular case, it holds that $\langle C, \gamma\rangle_{\Sigma}$ defeats $\langle A, \alpha\rangle_{\Sigma}$ and $\langle A, \alpha\rangle_{\Sigma}$ defeats $\langle C, \gamma\rangle_{\Sigma}$.

As a consequence of the way the comparison criterion "$\succeq_{\Sigma}$" is defined, if B defeats A, then B will also defeat every super-argument of A, that is, B also defeats every argument C such that A is a sub-argument of C (see Proposition 1).

Observation 3 Let Σ be a stratified belief base. If $\langle B, \beta\rangle_{\Sigma}$ is a sub-argument of $\langle A, \alpha\rangle_{\Sigma}$ and $\langle C, \gamma\rangle_{\Sigma}$ is a sub-argument of $\langle A, \alpha\rangle_{\Sigma}$, then $\langle B, \beta\rangle_{\Sigma}$ cannot defeat $\langle C, \gamma\rangle_{\Sigma}$ and vice versa. (see Observation 2).

Proposition 2 *Let $\langle A, \alpha\rangle_{\Sigma}$ be an argument. If $A \subseteq \Sigma_0$ then there is no argument $\langle B, \beta\rangle$ such that $\langle B, \beta\rangle_{\Sigma}$ defeats $\langle A, \alpha\rangle_{\Sigma}$.*

Proof An argument B can only defeat A if there exists a sub-argument C of A such that B is not weaker than C. Since $A \subseteq \Sigma_0$, $str_{\Sigma}(A) = 0$, and any sub-argument C will be such that $str_{\Sigma}(C) = 0$. Suppose that there exists an argument B that is a defeater of A at sub-argument C, then B is not weaker than C, i.e., $str_{\Sigma}(B) = str_{\Sigma}(C) = 0$, but that means that $B \subseteq \Sigma_0$. But B is a defeater of A at C, that is, the conclusions of B and C are contradictory. Since Σ_0 is consistent the existence of B is not possible. □

Corollary 1 *Given an argument $\langle A, \alpha\rangle_{\Sigma}$. If $str_{\Sigma}(A) = 0$ then there exists no argument that can defeat $\langle A, \alpha\rangle_{\Sigma}$.*

Proof It suffices to observe that if $str_{\Sigma}(A) = 0$ then $A \subseteq \Sigma_0$, and in that case, by Proposition 2, argument A can have no defeater. □

3.3 Argumentative Inference

The argumentation formalism introduced above allows us to build the set of arguments that will be involved in the process of deciding what the implicit beliefs are. In argumentation terminology this is the set of beliefs supported by warranted arguments. For obtaining these beliefs, we will make use of the semantics defined for abstract argumentation frameworks.

An argumentation framework [25] is a pair $\langle \mathfrak{Args}, \mathbf{R}\rangle$, where $\mathfrak{Args}$ is a finite set of arguments and $\mathbf{R}$ is a binary relation between arguments such that $\mathbf{R} \subseteq \mathfrak{Args} \times \mathfrak{Args}$. The notation $(A, B) \in \mathbf{R}$ (or, equivalently, $A\,\mathbf{R}\,B$) means that A *attacks* B. It is interesting to note that the Dung's relation called attack in his original paper corresponds to the notion of defeat introduced in this work, i.e., in Dung's formalism every attack was successful, therefore we will consistently use defeat instead of attack.

An argumentation semantics is the formal definition of the argument evaluation process leading to decide which arguments are able to survive the attacks defined in the framework. These "survivors" will be considered as being able to support their conclusions. The research has produced two different ways of carrying out the evaluation, namely extension-based and labeling-based argumentation semantics. The first one provides a declarative definition, meanwhile the second one is procedural in nature. An extension is a subset of arguments contained in the framework, and the extension-based approach specifies how to obtain the subsets that form the set of extensions. Every extension contains a set of arguments that together can be acceptable in the context of the attack relation. The labeling-based approach provides a way of assigning a label to each argument in the framework choosing that label from an appropriated set, such as $\{in, out, undecided\}$. The assignment of labels yields a set of labelings that correspond to the extensions found through the declarative method.

Dung [25] introduces several argumentation semantics that provide a way of evaluating the status of the arguments in the framework constructing extensions, e.g., complete, grounded, stable, and preferred semantics. Other semantics have been proposed after the initial definition, e.g., stage, semi-stable, ideal, *CF*2, and prudent semantics. For further details on recent developments see [4].

Given a stratified belief base Σ and using the argumentation formalism defined above, an argumentation framework $\langle \mathfrak{Args}, \mathbf{R} \rangle$ can be obtained, and some skeptical semantics can be applied in order to obtain warranted conclusions. An argument will be skeptically warranted if and only if it is warranted in all the extensions produced by the chosen semantics. Grounded semantics is an example of skeptical semantics, and we will use it in the example below.

Example 5 Consider again Example 1. The argumentation framework $\langle \mathfrak{Args}, \mathbf{R} \rangle$ can be obtained, where $\{A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7\} \subseteq \mathfrak{Args}$ and $\mathbf{R}$ contains, for instance, the following tuples:

$$\{(A_3, A_2), (A_3, A_1), (A_4, A_1), (A_7, A_4), (A_7, A_5), (A_6, A_3)\}$$

If we choose to use grounded semantics [25], the arguments A_1, A_2, A_6 and A_7 are warranted, and so all of b, c, and $\neg f$ can be inferred from our stratified belief base, among others.

Note that the particular semantics chosen for obtaining the warranted arguments is a decision that will affect the behavior of the system. For more details regarding different aspects of skeptical semantics in abstract argumentation frameworks see [3, 5]. The reader may obtain a general perspective on argumentation in general and argumentation semantics in particular in [6], or in [10, 55] where deeper and detailed accounts are presented.

4 Multiple Revision Subsystem

A multiple revision operator is defined as a belief revision operator capable of accepting a set of sentences as epistemic input. The intention is to use this type of operator as part of the global framework we are introducing. To construct a multi-layer revision operator, first we need to discuss certain general ideas concerning revision operators on each layer of a stratified belief base seen in isolation. Although some related literature on the topic of multiple revision exists, such as the proposed in [51] where multiple revision is reduced to classical AGM sentence revision, or the work of [21] where revision and expansion operators of logic programs by answer set semantics were presented, our approach differs from these in that we consider multiple revision in a more general, logically classic environment. Further discussion on related work on this particular subject can be found in [28]. We will proceed by recalling the main ideas originally presented in that paper and introducing a generalization of its capabilities.

4.1 The General Setting

The layers in a stratified belief base Σ can be regarded as separated, consistent belief bases. This will allow us to conceptualize the problem as multiple belief base revision.

Let S be a consistent belief base, A and B be consistent sets of sentences, and $*$ be a binary multiple revision operator that takes a belief base and a set of sentences as inputs. The following postulates are proposed for multiple revision operations:

Inclusion:	$S*A \subseteq S \cup A$.
Weak Success:	If A is consistent then $A \subseteq S*A$.
Relative Success:	$A \subseteq S*A$ or $S*A = S$.
Consistency:	If A is consistent then $S*A$ is consistent.
Vacuity 1:	If A is inconsistent then $S*A = S$.
Vacuity 2:	If $S \cup A \nvdash \bot$ then $S*A = S \cup A$.
Uniformity 1:	Given A and B two consistent sets, for all subset X of S, if $(X \cup A) \vdash \bot$ if and only if $(X \cup B) \vdash \bot$ then $S \setminus (S*A) = S \setminus (S*B)$.
Uniformity 2:	Given A and B two consistent sets, for all subset X of S, if $(X \cup A) \vdash \bot$ if and only if $(X \cup B) \vdash \bot$ then $S \cap (S*A) = S \cap (S*B)$.
Relevance:	If $\alpha \in S \setminus (S*A)$ then there is a set C such that $S*A \subseteq C \subseteq (S \cup A)$, C is consistent with A but $C \cup \{\alpha\}$ is inconsistent with A.
Core-Retainment:	If $\alpha \in S \setminus (S*A)$ then there is a set C such that $C \subseteq (S \cup A)$, C is consistent with A but $C \cup \{\alpha\}$ is inconsistent with A.

Inclusion says that any change operator between two arbitrary sets is included in the (unrestricted) union of them. Weak Success gives priority to the new information only when it is consistent. This postulate was inspired by the AGM success postulate with an additional restriction on the input set: it must be consistent. Relative Success establishes that all sentences of the input set are added to the changed set or nothing is changed (except in the limit case in which some sentences of the input set are in the belief base). Consistency determines that the changed set is consistent whenever the input set is consistent. Vacuity 1 says that nothing is changed if the input set is inconsistent. Vacuity 2 establishes that if the input set is consistent with the original belief base, then the changed set is equal to the union of them. Uniformity 1 determines that if two consistent sets are consistent with the same subsets of the original belief base S then the respective erased sentences of S should be identical. Uniformity 2 determines that if two consistent sets are consistent with the same subsets of the original belief base S then the respective preserved sentences of S should be identical. Uniformity 2 is an adaptation of Uniformity [39] which is used in revisions where the epistemic input is just a single sentence. Uniformities 1 and 2 are equivalent and they can be indistinctly used in representation theorems [28]. Relevance and Core-Retainment were adapted from [41] and they express the intuition that nothing is removed from the original belief base unless its removal in some way contributes to making the new belief base consistent. It is easily seen that Relevance is stronger than Core-Retainment, i.e., Relevance implies Core-Retainment. The following lemma shows some relations among postulates [28].

Lemma 1

(a) *If an operator satisfies* relevance *then it satisfies* core-retainment.
(b) *If an operator satisfies* vacuity 1 *and* weak success *then it satisfies* relative success.
(c) *An operator satisfies* uniformity 1 *if and only if it satisfies* uniformity 2.
(d) *If an operator satisfies* inclusion, weak success *and* core-retainment *then it satisfies* vacuity 2.

In order to construct the multiple revision operators, we must define the concept of (A-inconsistent) kernels and (A-consistent) remainder sets.

Definition 9 Let S, A be sets of sentences, where A is consistent. The set of *A-inconsistent-kernels* of S, noted by $S \perp\!\!\!\perp_{\perp} A$, is the set of sets X such that:

1. $X \subseteq S$.
2. $X \cup A$ is inconsistent.
3. For any X', if $X' \subset X \subseteq S$ then $X' \cup A$ is consistent.

That is, given a consistent set A, $S \perp\!\!\!\perp_{\perp} A$ is the set of minimal S-subsets inconsistent with A.

Definition 10 Let S, A be sets of sentences, where A is consistent. The set of A-*consistent-remainders* of S, noted by $S \perp_{\top} A$, is the set of sets X such that:

1. $X \subseteq S$.
2. $X \cup A$ is consistent.
3. For any X', if $X \subset X' \subseteq S$ then $X' \cup A$ is inconsistent.

That is, $S \perp_{\top} A$ is the set of maximal S-subsets consistent with A.

Example 6 Suppose that $S = \{p, p \rightarrow q, q, \neg r\}$ and $A = \{\neg q, r\}$. Then we have that:

- $S \perp\!\!\!\perp_{\perp} A = \{\{p, p \rightarrow q\}, \{q\}, \{\neg r\}\}$.
- $S \perp_{\top} A = \{\{p\}, \{p \rightarrow q\}\}$.

4.2 Prioritized Multiple Kernel Revision

The first construction of multiple revision by a set of sentences is based on the concept of A-inconsistent-kernels. In order to complete the construction, we must define an incision function that cuts in every inconsistent-kernel.

Definition 11 Let S be a set of sentences. Then σ is a *consolidated incision function* for S ($\sigma : \mathbf{2}^{\mathbf{2}^{\mathcal{L}}} \longrightarrow \mathbf{2}^{\mathcal{L}}$) if and only if, for all consistent set A:

1. $\sigma(S \perp\!\!\!\perp_{\perp} A) \subseteq \bigcup S \perp\!\!\!\perp_{\perp} A$.
2. If $X \in S \perp\!\!\!\perp_{\perp} A$ then $X \cap (\sigma(S \perp\!\!\!\perp_{\perp} A)) \neq \emptyset$.

Definition 12 Let S, A be sets of sentences, A consistent, and σ a consolidated incision function for S. The *prioritized multiple kernel revision of S by A* that is generated by σ is the operator $*_{\sigma}$ such that ($*_{\sigma} : \mathbf{2}^{\mathcal{L}} \times \mathbf{2}^{\mathcal{L}} \longrightarrow \mathbf{2}^{\mathcal{L}}$):

$$S *_{\sigma} A = \begin{cases} (S \setminus \sigma(S \perp\!\!\!\perp_{\perp} A)) \cup A & \text{if } A \text{ is consistent} \\ S & \text{otherwise} \end{cases}$$

Observation 4 Let S, A be sets of sentences, A be consistent. Suppose that $\alpha \in S$ and $\alpha \in A$. Then $\alpha \notin \bigcup(S \perp\!\!\!\perp_{\perp} A)$ and, therefore, $A \cap \bigcup(S \perp\!\!\!\perp_{\perp} A) = \emptyset$.

From Observation 4 and Definition 11, it follows that all the sentences of A are *protected*, meaning they cannot be considered for removing by the consolidated incision function. That is, a consolidated incision function selects among the sentences of $K \setminus A$ that make $K \cup A$ inconsistent.

Theorem 1 *An operator* $*_{\sigma}$ *is a* prioritized multiple kernel revision operator *for S if and only if it satisfies* inclusion, consistency, weak success, vacuity (1 and 2), uniformity (1 and 2), *and* core-retainment.

Weak Success ensures the input set is accepted when it is consistent. Vacuity 1 ensures that the belief base remains unchanged when the input set is inconsistent. These two postulates resolve a controversial point of AGM revisions such as success which forces α to be accepted in the revision of S by α even though α might be inconsistent.

4.3 Prioritized Multiple Partial Meet Revision

The second construction of multiple revision by a set of sentences is based on the concept of A-consistent-remainders. In order to complete the construction, we must define a selection function that selects the 'best' consistent remainders.

Definition 13 Let S be a set of sentences. Then γ is a *consolidated selection function* for S ($\gamma : \mathbf{2}^{\mathbf{2}^{\mathcal{L}}} \longrightarrow \mathbf{2}^{\mathbf{2}^{\mathcal{L}}}$) if and only if, for all set A:

1. If $S \perp_\top A \neq \emptyset$ then $\gamma(S \perp_\top A) \subseteq S \perp_\top A$.
2. If $S \perp_\top A = \emptyset$ then $\gamma(S \perp_\top A) = \{S\}$.

Observation 5 Let S, A be sets of sentences, A be consistent. Suppose that $\alpha \in S$ and $\alpha \in A$. Then $\alpha \in X$ for all $X \in S \perp_\top A$ and, therefore, $\alpha \in \bigcap(S \perp_\top A)$.

From Observation 5 and Definition 13, it follows that all the sentences of $K \cap A$ are *protected*, meaning that they are included in the intersection of any collection of remainders. That is, a consolidated selection function selects a subset of the set $K \perp_\top A$ whose elements all contain the set $K \cap A$.

Definition 14 Let S, A be sets of sentences, A consistent, and γ a consolidated selection function for S. The *prioritized multiple partial meet revision of S by A* generated by γ is the operator $*_\gamma : \mathbf{2}^{\mathcal{L}} \times \mathbf{2}^{\mathcal{L}} \longrightarrow \mathbf{2}^{\mathcal{L}}$ such that:

$$S *_\gamma A = \begin{cases} \cap\gamma(S \perp_\top A) \cup A & \text{if } A \text{ is consistent} \\ S & \text{otherwise} \end{cases}$$

Theorem 2 *An operator* $*_\gamma$ *is a* prioritized multiple partial meet revision operator *for S if and only if it satisfies* inclusion, consistency, weak success, vacuity (1 and 2), uniformity (1 and 2) *and* relevance.

Corollary 2 *Let S be a belief base. If* $*$ *is a* multiple partial meet revision *for S then* $*$ *is a* multiple kernel revision *for S.*

From the definitions, Lemma 1, and Theorems 1 and 2, it is easily seen that multiple partial meet revision operators are multiple kernel revision operators [28].

The following example, adapted from a similar one presented in [41, p. 91], clarifies that the converse of the above corollary does not hold, i.e., that some multiple kernel revision operators are not multiple partial meet revision operators.

Example 7 Let p, q and r be logically independent sentences and $S = \{p, q, r\}$. Suppose that $A = \{\neg p \vee \neg q, \neg p \vee \neg r\}$. Suppose that we are applying a prioritized multiple kernel revision. Then we have that:

$$S \perp\!\!\!\perp_{\perp} A = \{\{p, q\}, \{p, r\}\}$$

Suppose that $\sigma(S \perp\!\!\!\perp_{\perp} A) = \{p, r\}$. Then, the prioritized multiple kernel revision of S with respect to A is:

$$S *_{\sigma} A = (S \setminus \sigma(S \perp\!\!\!\perp_{\perp} A) \cup A = \{q, \neg p \vee \neg q, \neg p \vee \neg r\}$$

Now, suppose that we are applying a prioritized multiple partial meet revision. Then we have that:

$$S \perp_{\top} A = \{\{p\}, \{q, r\}\}$$

If γ is a selection function for S, then $S *_{\gamma} A = \bigcap (S \perp_{\top} A) \cup A$. We have three cases:

$\gamma_1(S \perp_{\top} A) = \{\{p\}\}$ and $S *_{\gamma_1} A = \{p, \neg p \vee \neg q, \neg p \vee \neg r\}$.
$\gamma_2(S \perp_{\top} A) = \{\{q, r\}\}$ and $S *_{\gamma_2} A = \{q, r, \neg p \vee \neg q, \neg p \vee \neg r\}$.
$\gamma_3(S \perp_{\top} A) = \{\{p\}, \{q, r\}\}$ and $S *_{\gamma_3} A = \{\neg p \vee \neg q, \neg p \vee \neg r\}$.

Thus, we can immediately conclude that $*_{\sigma}$ is not a multiple partial meet revision.

5 Revision on Stratified Belief Bases

In this section we will introduce one of the main contributions of the paper: the definition of a revision operator on stratified belief bases and several postulates characterizing it. This operator will allow for a multi-level reuse of discarded beliefs from the revision of each stratum.

5.1 The Revision Operator

After having presented the necessary elements to handle belief change by a set of sentences on a single belief base in Section 4 we will now define expansion and revision operators on stratified belief bases.

Definition 15 Let $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$ be a stratified belief base and let A be a consistent set. Then the *expansion of* Σ *with respect to* A *at level* i, $0 \leq i \leq n$, noted by $\Sigma + (A, i)$, is equal to:

$$\Sigma' = (\Sigma_0, \ldots, \Sigma_{i-1}, \Sigma_i \cup A, \Sigma_{i+1}, \ldots, \Sigma_n)$$

Note that when $\Sigma'_i = \Sigma_i \cup A$ is inconsistent the resulting Σ' cannot be a stratified belief base because it will not satisfy the conditions of its definition, i.e., the expansion of a SBB might not be a SBB.

As stated in Section 1, now we are able to define a revision operator on stratified belief bases. The discarded beliefs from the revision of each stratum will be reused and, as a result of this, dynamically changing the value of beliefs.

Definition 16 Let $*^j$ be a prioritized revision operator (either kernel or partial meet) on every $\Sigma_j \in \Sigma$, for all $0 \le j \le n$. This introduces a family of revision operators of the same type: kernel or partial meet. Let A be a consistent set of beliefs, accordingly with the type, the *kernel or partial meet revision of* Σ *with respect to* A *at level* i, $0 \le i$, noted by $\Sigma*(A, i)$, is defined as:

- If $i > n$ then $\Sigma*(A, i) = \Sigma$.
- If $i \le n$ then $\Sigma*(A, i) = \Sigma' = (\Sigma'_1, \ldots, \Sigma'_m)$ where $m = n$ or $m = n + 1$ and:
  - $\Sigma'_j = \Sigma_j$ for all $j < i$.
  - $\Sigma'_i = \Sigma_i *^i A$ and $R_{i+1} = \Sigma_i \setminus \Sigma'_i$, and then:
    - a) $\Sigma'_{i+k} = \Sigma_{i+k} *^{i+k} R_{i+k}$ and $R_{i+k+1} = \Sigma_{i+k} \setminus \Sigma'_{i+k}$ for $1 \le k \le n - i$.
    - b) If $R_{n+1} \neq \emptyset$ then a new stratum is created as $\Sigma'_{n+1} = R_{n+1}$.

That is, our multiple revision operator can be defined by means of an iterative/recursive algorithm. We suppose that each $*^i$ is a prioritized multiple revision operator for Σ_i. The result of $\Sigma*(A, i)$ (the revision of Σ with respect to A at the level i) is $\Sigma' = (\Sigma'_1, \ldots, \Sigma'_m)$, such that in general, for all j, $1 \le j \le n$:

$$\Sigma'_j = \Sigma_j *^j R_j,$$

where the R_j's are defined as follows:

- $R_j = \emptyset$ for $0 \le j < i$, so $\Sigma'_j = \Sigma_j$ for all $j < i$;
- $R_j = A$ for $j = i$;
- if $\Sigma_j \setminus \Sigma'_j \neq \emptyset$ then $R_{j+1} = \Sigma_j \setminus \Sigma'_j$.

The revised stratified belief base can have an additional layer when R_{n+1} is not empty. Since every layer Σ_i $(1 \le i \le n)$ is consistent, then every subset of it is also consistent. Therefore, $R_{i+1} \subseteq \Sigma_i$ is always consistent and the revision of every layer is well defined.

We have assumed that all revision operators applied on the layers satisfy (at least) Inclusion, Consistency, Weak Success, Vacuity 1 and 2, Uniformity 1 (and 2), and Core-Retainment, according to Theorems 1 and 2. This work generalizes the proposal in [29] where only the part of the represented knowledge corresponding to undefeasible rules could be demoted transforming them in defeasible rules; here it is possible that any sentence could be demoted and stored at a lower level. We will illustrate this revision operator below with some examples.

Note that rejection of the new information is not only justified when such information is internally inconsistent: it would seem a natural behavior that an agent when faced with (consistent) information of extremely poor quality, may reasonably opt for *flat-out rejection*, without incorporating it to its stratified belief base at any layer. An interesting aspect of this approach is that the priority given to new information is relative to its value, in that manner it is possible to incorporate it in the proper place without disturbing the information stored in higher value levels.

Example 8 This example shows two cases of revision at level 0 of a stratified belief bases Σ. First, we show the starting Σ, and then Σ', the result of revising Σ by $A_1 = \{c\}$ at level 0. Finally, appears Σ'', the result of revising Σ' by $A_2 = \{\neg a, \neg a \rightarrow \neg c\}$ at level 0. In each case, we are making prioritized multiple kernel revision on every layer.

Consider the set $A_1 = \{c\}$ and the stratified belief base $\Sigma = (\Sigma_0, \Sigma_1, \Sigma_2)$:

$$\Sigma_0 = \begin{Bmatrix} a \\ a \rightarrow b \\ c \rightarrow \neg b \end{Bmatrix} \; \Sigma_1 = \begin{Bmatrix} \neg f \\ b \rightarrow f \end{Bmatrix} \; \Sigma_2 = \begin{Bmatrix} e \\ e \rightarrow \neg h \\ \neg f \rightarrow h \end{Bmatrix}$$

Here follows $\Sigma' = \Sigma *^0 A_1$ showing how Σ_0 is revised by $\{c\}$. The set of $\{c\}$-inconsistent kernels of Σ_0 is $\{\{a, a \rightarrow b, c \rightarrow \neg b\}\}$. Suppose that the consolidated incision function for Σ_0 selects $\{a \rightarrow b\}$. Therefore, Σ_1 must be revised with respect to $\{a \rightarrow b\}$. Since $\{a \rightarrow b\}$ is consistent with Σ_1, no sentences are deleted in the revision of Σ_1. The resulting SBB is $\Sigma' = (\Sigma'_0, \Sigma'_1, \Sigma'_2)$, where

$$\Sigma'_0 = \begin{Bmatrix} a \\ c \rightarrow \neg b \\ \mathbf{c} \end{Bmatrix} \; \Sigma'_1 = \begin{Bmatrix} \neg f \\ b \rightarrow f \\ \mathbf{a \rightarrow b} \end{Bmatrix} \; \Sigma'_2 = \begin{Bmatrix} e \\ e \rightarrow \neg h \\ \neg f \rightarrow h \end{Bmatrix}$$

Sentences added to each layer are depicted in boldface.

We now revise Σ' by $A_2 = \{\neg a, \neg a \rightarrow \neg c\}$ at level 0, i.e., $\Sigma'' = \Sigma' *^0 A_2$. The process begins by finding the set of $\{\neg a, \neg a \rightarrow \neg c\}$-inconsistent kernels of Σ'_0 obtaining $\{\{a\}, \{c\}\}$. The consolidated incision function for Σ'_0 must select $\{a, c\}$. Next, the current Σ'_1 must be revised with respect to $\{a, c\}$. For this case the set of $\{a, c\}$-inconsistent kernels of Σ'_1 is $\{\{a \rightarrow b, b \rightarrow f, \neg f\}\}$. Assuming the consolidated incision function for Σ'_1 selects $\{\neg f\}$, the current Σ'_2 must be revised with respect to $\{\neg f\}$. In the following step, the set of $\{\neg f\}$-inconsistent kernels of Σ'_2 contains $\{\{e, e \rightarrow \neg h, \neg f \rightarrow h\}\}$. Suppose that the consolidated incision function for Σ'_2 selects $\{e \rightarrow \neg h\}$. Finally, a new layer

Σ''_3 appears holding the sentence that was rejected from Σ''_2. The resulting SBB is $\Sigma'' = (\Sigma''_0, \Sigma''_1, \Sigma''_2, \Sigma''_3)$, where

$$\Sigma''_0 = \begin{Bmatrix} c \rightarrow \neg b \\ \boldsymbol{\neg a} \\ \boldsymbol{\neg a \rightarrow \neg c} \end{Bmatrix} \Sigma''_1 = \begin{Bmatrix} b \rightarrow f \\ a \rightarrow b \\ \boldsymbol{a} \\ \boldsymbol{c} \end{Bmatrix} \Sigma''_2 = \begin{Bmatrix} \neg f \rightarrow h \\ e \\ \boldsymbol{\neg f} \end{Bmatrix} \Sigma''_3 = \{\boldsymbol{e \rightarrow \neg h}\}$$

As before, sentences added to each layer are depicted in boldface.

Example 9 Consider the SBB $\Sigma = (\Sigma_0)$, where $\Sigma_0 = \{a\}$. This example shows how successive revisions at level 0 by $\{\neg a\}$, then $\{a\}$, and finally by $\{\neg a\}$ affect a simple stratified belief base.

$$\begin{array}{rllll} \Sigma: & \Sigma_0 = \{a\} & & & \\ \Sigma' = (\Sigma *^0 \{\neg a\}): & \Sigma'_0 = \{\neg a\} & \Sigma'_1 = \{a\} & & \\ \Sigma'' = (\Sigma' *^0 \{a\}): & \Sigma''_0 = \{a\} & \Sigma''_1 = \{\neg a\} & \Sigma''_2 = \{a\} & \\ \Sigma''' = (\Sigma'' *^0 \{\neg a\}): & \Sigma'''_0 = \{\neg a\} & \Sigma'''_1 = \{a\} & \Sigma'''_2 = \{\neg a\} & \Sigma'''_3 = \{a\} \end{array}$$

In the example above, as the revisions are effected on Σ_0 the resulting strata contain information that can be perceived as *redundant*. Consider Σ''', the sentence $\neg a$ is in Σ'''_0 and also in Σ'''_2, and the sentence a is in Σ'''_1 and also in Σ'''_3. It is possible to eliminate this redundancy by applying a process of contraction or compaction over the stratified belief base (see future works in Section 7). Nevertheless, as the system is defined here, there is no conflict with maintaining the redundancy in the stratified belief base. Observe that from Σ''' an argument $\langle \{\neg a\}, \neg a \rangle$ can be constructed, and $str_{\Sigma'''}(\{\neg a\}) = 0$. Hence, by Proposition 2, no argument for a can be obtained from Σ'''. This is a general fact, if it is the case that $str_\Sigma(\{a\}) = i$, and $str_\Sigma(\{\neg a\}) = j$, $i < j$, the $str_\Sigma(\{a\}) > str_\Sigma(\{\neg a\})$ and therefore the sentence a will prevail.

Example 10 This example shows how stratified belief bases are revised at an arbitrary level i, with $i > 0$. Let us start by making a revision at level 1 by $\{f\}$, applying prioritized multiple kernel revision in cascade as necessary on every successive stratum. The followup revisions will use the sentences that are rejected in the previous stratum until the process is finished. We will show the successive states of the stratified belief base as the revisions are performed. Please notice that below we will show Σ, Σ', Σ'' and Σ''', but only the initial state Σ and the final state Σ''' that shows the result of the revision, should be considered as proper states; all other intermediate states are presented as an illustration of how the process progresses.

Consider the set $A_3 = \{f\}$ and the stratified belief base $\Sigma = (\Sigma_0, \Sigma_1, \Sigma_2)$:

$$\Sigma_0 = \begin{Bmatrix} a \\ a \rightarrow b \\ c \rightarrow \neg b \end{Bmatrix} \Sigma_1 = \begin{Bmatrix} \neg f \\ b \rightarrow f \end{Bmatrix} \Sigma_2 = \begin{Bmatrix} e \\ e \rightarrow \neg h \\ \neg f \rightarrow h \end{Bmatrix}$$

If we revise Σ_1 by $\{f\}$ we obtain Σ', i.e., $\Sigma' = \Sigma *^1 A_3$. To perform this revision we consider the set of $\{f\}$-inconsistent kernels of Σ_1, that in this case is $\{\{\neg f\}\}$. The only possible consolidated incision function for Σ_1 must select $\{\neg f\}$. This leads to a revision of Σ_2 with respect to $\{\neg f\}$, obtaining the first intermediate state $\Sigma' = (\Sigma'_0, \Sigma'_1, \Sigma'_2)$:

$$\Sigma'_0 = \begin{Bmatrix} a \\ a \to b \\ c \to \neg b \end{Bmatrix} \; \Sigma'_1 = \begin{Bmatrix} b \to f \\ \boldsymbol{f} \end{Bmatrix} \; \Sigma'_2 = \begin{Bmatrix} e \\ e \to \neg h \\ \neg f \to h \end{Bmatrix}$$

Again, sentences added to each layer are depicted in boldface.

Now it is necessary to revise Σ''_2 by $\{\neg f\}$, i.e., $\Sigma'' = \Sigma' *^2 \{\neg f\}$. In this case the set of $\{\neg f\}$-inconsistent kernels of Σ'_2 is $\{\{e, e \to \neg h, \neg f \to h\}\}$. Suppose for the sake of the example that the consolidated incision function for Σ'_2 selects $\{\neg f \to h\}$. We obtain the next intermediate state $\Sigma'' = (\Sigma''_0, \Sigma''_1, \Sigma''_2)$, where:

$$\Sigma''_0 = \begin{Bmatrix} a \\ a \to b \\ c \to \neg b \end{Bmatrix} \; \Sigma''_1 = \begin{Bmatrix} b \to f \\ f \end{Bmatrix} \; \Sigma''_2 = \begin{Bmatrix} e \\ e \to \neg h \\ \boldsymbol{\neg f} \end{Bmatrix}$$

The sentence $\{\neg f \to h\}$ should be used to revise Σ'_3, that will be created empty, i.e., the new layer Σ'''_3 will hold the sentence $\{\neg f \to h\}$ that was rejected from Σ''_2, leading to the final result $\Sigma'' = (\Sigma'''_0, \Sigma'''_1, \Sigma'''_2, \Sigma'''_3)$, where:

$$\Sigma'''_0 = \begin{Bmatrix} a \\ a \to b \\ c \to \neg b \end{Bmatrix} \; \Sigma'''_1 = \begin{Bmatrix} b \to f \\ f \end{Bmatrix} \; \Sigma'''_2 = \begin{Bmatrix} e \\ e \to \neg h \\ \neg f \end{Bmatrix} \; \Sigma'''_3 = \{\boldsymbol{\neg f \to h}\}$$

Note that when Σ is revised by the new information $\{f\}$ at level 1, causes $\{\neg f\}$ to be demoted to the level 2, which in turn forces some of the beliefs at the level 2 to be demoted to the level 3. In this case, the revision process does not affect the beliefs at level 0 because the new information used to revise Σ was not good enough to cast doubt on that level.

Revision operators for stratified belief bases show a sort of *cascade effect*, in which information demoted from stratum i to layer $i+1$ is preserved in Σ_{i+1} and, when necessary, some other information contained in this stratum can be further demoted to avoid inconsistency. We handle each demotion at stratum i as a revision by an information of value i. We adopt this criteria as a generalization of the *one-level demotion* used in [29], trying to expand the argumentative inferences in this framework.

5.2 Postulates for Revision on SBBS

From the construction, and following some accepted rationality criteria, we propose a set of postulates for the new revision operator introduced in Definition 16. Let $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$ be a stratified belief base and A be a consistent set of sentences. Let $\Sigma' = (\Sigma'_0, \ldots, \Sigma'_m)$ with $m = n$ or $m = n+1$ be the stratified belief base which results from the revision of Σ at the level i

with respect to A, noted by $\Sigma{*}(A, i)$. We propose the following postulates for revision:

SBB-Conservativity:	$\Sigma{*}(A, i)$ is a stratified belief base.
SBB-Success:	$A \subseteq \Sigma'_i$ where $\Sigma'_i \in \Sigma' = \Sigma{*}(A, i)$.
SBB-Vacuity 1:	If $A \subseteq \Sigma_i$, $\Sigma_i \in \Sigma$ then $\Sigma{*}(A, i) = \Sigma$.
SBB-Vacuity 2:	If $A \cup \Sigma_i \nvdash \bot$ then $\Sigma{*}(A, i) = \Sigma{+}(A, i)$.
SBB-Consistency:	For all $\Sigma'_i \in \Sigma'$ it holds that Σ'_i is consistent.
SBB-Inclusion:	$\bigcup(\Sigma{*}(A, i)) \subseteq \bigcup(\Sigma{+}(A, i))$.
SBB-Strengthening:	If $\Sigma' = \Sigma{*}(A, i)$ then $str_{\Sigma'}(A) \leq i$.
SBB-Weakening:	Given $\Sigma_i \in \Sigma$ and $\Sigma'_i \in \Sigma{*}(A, i)$, if $\alpha \in \Sigma_i$ and $\alpha \notin \Sigma'_i$ then it holds that $\alpha \in \Sigma'_{i+1}$.

SBB-Conservativity establishes that the revision of a stratified belief base is a (possibly new) stratified belief base. SBB-Success determines that the revision is prioritized since the input set is included in the stratum i of the revised stratified belief base. This approach incorporates a notion of success far more articulated and less counterintuitive than most other versions present in the belief revision literature: nothing is changed in the strata of higher value. SBB-Vacuity 1 says that nothing changes when the input set is already included in the stratum i. SBB-Vacuity 2 says that nothing is removed when the input set is consistent with the stratum i. SBB-Consistency determines that every stratum of the revised stratified belief base is consistent. SBB-Consistency is implied by SBB-Conservativity and the definition of stratified belief bases. SBB-Inclusion establishes that the union of strata in the revision of Σ by A is always included in the union of strata in the expansion of Σ by A. However, observe that SBB-Inclusion does not imply the inclusion among strata: if $\Sigma = (\Sigma_0, \ldots, \Sigma_n)$ and $\Sigma' = \Sigma{*}(A, i) = (\Sigma'_0, \ldots, \Sigma'_m)$ with $m = n$ or $m = n + 1$ then it is not ensured that $\Sigma_i \subseteq \Sigma'_i$ for all $1 \leq i \leq n$. Finally, SBB-Strengthening and SBB-Weakening show how the value of beliefs may change after a revision process.

Proposition 3 *Let* $*$ *be a* multiple revision operator at the level i *defined according to Definition* 16*. Then* $*$ *satisfies* SBB-Conservativity, SBB-Success, SBB-Vacuity 1, SBB-Vacuity 2, SBB-Inclusion, SBB-Strengthening, *and* SBB-Weakening.

Proof

SBB-Conservativity: $\Sigma{*}(A, i)$ is a stratified belief base.

We must show that every stratum $\Sigma'_i \in \Sigma{*}(A, i)$ is consistent for all $0 \leq i \leq m$. For $0 \leq i \leq n$, we have two cases:

- $\Sigma'_i = \Sigma_i$. Straightforward because Σ is a stratified belief base.
- $\Sigma'_i = \Sigma_i {*}^i R_i$. Then, by ${*}^i$ satisfying *consistency*, it holds that Σ'_i is consistent.

If $m = n$ we are done. If $m = n + 1$ then $R_{n+1} \neq \emptyset$. Then, by Definition 16, $\Sigma'_m = R_{n+1}$ and, since Σ_n is consistent, then every subset of Σ_n is consistent. Since $R_{n+1} \subseteq \Sigma_n$ then Σ'_m is consistent and we are done.

SBB-Success: $A \subseteq \Sigma'_i \in \Sigma' = \Sigma*(A, i)$.
Straightforward due to $*^i$ satisfying *weak success*.

SBB-Vacuity 1: If $A \subseteq \Sigma_i \in \Sigma$ then $\Sigma*(A, i) = \Sigma$.
If $A \subseteq \Sigma_i \in \Sigma$ then A is consistent. Then, by $*^i$ satisfying *vacuity 2*, it holds that $\Sigma_i *^i A = \Sigma_i \cup A = \Sigma_i$. Therefore, $\Sigma'_i = \Sigma_i$ for all $0 \leq i \leq n$ and therefore $\Sigma*(A, i) = \Sigma$.

SBB-Vacuity 2: If $A \cup \Sigma_i \nvdash \bot$ then $\Sigma*(A, i) = \Sigma+(A, i)$.
If $A \cup \Sigma_i \nvdash \bot$ then, by $*^i$ satisfying *vacuity 2*, it holds that $\Sigma'_j = \Sigma_i *^i A = \Sigma_i \cup A$. By Definition 16, $\Sigma'_j = \Sigma_j$ for all $j \neq i$. Therefore, $\Sigma*(A, i) = \Sigma+(A, i)$.

SBB-Consistency It is already implied by SBB-Conservativity.

SBB-Inclusion: $\bigcup(\Sigma*(A, i)) \subseteq \bigcup(\Sigma+(A, i))$.
Straightforward from Definition 16 and $*^i$ satisfying *inclusion*.

SBB-Strengthening: $str_{\Sigma*(A,i)}(A) \leq i$.
By $*^i$ satisfying *weak success* it holds that $A \subseteq \Sigma'_i \in \Sigma*(A, i)$. By Definition 5, the strength of the set $A = \{x_1, \ldots, x_s\}$ is:

$$str_{\Sigma*(A,i)}(A) = \max\{str_{\Sigma*(A,i)}(x_1), \ldots, str_{\Sigma*(A,i)}(x_s)\}$$

By Definition 4, $str_{\Sigma*(A,i)}(x) = j$ if and only if $x \in \Sigma'_j$ and there is no Σ'_k $(k < j)$ such that $x \in \Sigma'_k$. Since the revision ensures that all $x_l \in A$ are in Σ'_i but also may be in more valued strata, then $str_{\Sigma*(A,i)}(A) \leq i$.

SBB-Weakening: If $\alpha \in \Sigma_i \in \Sigma$ and $\alpha \notin \Sigma'_i \in \Sigma*(A, i)$ then it holds that $\alpha \in \Sigma'_{i+1} \in \Sigma*(A, i)$.
Suppose that $\alpha \in \Sigma_i \in \Sigma$ and $\alpha \notin \Sigma'_i \in \Sigma*(A, i)$. Then $\Sigma_i \neq \Sigma'_i$ and $\Sigma'_i = \Sigma_i *^i A$. Hence $\alpha \in R_{i+1} = \Sigma_i \setminus \Sigma'_i$. By Definition 16, $\Sigma'_{i+1} = \Sigma_{i+1} *^{i+1} R_{i+1}$ and, by $*^{i+1}$ satisfying *weak success*, it holds that $R_{i+1} \subseteq \Sigma'_{i+1} = \Sigma_{i+1} *^{i+1} R_{i+1}$. Therefore $\alpha \in \Sigma'_{i+1} \in \Sigma*(A, i)$.

□

Proposition 3 shows that the well-behavedness of the revision operators used on each stratum induces a well-behavedness on the revision operator for stratified belief bases in a straightforward way. In the proposed stratified belief base revision, new information can be taken considering different values, and the reuse of sentences is multi-level, something very interesting for the modeling of dynamics in stratified belief bases. This point addresses a novel issue in belief revision. Such changes are not handled properly by modifying the retraction and addition of beliefs. The basic idea is that inconsistencies arising when new information is incorporated into the stock of beliefs can be eliminated not only by removing beliefs, but also by weakening them, i.e., storing them in a lower value level.

5.3 Argumentation and Belief Revision

The possibility of changing the value of defeasible beliefs endows revision operations with a new quality with some important consequences for the corresponding argumentation systems. Indeed, a connection between belief revision and argumentation is immediately established by considering which changes in the argumentation structures are caused by the revision of the stratified belief base that gives rise to these arguments. If A is an argument for some belief α, then when the revision $\Sigma*(A, i)$ is effected leading to Σ' from which a new set of arguments can be created and a new defeat relation among the arguments can be defined. The following examples will help to clarify these comments.

Example 11 Consider the following stratified belief base Σ:

$$\Sigma_0 = \begin{Bmatrix} a \\ \neg h \\ c \\ a \rightarrow b \end{Bmatrix} \qquad \Sigma_1 = \{c \rightarrow \neg b\}$$

Here, we can obtain the argument $\langle A_1, b\rangle$ where $A_1 = \{a, a \rightarrow b\}$ with $str_{\Sigma}(\langle A_1, b\rangle) = 0$. Note that no argument for "$\neg b$" can be obtained (see Proposition 2), and b is warranted. Consider now that the stratified belief base is revised with respect to the set "$\{a \rightarrow h\}$" at the level 0. Suppose that we will make a prioritized multiple kernel revision of every stratum. Then, the set of $\{a \rightarrow h\}$-inconsistent kernel of Σ_0 is $\{\{a, \neg h\}\}$. Suppose that the consolidated incision function for Σ_0 selects $\{a\}$. Then Σ_1 must be revised with respect to $\{a\}$. Since $\{a\}$ is consistent with Σ_1 then no sentences are deleted in the respective revised stratum. Let $\Sigma' = \Sigma*(\{a \rightarrow h\}, 0)$. The result of the revision is:

$$\Sigma'_0 = \begin{Bmatrix} \neg h \\ c \\ a \rightarrow b \\ a \rightarrow h \end{Bmatrix} \qquad \Sigma'_1 = \begin{Bmatrix} c \rightarrow \neg b \\ a \end{Bmatrix}$$

Observe that sentence "a" is now in Σ'_1 and therefore, the value of the argument $\langle A_1, b\rangle$ changes. As another consequence of this change, from Σ' an argument for "$\neg b$" can now be obtained, and therefore, b is not warranted.

$$\begin{array}{lll} \langle A_1, b\rangle & A_1 = \{a, a \rightarrow b\} & str_{\Sigma'}(A_1) = 1 \\ \langle A_2, \neg b\rangle & A_2 = \{c, c \rightarrow \neg b\} & str_{\Sigma'}(A_2) = 1 \end{array}$$

Example 12 Consider the stratified belief base Σ':

$$\Sigma'_0 = \{e\} \quad \Sigma'_1 = \begin{Bmatrix} \neg h \\ c \\ a \rightarrow b \\ a \rightarrow h \end{Bmatrix} \quad \Sigma'_2 = \begin{Bmatrix} c \rightarrow \neg b \\ a \end{Bmatrix}$$

Here it holds that:

$$\begin{array}{lll} \langle A_1, b\rangle & A_1 = \{a, a \rightarrow b\} & str_{\Sigma'}(A_1) = 2 \\ \langle A_2, \neg b\rangle & A_2 = \{c, c \rightarrow \neg b\} & str_{\Sigma'}(A_2) = 2 \end{array}$$

Consider now that Σ' is revised with respect to the set "$\{\neg a, \neg b, \neg c\}$" at the level 1. Again, suppose that we will make a prioritized multiple kernel revision of every stratum. Then, the set of $\{\neg a, \neg b, \neg c\}$-inconsistent kernels of Σ'_1 is $\{\{a \rightarrow b\}, \{c\}\}$. Since the consolidated incision function must select at least one sentence of each set of the above set (of sets), then it selects $\{a \rightarrow b, c\}$.

In the next step, Σ' will be revised with respect to the set "$\{a \rightarrow b, c\}$" at the level 2. Then, the set of $\{a \rightarrow b, c\}$-inconsistent kernels of Σ'_2 is equal to $\{\{c \rightarrow \neg b, a\}\}$. Suppose that the consolidated incision function for Σ'_2 selects $\{a\}$. Since Σ'_3 does not exists, then $\{a\}$ will be the new stratum of the revised stratified belief base. Let $\Sigma'' = \Sigma' * (\{\neg a, \neg b, \neg c\}, 1)$. The result of the revision is:

$$\Sigma_0 = \{e\} \quad \Sigma''_1 = \left\{\begin{array}{l} \neg h \\ a \rightarrow h \\ \neg a \\ \neg b \\ \neg c \end{array}\right\} \quad \Sigma''_2 = \left\{\begin{array}{l} c \\ a \rightarrow b \\ c \rightarrow \neg b \end{array}\right\} \quad \Sigma''_3 = \{a\}$$

Observe that sentence "a" is now in Σ'_3 and therefore, the value of the arguments change.

$$\begin{array}{lll} \langle A_1, b\rangle & A_1 = \{a, a \rightarrow b\} & str_{\Sigma'}(A_1) = 3 \\ \langle A_2, \neg b\rangle & A_2 = \{c, c \rightarrow \neg b\} & str_{\Sigma'}(A_2) = 2 \end{array}$$

6 Related Work

In this section we will give a brief summary of some related works in which stratified belief bases, belief revision and argumentation are studied (see also [30]). We start with comparing out approach to a paper that seems most closely related.

The research presented in [7] also sets up a framework for argumentative inferences from stratified belief bases, where the semantics of the different strata are to reflect reliability, and the modeling is based on possibility degrees. More precisely, knowledge bases are stratified, namely each formula in the knowledge base is associated with its level of certainty, or reliability, corresponding to the layer to which it belongs. At first sight, the way they define the derivation of conclusions, called π-consequence, via arguments presents some commonalities with our approach. However, the evaluation of arguments sanctioning the inference is substantially simpler, as only conflicts between heads of arguments are considered; the definition of conflict does not consider sub-arguments, i.e., just conflicts between conclusions are recognized. This may lead to consequences that are against intuition. We will clarify this using an example taken from [32] adapting it to our framework.

Example 13 The agent believes (with different degrees of reliability) that *chicken and penguins are birds* ($c \rightarrow b$, $p \rightarrow b$), that *both species do not fly* ($c \rightarrow \neg f$, $p \rightarrow \neg f$) although *birds usually fly* ($b \rightarrow f$), but the agent believes that *scared chicken may fly* ($cs \rightarrow f$). Moreover, it believes that *flying things usually nest in trees* ($f \rightarrow n$). At the moment, it is observing a *scared chicken* (c, s). The beliefs of the agent are encoded in the following stratified belief base:

$$\Sigma_0 = \begin{Bmatrix} c \\ s \\ c \rightarrow b \\ p \rightarrow b \end{Bmatrix} \quad \Sigma_1 = \begin{Bmatrix} p \rightarrow \neg f \\ cs \rightarrow f \end{Bmatrix} \quad \Sigma_2 = \begin{Bmatrix} c \rightarrow \neg f \\ f \rightarrow n \end{Bmatrix} \quad \Sigma_3 = \{b \rightarrow f\}$$

With $A_1 = \{c, c \rightarrow \neg f\}$, $A_2 = \{c, c \rightarrow b, b \rightarrow f, f \rightarrow n\}$, the arguments $\langle A_1, \neg f\rangle_\Sigma$ and $\langle A_2, n\rangle_\Sigma$ can be built. In our approach, $\langle A_1, \neg f\rangle_\Sigma$ defeats $\langle A_2, n\rangle_\Sigma$ at the sub-argument $\langle\{c, c \rightarrow b, b \rightarrow f\}, f\rangle_\Sigma$ and so inhibits the derivation of n. The argumentative inference of [7], however, derives n, since there is no argument for $\neg n$.

Thus, our notion of defeat among arguments is substantially more complex than theirs. When the arguments are built and the defeat relation is established, the dialectical evaluation of the status of these arguments is carried out applying one of the possible Dung's style semantics.

The second essential difference with the work Benferhat and colleagues is that we revise the stratified belief base to ensure each stratum to be consistent. The research introduced in [7], and in a closely related work [8], basically aim at treating inconsistencies caused by the use of multiple sources of information, and they use a restricted form of argumentation for paraconsistent reasoning. They suggest that it is not even necessary to restore consistency on the level of the strata in order to make sensible inferences from an inconsistent knowledge base. The problem with adopting that view is that adding more inconsistent beliefs in the same stratum would yield more contradictory arguments of equal strength, so less conclusions can be supported effectively. This would correspond to an agent which gets increasingly more confused by new information that it is incorporated just superficially. In our approach, conflicts caused by new information are resolved by revision, equipping the agent with a consistent set of beliefs at each level, i.e., at any layer of the stratified belief base from which it may obtain informative conclusions via argumentation.

In a recent work [9], an interesting and useful family of computationally effective strategies for conflict resolution which can be used for iterated belief revision and merging information from multiple sources is developed. The resulting strategies are favorably compared and contrasted with other existing approaches to the problem of handling inconsistent information. The information is assumed to be given as ordered consistent knowledge bases, i.e., a set of ranked information represented as logical sentences. The origins of the research can be traced to [61] where an *Adjustment procedure* is presented; this is a system able to deal with conflicts in knowledge bases. These conflicts are produced as new information is received possibly leading to inconsistencies. The procedure propagates as many highly ranked formulas as possible, ignoring information at the highest rank where an inconsistency is found and information below that rank. The main disadvantage of Adjustment is that, to restore the consistency after accepting new information in a context where the independence of information is not made explicit, it can eliminate more content of the knowledge base than it is necessary. An improvement on this procedure is another strategy called *Maxi-Adjustment* [62]. Maxi-Adjustment solves conflicts at each rank of priority in the knowledge base in an incremental manner, starting from the information with highest rank. When inconsistency is encountered in the knowledge base, all formulas in the rank responsible for the conflicts are removed. The other formulas are kept, and the process continues to the next rank. The approach used in [9] involves the weakening of the information involved in conflicts instead of discarding it, minimizing in that way the loss of information; this approach is called *Disjunctive Maxi-Adjustment* (DMA). DMA improves Maxi-Adjustment by solving the conflicts at each rank of priority in the knowledge base and doing that in an incremental manner, starting from the information with highest rank. The full description of this strategy is beyond the reach of this paper, suffices to say that this is a pure belief revision procedure that re-establishes consistency. Although this weakening process superficially reminds our approach in its goal of comply with the Minimal Change principle, their research unlike ours does keep a consistent knowledge base. By introducing an argumentation reasoning procedure, although we keep each stratum (or rank in their terminology) consistent the complete set of strata stores conflicting information. We solve these conflicts by using the aforementioned argumentation reasoner. In our view, these setup permits to keep more information since the conflicts could be solved by demoting.

In [15] preferred subtheories as a generalization of default theories are introduced. Basically, preferred subtheories are maximal consistent subsets of stratified belief bases, and arguments are built from them to support conclusions. Brewka studied revision of stratified belief bases in a further paper [14]. However, while Brewka realizes revision by expanding the corresponding layer and making use of the underlying nonmonotonic reasoning machinery, our approach aims at resolving conflicts between annotated beliefs by applying belief base revision operators and argumentation theory.

The research introduced in [16] proposes a way to deal with contradicting information by considering a support ordering on that information. For Cantwell, information received from different sources could be inconsistent. Even when the sources of information can be ordered on the basis of their trustworthiness, it turns out that extracting an acceptable notion of support for information is a non-trivial matter, as is the question what information a rational agent should accept. This approach is shown to be closely related to the epistemic entrenchment and Grove spheres studied in the belief revision community.

The dynamics of a belief revision system was studied in [53]. They considered the relations among beliefs from a "derivational approach" trying to derive a theory of belief revision from a more concrete epistemological theory. According to them, one of the goals of belief revision is to generate a knowledge base in which each piece of information is justified (by perception) or warranted by arguments containing previously held beliefs. The difficulty is that the set of justified beliefs can exhibit all kinds of logical incoherences because it represents an intermediate stage in reasoning. Therefore, they propose a theory of belief revision concerned with warrant rather than justification.

In a framework oriented to defeasible reasoning, Falappa et al. [29] proposed a non-prioritized revision operator based on the use of explanations (arguments). In this formalism epistemic states are represented as a set of undefeasible beliefs (called strict knowledge) and defeasible beliefs that support tentative (and potentially inconsistent) conclusions. The idea is that an agent, before incorporating information which is inconsistent with its knowledge, requests an explanation supporting it. One of the most interesting ideas of this work is the generation of defeasible conditionals from a revision process. This approach preserves consistency in the undefeasible knowledge and it provides a mechanism to dynamically qualify the beliefs as undefeasible or defeasible.

A theory to cope with the dynamics of possibly inconsistent information represented by a finite state is presented in [59]. Since the agent's information will always be finite, Tamminga proposes the use of finite valuations to represent it, ensuring economy of representation and avoiding redundant valuations. Finite operations for expansion and contractions are defined, without the presence of extra-logical information such as choice functions or an ordering over (sets of) sentences.

In a research that puts together argumentation and belief revision in the same conceptual framework, the important role played by Toulmin's layout of argument in fostering such integration is highlighted [50]. They consider argumentation as 'persuasion to believe' and this restriction is useful to make more explicit the connection with belief revision. They propose a model of belief dynamics alternative to the AGM approach: *Data-oriented Belief Revision* (DBR). Two basic informational categories, data and beliefs, are put forward in their model, to account for the distinction between pieces of information that are simply gathered and stored by the agent (*data*), and pieces of information that the agent considers (possibly up to a certain degree) truthful representations of states of the world (*beliefs*). Whenever a new piece of evidence is acquired through perception or communication, it affects directly the agent's data structure and only indirectly his beliefs. Belief revision is often triggered by information update either on a fact or on a source: the agent receives a new piece of information, rearranges his data structure accordingly, and possibly changes his beliefs.

A direct relationship between argumentation and belief revision is shown in [12]. They consider argumentation as persuasion to believe and that persuasion should be related to belief revision. More recently, Boella et al. [13] presented the interrelation between argumentation and belief revision on Multiagent Systems. When an agent uses an argument to persuade another one, he must consider not only the proposition supported by the argument, but also the overall impact of the argument on the beliefs of the addressee.

In [56] an abstract theory that captures the dynamics of an argumentation framework through the application of belief revision concepts is introduced. They define a *dynamic abstract argumentation* theory including dialectical constraints, and then present argument revision techniques to describe the fluctuation of the set of active arguments (the ones considered by the inference process of the theory). They claim that the theory should be abstract from two standpoints: (1) there is no restriction to any particular representation for arguments nor argumentative semantics, (2) the characterization of the change operators (specially contractions), which is not restricted to a particular implementation. They define expansion, contraction, and revision operators, where the latter can be expressed in terms of the other two, leading to an identity similar to the one defined by Isaac Levi [45]. Their abstract theory allows the introduction of an argument ensuring it can be believed afterwards. This is achieved by applying a revision, that is, an expansion followed by a contraction. The expansion operator is quite straightforward, but (as usual in any model for the theory change) the main complexity relies on the definition of the contraction operator, which allows a wide range of possibilities: from affecting unrestrictedly any number of arguments in the system to keeping this perturbation to a minimum. This choice is up to the minimal change principle followed by the specification of the contraction operation, which also has an indirect impact over the defeat relation among arguments. Moguillansky et al. [48, 49] proposed an instantiation of their operators to *Defeasible Logic Programming*, *DeLP* [32]. The main contribution is to define an argument revision operator that ensures warrant of the conclusion of the (external) argument being added to a defeasible logic program. When they revise a program by an argument $\langle A, \alpha \rangle$ (where A is an argument for α), the program resulting from the revision will be such that A is an undefeated argument and α is therefore warranted.

In a recent work [17], a proposal for handling the addition of an argument to a Dung-style abstract argumentation system was introduced. Several change operations were defined considering four possibilities: modifying the attack relation (in Dung's sense) by adding or removing an attack, adding an argument together with a set of interactions, and removing an argument together with all interactions related to it, where an interaction is an attack received or effected by the argument involved. They define an argumentation framework $\langle \mathbb{A}, \mathbb{R} \rangle$ where $\mathbb{A}$ is a finite set of arguments and $\mathbb{R}$ is an attack relation among arguments. The main issue of any argumentation system is the selection of acceptable sets of arguments. An acceptable set of arguments must be under certain criteria coherent. An argumentation semantic defines the properties required for a set of arguments to be acceptable. The selected sets of arguments under a given semantics are called *extensions* of that semantic. According to [17], the outcome of a revision process is the set of extensions under a given semantic. Then, by considering how the set of extensions is modified under the revision process, they propose a typology of different revisions: decisive revision and expansive revision. Narrowing the research to two particular semantics: the grounded and preferred semantics [26], the impact on the set of extensions that such changes effect on the outcome of the argumentation was described.

In a logic programming setup, the work by Delgrande et al. [20] addresses the problem of belief revision in (nonmonotonic) logic programming under answer set semantics: given logic programs P and Q, the goal is to determine a program R that corresponds to the revision of P by Q, denoted $P * Q$. They proposed formal techniques analogous to those of distance-based belief revision in propositional logic. They investigate two specific operators: (logic program) expansion and a revision operator based on the distance between the SE models of logic programs.

7 Conclusions and Future Work

In this work we have introduced a revision operator on stratified belief bases characterizing it through a set of postulates. The information removed from the revised stratum is reused to revise the next less valued stratum, and so on. This revision mechanism brings about the possibility of changing the belief's value. The reuse of the removed beliefs improves de possibility of satisfying the principle of minimal change as proposed by the belief change theory in general and in the AGM model in particular. Then, by using a formal framework for argumentation, we exemplified how after a revision process the status of arguments is modified leading to changes in the set of inferred beliefs. As future work, we are working in stating representation theorems for this new type of revision over stratified belief bases.

In this work we have not explored the possibility of *promoting* pieces of information (beliefs), i.e., moving it from one layer to another with higher value. The operator was defined in such a way that beliefs can only be *demoted* after being eliminated from a layer in the process of restoring consistency; that belief can only be reinserted in a higher layer through a new addition from external sources. However, we hypothesize that the *promotion* of beliefs could be prompted in other forms diverse from the ways demotion is; it is possible that beliefs can be promoted for *internal* reasons regardless of the arrival of new external information. We are exploring the characterization of a more complete framework, with dual operators for the promotion and demotion of beliefs.

Another line of work in this research will be the development of a more extended set of change operators. An operation will be a *one-level contraction*, taking a stratified belief base Σ, a set A, and a stratum i, and eliminating all sentences of A from the stratum i of Σ.[3] Another operation will be a *multi-level contraction* which takes a stratified belief base Σ, and a set A, and generates that all sentences of A are removed from each stratum of Σ. The last operation will be a *compaction* which takes a stratified belief base Σ and generates a new stratified belief base Σ' such that there is no strata i and j and no sentence α such that $i \neq j, \alpha \in \Sigma'_i$ and $\alpha \in \Sigma'_j$.

In some recent developments in argumentation theory, several ways of assigning strength to each argument have been introduced resulting in interesting argumentation frameworks. Using this results, and understanding strength as value, we can consider that many relatively weaker arguments may "accumulate" or "accrue" their relative strength overcoming a single argument that is stronger that any one of them individually [36, 37, 54, 60]. For instance, if we have one argument for α with a given strength and we have a set of arguments for $\neg\alpha$ with lower individual strength, we could consider the idea that all them working together could defeat the stronger argument. That is, when a number of (independent) reasons against a given conclusion accrue, this should have some effect on the strength of that conclusion, even when each of these reasons taken individually would not have the strength sufficient to produce such effect. This idea could also be used to promote information considered stronger when a set of arguments supports it.

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[3] Note that we are formulating a *package contraction*, where all the sentences of the epistemic input must be removed from the epistemic state. The other alternative is to define a *choice contraction*, where it is sufficient that at least one of the sentences is removed. For more details about package and choice contraction, see [31].

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