Refractive index calculation from echo interference in pulsed terahertz spectroscopy

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A method for calculating the average refractive index of a sample using transmission terahertz time-domain spectroscopy in a single measurement is presented. The refractive index is derived from the analysis of the frequency-domain interference caused by Fabry-Pérot reflections in the sample through the discrete Fourier transform of the spectrum module. This approach is simple and fast (not requiring a reference measurement), improves sensitivity over direct time-domain analysis and allows the derivation of the refractive index in a specific frequency window. The method can also be used to identify unwanted reflections in the experimental setup.

Introduction: The terahertz (THz) band remains an under-exploited region of the electromagnetic spectrum. However, this band holds signatures of a wide range of physical processes and many materials of industrial interest (e.g. polymers, paper, wood and crystals) are transparent in the THz showing refractive indices practically constant with the frequency. This allows their non-destructive inspection by analysing the changes in the refractive index value.

Usually, the refractive index is obtained from the phase of the transmission coefficient, which is the phase of the spectra ratio detected with and without the sample [1]. However, the need of a reference measurement makes the process slower, more complex and therefore difficult to apply online inspection.

Another approach is deriving the average refractive index directly from the time delay between main pulse and echoes [2]. However, these echoes can be difficult to measure due to noise and signal attenuation limiting the benefits of this approach.

In this Letter, a method for measuring the average refractive index with a transmission THz time-domain spectrometer (THz-TDS) in a single measurement is proposed. It is based on taking a time window that includes the main pulse and, at least, one of its echoes and processing the frequency-domain Fabry-Pérot interference caused by reflections in the sample. It is shown that by simply applying the absolute value of the discrete Fourier transform (DFT) of the THz spectrum module, it is possible to determine the refractive index with improved discrimination and without the need for a reference signal for materials with almost constant refractive indices in the THz band (polymers, paper, wood, silica etc.) [2].

Method: The data processing carried out to obtain the average refractive index is shown in Fig. 1. First, the main pulse and at least one echo are measured ($I(t)$). By applying the absolute value of the DFT, a modulation is obtained with a period that depends on the phase shift between the transmitted and reflected signals ($X(f)$), and finally, the absolute value of the DFT of the frequency signal is made to identify the components that caused the modulation, achieving $Y(t)$. From the temporal value k, the refractive index can be obtained. When multiple echoes are considered in $I(t)$, the final result $Y(t)$ will present harmonic peaks of K of decreasing amplitude.

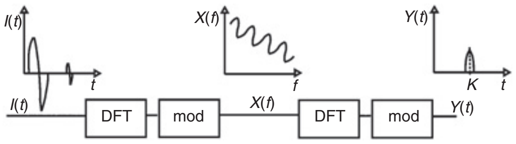

Fig. 1 *Block diagram of data analysis based on double DFT processing to estimate refractive index*

Theory: A continuous signal is considered in the analysis for the sake of simplicity. The spectrum of the electric field at the output of the sample can be written as [1]

$$E_T(\omega) = |E_i(\omega)| e^{(j\phi_i(\omega))} \left(\frac{4n(\omega)}{(n(\omega)+1)^2} \right) e^{(-jn(\omega)\omega d/c)} \times \sum_{m=0}^{+\infty} \left(\frac{(n(\omega)-1)^2}{(n(\omega)+1)^2} e^{(-j2n(\omega)\omega d/c)} \right)^m \tag{1}$$

where $|E_i(\omega)| e^{(j\phi_i(\omega))}$ is the incident electric field, c is the speed of light, d is the sample thickness, n is the refractive index and m is an integer representing the number of echoes produced by reflections in the material.

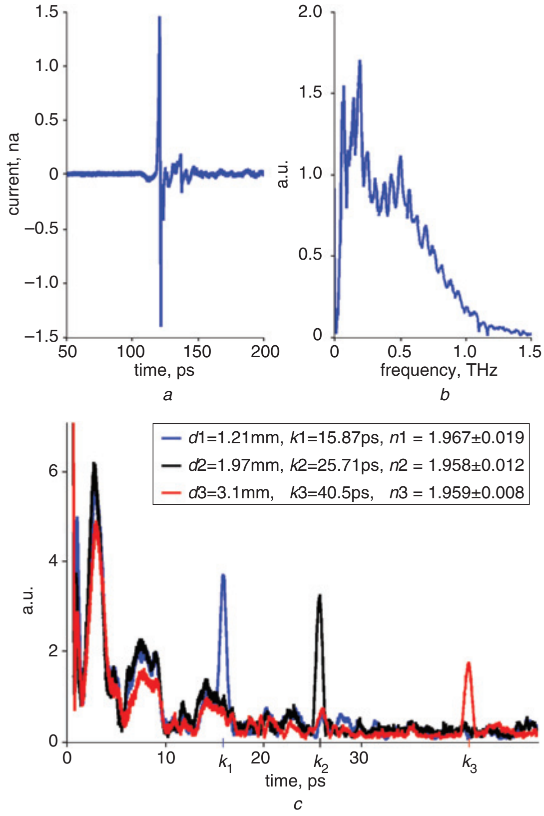

Fig. 2 *Measurements of fused silica plates*

a Temporal signal from spectrometer with fused silica width of 1.21 mm
b Frequency signal after applying absolute value of DFT from Fig. 1*a*
c Blue line corresponds with spectral analysis of Fig. 1*b*; black and red lines to same spectral analysis of samples of 1.97 and 3.1 mm, respectively

Assuming that both the refractive index and $|E_i(\omega)|$ are constant in the frequency domain, the steps of the processing described in Fig. 1 applied to (1), lead to

$$Y(t) = C_0\delta(0) + \sum_{m=1}^{\infty} C_m \left(\delta\left(t - \frac{2nd}{c} m \right) + \delta\left(t + \frac{2nd}{c} m \right) \right) \tag{2}$$

where C_m are constants that depend on the refractive index and the incident electric field amplitude. The Dirac deltas caused by the interference modulation are shifted by $k = \pm 2ndm/c$. Thus, the refractive index is

$$n = \frac{ck}{2dm}, \quad m = 1, 2, 3, \ldots \tag{3}$$

The finite signal length and small variations of the refractive index in the frequency range of interest result in a broadening of $Y(t)$. A material with a large variation of the refractive index will show a broader peak, providing an average refractive index value. This could be useful for example when an initial value is needed for iterative refractive index calculation [3].

Assuming a frequency independent refractive index, the error in the measurement can be estimated by performing the error propagation of (3)

$$E_n = \frac{k}{2dm} E_c + \frac{c}{2dm} E_T + \frac{ck}{2d^2\, m} E_d \tag{4}$$

where E_c is the error due to the light velocity approximation, E_T is the absolute error caused by the temporal resolution and E_d is the sample width error. Usually, the main contribution to the refractive index error is the sample thickness uncertainty. When the sample is wider, the value of k will increase, but the quadratic dependence with the sample width d will cause a lower error in the refractive index. In addition, (4) shows a decreasing error for echoes of higher order [1].

Experiment: A free space THz-TDS formed by a modelocked laser and photoconductive antennas has been used. The measurements were performed with a temporal sweep of 232 ps and a time resolution of 14.15 fs. Three plates of fused silica with 1.21, 1.97 and 3.1 mm width were used as samples.

In Fig. 2*a*, the measured THz time-domain pulse using a fused silica sample of 1.21 mm width is shown. The measurement shows that the echoes are not clearly discernible. In Fig. 2*b*, the absolute value of the DFT of Fig. 1*a* is calculated showing the modulation caused by the interference between the main pulse and its echoes. Fig. 2*c* shows the results from the second DFT for the three samples. The refractive indices values obtained are in agreement with previously reported values [2].

The spectral analysis of Fig. 1*b*, shown in Fig. 1*c*, depicts several components. In the case of a 1.21 mm-wide sample, any peak below 8 ps corresponds with a refractive index smaller than one (3). Beyond 8 ps, the echo time delay is given by the peak position. Thus, the delay of the peak marked as k_1 in Fig. 2*c* allows the derivation of the average refractive index of the sample. If there are peaks below 8 ps, as in Fig. 2*c*, they are caused by unwanted reflections in the spectrometer and therefore do not depend on the sample under test. Thus, these peaks can be used to find the sources of these reflections (which are not visible in Fig 2*a*) and improve the performance of the spectrometer. In the experimental setup used, the values obtained point to air gaps of 400 and 140 µm, respectively, revealing a possible reflection taking place in the union of the antenna with the silicon lens.

The discrimination improvement over direct observation in the time domain is due to the fact that random noise components usually do not produce a frequency modulation. Thus, after processing, even a low amplitude frequency component from the interference can be identified.

This method allows the determination of the refractive index in a given frequency window, by simply computing the absolute value of the DFT from the signal spectrum in a given frequency range. This can be of interest when the refractive index is only constant in a particular band or when only the region with the best dynamic range is of interest, unlike the refractive index calculation from the temporal trace which relies on the whole spectrum of the THz spectrometer.

Conclusion: A method to estimate the average refractive index from THz-TDS measurements without the need of comparison with a reference has been proposed. It can be used when echoes are immersed in noise or combined with spurious signals and therefore making very difficult a direct time delay analysis. This method allows the determination of the refractive index in a given frequency window and can also be used to identify unwanted reflections in spectroscopy instruments.

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One or more of the Figures in this Letter are available in colour online.

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