

PAPER

## Close Encounters with Heisenberg: uncertainty in secondary school

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# Close Encounters with Heisenberg: uncertainty in secondary school

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## Abstract

In this paper we present a pedagogical strategy to introduce the Heisenberg uncertainty principle to high school students. The basis on which this proposal relies is the Fourier transform, connecting the quantum function in the  $x$  domain,  $\Psi(x)$ , with the function in the wave number domain,  $A(k)$ . This mathematical relationship directly leads to the uncertainty principle: a state localized in the former domain will present an inversely proportional dispersion in the latter domain. Since in acoustic we have an analog situation we can carry out experiments with sounds leading students to the understanding of the uncertainty relation. In our approach a slightly modified musical theme from the Steven Spielberg movie ‘Close Encounters of the Third Kind’ is the starting point to reach the uncertainty relation.

## 1. Introduction

The uncertainty principle (UP) is one of the milestones, often used in didactics, marking the break-up with former theories describing phenomena at the macroscopic level. Its physical meaning has been deeply discussed along the quantum mechanics (QM) evolution and currently various approaches for describing the same principle have not yet come to a common ground. Investigations

and discussions about the mathematical features behind the UP, and the relation with its physical meaning, is probably the best ground where to bring students to a more conscious understanding of QM and, more generally, of physics as well. The UP arises from the wave nature of quantum systems which, in turn, emerges from the Schrödinger equation. The validity of the superposition principle leads to the possibility

to place side by side two functions related by a Fourier transform:  $\Psi(x)$  and  $A(k)$ . From a didactic point of view this fact is a striking foothold that cannot be missed. Since in acoustics we have a similar scenario (sounds in the time domain,  $\Psi(t)$ , related by a Fourier transform to functions in the frequency domain), according to the teaching with analogies (TWA) strategy introduced by Glynn, Duit, and Thiele [1], we will work with a musical theme to investigate the UP.

## 2. The teaching with analogies strategy

The term analogy refers to the process of transferring information or meaning from a particular object (the analogue or source) to another particular object (the target). The goal is to transfer ideas from a familiar concept (the analogue) to an unfamiliar one (the target) by mapping their relationship. In this framework we propose to carry out a simple experiment with sound to investigate consequences of the wave nature of quantum systems.

## 3. The research question

We have designed a method to teach the UP at the high school level based on the analogy between sound and quantum phenomena. As we will discuss below, the method explicitly avoids the association of the UP with measurement processes, as e.g. in the single slit experiment and in the perturbation induced by the measuring process approaches. We will explain how and why our didactic strategy should avoid common misconception about the Heisenberg principle [2] as well as how it opens up the possibility to introduce students to the core of QM.

## 4. The starting point

The Heisenberg UP refers not to the precision and disturbance of a measurement [3, 4], but to an intrinsic property of a quantum system. It holds regardless of any measurement on the system.

We thus think that any effort to introduce it involving a measuring process in order to define the position of a particle (for example the detection of a scattered photon) should be avoided. Some authors [5], although choosing the measurement approach, highlight the fact that also an ideal measuring device (sending a single photon

on the system), would cause a disturbance on the measured system. Even if this approach does not involve anymore the imperfection of the apparatus, it is still linking the UP to a measurement process, thus losing the possibility to reach the deep root of the principle itself.

The approach to the Heisenberg UP proposed in this work is not connected to the measurement disturbances. It goes straight to the core of the principle that involves the dispersion product of two wave-functions connected by a Fourier transform or, if we prefer, of two wave-functions of conjugate quantities. The experiment we propose simply investigates relations between dispersions of a classical system (a sound) in two different domains (time and frequency) and offers a way to move toward position and momentum dispersions.

The UP is also often addressed through the analysis of the single slit experiment. This approach gives the valuable possibility to engage students in hands-on activities performing a simple and low-cost experiment with a laser. However, the strategy does not fulfill the expected requirements. Again we lose the chance to introduce the principle as a quantum intrinsic aspect. The slit experiment shows properties arising from the interaction of the particle and the slit, beyond any doubt, is a scattering process. The formulation of the Heisenberg's Principle that we can achieve working on the electron diffraction pattern is a consequence of an interaction rather than an intrinsic property of the electron. Thus, at least, it should be pointed out that the obtained relation is not referring to the particle but to the system 'electron + slit'. In a sense the slit experiment is again a measuring process. Common assertions such as 'the spatial uncertainty  $\Delta x$  of the electron passing the slit is the slit width' sound bizarre. Students should accept that the  $\Delta x$  of the electron changes sharply when passing the slit. This could lead students to imagine a sudden change in the spatial dimension of the electron while passing the slit, without any given explanation.

## 5. Uncertainty from sound waves: the 'Close Encounter' approach

In acoustics, a sound  $\Psi(t)$  is connected by a Fourier transform to its spectrum  $\hat{\Psi}(f)$ . The

sound we have chosen is the five tone theme used by humans to communicate with the extra terrestrial intelligences in the Steven Spielberg's movie 'Close Encounters of the Third Kind'. A slight modification has been introduced: each of the five pure tones has been modulated in time with a Gaussian function and mathematically 'synthesized' with the software Mathematica (figure 1).

Five modulations, each with a different standard deviation in order to have a set of five differently dispersed sounds (figure 2).

Performing the spectrum (figure 3) of each tone and measuring, for each sound, the frequency dispersion  $\Delta f$  and the time dispersion  $\Delta t$ , we get a set of five data pairs showing the uncertainty relation  $\Delta t \cdot \Delta f = \text{const}$ . In the following section we show how to perform the analysis and which software was used.

## 6. Activities with the students: the analysis of the theme

We performed the analysis of the five-note tonal phrase (downloadable from the site PhE [6]) with the free software Praat [7]. Praat allows one to analyze sounds both in the time (figure 2) and frequency (figure 3) domains. The available tools for the time analysis allow one to measure the five time dispersions  $\Delta t$  of each note.

In figure 4, we zoomed in a single note and measured the time distance between the top of the intensity profile and the point 20 dB under the top level. We acted on the intensity line (lower plot in figure 4) expressed in dB. Clicking on any point of the plot returns the intensity level (dB) and the time value (s) of the point. In a similar way we zoomed in on a single spectrum peak and measured the frequency dispersion  $\Delta f$  as the frequency distance between the top of the spectrum and the point 20 dB below (figure 5).

This process, repeated five times, gives the following data table (table 1), showing the constant value of the dispersions product:  $\Delta t \cdot \Delta f = \text{const}$ .

Data was also fitted with a one parameter function  $y \cdot x = k$ ; the resulting plot (figure 6) shows the good agreement of the data set with the mathematical model (the parameter value in this case  $k = 0.365$ ).

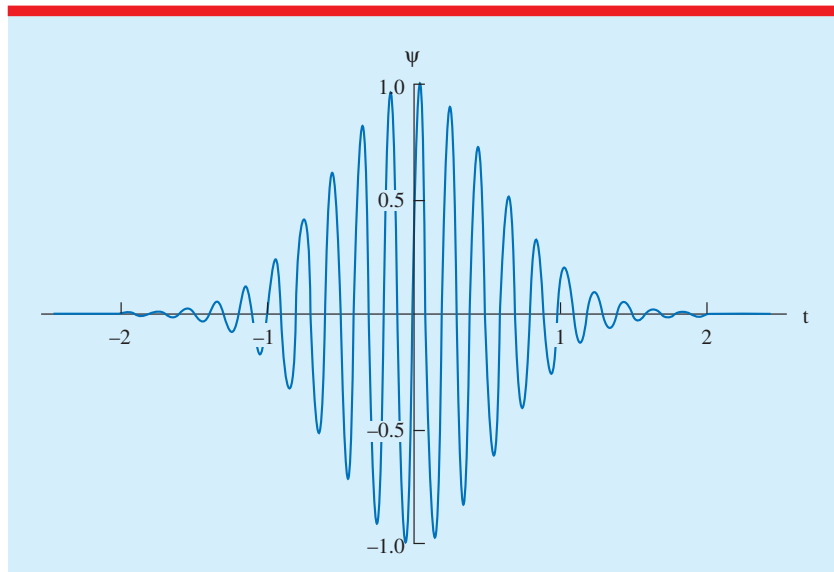
## 7. The 'Close Encounter' approach: dispersion, not precision

Introducing the UP with the 'Close Encounter' activity offers the possibility to engage students in critical thinking. In our opinion this is a crucial point in the learning process, in order to avoid a blind belief in what is taught and written in textbooks. In the proposed approach the deltas ( $\Delta t$ ,  $\Delta f$ ) may be considered as dispersions of the physical system instead of measurement uncertainties. We can see each note of the musical theme as a system spread over time. We do not have any problem in considering a system dispersed over space; think of a ball, for example, (figure 7). Thus, it is not difficult to understand that the measurement uncertainty  $\delta x$  in the definition of the position of such a system is not related to its spatial dispersion  $\Delta x$ . To find out the position of the ball we can choose a reference point P and measure it with a certain precision  $\delta x$ , which is conceptually separated from its dispersion  $\Delta x$ . In our experiment we may argue that we have the same situation described in figure 7, just transposed in time.

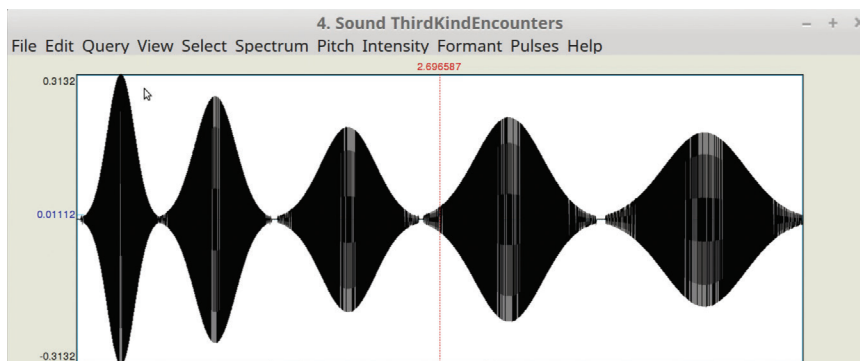
We are no longer dealing with measurement uncertainties but with dispersions in two different domains. The dispersion  $\Delta f$  of each sound has a meaning very close to that of the spatial dispersion of our spatial example: each sound may be considered as composed of many harmonics with frequencies spanning a range of the order of  $\Delta f$  (the same is true for the ball, composed of many parts displaced along  $\Delta x$ ). We thus have arrived to a meaning of the Heisenberg principle which differs from what is, almost everywhere, written: 'it is impossible to measure at the same time and with the desired precision the position and the momentum of a particle. These uncertainties are related by an inverse proportionality'. Our proposal rather leads to this statement: 'a quantum system has a dispersion in a position that is inversely proportional to the dispersion in momentum'.

## 8. The hydrogen atom as a clarifying example

We can consider the radial wave function of a hydrogen atom to show what we mean when we stress the difference between these two concepts:



**Figure 1.** Gaussian burst: a sine wave modulated by a Gaussian function.



**Figure 2.** The five-note theme as a function of time.

dispersion and measurement precision. For the sake of simplicity we will examine the ground state radial wave function ( $n = 1, l = 0$ ):

$$\Psi(r) \propto e^{-|r|}. \quad (1)$$

This plot (figure 8) clarifies the radial dispersion of the quantum state that we call the hydrogen atom, it is just a matter of patience to evaluate its dispersion in terms of the standard deviation. The quantum state has a spatial dispersion related to the momentum dispersion by the same connection linking the sound to its spectrum. Again, the smaller the spatial dispersion of the system the bigger the dispersion in momentum. At  $n = 2$  the hydrogen radial wave function suffers a larger spatial dispersion (figures 9 and

10) and as a consequence a smaller dispersion in the momentum. This is true regardless of any measurement on the atom, we even do not look at it; the property intrinsically belongs to the system.

We can also support these considerations evaluating the Fourier transform of the expression (1), which is a Lorentzian function (figure 11) in the momentum domain with a dispersion strictly related to the decay constant of the exponential radial wave function. A large value for the decay constant (small spatial dispersion) implies a huge dispersion of the Lorentzian function in the momentum. These computations are easily performed on-line with the computational knowledge engine Wolfram Alpha [8].

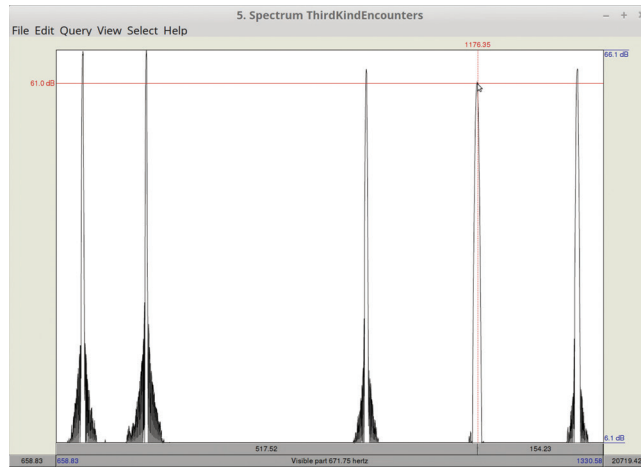


Figure 3. The spectrum of the five-note theme as a function of frequency.

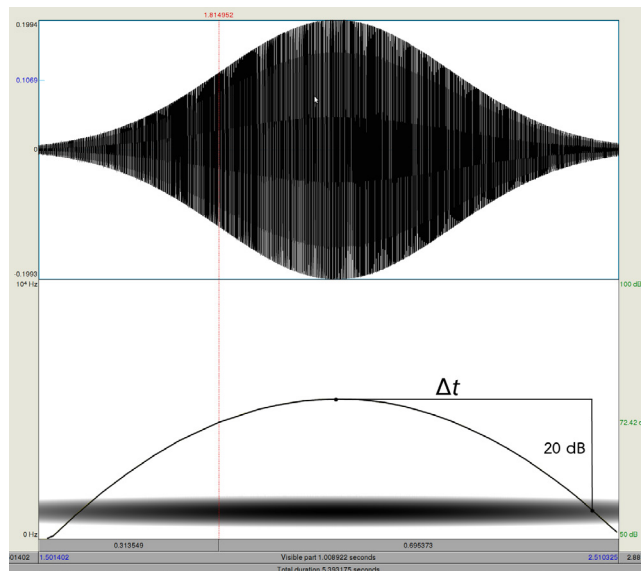
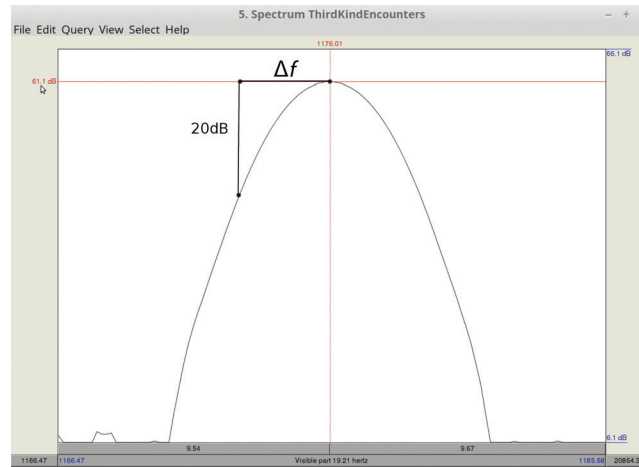


Figure 4. Time dispersion of a single note.

### 9. The ‘Close Encounter’ approach in the classroom context

Our approach to the UP relies on an acoustic analogy. The analogy in turn is based on the wavelike behavior of matter. Thus, before adopting this educational strategy in a classroom, students have to be familiar with some aspects related to the connection between waves and matter as well as to the physics of sound. Here we itemize the main concepts students have to be familiar with; also, we give some hints on how to reach these goals.

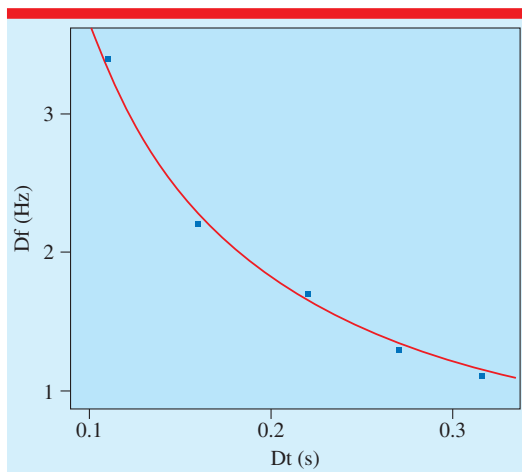
- *Wavelike behavior of matter.* In order to give plausibility to the acoustic analogy we need to give students clues about the relation between matter systems and waves. We briefly propose one possible approach to this issue. The double slit experiment performed with a laser provide the first step to familiarize students with the wavelike behavior of light (one possible approach to the explanation of this experiment is well described in [9] and is based on Feynman path integrals). Then, a discussion of the results of the



**Figure 5.** Frequency dispersion of a single note.

**Table 1.** The data set.

$\Delta t$ (s)	$\Delta f$ (Hz)	$\Delta t \cdot \Delta f$
0.11	3.4	0.37
0.16	2.2	0.35
0.22	1.7	0.37
0.27	1.3	0.35
0.32	1.1	0.35

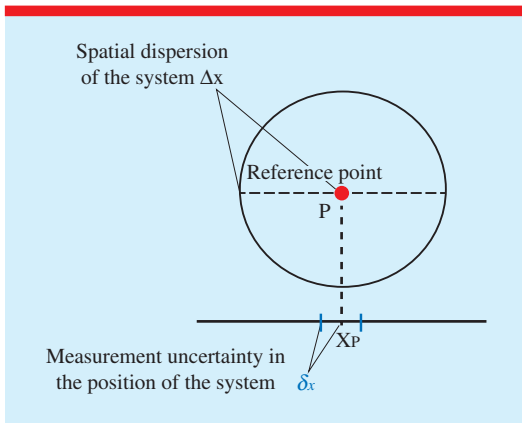


**Figure 6.** Plot showing the inverse proportionality relation between dispersions in time and frequency.

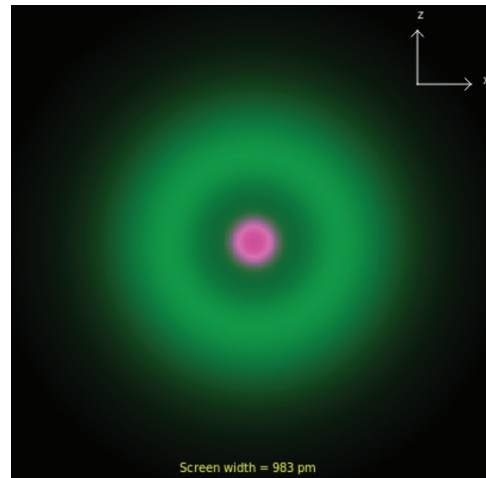
Davisson–Germer experiment (1928) [10] gives the first hint to establishing a relation between matter and waves and confirms the De Broglie hypothesis ( $\lambda = \frac{h}{p}$ ). Since it is easy and cheap to carry out a laser experiment directly showing interference patterns, the Davisson–Germer experiment, carried

out with electrons, builds a strong connection among these apparently distant fields.

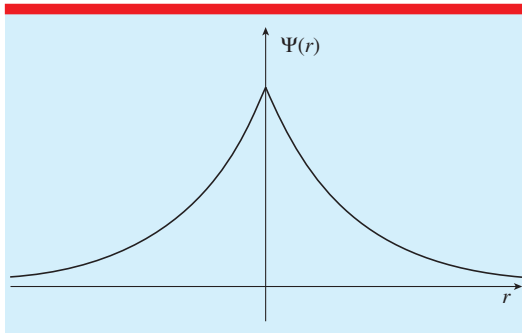
- *The spectrum of a sound.* The Fourier transform is the theoretical concept at the basis of the approach we are presenting and students will use it as an analysis tool. Still, we do not suggest the introduction of any mathematical formalism in the classroom. The only important pedagogical step that we propose is to make them familiar with the fact that a sum of harmonic sounds is a sound itself perceived as a whole. A huge set of apps [11] available for Android make it very simple to have as many sine waves as we want played with the chosen amplitude, duration and frequency. We can imagine each single student as a sine wave generator and the teacher as a recorder (again with a suitable app). Computing the spectrum of the recorded sound students will understand, through an engaging activity, that it is possible to reconstruct all the harmonic components they introduced before. Furthermore we can let students ‘play’ with the interactive simulation ‘Fourier: making waves’ designed by the University of Colorado [12].
- *From time-frequency to position-momentum.* Finally, it is necessary to switch from the time-frequency domain to the momentum-space domain, in order to reach the usual relation of the Heisenberg principle for quantum systems. One way to achieve this goal relies on the parallelism between time and position. As in the  $t$  domain we have



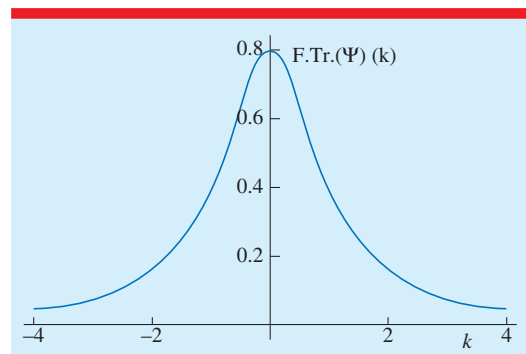
**Figure 7.** Difference between the measurement uncertainty in the position of the system ( $\delta x$ ) and the spatial dispersion of the system ( $\Delta x$ ).



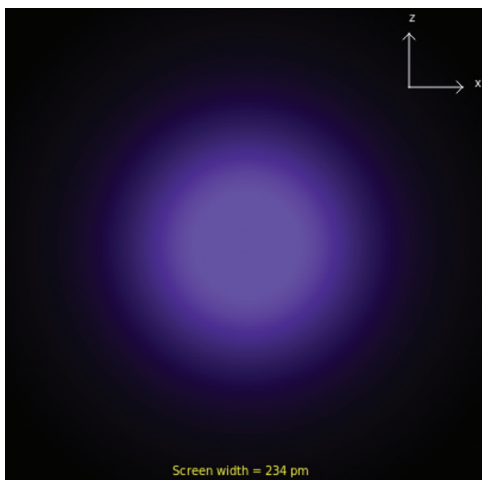
**Figure 10.** The 2s hydrogen orbital (radial scales in yellow).



**Figure 8.** Radial wave function for the Hydrogen atom; ground state ( $n = 1, l = 0$ ).



**Figure 11.** The Fourier transform of the 1s radial wave function (1) of the hydrogen atom.



**Figure 9.** The 1s hydrogen Orbital (radial scales in yellow).

a periodicity  $T$  strictly related to the frequency  $f \propto 1/T$ , in the  $x$  domain we have a periodicity  $\lambda$  strictly related to the wave number  $k \propto 1/\lambda$ . From a mathematical perspective there is no difference between periodic function in time or position. So, in  $x$  and  $k$ , we expect a similar uncertainty relation:

$$\Delta x \cdot \Delta k = \text{const.} \quad (2)$$

Since in QM the wave number is related to the momentum (De Broglie relation) we can easily reach the relation

$$\Delta x \cdot \Delta p \propto h. \quad (3)$$



## 10. Conclusions

We presented a didactic approach based on the sound waves analogy to introduce the UP. A set of Gaussian-shaped notes, based on the ‘Close Encounters’ theme, is given as the experimental dataset. This allows the students to directly test the UP with simple Fourier transform open-source software. This approach presents the UP as an intrinsic property of any quantum system and is given as an alternative to the slit experiment or measurement interactions methods. We think this approach highlights the fundamental properties of QM, i.e. the intrinsic dispersion of a quantum state. Furthermore this educational strategy offers the possibility to start an open discussion with students from the comparison of what is written on textbooks (at least in Italy) with the conclusions the experimental activity leads to.

At the moment the ‘Close Encounter’ method has not been tested with any sample of students. To know if this proposal is viable and produces the expected results, we will soon implement it in high school classrooms, then we will present the results.

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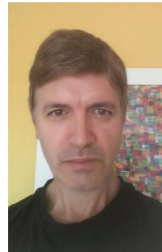
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## References

- [1] Glynn S M, Duit R and Thiele R B 1995 Teaching science with analogies: a strategy for constructing knowledge *Learning Science in the Schools: Research Reforming Practice* ed S M Glynn and R Duit (Mahwah, NJ: Erlbaum) pp 247–73
- [2] Bungum B, Henriksen E, Angell C, Tellefsen C and Be M 2015 ReQuant improving teaching and learning in quantum physics through educational design research *Nord. Stud. Sci. Educ.* **11** 153–68
- [3] Rozema L A, Darabi A, Mahler D H, Hayat A, Soudagar Y and Steinberg A M 2012 *Phys. Lett. Rev.* **109** 100404
- [4] Robertson H P 1929 *Phys. Rev.* **34** 163
- [5] Valentzas A and Halkia K 2011 *Res. Sci. Educ.* **41** 525539
- [6] PhE 2018 <https://goo.gl/Hw9AHB>
- [7] Boersma P and Weenink D Institute of Phonetic Sciences—University of Amsterdam ([www.fon.hum.uva.nl/praat/](http://www.fon.hum.uva.nl/praat/))
- [8] Wolfram Alpha ([www.wolframalpha.com/](http://www.wolframalpha.com/))
- [9] Fanaro M, Otero M R and Arlego M 2012 Teaching basic quantum mechanics in secondary school using concepts of Feynmans path integrals method *Phys. Teach.* **50** 156–8
- [10] Davisson C J and Germer L H 1928 Reflection of electrons by a crystal of Nickel *Proc. Natl Acad. Sci. USA* **14** 317322
- [11] Wave, Daniele Verducci 2012
- [12] Fourier: Making Waves, Colorado University Boulder (<https://phet.colorado.edu/en/simulation/fourier>)



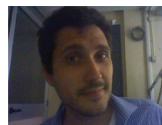
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