Evolution of Impatience:
The Example of the Farmer-Sheriff Game[†]

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The literature on the evolution of impatience, focusing on one-person decision problems, often finds that evolutionary forces favor the more patient individuals. This paper shows that in games where equilibrium involves threat of punishment there are forces generating an evolutionary advantage to the impatient. In particular, it offers a two-population example where evolutionary forces favor impatience in one group while favoring patience in the other. Moreover, efficiency may also favor impatient individuals. In our example, it is efficient for one population to evolve impatience and for the other to develop patience. Yet, evolutionary forces move the opposite direction. (JEL C73, C78)

W hy are we often more impulsive than we might like to be? To take one of many examples: although the "cost" of getting a copy of a new book or the last model of a computer decreases substantially with time, few people choose to wait—and in some cases there are even people that spend the night in line to be the first buyers. From the perspective of evolution this might pose a puzzle. Whenever evolution favors the very long run, given the great variation in patience and self-control in the population, will not evolutionary forces favor those more willing to wait? Should we not evolve towards ever-greater patience and absence of impulsivity in such environments? Indeed, Rogers (1994) argues that evolution would select subjective discount factors that are equal to 1 in decisions that affect the decision maker alone and not his descendants. Moreover, Blume and Easley (1992, 2006) and more recently Bottazzi and Dindo (2011) show in the context of a wealth accumulation

problem that evolution favors the patient so strongly that it favors the patient over the smart.[1] In different settings, other authors (see, for example, Hansson and Stuart 1990 and Robson and Wooders 1997) have also argued that evolution would favor a discount factor of 1.

One explanation, see for example Chowdhry (2011), is that we are impatient because we may not live to see tomorrow. However, this does not in itself explain why we should evolve to impatience: even a very patient individual will behave impatiently in the face of uncertain life. Robson and Samuelson (2009) find that aggregate uncertainty concerning survival rates leads to more impatience. Moreover, they find that when aggregate uncertainty differs across ages, the discount rate does not need to be constant.[2]

Here we explore an alternative explanation of the evolution of impatience. In particular, we present a game in which evolution and impatience are compatible. In an investment problem short-sightedness is dysfunctional, but the same is not necessarily true in a game: preferences can act as a form of commitment device. For example, in a repeated game a player who is patient can be coerced into taking actions that are disadvantageous by threats of future retaliation. This is why some repeated games admit equilibria that are Pareto inferior to static Nash equilibria. More concretely, imagine a player has a choice whether to give a costly gift to his opponent who must simultaneously choose whether to drown him in burning oil. A patient player can be coerced into giving the gift if a credible threat of future burning in oil is available. An impatient player is immune to this threat since he does not care about the possibility of future punishment. Hence, there is an equilibrium in which a patient player does badly—giving the expensive gift—while there is no equilibrium in which the impatient player does so. In effect, the impatient player commits to not giving the gift, so the opponent does not retaliate.

We explore the issue in the context of a simple game designed to illustrate both how impatience can emerge as an evolutionary outcome and also to understand how different social roles may result in different degrees of patience. Indeed, despite anecdotal evidence that the rich may be as impulsive as the poor, there is statistical evidence, for example in Cunha and Heckman (2009), that there is a strong connection between economically unsuccessful families and impatience and lack of self-control.

This paper is designed to further advance the literature on the evolution of preferences, in the path set out by Hirshleifer (1977). The evolution of altruism has been studied, for example, in Bowles (2001), and in the context of cultural evolution by Bisin, Topa, and Verdier (2004); Bisin and Verdier (2001) investigates the broader issue of cultural versus other forms of transmission. The relation between selection and kinship has been examined by Alger and Weibull (2010). Authors such as Ely and Yilankaya (2001) and Dekel, Ely, and Yilankaya (2007) have developed the theoretical underpinnings of evolutionary equilibrium when preferences evolve,

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[1] Specifically, Blume and Easley (2006) show that under complete markets, the rates of learning are irrelevant to survival when the discount factors differ: the consumption of the more impatient individual almost-surely vanishes. See their Theorem 8 and discussion therein.
[2] Robson and Samuelson (2010) is an excellent survey of this literature.

relating evolutionary outcomes to equilibria of the fitness game. In particular, the possibility that preferences that do not maximize fitness survive has been analyzed in Güth and Peleg (2001); Heifetz, Shannon, and Spiegel (2007); and Dekel, Ely, and Yilankaya (2007) as well. Robson (2001) surveys research on the evolution of preferences, including that of time preferences. However the evolution of impatience in interactive context has not been much studied.

There are a variety of subtle issues about impulsive behavior and self-control that have been explored in the behavioral economics literature—see for example Fudenberg and Levine (2006). However, the present paper is not focused on commitment, present bias, or time consistency—rather we focus on the simpler question of why intertemporal preferences with a low geometric discount factor might emerge in an evolutionary setting.

We also look at the inefficiency of equilibrium. In all of the above mentioned cases, the gains from impatience are private. However, there are also cases in which there are social gains from impatience. An example of this is provided in the literature on conflict.[3] In this literature people can satisfy their desires either by producing or by appropriating others' production (that is, through conflict). In general, resources spent in conflict are a social waste. Thus, it is best for society that people do not engage in appropriation by conflict; as a second best, it is best that those who do it be more impatient, so that they do not invest much in technologies that are detrimental to social welfare. This is an extreme case that can be explained in our model. An alternative, less extreme case, for example, is the case of speculators. They could have a social function, namely helping the alignment of prices, yet they do appropriate part of the gains from investments.

The rest of the paper is organized as follows. In Section I we develop the model. In Section II we analyze the equilibrium of the evolutionary process. In Section III we discuss efficiency issues. In Section IV we study the robustness of the results. Finally, we conclude in Section V.

I. The Matching Model

There is a continuum of players divided into two populations, *Farmers* who constitute a fraction ϕ of the population and *Sheriffs* who are the other $1 - \phi$ of the population. Each round Farmers and Sheriffs are randomly matched, so that the probability of a meeting between a Farmer and a Sheriff is $2\phi(1 - \phi)$. The remaining Farmers and Sheriffs are said to be unmatched. All players have an initial endowment of one bushel of wheat.

Players' preferences in rounds are characterized by discount factors δ_F, δ_S. In contrast, fitness—which is independent of preferences—will be assumed to be linear in wheat; put another way, its implicit discount factor is equal to 1.

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[3] See Hirshleifer (1991). Rent seeking is a particularly interesting special case of conflict that has received much attention at least since Tullock (1967) and Krueger (1974).

A round consists of either a one-person or two-person game that has three periods: in the first there is investment, in the second production, and in the third punishment may be imposed. Depending on matching, the following cases occur:

Unmatched Farmer [*Investment Game*].—

- Period 1: invest $k_I \in [0, 1]$, consume $1 - k_I$.
- Period 2: produce and consume output $y_I = Ak_I^\alpha$, where $\alpha A \leq 1$ and $0 < \alpha < 1, A > 0$.
- Period 3: nothing.

Unmatched Sheriff.—

- Period 1: consume endowment of 1.
- Period 2: nothing.
- Period 3: nothing.

Farmer-Sheriff Game.—

- Period 1a: Sheriff observes δ_F, invests $k_S \in [0, 1]$, consumes $1 - k_S$, and states a demand $d_S \geq 0$.
- Period 1b: Farmer observes d_S, invests $k_F \in [0, 1]$, consumes $1 - k_F$, and agrees to pay the Sheriff $d_F \geq 0$.
- Period 2: Farmer produces output $y_F = Ak_F^\alpha + G$, consumes $y_F - d_F$, and the Sheriff consumes d_F, where $G \geq 0$ is the "gain to trade" from the match.
- Period 3: if $d_F \geq d_S$, nothing; if $d_F < d_S$, the Sheriff issues a punishment that costs the Farmer ABk_S^α, where $B > 1$. This latter assumption implies that it is easier to destroy output than to produce it.

Discounting takes place between periods. In the Investment game the objective function of the Farmer is $1 - k_I + \delta_F y_I$. In the Unmatched Sheriff game the objective function of the Sheriff is 1. In the Farmer-Sheriff game the objective function of the Farmer is

$$1 - k_F + \delta_F (y_F - d_F) - \delta_F^2 \mathbf{1}_{d_F < d_S} AB k_S^\alpha,$$

where $\mathbf{1}_{d_F < d_S}$ is the indicator function equal to 1 when $d_F < d_S$ and 0 otherwise; and the Sheriff's payoff is

$$1 - k_S + \delta_S d_F.$$

Finally, notice the assumption that the Sheriff can observe the discount factor of the Farmer prior to investing and making a demand. It is this that makes impatience a possible device for commitment. Successful commitment requires two elements: credibility and publicity. Evolutionary forces, by building impatience into preferences, make impatient behavior credible. Publicity is guaranteed in this game by the

assumption of observability of preferences. On the other hand, if preferences were not directly observable, still there could be a form of "approximate" observability: if patience were hereditary, we could imagine that the Sheriff infers the Farmer's patience from the Farmer's parent's past behavior. How noisy a signal is would depend on how frequent mutation is.

A. *Comments and Interpretations*

The most visible aspect of the Farmer-Sheriff game is the presence of the delayed punishment. The resulting threat, as the reader will have guessed, is decreasing in the Farmer's impatience. But this is not the end of the story, for, as we will see, the more impatient Farmer will also produce less in each round, hence will be potentially less fit in the long run—it is by no means a foregone conclusion that evolutionary forces will favor the less patient Farmer. The tradeoff may be seen as the familiar one between the short-run share of the cake and the long-run size of it.

To see the economics captured by this game consider first the case $G = 0$. In this case, the Sheriffs do not contribute to social welfare beyond their own endowment: only Farmers are socially productive in the sense that they can make investments resulting in an increase in wheat. However, Sheriffs can appropriate some of the output of Farmers. In this sense, the model has a predator-prey flavor. However, the model is formulated so that there is no intrinsic distortion in the predation: the amount that the Sheriffs can appropriate is independent of how much is produced by Farmers. The predation takes place through threat of punishment: Farmers must choose whether or not to comply with the Sheriffs' demands. If Farmers fail to comply with the demand of the Sheriff then they are punished. The level of punishment depends on the investment made by the Sheriff. Notice that there is no commitment issue for the Sheriff; the more patient they are the more they will invest in punishment—and as we will see, Sheriffs will evolve towards a high degree of patience.

This game is unlike the Peasant-Dictator game[4] where the Dictator faces a commitment problem—but one that is not sensitive to patience. Here it is Farmers who face a commitment problem: punishment takes place with a delay. Because of the delay a less patient Farmer is less willing to give in to demands by the Sheriff, and if the Sheriff knows this, she will demand less.

In the case $G = 0$, Sheriffs have no social function and are merely predators or parasites. But if we think of the Sheriffs as landlords and the Farmers as peasants, generally landlords provide some services, ranging from protection to improvements to the capital stock. This we capture—somewhat crudely—through $G > 0$. This means that there is a positive surplus accruing to a match with a Sheriff. Notice that the output from the match accrues to the Farmer, not the Sheriff. Here the model becomes one of potentially beneficial trade—but the only mechanism the Sheriff has for appropriating some of the gains to trade is by threatening the Farmer. This mechanism is not related to the gain to trade: the amount the Sheriff can appropriate does not depend on how good the match is. This captures a situation that sometimes

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[4] See, for example, Van Huyck, Battalio, and Walters (1995).

occurs in practice: if one party owns the enforcement mechanism, why not appropriate the most that can be appropriated rather than some sort of amount determined by efficiency considerations? Why should a large politically connected monopolist merely appropriate what the market is willing to pay, when they can have a nice piece of tax revenue to go with it?

II. Equilibrium and Evolution

In this section, after spelling out the subgame perfect equilibria in each round, we compute fitness, define evolution as given by standard replicator dynamics, and then state the main result (Theorem 2) on existence of a stable steady state with a nonzero fraction of farmers with a discount factor less than 1. Some comparative statics results are given at the end of the section.

A. *Equilibrium of a Match*

We first study subgame perfect equilibria of the different matches.

In the investment game the objective function for the Farmer is $1 - k_I + \delta_F A k_I^\alpha$, and the solution is characterized by the first order condition $\alpha \delta_F A k_I^{\alpha-1} - 1 = 0$; hence the optimum is $k_I = (\alpha A)^{1/(1-\alpha)} \times \delta_F^{1/(1-\alpha)}$.

In the Farmer-Sheriff game the objective function of the Farmer is $1 - k_F + \delta_F \times (A k_F^\alpha - d_F + G)$ if $d_F \geq d_S$ or

$$1 - k_F + \delta_F(A k_F^\alpha - d_F + G) - \delta_F^2 A B k_S^\alpha$$

if $d_F < d_S$. Notice that this is rigged so that the optimal investment choice of the Farmer is independent of d_F, whether or not there is punishment, and is the same as when the Farmer is unmatched: $k_F = k_I = (\alpha A)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)}$. Notice that, as anticipated, more impatient Farmers produce less so are potentially less fit than more patient Farmers.

In choosing how much to pay, clearly the Farmer should choose either $d_F = 0$ and get $1 - k_F + \delta_F(A k_F^\alpha + G) - \delta_F^2 A B k_S^\alpha$ or $d_F = d_S$ and get $1 - k_F + \delta_F(A k_F^\alpha - d_S + G)$, whichever is larger.

The optimal play of the Sheriff is to choose the largest demand consistent with payment: $d_S = \delta_F A B k_S^\alpha$. The utility of the Sheriff is then $1 - k_S + \delta_S \delta_F A B k_S^\alpha$. Finally, k_S is chosen by the Sheriff to maximize his utility, so that $k_S = (\alpha A B)^{1/(1-\alpha)} \times (\delta_F \delta_S)^{1/(1-\alpha)}$. The corresponding demand is

$$d_S = \delta_F A B \left((\alpha A B)^{1/(1-\alpha)} (\delta_F \delta_S)^{1/(1-\alpha)} \right)^\alpha$$

$$= \alpha^{\alpha/(1-\alpha)} (A B)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)} \delta_S^{\alpha/(1-\alpha)}.$$

The amount demanded by the Sheriff is an increasing function of both the discount factor of the Sheriff—since a patient Sheriff will invest more—and the discount factor of the Farmer, since a patient Farmer is more susceptible to a threat. In contrast to

the Rubinstein (1982) bargaining setup where the patient player is stronger because he is happy enough receiving a reward later in time, here the patient player is weaker because he feels more strongly the bite of a punishment coming later in time.

B. *The Evolutionary Process: Two Types*

We now consider the coevolution of preferences as measured by the discount factors and the number of Farmers and Sheriffs. In the analysis, overall fitness of a particular population does not depend on preferences, and it is defined as the total, undiscounted expected utility over the life of the individual.[5]

For simplicity we consider first the case where there are two possible preferences: either patient preferences with discount factor equal to 1—corresponding to maximizing the same total fitness objective function as evolutionary fitness—or impatient preferences with some $0 < \delta < 1$. Thus $\delta_F, \delta_S \in \{\delta, 1\}$.

Recall that ϕ is the fraction of the population who are Farmers; let ψ denote the fraction of Farmers who are impatient;[6] and let ψ_S denote the fraction of the Sheriffs who are impatient. Let $V_F(\delta_F, \delta_S)$, $V_S(\delta_F, \delta_S)$ denote the evolutionary fitness of Farmers and Sheriffs as a function of preferences. We compute fitness in the different matches. The fitness of an unmatched Farmer is

$$V_F^U(\delta_F) = 1 + \alpha^{\alpha/(1-\alpha)} A^{1/(1-\alpha)} \delta_F^{\alpha/(1-\alpha)} - (\alpha A)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)},$$

while in the Farmer-Sheriff game her fitness is

$$V_F^{FS}(\delta_F, \delta_S) = V_F^U(\delta_F) - d_S + G$$

$$= V_F^U(\delta_F) - \alpha^{\alpha/(1-\alpha)} (AB)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)} \delta_S^{\alpha/(1-\alpha)} + G.$$

The fitness of an unmatched Sheriff is equal to 1, while in the Farmer-Sheriff game it is

$$V_F^{FS}(\delta_F, \delta_S) = 1 - k_S + d_S$$

$$= 1 + \alpha^{\alpha/(1-\alpha)} (AB)^{1/(1-\alpha)} \delta_F^{1/(1-\alpha)} \left(\delta_S^{\alpha/(1-\alpha)} - \alpha \delta_S^{1/(1-\alpha)} \right).$$

Our model of evolution is the standard replicator dynamics based on evolutionary fitness. If θ_j is the population fraction of group j, V_j is the fitness of the group and $\overline{V}$ is the average fitness of the population, then

$$\dot{\theta}_j = \theta_j \left(V_j - \overline{V} \right).$$

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[5] Fitness is meant to be what evolution favors, an objective measure independent of preferences. It is in general an elusive concept, but in our case preferences only enter as discount factors, hence removing them yields the desired measure of fitness.

[6] Anticipating, we omit the subscript F for the Farmers on ψ.

Our analysis is greatly aided by the observation that Sheriffs evolve strictly towards greater patience:

PROPOSITION 1: $\dot{\psi}_S < 0$.

PROOF:

It suffices to show that $V_S^{FS}(\delta_F, \delta_S)$ is increasing in δ_S. We compute

$$\frac{\partial}{\partial \delta_S} V_S^{FS}(\delta_F, \delta_S)$$

$$= \alpha^{\alpha/(1-\alpha)}(AB)^{1/(1-\alpha)}\delta_F^{1/(1-\alpha)} \frac{\alpha}{1-\alpha} \delta_S^{-1}\left(\delta_S^{\alpha/(1-\alpha)} - \delta_S^{1/(1-\alpha)}\right) > 0. \blacksquare$$

The interesting case in the long run, therefore, has only three types: patient Sheriffs, and both patient and impatient Farmers. In this case, on which we now focus, we can compute the overall fitness of a (patient) Sheriff to be

$$V_S = 1 + \alpha^{\alpha/(1-\alpha)}(AB)^{1/(1-\alpha)}(1-\alpha)\left\{(1-\psi)\phi + \psi\phi\delta^{1/(1-\alpha)}\right\}$$

and that of a Farmer to be

$$V_F(\delta_F) = 1 + \alpha^{\alpha/(1-\alpha)}A^{1/(1-\alpha)}\delta_F^{\alpha/(1-\alpha)}(1-\alpha\delta_F)$$

$$- (1-\phi)\alpha^{\alpha/(1-\alpha)}(AB)^{1/(1-\alpha)}\delta_F^{1/(1-\alpha)} + (1-\phi)G.$$

Notice that this depends on the Farmer's own type and on how many Farmers there are in total, but not, of course, in how many of each type there are. The replicator dynamics can now be summarized by two equations:

$$\dot{\psi} = \psi(1-\psi)[V_F(\delta) - V_F(1)]$$

$$\dot{\phi} = \phi(1-\phi)\left\{[V_F(\delta) - V_S] - (1-\psi)[V_F(\delta) - V_F(1)]\right\}.$$

THEOREM 1: *Suppose* $B^{1/(1-\alpha)}\alpha < (1-\alpha)(B^{1/(1-\alpha)} - 1)$. *Then for any* $0 < \delta < 1$ *there exists an open set of Gs such that there is a unique interior steady state and it is dynamically stable. At the steady state*

$$\phi = \phi^* \equiv 1 - \frac{1 - \alpha - \delta^{\alpha/(1-\alpha)}(1-\alpha\delta)}{B^{1/(1-\alpha)}(1 - \delta^{1/(1-\alpha)})}.$$

PROOF:

In Appendix A.

Notice that ϕ^* does not depend on G. Notice also that the hypothesis $B^{1/(1-\alpha)}\alpha < (1-\alpha)(B^{1/(1-\alpha)} - 1)$ is not vacuous since for any $B > 1$ it is satisfied for sufficiently small α. We can also compute

$$\frac{\partial \phi^*}{\partial \delta} \equiv \frac{\alpha}{1-\alpha}\, \delta^{\alpha/(1-\alpha)}\, \frac{\delta^{-1} - 1}{B^{1/(1-\alpha)}\left(1 - \delta^{1/(1-\alpha)}\right)}$$

$$+ \frac{\phi^*}{(1-\alpha)\left(1 - \delta^{1/(1-\alpha)}\right)}\, \delta^{\alpha/(1-\alpha)} > 0$$

so that if the impatient Farmers are less impatient there will be more of them at the steady state.

The key result here is that at a stable interior steady state in the long run there is a positive fraction of Farmers who are impatient: evolution leads to impatience. Furthermore, Appendix A shows that if the fraction of the population who are Farmers falls below ϕ^*, the fraction of Farmers who are impatient grows; and if the fraction of the population of Farmers rises above ϕ^*, the fraction of Farmers who are patient grows. That is: Many Sheriffs favor the impatient since impatience reduces the demands of the Sheriffs, while few Sheriffs favor the patient since patience leads to more productive investment. The problematic aspect of this analysis is that with only two possible discount factors the level of impatience δ is specified exogenously. A more satisfactory analysis would allow many different possible levels of impatience and ask which level emerges endogenously. We turn to this next.

C. *The Evolutionary Process: Many Types*

We now relax the assumption that the only possible preferences are given by two discount factors δ, 1. Suppose then that there are individuals with every discount factor in the interval $\delta \in [0, 1]$. Since the general case is intractable we use the mean field approximation widely used in the physical sciences, and introduced into game theory by Jovanovic and Rosenthal (1988). This enables us to determine a steady state value of δ.

First observe that as with the case with two types, Sheriffs with $\delta = 1$ always have higher fitness than those with lower discount factors, so in the long run the Sheriffs will evolve towards patience. As before, the interesting case is where there is a single group of patient Sheriffs, and we will focus on this case. The mean field approach is to notice that near an interior steady state the density function over discount factors ψ_δ can be approximated by a spike in which every type of Farmer evolves towards the optimal discount factor. The replicator dynamic is given by

$$\dot{\psi}_\delta = \psi_\delta\big(V_F(\delta) - \overline{V}_F\big),$$

where $\overline{V}_F$ is the mean fitness of farmers. Since the distribution of types is very concentrated near the mean value, which we denote by δ_F, we approximate the mean

fitness $\overline{V}_F$ by the fitness evaluated at the mean discount factor δ_F, which we denote by V_F:

$$\dot{\psi}_\delta = \psi_\delta\left(V_F(\delta) - \overline{V}_F\right)$$

$$\approx \psi_\delta\left(V_F + DV_F[\delta - \delta_F] - V_F\right)$$

$$= \psi_\delta DV_F[\delta - \delta_F],$$

where $DV_F \equiv V_F'(\delta_F)$. After a short interval of time τ the system will evolve according to

$$\psi_\delta(t + \tau) \approx \psi_\delta(t) + \dot{\psi}_\delta(t)\tau$$

$$\approx \psi_\delta(t) + \psi_\delta(t)DV_F[\delta - \delta_F]\tau.$$

We then compute the mean discount factor by integrating:

$$\delta_F(t + \tau) = \int \delta\psi_\delta(t + \tau)\,d\delta$$

$$\approx \int \delta\left[\psi_\delta(t) + \psi_\delta(t)DV_F[\delta - \delta_F]\tau\right]d\delta$$

$$= \int \delta\psi_\delta(t)\,d\delta + \int \delta\psi_\delta(t)DV_F[\delta - \delta_F]\tau\,d\delta$$

$$= \delta_F(t) + DV_F\tau \int \delta\psi_\delta(t)[\delta - \delta_F]\,d\delta$$

$$= \delta_F(t) + \sigma^2(t)DV_F\tau.$$

This then gives the mean field dynamic for discount factor of the Farmers as

$$\dot{\delta}_F \approx \sigma^2(t)DV_F.$$

The fact that the variance σ^2 is time varying does not matter for our stability analysis, so we hold it fixed, and study the dynamic equation

$$\dot{\delta}_F = \sigma^2 DV_F,$$

which is simply the continuous time best response dynamic—that is the mean moves in the direction of increasing fitness. The dynamics of ϕ is again the replicator dynamics, now based on the mean discount factor, so

$$\dot{\phi} = \phi(1 - \phi)(V_F - V_S).$$

THEOREM 2: *Assume $G > (\alpha AB)^{1/(1-\alpha)}$. Then there is a unique interior steady state and it is dynamically stable.*

PROOF:

In Appendix B.

Notice that like Theorem 1, for stability Theorem 2 requires that G not be too small. However, unlike Theorem 1 it does not place an upper bound on G. From the proof of Theorem 1 in Appendix A it transpires that the reason for the upper bound on G does not involve stability, but rather is needed to insure the existence of an interior steady state. To understand what is going on, recall that by Proposition 1 the value ϕ^* does not depend on G. As we increase G holding fixed the other parameters this increases the utility of the Farmers, while not changing the utility of the Sheriffs. Hence, once G is big enough at ϕ^*, regardless of the value of ψ Farmers of both types will do better than Sheriffs, and so the number of Farmers will be increasing. This implies that there is no interior steady state: to the right of ϕ^* patient Farmers are favored over impatient ones. However, this is an artifact of the fact that there are only two types. If the impatient Farmers were less impatient—that is to say, if δ were larger, we saw that this would shift ϕ^* to the right, and so for this higher value of δ there could be a steady state. Once we endogenize δ_F, Theorem 2 shows that this is the right intuition: regardless of how large G is there is always a steady state.

We now establish some comparative statics results concerning the steady state.

THEOREM 3:

   (i) *The steady state value of ϕ is larger than $1/2$, and larger the larger is G.*

   (ii) *The comparative statics with respect to G and B are the following: $D_G\delta_F > 0$, $D_G\phi > 0$, $D_B\delta_F < 0$, and for sufficiently large G, $D_B\phi < 0$.*

PROOF:

In Appendix B.

III. Efficiency

We now turn to the issue of welfare, measured by the average fitness of the whole population. Our goal is to show how an inefficient steady state arises in which the wrong population is impatient.

To compute the average fitness of the entire population, observe that: there is a fraction ϕ^2 of unmatched Farmers with fitness $V_F^U(\delta_F)$, a fraction $(1 - \phi)^2$ of unmatched Sheriffs with fitness 1, and a fraction $2\phi(1 - \phi)$ of matched Farmers and Sheriffs who share a total fitness of $V_F^{FS}(\delta_F, \delta_S) + V_S^{FS}(\delta_F, \delta_S)$. Therefore expected average fitness is

$$(1) \quad V = \phi^2 \int_0^1 V_F^U(\delta_F) f_F(\delta_F) \, d\delta_F + (1 - \phi)^2 1$$

$$+ \phi(1 - \phi) \int_0^1 \int_0^1 \left(V_F^{FS}(\delta_F, \delta_S) + V_S^{FS}(\delta_F, \delta_S) \right) f_F(\delta_F) f_S(\delta_S) \, d\delta_F d\delta_S.$$

We think of the social planner as choosing a distribution over discount factors for Farmers and Sheriffs, $f_F(\delta_F)$, $f_S(\delta_S)$ respectively (which may and in fact will be Dirac delta functions), and the fraction ϕ of the population that is assigned the role of a Farmer, in order to maximize fitness. In turn, each individual chooses his optimal level of investment. Since the planner is constrained to choose discount factors, we refer to this as the *second best*.[7]

THEOREM 4: *The second best distribution is given by*

$$\phi \;=\; \min\left\{ 1,\; \frac{1}{2} + \frac{1}{2}\, \frac{A^{\frac{1}{1-\alpha}}\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{G} \right\},$$

and f_F and f_S assign point mass at $\delta_F = 1$ and $\delta_S = 0$, respectively.

PROOF:
The social planner chooses the investment levels k_F and k_S indirectly, by choosing the discount factors. The implemented investment satisfies:

$$k_F \;=\; k_I \;=\; (\alpha A)^{1/(1-\alpha)}\, \delta_F^{1/(1-\alpha)}$$

$$k_S \;=\; (\alpha AB)^{1/(1-\alpha)} (\delta_F \delta_S)^{1/(1-\alpha)}.$$

In terms of investments, fitness is given by:

$$V_F^U(\delta_F) \;=\; 1 \,+\, Ak_F^\alpha - k_F$$

$$V_F^{FS}(\delta_F, \delta_S) \,+\, V_S^{FS}(\delta_F, \delta_S) \;=\; 2 \,+\, Ak_F^\alpha \,+\, G - k_F - k_S.$$

Given that fitness is strictly decreasing in k_S, the optimal distribution assigns point mass to the value of δ_S, which implements $k_S = 0$, namely $\delta_S = 0$. Similarly, fitness is maximized when Farmers choose to maximize net output, which they do if $\delta_F = 1$. Both conclusions hold irrespective of ϕ. Hence, we may find the optimal value of this latter parameter by maximizing equation (1) when f_F and f_S are evaluated at their optimal values, that is, they assign point mass at $\delta_F = 1$ and $\delta_S = 0$, respectively. Thus, the objective becomes

$$V \;=\; \phi^2(1 + Ak_F^\alpha - k_F) + (1 - \phi)^2 + \phi(1 - \phi)(2 + Ak_F^\alpha - k_F + G - k_S),$$

which is maximized as asserted. ∎

The intuition for the optimal discount factors is simple: Sheriffs' investments are a social waste, which they would not generate if they become extremely impatient. On the other hand, Farmers are productive, and they would choose the optimal

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[7] We refer to this as second best because of the constraint on the planner. However, the constraint does not bind, so the second best plan is in fact first best.

investment if they were extremely patient. In fact, in the language of Hirshleifer (1991) Sheriffs obtain their wealth through conflict; in the language of Tullock (1967) and Krueger (1974), Sheriffs are rent-seekers. In contrast, Farmers obtain their wealth through production.

As for the optimal fraction of Farmers, it is less than 1 whenever a meeting between a Farmer and a Sheriff produces a high enough social gain G. The fraction of the matched population is maximized at

$$\phi = \frac{1}{2}.$$

In the spirit of the rent-seeking literature this is saying that societies would optimally have rent seekers only if when matched to productive agents they were to increase "social output" (that is, G sufficiently high). Otherwise, if G is too small, it would be optimal not to have rent seekers.

A related question has to do with the optimal mix of Farmers and Sheriffs when the social planner does not choose their discount factors, but instead when they are at their equilibrium values. The first order condition for this constrained maximization problem gives

$$\phi = \min\left\{1, \frac{1}{2} + \frac{1}{2}\frac{Ak_F^\alpha - k_F}{G - k_S}\right\},$$

which is sufficient provided $G - k_S > 0$.[8] The value of ϕ is less than 1 for G large enough, and tends to $1/2$ as G grows.

The fact that steady state $\phi > 1/2$ (see Theorem 3) implies that if G is large enough, in the steady state there are inefficiently many Farmers, and too few Sheriffs. The intuition is that this arises because the Sheriff's have to pay to collect a share of G.

IV. Robustness and Scope

One of the striking features of the Farmer-Sheriff game is the survival of the impatient, in spite of the fact that patient players effectively maximize fitness (see Theorem 1). In this subsection we show that observability of types is a necessary condition for this result to hold. This argument is in line with the literature on the necessity of the observability of preferences for nonfitness maximizing preferences to evolve (e.g., Ely and Yilankaya 2001; and Dekel, Ely, and Yilankaya 2007).

Let $a(x, y)$ be the optimal action of a player of type x in a match with another of type y; $v(x, y)$ the payoff of a player of type x in a match with a player of type y; and, as before, $V(x)$ the expected payoff of a player of type x.

Consider first the two-type case and two player roles, the patient and the impatient, where the first role is filled by a patient player and the second by an impatient player. Replicator dynamics implies that if one type has a higher fitness than the other for all population compositions, the latter type will eventually become extinct.

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[8] When the second order condition does not hold (that is $G - k_S < 0$), the optimal solution is $\phi = 1$.

A necessary condition for the (nontrivial) survival of the impatient is that for some interior composition of the population, $V_I > V_P$. Computing the expected fitness difference we get:

$$V_I - V_P = \mu\big(v(I, P) - v(P, P)\big) + (1 - \mu)\big(v(I, I) - v(P, I)\big),$$

where μ is the fraction of type-P players. Hence, if $V_I > V_P$ for some m it is necessary and sufficient that either $v(I, P) > v(P, P)$ or $v(I, I) > v(P, I)$, that is,

$$v\big(a(I, P), a(P, I)\big) > v\big(a(P, P), a(P, I)\big)$$

or

$$v\big(a(I, I), a(I, I)\big) > v\big(a(P, I), a(I, P)\big).$$

If $a(P, P) = a(P, I)$ then $v\big(a(I, P), a(P, I)\big) > v\big(a(P, P), a(P, I)\big)$ implies that $a(P, P)$ is not a best reply to itself—a contradiction. It follows that $a(P, P) \neq a(P, I)$, that is, the patient player must act differently against a patient than against an impatient player. Similarly, if $a(I, I) = a(I, P)$, then $v\big(a(I, I), a(I, I)\big) > v\big(a(P, I), a(I, P)\big)$ implies that $a(P, I)$ is not a best reply against $a(I, P)$, for $a(I, I)$ is better against it. Then $a(I, I) \neq a(I, P)$: the impatient must behave differently against an impatient opponent than against a patient opponent.

For either $a(P, P) \neq a(P, I)$ or $a(I, I) \neq a(I, P)$ to hold, the opponent's type must be observable, as asserted. It is readily seen that the same argument applies to an arbitrary finite number of types. The intuition is simple. Since the patient player maximizes his utility, and given that for a patient player utility is equal to fitness, the only way that the impatient can have a higher fitness than the patient is if the opponent's action depends on the patience of the rival. Thus, the opponent's type must be observable.

The second feature of the Farmer-Sheriff example we want to highlight is that within the group of Farmers, not only the impatient players survive, but also their survival acts against efficiency. In contrast, Dekel, Ely, and Yilankaya (2007) find that "When preferences are observable—as in our case—, a stable outcome must be efficient." Indeed, at first glance their result would appear to contradict ours in this respect. However, their result does not apply to our model. A crucial assumption in their analysis is that all subjective payoffs (namely, all possible preferences over action profiles) are represented in the population. In particular, there is a type for which each action is a dominant strategy. This assumption is not satisfied in our model. Even if we were to allow for a continuum of types, we would still be restricting attention to payoffs that are linear combinations of the flow-objective payoffs. Since the set of preferences we consider is not as rich as that of Dekel, Ely, and Yilankaya (2007), we can and indeed do get inefficiency.

V. Conclusion

We have shown that impatience survives evolutionary forces when it keeps down punishment by the opponents. There are alternative characterizations, and real world situations, where in games less patient people do better than patient people, in contrast

to the single-person investment context where (as in Blume and Easley 1992, 2006) the patient beats the informed. In Blaydes (2004) a version of Fearon's (1998) model is used to explain the division of cartel profits within the Organization of Petruleum Exporting Countries (OPEC). In that model there is a first step in which bargaining determines the payoffs of a static game that is infinitely repeated. To enforce the "efficient" outcome in the infinitely repeated game, more impatient players need a higher "static" payment. Here impatience is a source of bargaining strength.

The general theme of the paper is that although more impatient agents invest less and hence are potentially less fit in the long run in contexts or games where enforcing a norm may involve the threat of punishment impatient players have the advantage of being less prone to retaliation. Thus, in this game evolution and impatience are compatible.

In our environment patient Farmers/producers are more susceptible to threats. Hence, more impatient Farmers do better than patient Farmers. On the one hand, this has the socially beneficial effect that Sheriffs do not waste resources by investing in retaliation. On the other hand the equilibrium fraction of Farmers is inefficiently high, because in a more balanced population there would be more Farmer-Sheriff matches generating more gains from trade.

Although the point is made in a specific example, the principle (which to our knowledge is new in the literature) seems robust and susceptible to analysis in more general contexts.

Appendix A: Proof of Theorem 2

From the text, the dynamical system is given by

$$\dot{\psi} = \psi(1 - \psi)[V_F(\delta) - V_F(1)]$$

$$\dot{\phi} = \phi(1 - \phi)\{[V_F(\delta) - V_S] - (1 - \psi)[V_F(\delta) - V_F(1)]\}.$$

From the fitnesses in the text, we can compute the fitness differences

$$V_F(\delta) - V_S = (1 - \phi)G + \alpha^{\alpha/(1-\alpha)}A^{1/(1-\alpha)}$$
$$\times \left\{ \delta^{\alpha/(1-\alpha)}(1 - \alpha\delta) - B^{1/(1-\alpha)}\delta^{1/(1-\alpha)} \right.$$
$$\left. + \phi B^{1/(1-\alpha)}\left[\delta^{1/(1-\alpha)} - (1 - \alpha)(1 - \psi + \psi\delta^{1/(1-\alpha)})\right] \right\}$$

$$V_F(\delta) - V_F(1) = \alpha^{\alpha/(1-\alpha)}A^{1/(1-\alpha)} \times \left\{ \left(\delta^{\alpha/(1-\alpha)}(1 - \alpha\delta) - 1 + \alpha\right) \right.$$
$$\left. + (1 - \phi)B^{1/(1-\alpha)}\left(1 - \delta^{1/(1-\alpha)}\right) \right\}.$$

LEMMA A.1: *For* $1 > \psi > 0$ *we have* $\dot{\psi} >=< 0$ *as* $\phi <=> \phi^*$ *where*

$$\phi^* = 1 - \frac{1 - \alpha - \delta^{\alpha/(1-\alpha)}(1 - \alpha\delta)}{B^{1/(1-\alpha)}(1 - \delta^{1/(1-\alpha)})}$$

lies between 0 *and* 1.

PROOF:

The computation of ϕ^* comes from solving $V_F(\delta) - V_F(1) = 0$, and we may also compute

$$\frac{\partial}{\partial \phi} \left[V_F(\delta) - V_F(1) \right] \propto - \phi B^{1/(1-\alpha)} \left(1 - \delta^{1/(1-\alpha)} \right) < 0$$

from which the signs follow.

Rewriting

$$1 - \phi^* = \frac{1 - \delta^{1/(1-\alpha)} - \left[\delta^{\alpha/(1-\alpha)}(1 - \delta) + \alpha \left(1 - \delta^{1/(1-\alpha)} \right) \right]}{B^{1/(1-\alpha)} \left(1 - \delta^{1/(1-\alpha)} \right)},$$

we can see that since $B \geq 1$, the numerator of the right-hand side is smaller than the denominator implying $1 - \phi^* < 1$, so that ϕ^* cannot be negative. We may also write the numerator of $1 - \phi^*$ as

$$f(\delta) \equiv 1 - \alpha - \left(\delta^{\alpha/(1-\alpha)} - \alpha \delta^{1/(1-\alpha)} \right).$$

We then compute

$$f(0) \equiv 1 - \alpha$$

$$f(1) \equiv 0$$

$$f'(\delta) \equiv -\frac{\alpha}{1 - \alpha} \, \delta^{-1} \delta^{\alpha/(1-\alpha)} (1 - \delta) < 0$$

from which it follows that $f(\delta) \geq 0$, and so $\phi^* \leq 1$. ∎

LEMMA A.2: $\dot{\phi} \propto a + b\phi + c\psi + d\phi\psi$ *where the factor of proportionality is* $A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)}$ *and*

$$a = \tilde{G} - \alpha - \left(B^{1/(1-\alpha)} - 1 \right)$$

$$b = \left(B^{1/(1-\alpha)} \alpha - \tilde{G} \right)$$

$$c = \delta^{\alpha/(1-\alpha)}(1 - \alpha\delta) - 1 + \alpha + B^{1/(1-\alpha)} \left(1 - \delta^{1/(1-\alpha)} \right)$$

$$d = -B^{1/(1-\alpha)} \alpha \left(1 - \delta^{1/(1-\alpha)} \right)$$

with $\tilde{G} = G/A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)}$.

PROOF:

Direct computation using the fitness differences. ∎

COROLLARY A.1: $d < 0$, $c + d \geq 0$.

PROOF:

$d < 0$ is immediate. For $c + d$ we compute

$$c + d = f(\delta)$$

$$\equiv \delta^{\alpha/(1-\alpha)}(1 - \alpha\delta) + (1 - \alpha)\left[B^{1/(1-\alpha)}\left(1 - \delta^{1/(1-\alpha)}\right) - 1\right]$$

$$f(0) = (1 - \alpha)\left[B^{1/(1-\alpha)} - 1\right] > 0$$

$$f(1) = 0$$

and the derivative

$$f'(\delta) = \delta^{\alpha/(1-\alpha)} \times \left\{\frac{\alpha}{1 - \alpha}\left(\delta^{-1} - 1\right) - (1 - \alpha)B^{1/(1-\alpha)}\delta\right\}.$$

The part in brackets is decreasing, and this implies that $f(\delta)$ is single peaked. Hence, it follows from the boundary conditions that $f(\delta) \geq 0$. ∎

LEMMA A.3: *An interior steady state exists if and only if*

$$X \equiv (1 - \phi^*)\tilde{G} + (1 - \alpha) - B^{1/(1-\alpha)}[1 - \alpha\phi^*] < 0$$

$$Y \equiv (1 - \phi^*)\tilde{G} + (1 - \alpha) - B^{1/(1-\alpha)}\left[1 - \phi^* + (1 - \alpha)\phi^*\delta^{\alpha/(1-\alpha)}\right] > 0$$

and if it exists, it is unique.

PROOF:

If there is an interior steady state by Lemma A.1, it must occur for $\phi = \phi^*$. This implies that the fitness of both types of farmers is equal, so that the sign of $\dot{\phi}$ is determined by

$$V_F(1) - V_S \propto f(\psi) \equiv (1 - \phi)\tilde{G} + (1 - \alpha) - (1 - \phi)B^{1/(1-\alpha)}$$

$$- B^{1/(1-\alpha)}(1 - \alpha)\left\{\phi^* + \psi\phi^*\left(\delta^{1/(1-\alpha)} - 1\right)\right\}.$$

This is linear and increasing in ψ. Hence, there is an interior steady state if and only if $f(0) < 0$, $f(1) > 0$, and in that case because $f(\psi)$ is linear, it is unique. The conditions in the Lemma follow from the expression for $f(\psi)$. ∎

LEMMA A.4: *A sufficient condition for an interior steady state ϕ^*, ψ^* to be stable is $b < 0$.*

PROOF:

It is sufficient that in the system linearized at the steady state the trace be negative and the determinant positive. Disregarding irrelevant factors, the matrix of the linearized system is

$$
\mathbf{M} = \begin{bmatrix} 0 & e \\ c + d\phi^* & b + d\psi^* \end{bmatrix},
$$

where

$$
e = -B^{1/(1-\alpha)}\left(1 - \delta^{1/(1-\alpha)}\right) < 0.
$$

Hence, the sufficient condition is $c + d\phi^* > 0$ and $b + d\psi^* < 0$. By Corollary A.1 $d < 0$, $c + d \geq 0$, and $\phi^* < 1$ implies $c + d\phi^* > 0$, so the remaining condition is sufficient. Since $d < 0$, it is in turn sufficient that $f(0) = -\Gamma < 0$. ∎

THEOREM 2: *Suppose $B^{1/(1-\alpha)}\alpha < (1 - \alpha)\left(B^{1/(1-\alpha)} - 1\right)$. Then for any $0 < \delta < 1$ there exists an open set of G's such that there is a unique interior steady state and it is dynamically stable. At the steady state*

$$
\phi = \phi^* \equiv 1 - \frac{1 - \alpha - \delta^{\alpha/(1-\alpha)}(1 - \alpha\delta)}{B^{1/(1-\alpha)}\left(1 - \delta^{1/(1-\alpha)}\right)}.
$$

PROOF:

The characterization of ϕ^* is in Lemma A.1. For sufficiently small $\varepsilon > 0$ we can choose

$$
\tilde{G} = \frac{B^{1/(1-\alpha)}[1 - \alpha\phi^*] - (1 - \alpha) - \varepsilon}{(1 - \phi^*)} > 0.
$$

The first condition from Lemma A.3 for an interior steady state is

$$
X \equiv -\varepsilon < 0.
$$

Moreover,

$$
Y = X + B^{1/(1-\alpha)}(1 - \alpha)\phi^*\left(1 - \delta^{1/(1-\alpha)}\right)
$$

$$
= -\varepsilon + B^{1/(1-\alpha)}(1 - \alpha)\phi^*\left(1 - \delta^{1/(1-\alpha)}\right),
$$

which is positive for ε sufficiently small. Hence, for such choices of $\tilde{G}$, an interior steady state exists.

Turning to stability, by Lemma A.4, we require $b < 0$, by Lemma A.2 this condition is

$$B^{1/(1-\alpha)}\alpha < \tilde{G}.$$

Notice that

$$\tilde{G} = \frac{B^{1/(1-\alpha)}[1 - \alpha\phi^*] - (1 - \alpha) - \varepsilon}{(1 - \phi^*)}$$

$$> (1 - \alpha)\left(B^{1/(1-\alpha)} - 1\right) - \varepsilon.$$

By the assumption that $B^{1/(1-\alpha)}\alpha < (1 - \alpha)\left(B^{1/(1-\alpha)} - 1\right)$ this implies that $\tilde{G} > B^{1/(1-\alpha)}\alpha - \varepsilon$, so that $b < 0$ for ε sufficiently small. ∎

APPENDIX B: PROOF OF THEOREMS 3 AND 4

As in the model with two types we can compute the fitnesses

$$V_S = 1 + (AB)^{1/(1-\alpha)}(1 - \alpha)\alpha^{\alpha/(1-\alpha)}\phi\,\delta_F^{1/(1-\alpha)}$$

$$V_F = 1 + (1 - \phi)G + \alpha^{\alpha/(1-\alpha)}A^{1/(1-\alpha)}\delta_F^{\alpha/(1-\alpha)}\left\{1 - \alpha\delta_F - (1 - \phi)B^{1/(1-\alpha)}\delta_F\right\}.$$

Define $\tilde{\alpha} = \alpha/(1 - \alpha)$, $\tilde{B} = B^{\tilde{\alpha}+1}$ and as in Appendix A, $\tilde{G} = \tilde{\alpha}G/(\alpha A)^{\tilde{\alpha}+1}$. Note since $B > 0$, $\alpha > 0$ that $\tilde{B} > 1$. Normalizing $\sigma^2 = 1$[9] this enables us to write the dynamical system as

$$\dot{\delta}_F = \frac{(\alpha A)^{\tilde{\alpha}+1}}{1 - \alpha}\delta_F^{\tilde{\alpha}-1}\left[(1 - \delta_F) - \alpha^{-1}(1 - \phi)\tilde{B}\,\delta_F\right]$$

$$\dot{\phi} = \phi(1 - \phi)h(\phi, \delta_F)$$

$$h(\phi, \delta_F) \equiv (\alpha A)^{\tilde{\alpha}+1}\delta_F^{\tilde{\alpha}}\left[\alpha^{-1} - \delta_F - \alpha\tilde{B}\,\delta_F + \phi\tilde{B}\,\delta_F\right] + (1 - \phi)G.$$

LEMMA B.1: *There is a unique interior steady state.*

PROOF:
Combining $\dot{\phi}/(\phi(1 - \phi)) = 0$ and $\dot{\delta}_F = 0$ yields

$$f(\delta_F) \equiv \tilde{B}\left(1 + \alpha^{-1}\tilde{B}\right)\delta_F^{\tilde{\alpha}+2} - \left(1 + \alpha^{-1}\right)\tilde{B}\,\delta_F^{\tilde{\alpha}+1} + \tilde{G}\delta_F - \tilde{G} = 0$$

---

[9] This is relevant only to the stability analysis, and since that is based on a sign argument, the magnitude does not matter.

and letting $\xi = 1 - \phi$

$$g(\xi) \equiv \xi\left[(1 + \alpha^{-1})\tilde{B} + \tilde{G}(1 + \alpha^{-1}\tilde{B}\xi)^{\tilde{\alpha}+1}\right] - (\tilde{B} - 1) = 0.$$

We show that each has a unique zero in $(0, 1)$. Examining g first, we have $g(0) = -(\tilde{B} - 1) < 0$ and $g(1) = \alpha^{-1}\tilde{B} + \tilde{G}(1 + \alpha\tilde{B})^{\tilde{\alpha}+1} + 1 > 0$. Moreover g is the sum of a constant and two increasing functions, so it is increasing, and hence has a unique zero in $(0, 1)$.

Turning to f, we see that $f(0) = -\tilde{G} < 0$ and $f(1) = \alpha^{-1}\tilde{B}(\tilde{B} - 1) > 0$, so that there is at least one solution by continuity. To prove uniqueness, observe that

$$f'(\delta_F) = (\tilde{\alpha} + 2)\tilde{B}(1 + \alpha^{-1}\tilde{B})\delta_F^{\tilde{\alpha}+1} - (\tilde{\alpha} + 1)(1 + \alpha^{-1})\tilde{B}\,\delta_F^{\tilde{\alpha}} + \tilde{G}.$$

Hence, $f'(0) = \tilde{G} > 0$,

$$f'(1) = (\tilde{\alpha} + 2)\tilde{B}(1 + \alpha^{-1}\tilde{B}) - (\tilde{\alpha} + 1)(1 + \alpha^{-1})\tilde{B} + \tilde{G}$$

$$= (1 + \alpha^{-1})\tilde{B} + \alpha^{-1}(\tilde{\alpha} + 2)\tilde{B}(\tilde{B} - 1) + \tilde{G} > \tilde{G}.$$

The second derivative is

$$f''(\delta_F) = \tilde{B}\,\delta_F^{\tilde{\alpha}-1}\left[(\tilde{\alpha} + 2)(\tilde{\alpha} + 1)(1 + \alpha^{-1}\tilde{B})\delta_F - (\tilde{\alpha} + 1)\tilde{\alpha}(1 + \alpha^{-1})\right].$$

This is negative below $\delta^0 \equiv \tilde{\alpha}(1 + \alpha^{-1})/(\tilde{\alpha} + 2)(1 + \alpha^{-1}\tilde{B}) < 1$ and positive above. So f' decreases to its minimum at δ^0 then increases. There are two possibilities: $f'(\delta^0) \geq 0$ or $f'(\delta^0) < 0$. In the first case f increases from $f(0) < 0$ to $f(1) > 0$ so has a unique zero. In the second case it increases to a local maximum at $\delta^1 \in (0, \delta^0)$, then decreases; then, since $f'(1) > 0$ increases again to $f(1) > 0$. A unique zero follows provided that $f(\delta^1) < 0$. Since from 0 to δ^0, and in particular from 0 to δ^1, f is concave, it follows that $f(\delta^1) < f(0) + f'(0)\delta^1 = -\tilde{G} + \tilde{G}\delta^1 = -\tilde{G}(1 - \delta^1) < 0$. ∎

LEMMA B.2: *If $G > (\alpha AB)^{1/(1-\alpha)}$ then the interior steady state is stable.*

PROOF:

As in the proof of Lemma A.4 it is sufficient that in the system linearized at the steady state the trace be negative and the determinant positive. Disregarding irrelevant factors, the matrix of the linearized system is

$$\mathbf{M} = \begin{pmatrix} \partial\dot{\delta}_F/\partial\delta_F & \partial\dot{\delta}_F/\partial\phi \\ \partial h/\partial\delta_F & \partial h/\partial\phi \end{pmatrix}.$$

Consequently, it is sufficient that $\partial \dot{\delta}_F / \partial \delta_F$, $\partial h / \partial \phi < 0$, and $\partial \dot{\delta}_F / \partial \phi > 0$, $\partial h / \partial \delta_F < 0$. We compute

$$\frac{\partial \dot{\delta}_F}{\partial \delta_F} = \frac{(\alpha A)^{\tilde{\alpha}+1}}{1 - \alpha} \left[(\tilde{\alpha} - 1)\delta_F^{\tilde{\alpha}-2}(1 - \delta_F) - \delta_F^{\tilde{\alpha}-1} - \tilde{\alpha}\alpha^{-1}(1 - \phi)\tilde{B}\delta_F^{\tilde{\alpha}-1} \right].$$

Using the fact that when $\dot{\delta} = 0$ we have $\alpha^{-1}(1 - \phi)\tilde{B} = (1 - \delta)/\delta$, from which one obtains

$$\frac{\partial \dot{\delta}_F}{\partial \delta_F} = -\frac{(\alpha A)^{\tilde{\alpha}+1}}{1 - \alpha} \delta_F^{\tilde{\alpha}-2} < 0.$$

Next,

$$\frac{\partial \dot{\delta}_F}{\partial \phi} = \frac{(\alpha A)^{\tilde{\alpha}+1}}{1 - \alpha} \alpha^{-1} \tilde{B}\delta_F^{\tilde{\alpha}} > 0.$$

Using the definition of h we have

$$\frac{\partial h}{\partial \delta_F} = \alpha^{\tilde{\alpha}} A^{\tilde{\alpha}+1} \tilde{\alpha} \delta^{\tilde{\alpha}-1} \left\{ 1 - \left(1 + \left(\alpha^{-1} - \phi \right)\tilde{B} \right)\delta_F \right\}.$$

Using the steady state condition

$$\delta_F = \frac{1}{1 + \alpha^{-1}\tilde{B}(1 - \phi)},$$

the expression in brackets becomes $-\delta\phi\tilde{B}\tilde{\alpha}^{-1}$, so that

$$\frac{\partial h}{\partial \delta_F} = -\alpha^{\tilde{\alpha}} A^{\tilde{\alpha}+1} \tilde{B}\delta^{\tilde{\alpha}} < 0.$$

Finally, compute

$$\frac{\partial h}{\partial \phi} = (\alpha A)^{\tilde{\alpha}+1} \tilde{B}\delta_F^{\tilde{\alpha}+1} - G.$$

Since $\delta < 1$, $(\alpha A)^{\tilde{\alpha}+1} \tilde{B}\delta_F^{\tilde{\alpha}+1} - G < (\alpha A)^{\tilde{\alpha}+1}\tilde{B} - G$, which is negative for $G > (\alpha A)^{\tilde{\alpha}+1} \tilde{B}$, that is to say for the condition of the Lemma $G > (\alpha AB)^{1/(1-\alpha)}$.
   Theorem 3 follows directly from Lemmas B.1 and B.2. ∎

LEMMA B.3: *The steady state* $\phi > 1/2$.

PROOF:
   Using $\left(1 + \alpha^{-1}\tilde{B}\xi \right)^{\tilde{\alpha}+1} > 1 + \alpha^{-1}\tilde{B}\xi$, it is easily checked that $g(1/2) > 0$ and g is increasing in ξ, which implies that if $\xi \geq 1/2$, then $g(\xi) > 0$. It follows that the steady state value of ξ is less than $1/2$, so that the steady state value of $\phi = 1 - \xi$

is greater than $1/2$. The last assertion follows from the fact that g is larger for all ξ the larger is G. ∎

LEMMA B.4: $D_G \delta_F > 0, D_G \phi > 0.$

PROOF:

It suffices to show this for $\tilde{G}$ as given the other parameters $\tilde{G}$ is an increasing linear function of G. From the definitions of f, g the former is decreasing and the latter increasing in $\tilde{G}$. In the proof of Lemma B.1 we showed that both f, g cross the horizontal axis from below. The implicit function theorem then gives the desired result. ∎

LEMMA B.5: $D_B \delta_F < 0,$ *for sufficiently large* $G, D_B \phi < 0.$

PROOF:

It suffices to show the result with respect to $\tilde{B}$ as this is an increasing function of B. By inspection $D_{\delta_F} f > 0$ and $\partial_\xi g > 0$, so $D_\phi g < 0$. We compute

$$D_{\tilde{B}} f = \delta_F^{\tilde{\alpha}+1}\left(2\alpha^{-1}\delta_F\tilde{B} + \delta_F - \left(1 + \alpha^{-1}\right)\right)$$

$$> 2\alpha^{-1} + \delta_F - \left(1 + \alpha^{-1}\right) > \delta_F > 0.$$

It is also the case that $\delta_F\tilde{B} > 1$ in the steady state. This follows from the fact that

$$f\left(1/\tilde{B}\right) = \tilde{B}^{-(\tilde{\alpha}+1)}\left(1 - \tilde{B}\right)\left(1 + \tilde{G}\tilde{B}^{\tilde{\alpha}}\right) < 0.$$

Hence, $D_{\tilde{B}} \delta_F < 0$.

Finally

$$D_{\tilde{B}} g = \xi\left[\left(1 + \alpha^{-1}\right) + \tilde{G}\xi\alpha^{-1}(\tilde{\alpha} + 1)\left(1 + \alpha^{-1}\tilde{B}\xi\right)^{\tilde{\alpha}}\right] - 1.$$

We can write $g(\xi) = 0$ as

$$\xi\left[\frac{\left(1 + \alpha^{-1}\right)\tilde{B}}{\tilde{G}} + \left(1 + \alpha^{-1}\tilde{B}\xi\right)^{\tilde{\alpha}+1}\right] = \frac{\tilde{B} - 1}{\tilde{G}}.$$

The expression in brackets is bounded below by 1, so that as $\tilde{G} \to \infty$ it must be that one $\xi \to 0$. Rewriting the expression as

$$\tilde{G}\xi = \frac{\tilde{B} - 1}{\dfrac{\left(1 + \alpha^{-1}\right)\tilde{B}}{\tilde{G}} + \left(1 + \alpha^{-1}\tilde{B}\xi\right)},$$

we see that as $\tilde{G} \to \infty, \xi \to 0$, the RHS approaches $\tilde{B} - 1$, and so $\Gamma\xi \to \tilde{B} - 1$. Hence, as $\tilde{G} \to \infty$, we have $D_{\tilde{B}} g \to -1$. The implicit function theorem then gives the second result. ∎

Theorem 4 now follows directly from Lemmas B.3, B.4, and B.5.

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