CORRIGENDUM TO "CLUSTER VALUES FOR ALGEBRAS OF ANALYTIC FUNCTIONS"

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ABSTRACT. While the article was in publication process, we found a mistake in a key tool for the proof one of the main results. As a consequence, our result on the ball $A_u(B_X)$ algebra remains open. For the algebra $H_b(X)$ we obtain a weaker statement, which still extends previous work in the subject. In this note we enumerate those results which should be omitted or modified.

The proof of the implication ii) $\Rightarrow iii$) in [2, Lemma 2.2] is incorrect. As a consequence, Theorem 1.1 and Corollary 1.2 do not follow from our arguments. Therefore, the validity of the Cluster Value Theorem for $A_u(B_X)$ is, up to our knowledge, proved only for trivial cases (where the spectrum is formed only by evaluation functionals) and for the infinite dimensional Hilbert space.

Lemma 2.2 remains correct for bounded and absolutely convex sets, but unfortunately, it is not enough to provide a proof of Theorem 1.1. A correct version of Lemma 2.2 would read as follows.

Lemma 1. Let X be a Banach space and $U \subset X$ a bounded and absolutely convex open set. The following are equivalent:

i) for each U-bounded set A, each $z \in X^{**}$ and each $f \in H_b(U)$, there exists a U-bounded set \tilde{A} such that

$$\hat{f}(\mathcal{M}_{z,A}) \subset Cl_{\tilde{A}}(f,z)$$

ii) for each 0 < r < 1, each $z \in X^{**}$ and each $f \in H_b(U)$ we have

$$\hat{f}(\mathcal{M}_{z,rU}) = Cl_{rU}(f,z).$$

Regarding the algebra $H_b(X)$ of holomorphic functions of bounded type, Theorem 2.1 must be restated as the following theorem, which extends Theorem 1.8 in [1].

Theorem 2. Let X be a Banach space whose dual has the bounded approximation property. Then for each r > 0 there exists R > 0 such that

$$Cl_{rB_X}(f,z) \subset \hat{f}(\mathcal{M}_{z,rB_X}) \subset Cl_{RB_X}(f,z)$$
 for every $f \in H_b(X), z \in X^{**}$.

This work was partially supported by projects CONICET PIP 11220130100329CO, ANPCyT PICT 2015-2299, ANPCyT PICT-2015-3085, ANPCyT PICT-2015-2224, UBACyT 20020130300057BA, UBACyT 20020130300052BA, UBACyT 20020130100474BA and MINECO and FEDER Project MTM2014-57838-C2-2-P.

Theorems 2.5 and 2.6 use Lemma 2.2, so their proofs are incorrect. A correct version of these two results, dealing with balls instead of arbitrary subsets of X, can be stated as follows.

Theorem 3. Let X be a Banach space. The following statements are equivalent:

- i) For every r > 0, every $z \in X^{**}$ and every $f \in H_b(X)$ we have $\hat{f}(\mathcal{M}_{z,rB_X}(H_b(X))) = Cl_{rB_X}(f,z)$.
- ii) For every $z \in \overline{B_{X^{**}}}$ and every $f \in A_u(B_X)$ we have $\hat{f}(\mathcal{M}_z(A_u(B_X))) = Cl(f, z)$.
- iii) For every 0 < r < 1, every $z \in B_{X^{**}}$, and every $f \in H_b(B_X)$ we have $\hat{f}(\mathcal{M}_{z,rB_X}(H_b(B_X))) = Cl_{rB_X}(f,z)$.

In Proposition 3.1 and Corollary 3.2 the hypothesis of "the dual has the bounded approximation property" has to be changed by "X satisfies the Cluster Value Theorem". Then, we cannot guarantee the validity of the list of examples after Corollary 3.2.

The remaining results of Section 3 are correct, but let us avoid ambiguity in the definition of Cluster Value Theorem for arbitrary open sets U (since we lost Lemma 2.2, we do not have equivalence between different definitions). We say that X satisfies the Cluster Value Theorem for $H_b(U)$ if there exists a fundamental sequence $\{A_n\}_n$ of U-bounded sets such that for every $n \in \mathbb{N}$, every $z \in X^{**}$ and every $f \in H_b(U)$ we have

$$\hat{f}(\mathcal{M}_{z,A_n}(H_b(X))) = Cl_{A_n}(f,z).$$

References

- Richard M Aron, Daniel Carando, Silvia Lassalle, and Manuel Maestre. Cluster values of holomorphic functions of bounded type. Trans. Amer. Math. Soc., 368:2355-2369, 2016.
- [2] Daniel Carando, Daniel Galicer, Santiago Muro, Pablo Sevilla-Peris. Cluster values for algebras of analytic functions Adv. Math., 329:157-173, 2018.

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