Utility of mathematical knowledge as a motivating factor in students' learning of Adult Secondary Education

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ABSTRACT
This paper analyzes the utility of the mathematics for Adult Secondary Education (ASE) as a factor to recover motivation in learning. The results of a survey with N=820 ASE students about their perspectives of the utility of mathematics are presented. The results show that teaching mathematics based on its inherent formative utility for daily life and work, could reduce much of adults' difficulty to study mathematics. For mathematics to be useful for students, mental calculation and numerical operations should take an important place in the school curriculum. Mathematics problems should be developed in a similar way to those that adults face daily, regaining the arithmetic knowledge which adults master. The current way Algebra is taught should be reconsidered, students admit it is unhelpful, despite its potential. This phenomenon suggests a new research question: What type of school algebra should be taught in ASE?

KEYWORDS
Adult secondary education, teaching of mathematics, utility

RÉSUMÉ
Cet article analyse l'utilité des mathématiques dans l'enseignement Secondaire des Adultes (ESA) comme un facteur qui permet de récupérer l'intérêt par apprendre les maths dans cette institution. Nous proposons les résultats d'une enquête sur l'utilité des mathématiques, qui a été répondu par N=820 étudiants de l'ESA les résultats montrent que l'enseignement des mathématiques basé sur son utilité formative inhérente, notamment dans la vie quotidienne et le monde du travail, pourrait réduire grandement les difficultés d'étude des adultes. Les mathématiques seraient utiles aux étudiants si le calcul mental et les opérations avec les nombres auraient une place importante dans les programmes scolaires. Les types de problèmes traités dans la classe de mathématiques devraient être liées auxquels les adultes sont confrontés quotidiennement, en récupérant les connaissances arithmétiques qu'ils dominent très bien. La manière actuelle d'enseigner l'algèbre dans l'ESA devrait être réexaminée, donc les étudiants expriment qu'elle n'est pas utile, malgré son potentiel. Ce phénomène dirige la recherche vers la question : quelle algèbre scolaire enseigner dans l'ESA?

MOTS-CLÉS
Éducation secondaire des adultes, enseignement des mathématiques, utilité

INTRODUCTION

Mathematics is present in many situations of adult life and its use (or not) could affect the quality and lifestyle of people. Notwithstanding, it is a known fact that in Adult Secondary Education (ASE) teaching of mathematics as a useful tool does not occur. Adult students present serious difficulties to study and learn mathematics. Such difficulties lead to reflect about the contents of the curriculum and the motivations to study them. Adults generally assume that mathematical school knowledge (traditionally taught) is not related to the mathematics used in adult life (Díez-Palomar, 2004). This duality, between academic mathematics and the ones used in their daily life, results in a resistance to learn mathematics associated with the lack of motivation to study it (Wedege & Evans, 2006). According to Lukianova (2016), to achieve the efficiency of learning activities in adult students, it would be necessary to thoroughly analyze their motives to learn, to determine their needs, interests, attitudes, and inclinations.

In Argentina, as in other countries, ASE students make up a heterogeneous population group (different social realities, occupations, ages, prior knowledge, etc.). In this country there are around 600.000 native students, over 16 years old who have been left out of the traditional education system. These students return to school as adults, with specific needs and aspirations. However, since 1968 researches have not been carried out to know the singularities of the population to whom the study plans are directed. Consequently, despite its great importance, the development of a curriculum, which meets adults' interest and needs, has not yet been achieved.

This work assumes hypothetically that the motivation of students to learn mathematics could focus on the formative utility for adult citizens. In this sense, it is convenient to adopt the Anthropological Theory of Didactics (ATD) as a theoretical framework. The ATD characterizes the current school paradigm of teaching mathematics as *Monumental paradigm* or of *visiting the works* (Chevallard, 2013, 2017; Otero et al., 2013). In this paradigm, mathematical contents are presented as unquestionable objects, valuable in themselves, like the works of a museum to which we must honor, revere and study. Thus, the reasons for studying school mathematics are: either the curriculum establishes it as important, or it has a supposed transcendent formative utility (Chevallard 2017; Kim, 2015), such as: "train the mind". Consequently, since knowledge has been originated from questions and problems that remain hidden, its study is meaningless for students, it is unmotivated and they do not know why they study mathematics. This is a general problem that is aggravated in ASE because students do not find reasons related to the interests of adult life and inherent in mathematics, that justify its study.

This work is part of an investigation which objective is to reveal which mathematical contents should be taught in ASE in order to be useful for adult students. In this way, five focus groups were carried out, in which students' opinion (so-called doxa) were explored in depth. Then, based on the results, a survey was designed to carry out large-scale research. In this work, a part of the results of the research related to the usefulness of mathematical knowledge for adult life are described. Four items of the aforementioned survey are presented analyzing the importance of studying school mathematics for its utility. Also, this work analyzes which is the most useful knowledge from the viewpoint of adult students.

THEORETICAL FRAMEWORK

In the axiology assumed by the ATD, utility is a remarkable epistemic value. The ATD adopts a functionalist and instrumental vision of mathematics, based on the fact that mathematical activity, as other human activities, is a product of culture and the need to solve and answer vital questions. The ATD emphasizes the fact that mathematical knowledge has two types of formative utility: Transcendent and Inherent (Kim, 2015; Chevallard, 2017). This value is very important when thinking about a curriculum.

The *inherent formative utility* is a utility intrinsic, or immanent, of a mathematical knowledge. Inherent is what is inseparably linked to something, although it can be rationally distinguished from it. For Kim (2015, p. 274) the *inherent formative utility* "is the domain of knowledge so it can be used in situations of social life". Thus, for example, the resolution of exponential equations is studied, because it would allow to solve problems such as determining the time needed to obtain certain capital with a fixed term.

On the other hand, the *transcendent* term denotes the existence of a limit (to which immanence subscribes) and the overcoming of it. The *transcendent formative utility*, as opposed to the inherent one, goes beyond what is intrinsically served by knowledge. For example, the resolution of exponential equations (as many other cases) are studied because by doing so students would develop logical-deductive thinking and mathematical rigor.

According to Chevallard (2017) and Kim (2015) the secondary mathematics was thought for the experts. In other words, it is the mathematics in which only a small group of the population P needs: P_1, formed by the mathematicians; and P_2, made up by people who use mathematics at a university level (professors, engineers, doctors, etc.). The complement of these subpopulations consisting of a subgroup P_3, which are all people who once completed secondary school, do not return to study mathematics, although they study at tertiary education (understanding it as post-secondary education). For Chevallard, the interests of P1 and P2 are clearly represented in the secondary school curriculum, while those of P_3 are not. The question "What is the mathematical training for these citizens?" has not yet been studied in depth and still remains open (Chevallard, 2017). To contribute to the curricular guidelines of mathematics in ASE, beyond its transcendent utility, it is necessary to investigate what mathematics is inherently useful for these citizens.

METHODOLOGY

We use qualitative and quantitative techniques to analyze the following question: What are the interests and motivations of ASE adults in Argentina to study mathematics? The research followed a four-stage scheme. In Stage 1, Focus Group (FG) was developed to explore in depth the interests of adults and their opinions about school mathematics. Five FG were performed with N=29 participants, making an inductive categorization of the answers until assuming the theoretical saturation (Gürtler & Huber, 2007). In stage 2, a survey was designed based on the results of the FG (Donvito, Fanaro & Otero, 2017; Donvito, Otero & Fanaro, 2017). In stage 3, the instrument was validated, a pilot test was carried out, the type of sample was decided (stratified by provinces) and administered to 820 ASE students in Argentina. In the fourth stage, the results of the survey were analyzed with univariate, bivariate and multivariate statistics techniques.

In this paper we present the results of the univariate analysis of four items of the survey. The first is related to the difficulty to study mathematics in ASE. This item is a closed question and

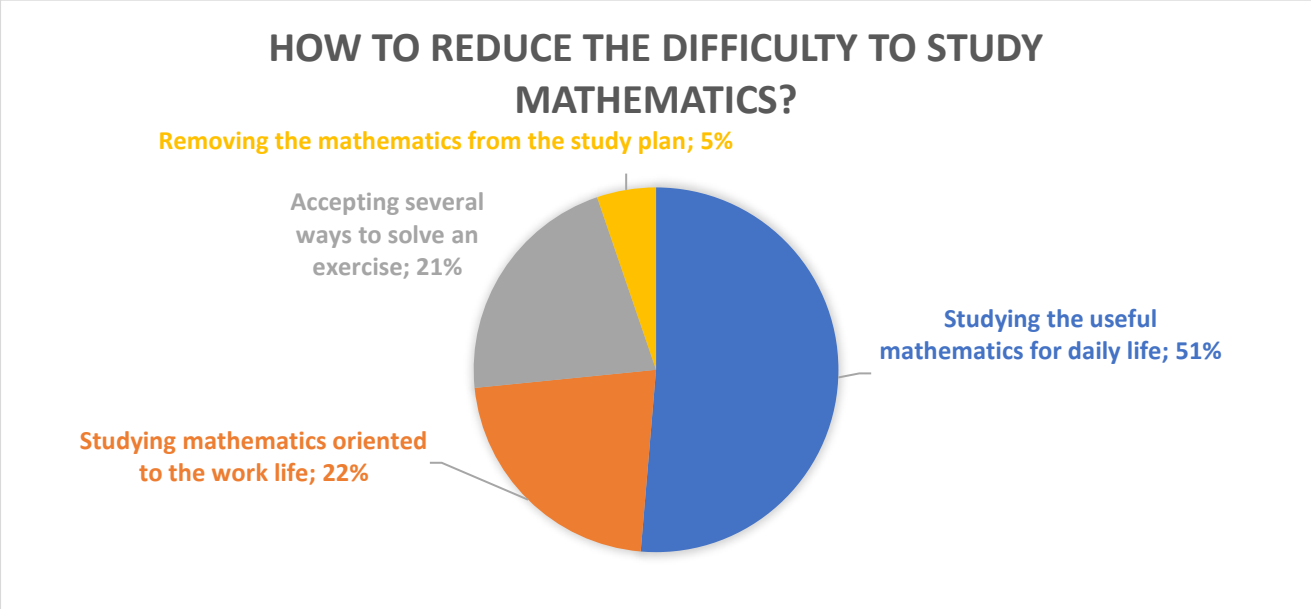
the answers comes from the analysis of the FG. The remaining three questions refer to the types of utility students consider to have (or should have) ASE mathematics (Donvito, Fanaro & Otero, 2017): *Utility for daily life, for the working life, or to study a career*. These are also closed questions, but the possible answers are mathematical contents of the curriculum.

RESULTS AND DISCUSSION

Difficulties to study mathematics

The question Q1: "How could the difficulty in studying mathematics be reduced?" has five possible answers: (a) *Studying the useful mathematics for daily life*: this answer is associated to students who consider school mathematics is unrelated to their daily life (unless they want to study a tertiary career). For example, a student argues: "(…) ASE contents are advanced mathematics but, when do you apply it? I do not mean about people who plan to study a career with mathematics, I mean about daily life. This kind of mathematics do not work" (Donvito, Otero & Fanaro, 2017, p. 110). (b) *Studying mathematics oriented to the work life*: as the previous one, it is associated to students who consider school mathematic is not useful for their jobs. (c) *Accepting several ways to solve an exercise*: It is associated with students having mathematical techniques learned outside of school. Sometime, these techniques work more efficiently than the ones other teachers insist on developing. For example, a student is forced to memorize a formula to obtain the area of a trapezium, but he argues it is easier to calculate it as the sum of the areas of a rectangle and a triangle, because he does it daily in his work. Unfortunately, the school disregards this knowledge. (d) *Removing the mathematics from the study plan*: This answer is associated with students' opinions about mathematics only serves to "train the mind", for which, as a student say, mathematics could be replaced with chess class (Donvito, Otero & Fanaro, 2017, p. 110). The results of the survey are summarized in figure 1.

FIGURE 1



How to reduce the difficulty to study mathematics? Student's viewpoint

The answer *studying the useful mathematics for daily life* is the mode, with 51% of the responses. This confirms not only the distance between *academic mathematics* and *daily life* (Díez-Palomar, 2004), also the greatest difficulty in learning is found here. The second most chosen answer is *studying mathematics oriented to the work life*. Both answers indicate that school mathematics is taught without considering that many students return to school seeking to improve their employment status. On the other hand, it is noteworthy that although adults do not find motivation to study mathematics, it is very low the percentage of those who would be willing to take it out of the curriculum (5%).

According to these results, teaching mathematical knowledge based on its *inherent formative utility* for questions of daily life and work life is an interesting way to regain motivation in the study. From students' perspective, for 73%, it could reduce the difficulties they encounter in learning. Adults do not dismiss mathematics and are motivated to study useful things, although doing so requires more effort.

Utility of mathematical knowledge

The other questions are Q2: *Which mathematical knowledge is the most useful for daily life?* Q3: *Which mathematical knowledge is the most useful for the working life?* And Q4: *Which mathematical knowledge is the most useful to study a tertiary career?* All of them are closed questions with seven possible answers. The answers *Numerical operations*, *Probability and Statistics*, *Geometry*, *and Functions and Equations* are part of the ASE curricular designs as areas of this discipline. In *Numerical operations*, addition, subtraction, multiplication, division, power and root are studied with different numerical sets: Natural, Integer, Rational, Irrational and Real. In *Probability and Statistics* notion of probability is studied, also, reading and construction of graphs (bar and pie charts) and the notion of statistical parameters such as mean, median, and mode. In *Geometry*, relations such as Pythagorean, trigonometric and similarity ratios are studied, similar figures are also analyzed and perimeters, surfaces and volumes of the most common shapes and solids are calculated. In *Functions and Equations*, algebraic operations, notions of function, and linear, quadratic, exponential and logarithmic functions are studied, also, the notions of equation, inequation and the system of two equations.

About the other answers, *Logical reasoning* and *Mathematics problems* were considered because ASE curriculum stands out both of them for its *transcendental formative utility*, as abilities to solve problems and reason with logical thinking. The *mental calculation* is not found in the school secondary curriculum, but it has been included because the students mention "we are doing calculations all day", or even "we are able to calculate faster than the calculator". The mental calculation involves techniques to operate with a great *inherent formative utility* to daily decisions of an adult. The results of questions Q2, Q3 and Q4 are summarized in figure 2.

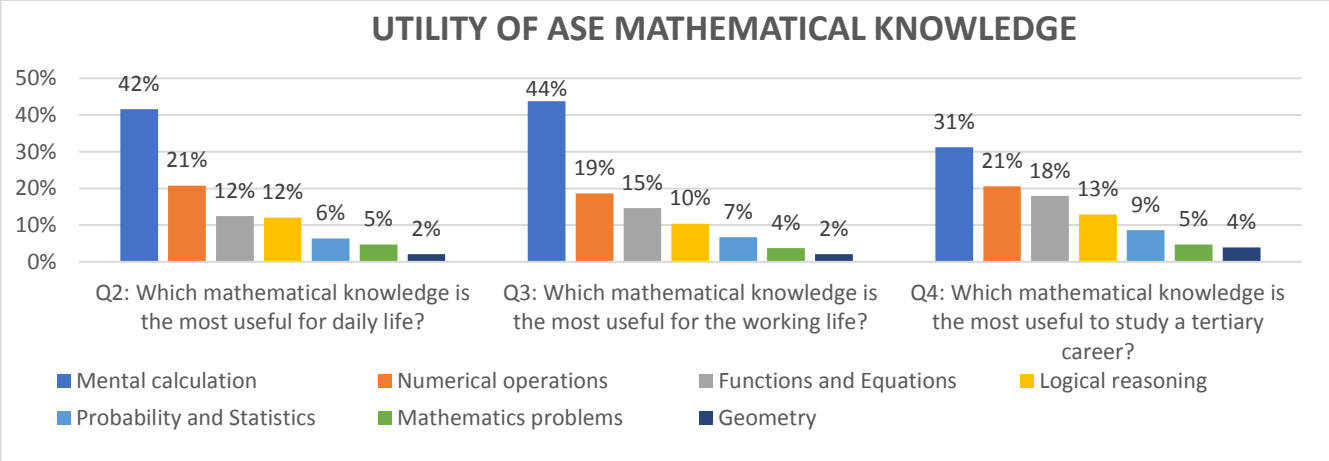
The results of these three questions are very similar. Although the percentages vary, mathematical knowledges share the same order. Mode is *mental calculation*, which for work and for daily life doubles in percentage to its consecutive, *numerical operations*. These first two knowledge points out that for adults, calculating is the most useful of the ASE scholar mathematics. *Functions and Equations* obtains a low percentage (12%, 15% and 18%), although it takes a large part of the mathematics classes. Then, there are *logical reasoning* and *probability and statistics*.

Finally, with very low percentages, appear *mathematics problems* and *geometry*.

It is noteworthy that students do not discriminate among different types of utility. This reveals a social belief which implicitly assumes the existence of a single mathematic whose utility is universal, regardless of the social context, work and the profession they aspire to develop. In all three cases, students emphasize knowledge directly related to calculating: *mental calculation* and

numerical operations. It is remarkable that *numerical operations* are part of any curriculum and are generally practiced while defining the numerical sets. However, unlike the common primary and secondary school, no time is assigned in classes to develop and validate *mental calculation* techniques, although adults consider it very useful.

FIGURE 2



Utility of ASE mathematical knowledge

Mental calculation is the most emphasized by adult learners because in many situations they must make quick decisions, without enough time to operate with pencil and paper. Adults use mental calculation every day, for example, when deciding which product to buy in a supermarket when those are presented in different sizes and prices, or also, when making instant decisions as in a negotiation. Both in daily life and in different jobs, certain margins of error are acceptable for a quick and effective response allowing to deal with a situation. This type of situations is highlighted by adults from their experience. In this sense, it would be beneficial for students to include tasks to develop techniques in order to mentally solve operations, based on mathematical properties.

School mathematics is socially conceived as a reduction of the mathematics studied by teenagers. There is less time and, therefore, less contents. But this reduction, without a deep reflection of the adults' characteristics, only exacerbates the effects of the *Paradigm of visiting the works*. This is reflected in the survey because adults recognize little use for certain mathematical knowledge. For example, *geometry* barely reached 2% of the responses, although paradoxically synthetic geometry is useful in many trades and situations of adult life. A student, who is a bricklayer, knows numerical techniques to draw lines to square, level and plumb, or even at different angles, but in mathematic classes this knowledge is not recovered, nor taught from it (Donvito, Otero & Fanaro, 2017, p. 108-109).

Regarding ASE algebra, the curriculum does not recover advances in didactic knowledge, as it does in secondary school for adolescents. Thus, introduction to algebra is mistakenly posited as the generalization of numbers to solve a problem. However, adults have effective numerical resources to solve school mathematic problems, using arithmetic and avoiding algebraic treatment. Adults know from experience that not all “problems” require algebra. The reason of being of school algebra is blurred because it is expected they use certain algebraic objects (relatively complex) to solve simple problems. For example, item 3 of the *Aprender 2016* test (Secretaría de Evaluación Educativa, 2016), which objective is to evaluate the quality of teaching in Argentina, given: For

the equation -3x + 6 = 18 What should be the value of x for equality to be met? A) 8; B) 4; C) -4; D) -8. According to this text, the exercise was designed to evaluate the cognitive capacity to solve operations using mathematic properties. However, it is easier and faster to solve it numerically, even the result is suggested. Problems like this produce the loss of meaning in the study of school algebra and the lack of motivation in learning.

Functions have taken much of the classes time, replacing many other algebraic objects. Linear, Affine, Quadratic, Exponential and Logarithmic Functions are defined, analyzing in each case x-intercept, y-intercept, positivity intervals, relatives minimum (or maximum) and its graphic representation. Thus, study of *Functions* is considered as an aim and not as an instrument to solve problems. On the other hand, *mathematics problems* and *equations* are proposed in the school curriculum from a translation perspective of *algebra*: symbolically expressing a statement; writing an algebraic expression in colloquial language; finding an equation equivalent to a given one, etc. In other words, these *mathematics problems* are not genuine problems of adult life, so it lacks utility according to the students (4%). Algebra would gain more value for adults if it abandons its traditional study as the "algebraization of arithmetic", in order to advance on the teaching of algebra as a "modeling tool" (Ruiz Munzón, 2010). In this sense, a set of situations interesting to adults could be considered, for example, situations linked to capitalization and savings (Donvito, Otero & Sureda, 2017; Donvito, Sureda & Otero, 2013).

Statistics and probability are increasingly present in adult life. According to Gravemeijer, et al., (2017), this is due to an increasing field called "Big Data", product of an unprecedented access to data and the power of computers and *machine learning*. From this point of view, statistics is an important knowledge for the citizens of the 21st century, both for the workplace and for exercising a critical citizenship. Paradoxically, students assign scarce utility to ASE's school statistics for daily life (6%) and for the world life (7%). Adults seem to disregard the fact that the large volumes of data presented in society are statistically synthesized. Citizen need to know how reading, interpreting and making decisions based on the different systems of statistical representations offered by newspapers and state institutions. However, works such as that Eudave Muñoz (2009) carried out, point out the difficulties of ASE adults for understanding statistical information from tables and graphics. On the other hand, adults also need understanding and interpreting concepts such as variability, sampling, error and bias to assume a critical position when reading unreliable information.

In Argentina, the latest curricular guidelines at national level for ASE have been published in 2015 (CFE, 2015). This document points out the need to focus mathematic teaching for work matters and for daily decisions. However, this indication remains in a nominal framework if no further guidelines on how to do so are included. Mathematics teachers do not find resources to teach the contents proposed by the ministry in a functional way to adult life. There are also no mathematic books or manuals for ASE to extract some ideas. The few suggestions by school curriculum about vital issues of adult life, exhibit a superficial view of adults and show the lack of an approach and exploration of adult reality: "Their acquired knowledges are quite heterogeneous and were generally learned in non-formal areas, in activities linked to work or daily life. For example, adults use real numbers every time they take measurements; use maps or plans on a certain scale; interpret charts of newspapers and magazines; listen the possibility of rains in the weather forecast" (CFE, 2015 p.11).

Adults are not only exposed to these types of situation, they make decisions based on more or less explicit calculations every day: they wonder what payment method is more convenient to buy an appliance or clothes with discount, analyze (to a certain degree) the convenience to take a loan; entrepreneurs calculate what price to sell their products or services and what sales they can

perform; those who develop trades calculate working budgets using several parameters such as fixed and variable (proportional to work) costs. In other words, if a design of a curriculum which allowed adults to regain their motivation to study mathematics is wanted, an approach to their reality should be made. Studying scholar mathematics recovering the knowledge available from students, and teaching new knowledge based on its *inherent formative utility* in adult issues, could be an important factor to regain the students' interest to learn mathematics.

CONCLUSION

ASE mathematics curriculum should be modified to suit the motivations of the students. One way to do this would be to focus it on the utility of mathematical knowledge for adult life. The results, based on a survey with N=820 students of the ASE, show that one of the strongest difficulties identified to learn mathematics is the lack of association between school mathematics and the one adults use in daily life and at work. For mathematics teaching to meet the students' needs, mental calculation and numerical operations should take an important place in the school curriculum. Mathematics problems should be developed in a similar way to those that adults face daily, especially situations which are solved with arithmetic. The current way Algebra is taught should be reconsidered, students consider it is unhelpful, despite its potential. This phenomenon suggests a new research question: What type of school algebra should be taught in ASE?

REFERENCES

Chevallard, Y. (2013). Enseñar matemáticas en la sociedad de mañana: Alegato a favor de un contraparadigma emergente. *Journal of Research in Mathematics Education, 2*(2), 161-182.

Chevallard, Y. (2017). ¿Por qué enseñar matemáticas en secundaria? Una pregunta vital para los tiempos que se avecinan. *Gaceta de la Real Sociedad Matemática Española. 20*(1), 159-169.

CFE. (2015). *Resolución 254/15. Marcos de Referencia para la Modalidad de Educación Permanente de Jóvenes y Adultos – Lineamientos para la construcción de diseños y/o planes de estudio jurisdiccionales*. Consejo Federal de Educación, Buenos Aires, Argentina.

Díez-Palomar, J. (2004). *El aprendizaje de las matemáticas en la educación de personas adultas. Un modelo dialógico*. Tesis Doctoral, University of Barcelona, Spain. Retrieved from http://hdl.handle.net/2445/41425.

Donvito, A., Sureda, P., & Otero, M. R. (2013). REI bidisciplinar en tres escuelas secundarias. In M. R. Otero, M. A. Fanaro, A. R. Córica, V. C. Llanos, P. Sureda & V. Parra (Eds.), *La teoría antropológica de lo didáctico en el aula de matemática* (pp. 61-72). Buenos Aires: Editorial Dunken.

Donvito, A., Fanaro, M. A., & Otero, M. R. (2017). Estudiantes adultos y sus representaciones sociales acerca de la escuela secundaria de adultos: Una exploración utilizando la técnica de grupos de enfoque. *Praxis Educativa, 21*(3), 68-76.

Donvito, A., Otero, M. R., & Fanaro, M. A. (2017). La utilidad de la matemática y su enseñanza en la escuela secundaria de adultos: El punto de vista de los estudiantes. *Perspectiva Educacional, 53*(3), 98-122.

Donvito, A., Otero, M. R., & Sureda, P. (2017). *Enseñanza por investigación en la Escuela Secundaria Explorando la relación entre la matemática y las finanzas personales*. Tandil, Buenos Aires: Universidad Nacional del Centro de la Provincia de Buenos Aires.

Eudave Muñoz, D. (2009). Niveles de comprensión de información y gráficas estadísticas en estudiantes de centros de educación básica para jóvenes y adultos de México. *Educación Matemática, 21*(2), 5-37.

Gravemeijer, K., Stephan, M., Lin, F., Julie, C., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal for Science and Mathematics Education, 15*(1), 105-123.

Gürtler, L., & Huber, G. L. (2007). Modos de pensar y estrategias de la investigación cualitativa. *Liberabit, 13*(13), 37-52.

Kim, S. (2015). *Les besoins mathématiques des non-mathématiciens quel destin institutionnel et social? Études d'écologie et d'économie didactiques des connaissances mathématiques*. Thèse doctorale, Université Aix-Marseille, France.

Lukianova, L. (2016). Motivation factors of adult learning. *The New Educational Review, 44*(2) 223-229.

Otero, M. R., Fanaro, M. A., Córica, A. R., Llanos, V. C., Sureda, P., & Parra, V. (2013). *La Teoría Antropológica de lo Didáctico en el aula de Matemática*. Buenos Aires: Editorial Dunken.

Ruiz Munzón, N. (2010). *La introducción del álgebra elemental y su desarrollo hacia la modelización funcional*. Tesis doctoral, Universitat Autònoma de Barcelona, Spain.

Secretaría de Evaluación Educativa (2016). *Aprender 2016 – Matemática – 2º / 3º año de Educación Secundaria. Ítems liberados*, Argentina.

Wedege, T., & Evans, J. (2006). Adults' resistance to learning in school versus adults' competences in work: The case of Mathematics. *Adults Learning Mathematics – an International Journal, 1*(2), 28-43.