Review of Derivatives Research, 8, 49–60, 2005

Pricing of Defaultable Bonds with Log-Normal Spread: Development of the Model and an Application to Argentinean and Brazilian Bonds During the Argentine Crisis

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Abstract. In this paper we describe a two-factor model for a defaultable discount bond, assuming log-normal dynamics with bounded volatility for the instantaneous short rate spread. Under some simplified hypothesis, we obtain an explicit barrier-type solution for zero recovery and constant recovery. We also present a numerical application for Argentinean and Brazilian Sovereign Bonds during the default crisis of Argentina.

Keywords: credit risk, defaultable bonds, log-normal spread.

JEL Classification: G 13

Introduction

The approaches to modeling credit risk can be broadly classified into two types. The first includes the so called structural models, based on the firm's value approach introduced in Merton (1974), extended in Black and Cox (1976), and Longstaff and Schwartz (1995) among others.

More recent is the type of the generally termed reduced-form models, in which the assumptions on a firm's value are dropped, and the default is modeled as an exogenous stochastic process. Reduced-form models have been proposed in Jarrow and Turnbull (1995), Duffie and Kan (1996), Jarrow, Lando, and Turnbull (1997), Schonbucher (1998), Cathcart and El Jahel (1998), Duffie and Singleton (1999), Duffie, Pedersen, and Singleton (2000), Schonbucher (2000), and others.

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A survey of both classes can be seen in Schonbucher (2000), and in Bohn (1999) (models published before 1998). For a detailed overview of reduced-form models published before 1997 see Lando (1997).

The objective of this paper is to describe a two-factor model where the price of a risky bond is derived as a function of the risk-free short rate and the instantaneous short spread, and the requirement is that the short spread must be positive. The dynamics of the spread is assumed to satisfy a log-normal diffusion with bounded volatility, and the default occurs if the spread reaches an upper barrier.

Our approach is motivated by a remark in Schonbucher (2000) stating that an alternative to his model of the term structure of defaultable bonds, based on the Heath-Jarrow-Morton (HJM) model (cf. Heath, Jarrow, and Morton, 1992), would be a two-factor model using an arbitrage-free model for the risk-free rate and a model for the forward spread that generates a positive short rate spread.

Our model relates to the one presented in Cathcart and El Jahel (1998), since it is also a reduced-form model, solved by a structural approach, that leads to a barrier-type solution; in their model they assume that the default occurs when a signaling process hits some predefined lower barrier.

An extension of Cathcart and El Jahel model is proposed in Lo and Hui (2000), where foreign exchange rates are chosen as the signaling barrier and the dynamics of the default barrier depends on the volatility and drift of the signaling barrier.

Blauer and Wilmott (1998) also use the Black and Scholes option pricing technique to develop a two-factor model applied to Brady bonds, but they took expectation on the risk of default instead of hedging it, so our pricing equation and its solution are different from theirs.

The remainder of the paper is organized as follows. The bond pricing equation is derived in Section 1. In Section 2 we obtain the solution for a log-normal dynamics of the short spread without recovery and with constant recovery. Section 3 shows the numerical results of expected implied recovery rates for Argentina and Brazil during the first months of the Argentinean default crisis. The last section contains the conclusions and comments on future work.

1. The Pricing Equation

We work in a continuous time framework, in which $r_d(t)$ is the defaultable short rate if a default event has not occurred until t, $r(t)$ is the risk-free short rate, and the spread $h(t)$ is defined as

$$h(t) = r_d(t) - r(t)$$

Our assumptions are:

(i) at any time t, risk-free discount bonds and defaultable discount bonds of all maturities are available,

(ii) the dynamics of $r(t)$ and $h(t)$ are governed by diffusion equations

$$dr(t) = \mu_r(r,t)dt + \sigma_r(r,t)dW_1,$$
$$dh(t) = \mu_h(h,t)dt + \sigma_h(h,t)dW_2,$$

where W_1 and W_2 are uncorrelated standard Brownian motions,

(iii) the spread is positive, $h(t) > 0$.

To derive a general equation for the defaultable bond, we set a portfolio Π containing a defaultable bond $P(r,h,t,T)$, of maturity T, a number Δ of risk-free bonds $B(r,t,T_1)$, of maturity T_1, and a number Δ_1 of defaultable bonds $P_1(r,h,t,T_2)$ of maturity T_2,

$$\Pi = P(r,h,t,T) - \Delta B(r,t,T_1) - \Delta_1 P_1(r,h,t,T_2),$$

and look for values of Δ and Δ_1 that eliminate the randomness in $d\Pi$.

From Itô's lemma it follows that

$$\Delta_1 = \frac{\frac{\partial P}{\partial h}}{\frac{\partial P_1}{\partial h}}, \quad \Delta = \frac{1}{\frac{\partial B}{\partial r}}\left[\frac{\partial P}{\partial r} - \frac{\frac{\partial P}{\partial h}}{\frac{\partial P_1}{\partial h}}\frac{\partial P_1}{\partial r}\right],$$

and by non-arbitrage arguments we arrive at the pricing equation of the defaultable bond

$$\frac{\partial P}{\partial t} + \frac{1}{2}\sigma_r^2(r,t)\frac{\partial^2 P}{\partial r^2} + \frac{1}{2}\sigma_h^2(h,t)\frac{\partial^2 P}{\partial h^2} + \phi(r,t)\frac{\partial P}{\partial r} + \psi(h,t)\frac{\partial P}{\partial h} - rP = 0,$$

where

$$\phi(r,t) = \mu_r(r,t) - \lambda_r(r,t)\sigma_r(r,t), \quad \psi(h,t) = \mu_h(h,t) - \lambda_h(h,t)\sigma_h(h,t)$$

are the risk adjusted drifts, and $\lambda_r(r,t)$ and $\lambda_h(h,t)$ are, respectively, the market prices of rate risk and the risk associated with the spread.

Since r and h were not correlated, a solution

$$P(r,h,t,T) = Z(r,t,T)S(h,t), \tag{1}$$

where $Z(r,t,T)$ is the solution of a risk free bond[1] (e.g. Hull & White), separates the problem, and leads to

$$\frac{\partial S}{\partial t} + \frac{1}{2}\sigma_h^2(h,t)\frac{\partial^2 S}{\partial h^2} + \psi(h,t)\frac{\partial S}{\partial h} = 0, \tag{2}$$

with the final condition

$$S(h,T) = 1$$

if a default has not occurred until maturity.

2. Modeling the Spread

The log-normal assumption for the dynamics of $h(t)$ is the natural and simplest way to assure its positivity. In Hogan (1993) it has been shown that this assumption is not suitable for continuously compounded interest rates, since it implies that expected accumulation factors over any finite time interval are infinite with positive probabilities. This problem has been addressed, e.g., in Sandmann and Sondermann (1994), Miltersen, Sandmann, and Sondermann (1994), and Goldys, Musiela, and Sondermann (1996), where alternative log-normal type term structures that preclude explosion of rates are proposed.

However, for a log-normal term structure model with bounded volatility, the spread is positive and remains finite. As it becomes (or it is perceived to become) more likely that the bond may default the spread increases. But it does not increase unboundedly; in practice there is a finite upper barrier, even when it is not known in advance.

For a log-normal diffusion, imposing an upper bound H_b to the short spread, $0 < h \leq H_b < \infty$, is equivalent to defining a bounded volatility process, i.e.

$$dh(t) = \mu_h(h,t)dt + \sigma(h,t)dW_2,$$

with

$$\sigma_h(h,t) = \min(H_b, h(t))\sigma_h(t),$$

where $\sigma_h(t)$ is a deterministic function and, as shown in HJM (1992) this volatility process gives finite positive rates (spread in this case).

The simplified assumption that

$$k = 2\left(\frac{\mu_h(t)}{\sigma_h^2(t)} - \lambda_h(t)\sigma_h(t)\right) \tag{3}$$

is a positive constant, allows us to obtain a closed-form solution.

With the above choices for h, equation (2) reduces to

$$\frac{\partial S}{\partial t} + \frac{1}{2}\sigma_h^2 h^2 \frac{\partial^2 S}{\partial h^2} + [\mu_h - \lambda_h \sigma_h] h \frac{\partial S}{\partial h} = 0, \quad 0 \leq t < T, 0 < h < H_d, \tag{4}$$

where H_d $(<H_b)$ is the default boundary, with the final condition

$$S(h,T) = 1$$

if default has not occurred until T.

Requiring that, for spread tending to zero, $P(r,h,t,T)$ should approximate to the solution of a risk-free discount bond, gives us the first boundary condition, namely

$$\lim_{h \to 0} S(h,t) = 1$$

The second boundary condition arises from the assumption that default occurs if h ever reaches the barrier H_d. Therefore, for zero recovery we must have

$$P(r, H_d, t, T) = 0,$$

which implies $S(H_d, t) = 0$.

With the usual change of variables

$$h = e^x, \quad t = T - \frac{2\tau}{\sigma_h^2}, \quad S(h,t) = e^{\alpha x + \beta\tau} u(x,\tau),$$
$$\text{for } \alpha = -\frac{1}{2}(k-1),\ \beta = -\frac{1}{4}(k-1)^2, \tag{5}$$

where k is given by (3), the problem (4) becomes

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}, \quad \tau > 0,\ -\infty < x < \ln H_d, \tag{6}$$

with initial condition

$$u(x, 0) = e^{\frac{1}{2}(k-1)x},$$

and boundary conditions

$$u(\ln H_d, \tau) = 0, \quad \lim_{x \to -\infty} u(x,\tau) = e^{\frac{1}{2}(k-1)x + \frac{1}{4}(k-1)^2\tau}.$$

The solution to (6) is

$$u(x,\tau) = e^{\frac{1}{4}(k-1)^2\tau}\left[e^{\frac{1}{2}(k-1)x}N(d_1) - e^{\frac{1}{2}(k-1)(2\ln H_d - x)}N(d_2)\right],$$

where

$$d_{1,2}(x,\tau) = (+,-)\frac{\ln H_d - x}{\sqrt{2\tau}} - \frac{1}{2}(k-1)\sqrt{2\tau},$$

and $N(x)$ is the cumulative probability distribution function for a normally distributed variable with mean zero and variance 1.

Going back to (5), we can write the solution to the problem (2) with zero recovery, in financial variables, denoted by $S_0(h,t)$, as

$$S_0(h,t) = N(d_1) - \left(\frac{H_d}{h}\right)^{(k-1)} N(d_2), \tag{7}$$

where

$$d_{1,2}(h,t) = (+,-)\frac{\ln(\frac{H_d}{h})}{\sigma\sqrt{(T-t)}} - \frac{1}{2}(k-1)\sigma\sqrt{(T-t)}, \tag{8}$$

It is easy to see that the final condition and the boundary condition at H_d are verified by construction.

For $t = T$, $N(d_1) = 1$ and $N(d_2) = 0$. Hence $S_0(h, T) = 1$. At $h = H_d$, $d_1 = d_2 = -\frac{1}{2}(k-1)\sigma\sqrt{(T-t)}$, which yields $S_0(H_d, t) = 0$.

The boundary condition for $h \to 0$

$$\lim_{h\to 0} S_0(h,t) = \lim_{h\to 0}\left[N(d_1) - \left(\frac{H_d}{h}\right)^{(k-1)} N(d_2)\right].$$

remains to be checked, Since for $h \to 0$, $d_1 \to \infty$, then $\lim_{h\to 0} N(d_1) = 1$.

Using the asymptotic expression for the cumulative normal probability distribution function (c.f. Abramovitz and Stegun, 1970) it is easy to show that

$$\lim_{h\to 0}\left(\frac{H_d}{h}\right)^{(k-1)} N(d_2) = 0,$$

and, therefore, $\lim_{h\to 0} S_0(h, t) = 1$

Introducing a recovery is equivalent to specifying a boundary condition

$$S(H_d, t) = Q(t),$$

and due to this contribution, there will be an extra term $S_Q(h, t)$ added to the solution (7).

For the particular case of a recovery paid in cash, or when it is a fraction of the face value, $Q(t) = Q$ is constant; this makes the problem mathematically equivalent to the modeling of a constant rebate for an up-and-out barrier. The additional term takes the form

$$S_Q(h,t) = Q\left[N(-d_2) + \left(\frac{H_d}{h}\right)^{(k-1)} N(-d_1)\right]. \tag{9}$$

The factor related to the spread, $S(h, t)$, given by the sum of (7) and (8), can be written as

$$S(h,t) = Q + (1-Q)\left[N(d_1) - \left(\frac{H_d}{h}\right)^{(k-1)} N(d_2)\right], \tag{10}$$

with $d_{1,2}(h, t)$ given by (8).

3. An Application of the Model: Argentina and Brazil's Sovereign Bonds During the Argentine Crisis

The market data of the sovereign debt of Argentina and Brazil were fitted into the pricing model developed in previous sections of this paper with the aim of obtaining the implied market expectations over the recovery rate of these bonds and studying their dynamics during the period of unfolding of Argentina's Debt Crisis. To this end, we fed daily market data from Argentinean and Brazilian bonds belonging to JP Morgan's EMBI+ Argentina

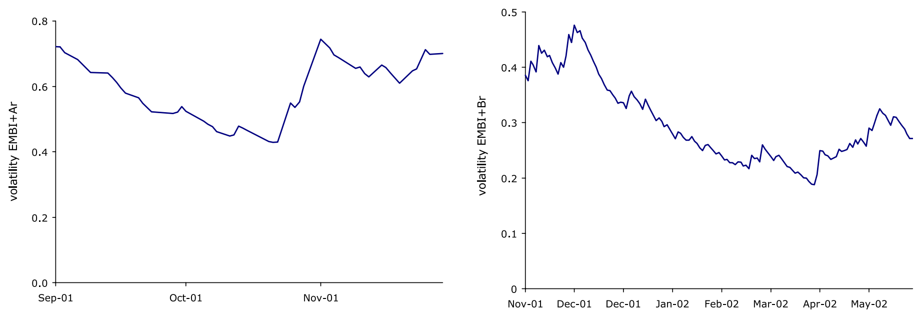

Figure 1. This figure plots the Exponentially Weighted Moving Average volatility for the EMBI+ Indexes of Argentina and Brazil. The left panel shows the log-volatility of the EMBI+ Argentina, σ_{AR}, from September 2001 to the end of November 2001. The right panel shows the log-volatility of the EMBI+ Brazil, σ_{BR}, from November 2001 to the end of May 2002.

Index and JP Morgan's EMBI+ Brazil Index, respectively, to the specification of the model given by Equation (1).

In Equations (10) and (8), $h(t)$is the EMBI+ time series, σ and μ, are the log-volatility and drift of the process, and the spread value at default is $H_d = 0.4722$ for Argentina, and was set to $H_d = 0.5$ for Brazil.

Instead of modeling the risk-free term structure we use the present value of risk-free cash flows, and we take as T_i the average life of each bond.

An additional hypothesis, needed for our calculations, is an equal recovery factor Q for all the bonds of the same issuer.

The log-volatilities were estimated by the Exponentially Weighted Moving Average (EWMA) method. Figure 1 shows the behavior of the volatilities for Argentina and Brazil.

As the model assumes constant volatility, we partitioned the time interval studied and used the set of periods and average values shown in Table 1.

In order to completely specify the model, we used synchronous values of the EMBI+ bonds to estimate cross-sectionally the parameters k and Q, for a series of days, and then examined the time series of parameters produced by the estimation procedure to test whether the empirical results validated or rejected the model.

Let us consider the sum of the squares of the deviations between model and market bond prices

$$x^2 = \sum_{i=1}^{N} n_i [B_i - Z_i S_i(Q, k)]^2, \qquad (11)$$

where N is the number of bonds used in the calculation of the EMBI+, n_i is the weight of bond i in the EMBI+, B_i is the observed daily mid-market bond price, and Z_i is the risk-free price. We look for the set of parameters k and Q that minimizes (11). Since one of the parameters appears as argument of an exponential function, the minimization problem is strongly non-linear, and we must search for an adequate local minimum.

Table 1. Average volatilities for each period

Panel A: Argentina		Panel B: Brazil	
Period	EMBI+AR volatility	Period	EMBI+BR Volatility
From 05-Sep-01 to 04-Oct-01	0.60	From 05-Nov-01 to 13-Dec-01	0.42
From 05-Oct-01 to 31-Oct-01	0.48	From 14-Dec-02 to 10-Ene-02	0.35
From 01-Nov-01 to 30-Nov-01	0.67	From 11-Jan-02 to 31-Jan-02	0.30
		From 01-Feb-02 to 07-Mar-02	0.24
		From 08-Mar-02 to 17-Apr-02	0.22
		From 18-Apr-02 to 30-May-02	0.27

Notes: This table shows the average log-volatility used in the calculations for each period of constant volatility considered. Panel A, shows the periods and averages volatilities used for the EMBI+ Argentina Index, while Panel B does the same for the EMBI+ Brazil Index.

Let us recall that k does not have an obvious economic meaning, but is only a dimensionless parameter defined for the convenience of the solution of the partial differential equation, while Q, also a dimensionless parameter, expresses the recovery as a fraction of the risk-free price.

From the parameters k and Q obtained through minimization we derived the series of daily implied average expected recovery rates R.

In Figures 2 and 3 we present the plot of the series of the two mentioned implied parameters for Argentina, and the expected recovery rates coupled with the spread of the EMBI+ Argentina Index, during the period running form the beginning of September 2001 to the end of November 2001. After this period, the worst of the crisis, the model ceases to provide a good fit to the market data.

In Figures 4 and 5 we present the plot of the same set of parameters for Brazil, and the recovery rates coupled with the EMBI+Brazil Index, during the period running from November 2001 to May 2002.

The first step to test the validity of the model is to examine the daily series of the estimated parameters. In our model k and Q are not functions of time, hence if the model is correct, the estimation procedure should produce the same estimates over time. In fact, it is to be expected that the parameters will not be exactly constant but fluctuating within a statistical noise.

As it is apparent from Figure 4, the minimization parameters for Brazil lie in a plausible range. k_{BR} oscillates around different constant values, its jumps corresponding to the changes of the average volatility, while Q_{BR} only exhibits a small jump, on January 11. Furthermore, it fluctuates around two very close average values: 0.42 at first, and later 0.39.

In Figure 2 the Argentinean parameters show a similar behavior in the first two periods. However, in the third period, as the EMBI+AR Index increases approaching to the default value, the oscillations of k_{AR} and Q_{AR} become more significant. After November 30th. 2001, the lack of stability of the minimization parameters or the inability of the minimization procedure to produce an adequate set of parameters, leads to believe that the model could no longer be validated as a reasonable description of the real process.

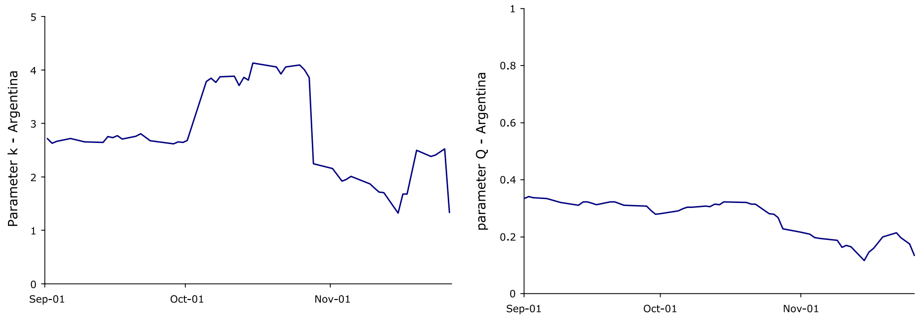

Figure 2. This figure plots the series of the dimensionless parameters k and Q that minimize Equation (11) based on the data of Argentina from September 2001 to the end of November 2001. The left panel shows the evolution of the parameter k_{AR}, while the right panel shows Q_{AR}, which expresses the market implied recovery as a function of the risk-free price.

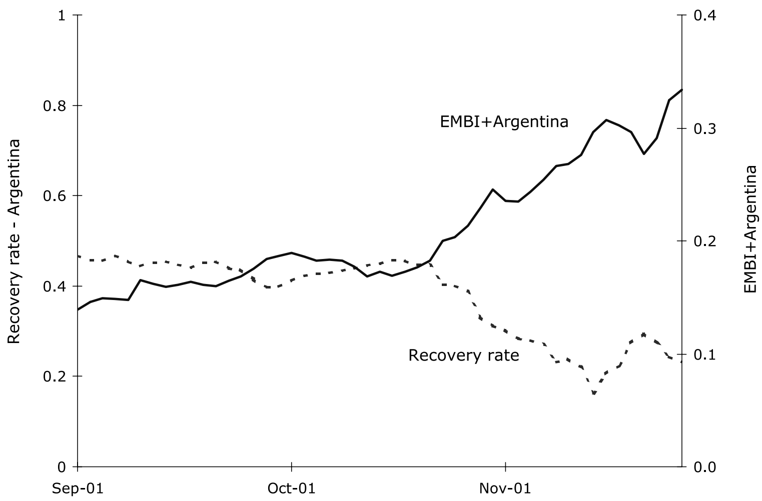

Figure 3. The solid line plotted in this figure shows the evolution of the EMBI+Argentina Index, while the dotted line corresponds to the expected Recovery Rate for the Argentinean sovereign bonds, R_{AR}. Both series are shown from September 2001 to the end of November 2001.

Looking at Figures 3 and 5 we find, as expected, a significant negative correlation between the EMBI+ Index for each country and the corresponding expected recovery rate, suggesting that the recovery rate is largely determined by the evolution of the credit spread. In closer examination, we find that the correlation coefficient between these two variables is, in absolute terms, greater in Argentina than in Brazil (−0.96 vs. −0.73). We believe the explanation is based on the fact that although the credit spread represents essentially the risk of default, it also has a component that depends on the expectations regarding payoff in case of default. As the probability of default approaches 1, this component becomes more important, given the fact that investors are nearly certain of the upcoming default

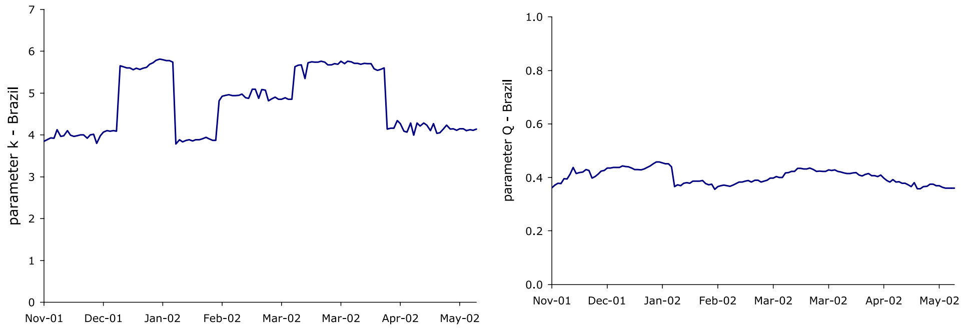

Figure 4. This figure plots the series of the dimensionless parameters k and Q that minimize Equation (11) based on the data of Brazil from November 2001 to the end of May 2002. The left panel shows the evolution of the parameter k_{BR}, while the right panel shows Q_{BR}, which expresses the market implied recovery as a function of the risk-free price.

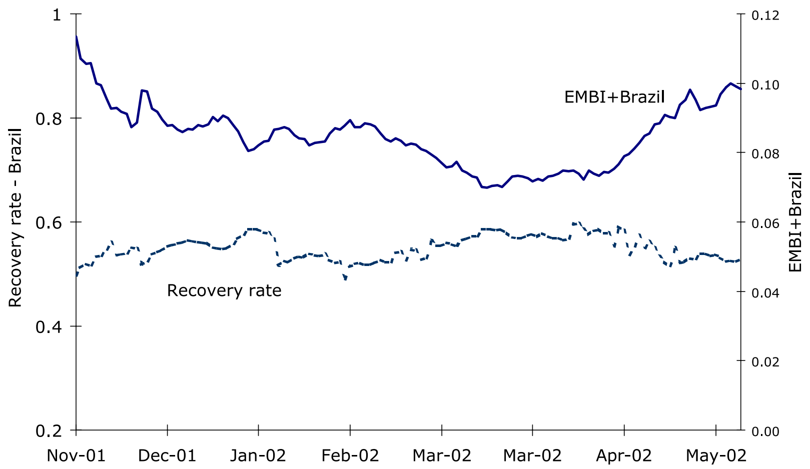

Figure 5. The solid line plotted in this figure shows the evolution of the EMBI+ Brazil Index, while the dotted line corresponds to the expected Recovery Rate for the Brazilian sovereign bonds, R_{BR}. Both series are shown from November 2001 to the end of May 2002.

and the only source of value left is the expected recovery rate after default. We found this explanation also consistent with the fact that the absolute value of the mentioned coefficient of correlation for Argentina grows constantly as the country approaches default.

The values observed for R in Figures 3 and 5 also provide interesting information about the levels of the expected recovery rate. At the beginning of the dataset, Argentina shows an average expected rate of recovery of approximately 47% of the face value of the bonds. This level is similar to the worst expected recovery rate showed by Brazil (49%) and not very far from Brazil's average of 55% that remains pretty stable during the whole sample. Later on, as the scenario worsens for Argentina,[2] from the last days of October onward,

R drops and finally stabilizes around a level of 25%. This level is significantly lower than the evidence for recovery rates from Altman and Eberhart (1994) for US corporate debt (50%), Altman, Cooke, and Kishoe (1999) (40%) and Merrick (1999) for sovereign issues of Argentina during the period of Russia's GKO default crisis in 1998 (50%).

Summarizing the findings of the empirical application of the model, we found that the implied recovery rate level for Brazilian sovereign bonds has persisted during the period of study around 55%, a value not completely out of line with Merrick's findings for Argentina during the Russian crisis. In the case of Argentina the results are quite different, since the model shows very low expected recovery values in comparison with previous cases of default, and very much in line with the proposals of record haircut made by the Argentinean government to the bondholders.

4. Conclusions

Under simplified assumptions, and modeling the spread as a log-normal random walk with bounded volatility, we have obtained a barrier type closed-form solution for a two-factor model of a defaultable discount bond. Furthermore, the model has proved useful for the analysis of the expected recovery rates of Argentina and Brazil during the development of Argentina's financial breakdown.

This log-normal type model for the spread is the simplest one that satisfies the requirement of positivity, and by relaxing some of the hypothesis it may be improved to better agree with observed phenomenological facts. In particular, in Duffie (1999) it is pointed out that the empirical instantaneous risk of default is mean-reverting under the real measure. Therefore, our next step shall be to consider a mean-reverting log-normal type random walk for the spread, and preliminary calculations show that, in this case, a quasi-closed-form solution may be obtained in terms of the confluent hypergeometric functions.

Acknowledgments

We thank Dr. Kenneth Garbade for his thorough reading and constructive criticism, which has greatly contributed to improve this work, and Prof. Menachem Brenner for his helpful comments and suggestions.

Two of the authors, E. Cortina and C. Ferro Fontán are fellows of the Consejo Nacional de Investigaciones Científicas y Técnicas de la Argentina.

Notes

1. For a full description of interest rate models see Rebonato (1998).
2. On October 17th, Standard & Poor's, Moody's Investor Service and Fitch warned that they would rate Argentina in technical default if bondholders lost any money in a planned domestic swap. Shortly afterwards, Fitch stated that bondholders could be facing losses of UDS 10 billion. Finally, on November lst., the president of Argentina and his ministry of economy gave confirmation of the details of a debt swap that resulted in a significant loss of value for domestic investors.

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