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A new formulation of heat dissipation in a rocking Büttiker-Landauer ratchet model

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Abstract. Thermal ratchets achieve net particle transport through rectification of thermal fluctuations, which arise from one or more heat baths in the system. We propose a new formulation of heat dissipation from the ratchet to the thermal baths, using a rocking Büttiker-Landauer ratchet model. We found that heat transport between the ratchet and the heat baths is related to the effective temperature through the generalization of the fluctuation-dissipation theorem for systems far from equilibrium. We showed that the net heat transport between the ratchet and the heat baths is different from Fourier's law and is the sum of two terms which are proportional to the nth power of the difference between the effective temperature of ratchet and the temperature of the baths. The power n depends only on the temperature of the bath, while the thermal conductivity also depends on the ratchet potential. These findings suggest that anomalous heat dissipation can be a non-equilibrium measure for systems far from equilibrium.

1. Introduction

Fluctuations-induced transport has been extensively studied both experimentally and theoretically [1, 2, 3]. This process appears in simple stochastic models, such as thermal ratchets in which some of the energy in a nonequilibrium bath is transformed into work or into a current of particles at the expense of increased entropy [4, 5]. Ratchets or Brownian motors are kept away from equilibrium by external fluctuations or temperature gradients, which make them completely different from Carnot engines. Studies on the thermodynamics of these ratchet transport have been mainly focused on efficiency [6, 7, 8]. In this study, we focus on heat dissipation between the ratchet and the heat baths, because high efficiency may be achieved with considerable heat dissipation losses. Another reason for our focus on heat dissipation is that the entropy plays such an important role in fluctuations-induced transport processes that they have been associated with Maxwell's demon performance [9]. Heat transport is usually associated to the empirical Fourier's law, which states that the time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles

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to that gradient, through which the heat flows. The Fourier's law has been experimentally examined in virtually all 3D thermal conductors. Nevertheless, anomalous heat conduction was found in one dimensional (1D) momentum-conserving model systems, such as Fermi-Pasta-Ulam (FPU)- lattices [10] and the 1D diatomic Toda lattices [11]. Similar anomalous heat conduction behavior has also been predicted in momentum-conserving physical materials, such as in carbon nanotubes [12], silicon nanowires [13] and in polymer chains [14]. Experimental evidence of the violation of Fourier's law in nanotube thermal conductors has been obtained in [15]. On the other hand, normal heat conduction obeying Fourier's law has been obtained for 1D nonlinear lattice systems where the total momentum is not conserved, such as 1D nonlinear Frenkel-Kontorova [16] lattices and ϕ^4 lattices [17].

We propose in this work to relate the process of heat transport to the effective temperature through the generalization of the fluctuation-dissipation theorem on far from equilibrium systems. The effective temperature is defined by the comparison between induced and spontaneous fluctuations. Recently Harada [18] has investigated the energetic efficiency of a rocking ratchet at a nonequilibrium steady state as a function of the effective temperature. In that work, a rise of the effective temperature with respect to the bath temperature has been found that causes a decrease in the efficiency of the ratchet through irreversible heat dissipation to the bath. The effective temperature of these systems gives us relevant information about the distribution of the input energy and how much work can be extracted from the free energy.

We study the heat transfer on a thermal Büttiker-Landauer (BL) [19, 20] ratchet, with a rocking driving force and transport reversal. To calculate the effective temperature [21], we focus on the spontaneous dynamic fluctuations and the response (induced) fluctuations to a weak constant force on the ratchet. The current reversal is due to the presence of two heat reservoirs in the B-L ratchet model [22]. The rest of the paper is organized as follows. We describe the model in section 2 in detail. In section 3, the results are divided into two parts: the analysis of the mean current, power and effective temperature (subsection 3.1) and the heat conduction of the ratchet to the reservoirs (subsection 3.2). Finally, section 4 consists of concluding remarks.

2. The model

We consider an overdamped Brownian particle in a sawtooth potential subjected to an unbiased external force that pushes it left and right periodically (rocking ratchet). This model has been first proposed by Büttiker [19] and Landauer [20] to describe a molecular motor in biological systems [23]. The stochastic differential equation (Langevin equation) for such a particle is given by

$$\gamma \frac{dx}{dt} = -\frac{dV(x)}{dx} + \sqrt{2\gamma k_B T(x)}\xi(t) + F(t), \qquad (1)$$

where the left-hand side describes a frictional force experienced by the particle when it is moving relative to its environment, with γ being the drag coefficient. The first term of the right-hand side is the force due to the sawtooth potential $V(x)$, with period L such that $V(x) = V(x + L)$ described by

$$V(x) = \begin{cases} \frac{V_0}{L_1}x, & 0 < x < L_1 \\ \\ \frac{V_0}{L_2}(L - x), & L_1 < x < L \end{cases} \qquad (2)$$

where V_0 is the barrier height and L_1 and L_2 are the projections over the x-axis of both sides of the sawtooth as shown in Fig. 1(a). The second term of the right-hand side of Eq. (1) describes

the fluctuations of the ratchet with the surrounding through $\xi(t)$, a zero-mean, delta-correlated, Gaussian white noise i.e., $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$. Here, $\langle...\rangle$ denotes an ensemble average over the distribution of the fluctuating force $\xi(t)$. These fluctuations are imposed onto the particle by the interaction with two heat baths at different temperatures, one for each side of the potential given by

$$T(x) = \begin{cases} T + \delta, & 0 < x < L_1 \\ T, & L_1 < x < L \end{cases} \tag{3}$$

where $T(x)$ has the same period as the potential $V(x)$, that is $T(x) = T(x + L)$.
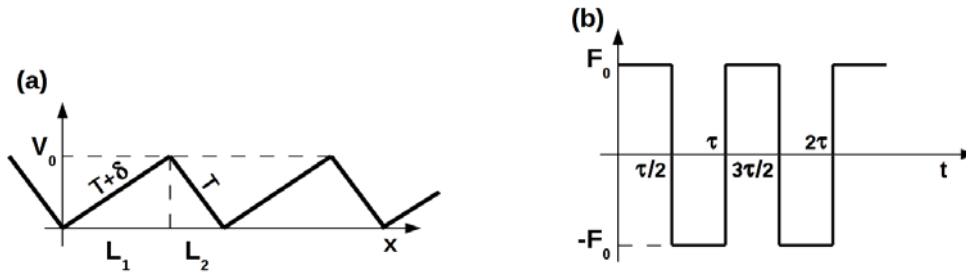

Figure 1. (a) Asymmetric potential $V(x)$ with period length $L = L_1 + L_2$ ($V(x + L) = V(x)$) amplitude V_0 and asymmetry $\Delta = L_1 - L_2$; $T + \delta$ and T indicate the temperature profiles. (b) Driving force $F(t)$ that pushes the particle left and right periodically, with temporal period τ and amplitude F_0.

Ultimately, $F(t)$ in Eq. (1) is an external driving force with a temporal period τ and amplitude F_0, that satisfies the condition $\int_0^\tau F(t)dt = 0$ (see Fig. 1(b)).

For simplicity and without loss of generality, we assume the drag coefficient $\gamma = 1$ and measure the energy in units of k_B ($k_B = 1$) and length in units of L ($L = 1$), respectively.

3. Results
3.1. Current, input power and effective temperature for the rocking B-L ratchet
According to the Einstein relation the ratio between independent measurements of fluctuations and the response to a weak external force on a particle such as, the diffusivity D and the mobility μ is equal to the thermal temperature T when the system is kept close to thermal equilibrium ($D/\mu = T$). The mobility and the fluctuations must be related, because both come from the same origin, the interaction of the particle with its surroundings.

The Einstein relation is a form of the fluctuation-dissipation theorem, strictly valid for an equilibrium thermal system or close to it. However, many out-of-equilibrium systems have shown that this relation gives rise to a well-defined effective temperature which is different from the bath temperature and governs the heat flow [24, 25, 26]. Indeed, in [21] have introduced the notion of effective temperature T_{eff} for glassy systems from a modification of the fluctuation-dissipation theorem (FDT) as

$$\frac{\chi}{MSD} = \frac{1}{2T_{eff}}, \tag{4}$$

where MSD and χ are the mean-squared displacement of the particle and the response function, respectively. The response function is the average displacement of the particle due to an weak

constant force divided by the amplitude of this force. Thus, the expressions for MSD and χ are given by

$$
\begin{aligned}
MSD &= \left\langle |x(t) - \langle x(t) \rangle|^2 \right\rangle \\
\chi &= \langle |x_h(t) - x_{h=0}(t)| \rangle / h
\end{aligned}
\tag{5}
$$

where h is the amplitude of the force.

Modified and ordinary FDT are equivalent, for systems in equilibrium (see Eq. (4)). Indeed, $MSD = 2Dt$ for a Brownian particle at a temperature T. On the other hand, if we pull gently on the particle with a constant force, its surrounding responds with a viscous, dissipative force and its average displacement divided by the force after a large time is $\chi = \mu t$. Then, the ratio $\chi/MSD = \mu/2D$. From the Einstein relation and Eq. (4), we arrive at $T_{eff} = T$, in thermal equilibrium. In general, for an out-of-equilibrium system in contact with a thermal bath, $T_{eff} \geq T$, when the system reaches a steady state [27]. However, for a very confined system, $T_{eff} < T$. In this model, confinement means that the particle cannot get out from the two nearest barriers of the ratchet potential. We will only focus on the case where the Brownian particle is not confined, that is when the particle goes through different barriers.
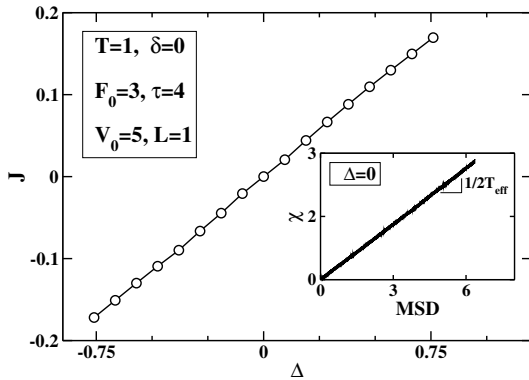

Figure 2. Current J as a function of the asymmetry parameter Δ of the potential for a single heat bath at both sides of it. The inset shows χ versus C, using the same parameter values as in the main plot and for $\Delta = 0$. All results in this paper are obtained for $\tau = 4$, chosen under the criterion described in the text above Eq. 6.
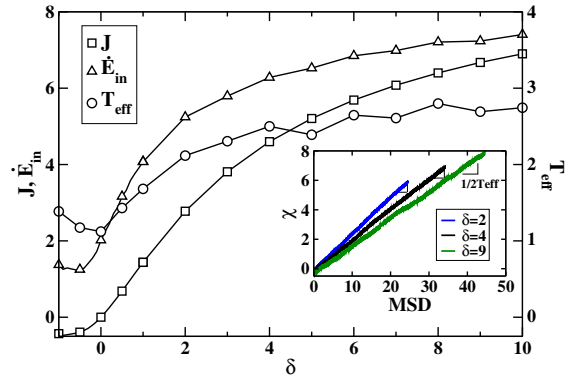

Figure 3. Current J, input power $\dot{E}_{in}$ and effective temperature T_{eff} versus temperature difference δ between the sides of a symmetric potential $\Delta = 0$, for $T = 1$, $F_0 = 3$ and $V_0 = 5$. (The left (right) scale corresponds to J and $\dot{E}_{in}$ (T_{eff})). Error bars are approximately the same size as the symbols except for T_{eff}, which they increase with the absolute value of δ (two error bars are shown as references). The inset shows χ versus C, using the same parameter values as in the main plot and for $\delta = 2, 4$ and 9.

In order to gain some insight on the fluctuations-induced transport of the model, in addition to effective temperature, we also compute the average current J and the input mean energy per unit of time $\dot{E}_{in}$, as functions of the different parameters of the model. The current J and the input mean power $\dot{E}_{in}$ have been obtained in [22, 28]. The average current over a fluctuation period τ of $F(t)$ is $J = \frac{1}{\tau} \int_0^\tau J(t) dt$. If the period τ of the external force is longer than any other

time scales of the system, there exists a quasi-steady state which leads to

$$
\begin{aligned}
J &= \frac{1}{\tau}\left(\int_0^{\tau/2} J(t)dt + \int_{\tau/2}^{\tau} J(t)dt\right) \\
&= \frac{1}{2}\left(J_+ + J_-\right)
\end{aligned}
\tag{6}
$$

where J_+ (J_-) is the current induced by the force $F(t)$ when it is equal to F_0 ($-F_0$) (see Fig.1(b)).

The input mean energy per unit of time or power $\dot{E}_{in}$ comes from the fluctuations generated by the external driving force $F(t)$ to the ratchet and is given by [8]:

$$
\dot{E}_{in} = \frac{1}{\tau}\int_0^{\tau} J(t)F(t)dt = \frac{F_0}{2}\left(J_+ - J_-\right).
\tag{7}
$$

Figure 2 shows the current J in the quasi-steady state as a function of the asymmetry parameter Δ of the potential, for a single heat reservoir ($\delta = 0$). J is an anti-symmetric function as expected, negative for $\Delta < 0$ and positive for $\Delta > 0$. When $\Delta = 0$, the current $J = 0$ however, the input mean energy per unit of time $\dot{E}_{in} \neq 0$. Moreover, we found constant $\dot{E}_{in}$ for every value of the asymmetry parameter Δ. Therefore, the energy consumption per unit of time is independent of the current.

The inset of Fig. 2 shows the response function χ versus the mean-squared displacement MSD, using the same parameter values as in the main plot and for $\Delta = 0$. χ and MSD are calculated from Eq. (5). To compute χ, we simulate two replicas of the system, with the perturbative force $h = 10^{-3}$ applied to one of them. The replicas evolve using the same random numbers. Then, the results are obtained averaging over $1.5 - 3 \times 10^5$ replicas. According to Eq. (4), the slope of χ vs. MSD in the inset of Fig. 2 is equal to $1/2T_{eff}$, when the ratchet reaches the steady state. Thus, we obtain $T_{eff} = 1.12 \pm 0.01$, for $\Delta = 0$, $T = 1$, $\delta = 0$, $V_0 = 5$ and $F_0 = 3$. Moreover, we find that T_{eff} is independent of the asymmetry parameter Δ of the potential, with one bath. We will return to this result in the next section.

Figure 3 shows the current J, the input power $\dot{E}_{in}$ and the effective temperature T_{eff} versus the temperature difference δ between the sides of a symmetric potential $\Delta = 0$, for $T = 1$, $F_0 = 3$ and $V_0 = 5$. The temperature difference δ controls the magnitude of these three variables and the direction in the case of the current ($\dot{E}_{in}$ and T_{eff} are by definition always positives). When $\delta = 0$, there is no current and both the input power and effective temperature are non-zero. Then, as δ increases, the three variables increase indefinitely, as shown in Fig. 3. Note that, T_{eff} appears to converge to an asymptotic value; however, it keeps increasing as δ increases.

The inset of Fig. 3 shows the response function χ versus the mean-squared displacement MSD, using the same parameter values as in the main plot and for three positive values of $\delta = 2, 4$ and 9. As δ increases, the straight line slopes in the inset decrease, revealing the increase of T_{eff} (see Eq. (4)).

Finally, we study the current J, the input power $\dot{E}_{in}$ and the effective temperature T_{eff} as a function of temperature T, for a combination of Δ and δ. As we mentioned previously, the current J for this model has been extensively studied in [22, 28]. Positive current, single or double reversals have been obtained for different combinations of Δ and δ. Two particular cases, $\Delta = 0.1, -0.8$, and fixed $\delta = 0.1$ with direct and single reversal current, respectively are shown in Fig 4(a). For $\Delta = 0.1$, $J > 0$, the motor always moves to the right (see Fig. 1(a)) following the regular behavior of a rocking ratchet, while for $\Delta = -0.8$, J may reverse its direction. Despite the differences between their currents, the input power $\dot{E}_{in}$ is the same for both cases as shown in Fig 4(a). $\dot{E}_{in}$ increases with T, up to a limit value in which the
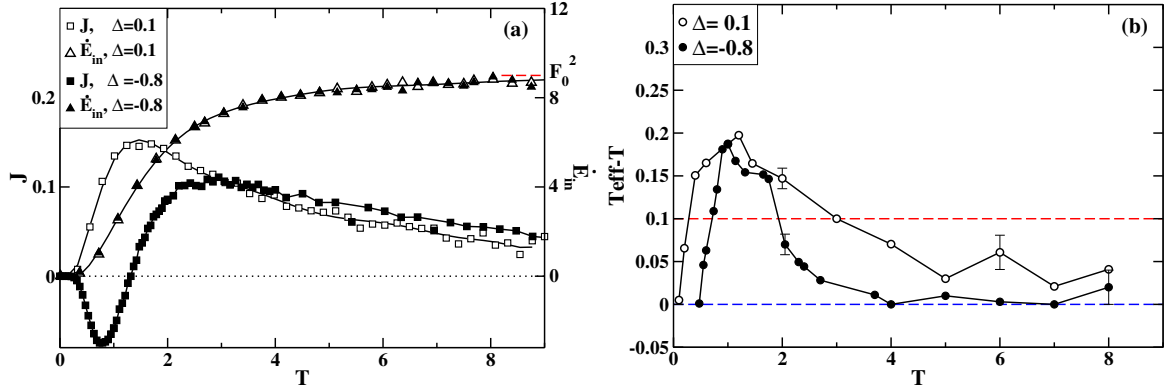
Figure 4. J, $\dot{E}_{in}$ and (b) $(T_{eff} - T)$ as functions of T, for $\delta = 0.1$, $F_0 = 3$, $V_0 = 5$, and two values of the asymmetry parameter Δ. The left (right) scale in (a) corresponds to J ($\dot{E}_{in}$). F_0^2 denotes the maximum asymptotic of $\dot{E}_{in}$. When $\Delta = 0.1$, $J > 0$, the ratchet behaves as a refrigerator, while when $\Delta = -0.8$, the ratchet behaves as heat machine (see section 3.2.2 for further details). The bottom (up) dashed line in (b) represents the cold (hot) bath at T $(T + \delta)$. Error bars are approximately the same size as the symbols in (a) and increase with the value of T in (b) (two error bars are shown as references for $\Delta = 0.1$ and -0.8, respectively).

motor moves as an overdamped Brownian particle periodically pushed to the left and right by the force F_0 (see Fig. 1(b)). Certainly, when the sawtooth potential does not offer resistance to the ratchet movement, $J = 0$ and $\dot{E}_{in} = F_0^2$.

Plots of the difference $(T_{eff} - T)$ versus T, for the two case studies, $\Delta = 0.1, -0.8$, and fixed $\delta = 0.1$ are shown in Fig. 4(b). In general, T_{eff} can be between the bath temperatures or can be greater than both of them (see $(T_{eff} - T)$ data between the dashed lines or above them in Fig. 4(b), where the bottom and upper dashed lines represent the cold and hot baths at T and $T + \delta$, respectively). As we have mentioned before, the motor always transfers a net heat in a steady state. The heat conduction of a system far from equilibrium mostly does not obey a linear transfer law with the temperature gradient. In the next section we will study the heat conduction starting with the simplest case in which there is only one thermal reservoir exchanging heat with the motor and continuing on the more general case of two thermal reservoirs. We will show that the Brownian particle can absorb heat from the cold reservoir and transfer heat to the hot one o vice versa, even when the effective temperature of the Brownian particle is greater than both reservoir temperatures, working as refrigerator or heat engine, respectively. In fact, the system is far from equilibrium continuously excited by the external agent $F(t)$, which may result in a heat flux against the temperature gradient concerned.

Note that, $\dot{E}_{in}$ is the same function of T for both Δ values, while T_{eff} is always greater for $\Delta = 0.1$ than for $\Delta = -0.8$ (see Figs. 4(a) and (b)). This last result is due to the difference of the relative contact time between the motor and the reservoirs. Indeed, in a steady state, the motor spends a longer time on the long side of the potential. Thus, for $\Delta = 0.1$, the motor spends more time in contact with the hot bath than the cold one and vice versa, for $\Delta = -0.8$, reaching a higher effective temperature in the first case.

3.2. Heat conduction for the rocking B-L ratchet

In general, a Brownian motor exchanges energy with three types of systems: the thermal baths, a load that transports, and the potential or external agent that is the source of nonequilibrium [7, 8, 29]. The flow of energy is presented in Fig. 5. The first law of thermodynamics leads to

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$$\dot{E}_{in} = \dot{W} + \dot{Q}_T + \dot{Q}_{T+\delta}, \tag{8}$$

where $\dot{E}_{in}$ is the input power, $\dot{W}$ is the work done by the ratchet against the load per unit of time and $\dot{Q}_T$ and $\dot{Q}_{T+\delta}$ are the transfers of energy per unit of time from the motor to the thermal baths at the temperature T and $T + \delta$, respectively. All these magnitudes are taken averaging over a fluctuation period τ (see previous section). In the stationary regime, $\dot{E}_{in}$ is given by the Eq. (7) and $\dot{Q}_T$ and $\dot{Q}_{T+\delta}$ can be positives or negatives, depending on the heat transferred to or from the bath, respectively.
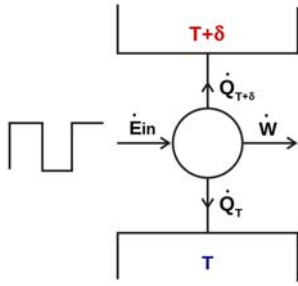

Figure 5. Energy flows in the motor (circle) subjected to a load and an external agent (square wave) in contact with two thermal baths at temperatures T and $T + \delta$.

In many experimental systems the heat transport responds to a linear expression with the temperature gradient, known as Fourier's law. We propose here, that the heat transport is a power-law of the temperature difference between the motor and the thermal bath,

$$\dot{Q}_T = c|T_{eff} - T|^{n_T}, \tag{9}$$

with c the thermal conduction coefficient and n_T an exponent that only depends on T.

Next, we will test this expression operating with one heat bath and between two different heat baths and also its valid range.

3.2.1. Operating with one heat bath In order to determinate the behavior of the heat transport with the temperature gradient, we begin with the simplest case: a particle without load moving in a symmetric potential, with only one heat reservoir (i. e. $\dot{W} = 0$, $\Delta = 0$ and $\delta = 0$). In this case, the particle transfers heat to the reservoir and taking into account Eqs. (7) and (8), the heat flux is given by $\dot{Q}_T = \frac{F_0}{2}(J_+ - J_-)$. Fig. 6 (a) shows $\dot{Q}_T$ as a function of $(T_{eff} - T)$, in log-log scales, for three different temperatures. The heat flux from the particle to the reservoir follows a power-law as predicts Eq. (9) with an exponent n_T decreasing with the bath temperature T (n_T is provided in the inset of Fig. 6(a), for different T). This exponent is independent of the barrier height, while the coefficient c in Eq. (9) depends on T and V_0 as shown in Fig. 6(b). The Eq. (9) is valid for low T and $F_0 \leq 2V_0$, that is when the B-L ratchet is working (i.e. $F_0 \leq |dV/dx|$, for all x). The range of T within which the Eq. (9) is valid is limited by the condition that the B-L ratchet is working.

3.2.2. Operating between two different heat baths When the system is working with two different heat baths the net heat transferred from a Brownian particle to two reservoirs at T and $T + \delta$ is

$$\begin{aligned} \left(\dot{Q}_T + \dot{Q}_{T+\delta} \right) = \quad & c_1 \quad |T_{eff} - T|^{n_{T+\delta}} + \dot{Q}_V \\ + \quad & c_2 \quad |T_{eff} - T|^{n_T} - \dot{Q}_V, \end{aligned} \tag{10}$$
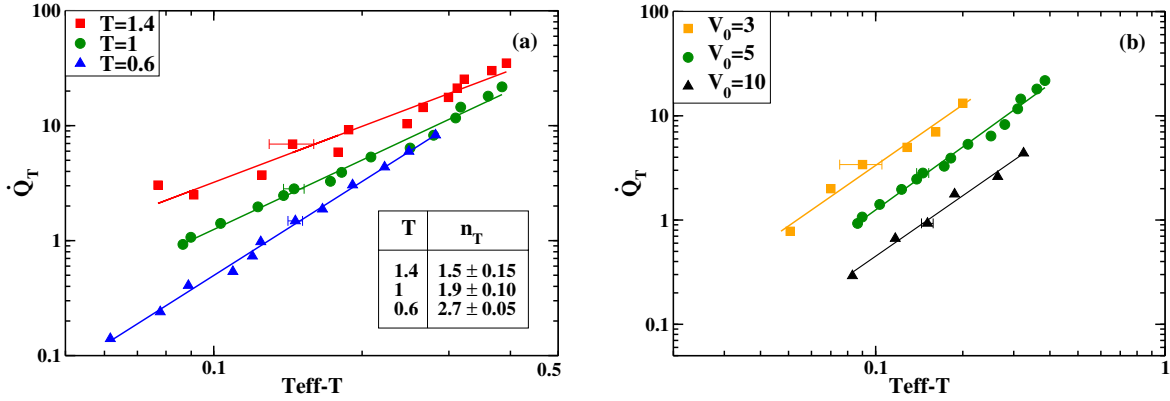
Figure 6. $\dot{Q}_T$ against $(T_{eff} - T)$, in log-log scales, for a ratchet operating with one heat bath, $\delta = 0$, $\Delta = 0$ and different values of V_0 and T: $V_0 = 5$ (a) and $T = 1$ (b). The inset in panel (a) shows the exponents n_T obtained from the slopes of straight lines. The vertical error bars are approximately the same size as the symbols while the horizontal ones increase with the value of T in (a) and decease with the value of V_0 in (b) (three error bars are shown as references in each panel).

where c_1 and c_2 depend on the bath temperatures and the sawtooth potential parameters, $\dot{Q}_V$ is the heat flux due to the ratchet force at the $T + \delta$ side and $-\dot{Q}_V$ is the heat flux due to the ratchet force at the T side. These two heat fluxes cancel each other to obtain the net heat transfer [29].

Fig. 7 shows the net heat flux $\left(\dot{Q}_T + \dot{Q}_{T+\delta} \right)$ from a motor without load to two thermal reservoirs versus $(T_{eff} - T)$, in log-log scales, for $T = 0.9$ and $T + \delta = 1$. The solid line corresponds to the best fit of the data using Eq. (10), with the exponents $n_{0.9} = 2.5$ and $n_1 = 1.9$. A good agreement between simulated data and Eq. (10) is achieved. It must be pointed out that these same exponents are obtained for a ratchet operating with one heat bath at the temperatures $T = 0.9$ and 1, respectively (for the sake of space, only the exponent n_1 is shown in Fig. 6(a)).
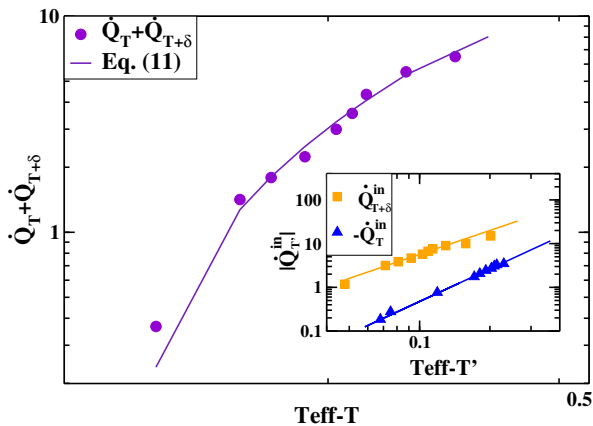

Figure 7. $\dot{Q}_T + \dot{Q}_{T+\delta}$ against $(T_{eff} - T)$, in log-log scales, for a ratchet operating between two different heat baths, $T = 0.9$, $\delta = 0.1$, $\Delta = -0.8$ and $V_0 = 5$. Symbols correspond to model results and solid line corresponds to the best fit of the results using Eq. (10) and subject to the exponents $n_{0.9} = 2.5$ and $n_1 = 1.9$. The inset shows $|\dot{Q}_{T'}^{in}|$ versus $(T_{eff} - T')$, in log-log scales, for $T' = 0.9$ and $T' = 1$. Symbols are obtained using the currents induced in each side of the sawtooth potential (see text). The slope of the blue (red) straight line is equal to $n_{0.9}$ (n_1).

The net flux $(\dot{Q}_T + \dot{Q}_{T+\delta})$ is obtained using the induced currents in each side of the sawtooth

potential. According to Eq. (8), the input power due to the external agent $\dot{E}_{in}$ is directly the sum of the heat fluxes from the motor without load to the thermal reservoirs. On the other hand, the integral of Eq. (7) can be splitted into two parts related to the external driving force on each side of the sawtooth potential and thus arrive to the expression $\dot{Q}_{T'}^{in} = \frac{F_0}{\tau}\left(\tau_+^{T'} J_+^{T'} - \tau_-^{T'} J_-^{T'}\right)$, where $T' = T, T + \delta$, $J_+^{T'}$ $(J_-^{T'})$ is the current in the side of the sawtooth potential at temperature T' induced by the external agent $F(t)$ when it is equal to F_0 $(-F_0)$ and $\tau_+^{T'}$ $(\tau_-^{T'})$ is the time it takes for the particle to travel on the side of the sawtooth potential at T' while $F(t) = F_0$ $(-F_0)$. Note that, the currents $J_\pm$ are recovered through the weighted means $J_\pm = \frac{\tau_\pm^T}{\tau} J_\pm^T + \frac{\tau_\pm^{T+\delta}}{\tau} J_\pm^{T+\delta}$.

The inset of Fig. 7 shows $|\dot{Q}_{T'}^{in}|$ calculated from the induced currents versus $(T_{eff} - T')$, in log-log scales, for the ratchet operating between two different heat baths at $T' = 0.9$ and $T' = 1$ (see previous paragraph). This figure confirms that, indeed, the heat fluxes related to the external driving force follow a power-law to each side of the sawtooth potential namely, $\dot{Q}_{T'}^{in} = \frac{F_0}{\tau}\left(\tau_+^{T'} J_+^{T'} - \tau_-^{T'} J_-^{T'}\right) = c'|T_{eff} - T'|^{n_{T'}}$, with $T' = T, T + \delta$ and the sum $\left(\dot{Q}_T + \dot{Q}_{T+\delta}\right)$ is given by Eq. (10). This expression is valid for two reservoirs at temperatures close to each other and under the conditions of Eq. (9).

Note that, $\dot{Q}_{T=0.9}^{in} < 0$ while $\dot{Q}_{T+\delta=1}^{in} > 0$. Nevertheless, when the heat transfer related to the ratchet force is considered, the ratchet absorbs heat from the reservoir at $T + \delta = 1$ and transfers heat to the other one at $T = 0.9$, working as a heat engine. The ratchet can also work as a refrigerator or transferring heat to both reservoirs. In Fig. 4(b) the different behaviors of the ratchet depend on the asymmetry parameter Δ of the ratchet: Heat engine for $\Delta = -0.8$ and refrigerator for $\Delta = 0.1$.

4. Conclusions

In this work, we study numerically the heat transfer from a Büttiker-Landauer rocking ratchet to the heat reservoirs, and find that the net heat transfer to both heat sources depends on the difference of the effective temperature T_{eff} and each reservoir temperature T. We examine different setting of the parameters that govern the ratchet transport. The B-L ratchet in an asymmetric potential with an external driving force with two thermal baths can reach a T_{eff} that is between the bath temperatures or greater than both of them. This higher effective temperature is possible because of the external agent that sustains the system far from equilibrium. Moreover, the ratchet motor at T_{eff} can absorb heat from a reservoir at temperature $T < T_{eff}$ or transfer heat to a reservoir at $T > T_{eff}$. Interestingly, the directions of the heat fluxes between the ratchet and the two reservoirs depend on the asymmetry parameter of the potential, independent of the value of T_{eff}. The net heat transfer to the heat reservoirs is the sum of the heat fluxes related to the external force for both sources $\dot{Q}_T^{in}$ and $\dot{Q}_{T+\delta}^{in}$. The directions of these heat fluxes are independent of T_{eff}, which only regulates their magnitudes. We found that the magnitudes of these heat transports from the motor to the reservoirs are proportional to the n_T power of the difference between T_{eff} of the motor and the temperature of the bath concerned. The power n_T depends only on the temperature of the bath, while the thermal conductivity coefficient also depends on the ratchet potential (the barrier height of the potential and its asymmetry for the case of two reservoirs). The exponent n_T of this power-law increases as the bath temperature decreases, reaching values much greater than one. Thus, the net heat conduction, far from being linear with the temperature gradient as is the phenomenological Fourier's law is a non-equilibrium measure provided by the value of the exponent. Finally, this expression for the net heat conduction of the ratchet still holds true when the temperatures of the thermal baths are low enough and close to each other, and when amplitudes of external energy input are lower than or in the same order of magnitude as the barrier height of the potential.

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