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Students' Interpretations of Quantum Mechanics Concepts from Feynman's Sum of all Paths Applied to Light

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Abstract

We analyse part of the implementation of a didactic sequence to teach different aspects of light in a unified non traditional framework. The goal was to propose the quantum theory of light as a universal framework to describe different phenomena observed. The laws of quantum mechanics for light using Feynman´s "Sum of all Paths" approach adapted to the mathematical level of the students was proposed as a model to explain the experiences. This particular denomination of Feynman´s approach is an intentional choice to avoid the language of "integral" because the students haven't had calculus. Graphic representations and basic operations with vectors capturing the essential aspects of the theory, were used. Simulations made with the software GeoGebra(R) and Modellus were created to help students visualize the formulation. The sequence was carried out in four courses (aged 15-16). For the data analysis, an answer categorization was formulated, considering among other aspects the quantum reformulation of experiment shown herein. This analysis seeks to understand the student´s conceptualization process about quantum interpretation. The results support the conclusion that the conceptualization is complex, and slow, due to both the concepts involved and the representation systems demanded by the situations.

Keywords

Quantum, Light, Feynman, Double slit experiment, Conceptualization

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Introduction:

In the Science Education field, the teaching of quantum mechanics is a topic that has been present in the researchers' agenda and it is still open (Cabral de Paulo y Moreira, 2005; Cuppari et al. 1997; Fischler & Lichtfeldt, 1992; González, Fernández, & Solbes, 2000; Lobato & Greca, 2005; Greca, Moreira & Herscovitz, 2001; Hadzidaki, 2008; Henriksen et al., 2015; Henriksen, et al., 2014, 2015; Montenegro & Pessoa, 2002; Ireson, 2000; Müller y Wiesner, 2002; Mannila, Koponen, & Niskanen, 2002, Niedderer, 1996; Olsen, 2002; Ostermann & Ricci, 2004; Pessoa,1997; Pinto & Zanetic, 1999; Zollman, 1999). The work of Krijtenburg-Lewerissa, Pol, Brinkman & Joolingen, V. (2017) presents an updated literature review in this topic.

The Feynman approach, called "Paths Integrals" or "Feynman´s Multiple Paths formulation" (developed in 1949 by Richard Feynman) is an alternative to canonical approach that has been employed in different teaching contexts. It provides a formulation of time-dependent quantum mechanics, equivalent to the Schrödinger approach. Among its main advantages it can be mentioned that the Path Integral formulation provides a unified description of quantum mechanics and offers a natural way of obtaining the classical limit of quantum mechanics. Feynman showed the simplest version of the method in his book 'QED: The Strange Story of Light and Matter' (Feynman 1985). Based on this idea, a teaching line of quantum mechanics started with the works of Dowrick (1997) and Taylor et al (1998), for undergraduate courses for non-specialists. Subsequently proposals were made to include the Feynman formulation in high school curriculum (Cuppari et al. 1997, Ogborn Taylor, 2005; Ogborn, Hanck & Taylor, 2006; Hanck, & Tuleja, 2005). A major step forward in the popularization of the formulation at a basic level was the inclusion in the Advancing Physics AS (2000) project of an A-level physics course for the British high school system, where the quantum physics chapter was treated using the Sum Over Paths approach (Dobson, K, Lawrence, I & Britton, P, 2000). In general, these investigations and didactic proposals were based on computer simulations and focalized on the photon concept. On the other hand, the work of Beau (2012) presented an interesting pedagogical introduction to Feynman's formulation of quantum mechanics. It focused on electrons in the Double Slit Experiment and dealt with the foundations of quantum mechanics for university students.

The works of Malgieri, Onorato and De Ambrosis, (2014, 2015a, 2015b; 2016, 2017) in Italy, extended the variety of quantum mechanics topics that can be addressed using the Feynman formulation, including teacher training and implementation results analysis. These works started by reconsidering optics from the standpoint of the quantum nature of light, analyzing both traditional and modern experiments (Malgieri et al.2014). They focused on the connection between the wave phenomenology of light and the properties of the photon.

Finally, we would like to mention our contributions to the subject, which range from didactic sequences proposals based on the Feynman formulation to high school implementation analysis (Arlego, Fanaro, and Otero, 2012; Fanaro; Arlego, and Otero, 2014a; 2014b).

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Concerning the analysis of the difficulties that students face when approaching the study of phenomena associated with light, most of the studies consider classical wave and ray optics aspects. Among them it is worth mentioning the works of Ambrose (1999), Wosilait et al. (1999), Stamatis Vokos et al. (2000), Colin and Viennot (2001), Maurines (2010). However, both the difficulties in the approach and the conceptualization of the quantum character of light have been much less explored. In this work we address in part this last aspect by analysing the way that the students interpret the quantum mechanics concepts from the Feynman´s Sum of All Paths† applied to the Double Slit Experiment (DSE), in particular the light probability distribution on the detection screen. Our study is part of a longer-range project where we proposed the quantum theory of light as a general framework to describe different phenomena observed bypassing classical light models (see more details of the complete sequence in Fanaro, Elgue, M. and Otero, 2016).

Theoretical Framework

Physical foundations: Feynman´s Path Integral formulation and its adaptation to school

Here we briefly review the main characteristics of the time independent path integral formulation that we have adapted to high school level (Fanaro, Elgue, M. and Otero, 2014).

In classical physics the certainty of a given event is predicted, i.e, given an initial condition there is only one possible final state. On the contrary in quantum mechanics a fixed initial condition does not imply a unique final state, but a variety of final states are possible. The rules of quantum mechanics predict the relative probability of the different possible final states compatible with the initial state. For example, it predicts the probability that an electron or light emitted from a source is detected at a given point on the screen behind the slits in the DSE.

The way in which the probability of a given event is calculated in the adapted Feynman formulation presented here can be summarized in the following sequence of steps (Fanaro, Arlego and Otero, 2014 p.2).

1) Consider different paths connecting the initial state I with the final state

2) Associate to each path a unitary vector in the plane, whose direction is proportional to the length of the path. The proportionality constant depends on the "type" of light (red, green, infrared, etc.).

3) The resulting vector is obtained by adding the vectors corresponding to all paths, as shown schematically in Fig. 1(c).

4) The squared magnitude of the resulting vector is proportional to the probability of detecting at point F the light emitted at I.

† For educational purposes we decided to call the "Feynman´s Paths Integrals formulation", "Feynman´s sum of all paths", because it is more familiar for the students to speak about sums instead of integrals.

Figure 1 below shows some of them (A, B, C, D, E, F and G). For educational purposes, we decided to call the "Feynman´s Paths Integrals formulation", "Feynman´s sum of all paths", because it is more familiar for the students to speak about sums instead of integrals.

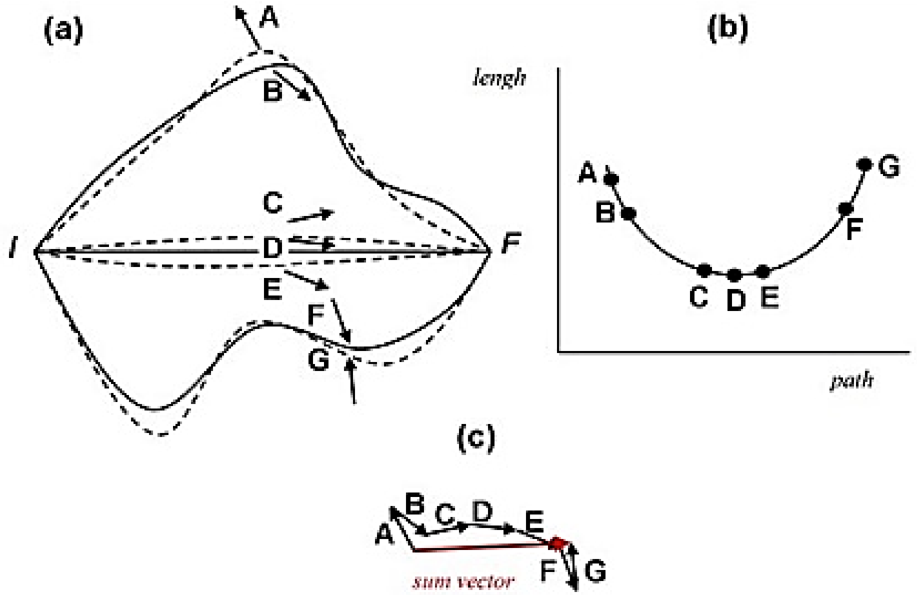

Figure 1. (a) Some paths connecting I with F, and their associated vectors. (b) Representation of the path length for different "paths" (c) Sum of the associated vectors.

The sum of all infinite contributions from the different paths represents a difficulty in Feynman's original formulation. In a version adapted for didactic purposes, this problem can be solved by analysing the special role that the 'classical' paths play, recognized as the technique of semiclassical approximation in quantum mechanics (e.g. Shankar, 1980). The key idea is that in many cases the shortest (classical) path along with those of its adjacent paths have a dominant contribution to the sum. This is due to the fact that arbitrary paths -whose lengths are not related- have associated vectors with arbitrary directions, thus the total effect of these paths will be a mutual 'statistical' cancellation. However, the associated vectors with the shortest path and its adjacent paths will have approximately the same direction and therefore they will contribute in phase to the sum. This is a consequence of the basic property of the extremes of a function: the variations around the minimum are smaller than around any other point. In this work we will use this prescription by considering only classical paths contributions.

The didactic-cognitive framework

Our key viewpoint consisted in teaching by means of questions facing the students with specific situations. These were genuine problems and tasks related to a specific domain that students must get engaged and solve in groups. Every situation was designed to trigger the specific concepts to be taught, considering possible lines of action and thought and the prior knowledge of students. To this end we adopted the Vergnaud Theory of Conceptual Fields (Vergnaud, 1990, 2013). This theory was not only the basis employed for the design of the situations, but also for the prior didactic analysis, which allowed to anticipate possible actions of the students and the teacher. Besides this theory allowed to analyse the potential advantages and obstacles in the conceptualization of the students when confronting the proposed situations.

Within the Vergnaud theory the conceptualization is defined as the process of the identification of concepts, their properties and their relations with other concepts. (Otero, et al., 2014). The concepts are well defined in this framework by a set of three interrelated elements: C=(S,I,R), where S represents the set of situations that make the concept useful and meaningful, I is the set of operational invariants that can be used by individuals to deal with these situations, and R the set of symbolic representations, linguistic, graphical or gestural that can be used to represent invariants, situations and procedures (see Vergnaud, 1990, 2013 for a more complete description of the Theory). The Operational invariants can be of two types: concepts-in-action and theorems-in-action. These invariants organize the action of the subject and make it operational. A concept-in-action is a category, a property, a predicate that is considered relevant to the situation. A theorem-in-action is a proposition that the subject considers true about that situation.

The operational invariants are implicit, and not comparable with concepts or scientific principles, because the latter are explicit and their relevance and validity can be discussed.

Method

We designed a didactic sequence composed by 14 situations, adapted to the mathematical and physical knowledge of the students based on the Feynman´s approach. The implemented sequence aimed to understand the process of conceptualization of students on reflection, refraction and the double slit experiment (DSE) from a quantum point of view. The logic of the designed didactic sequence (Fanaro, Elgue, M. and Otero, 2016) can be summarized as follows. The sequence began with the students' prediction of the results of the reflection, refraction, and double-slit experiment (DSE). After that these experiments were carried out in the classroom, using a laser light source. Later the results of the DSE were presented through a sequence of actual photographs taken from the results of the DSE with very low intensity light. This enabled to show the individual detection events on the screen which evolved into a definite pattern of alternated fringes.

The laws of quantum mechanics for light using Feynman´s "Sum of all Paths" approach was proposed as a model to explain these experiments. The main adaptation consisted in working with sums of vectors in the Cartesian plane, instead of integrals. Graphical representations and basic operations with vectors capturing the essential aspects of the theory were used. Simulations made with the software GeoGebra were developed to help students visualize the Feynman´s "Sum of all Paths" formulation results in the simple cases of light emission and detection, light reflection and refraction. In all three cases, the simulations generated with GeoGebra enabled to select different paths between the emission and light detection events, and graphically show the corresponding associated vectors (Elgue, M., 2015). Finally the Feynman´s "Sum of all Paths" formulation was applied to the DSE to describe the results obtained in the situation relative to the localized detection and the alternated fringes pattern. A simulation was created with Modellus to plot the probability function as a function of the distance to the centre of the screen, and dynamically show the sum of the two main vectors for each point of the light collector screen.

The sequence was carried out in four courses of two state secondary schools, with N=83 students aged 15-16, corresponding to the penultimate year in high school. The whole sequence was carried out during 23 school hours. The schools and courses were selected to control the variables: time, previous students' knowledge, articulation with other contents, and familiarity with the institutional conditions. The teacher had to know the didactic intention of each situation, and its management. The teacher proposed the situations to the students who must talk and solve in groups, and then agree on a written response. In each class there was also a non-participant observer who registered what happened. The written resolutions of the students were collected class after class. Thus we obtained N = 1245 resolutions with A1S1 ... A1S15 ... A83S15 (83 resolutions for each of the 15 situations, including the final test)

The qualitative analysis was based on all the students' productions, when they were faced with the situations. Thus, we analyzed the conceptualization process, based on Vergnaud theory to identify possible aids and obstacles in the conceptualization. A categorization was done aiming to describe the students´ conceptualization analyzing if the students´ operational invariants and the symbolic representations were according to the Sum of All Paths viewpoint.

In this work we only focus on how the students use the Feynman´s "Sum of all Paths" formulation and quantum reformulations about the DSE.

Results

In the last stage (fourth) of the didactic sequence, the calculation of probability of light detection in the DSE (**Figure 2**) was proposed.

Applying concepts previously studied in the sequence relative to the Feynman´s Multiple Paths formulation (Fanaro, Elgue and Otero, 2016) and doing simple mathematical operations, the expression of P(x) was constructed without difficulties with the students.

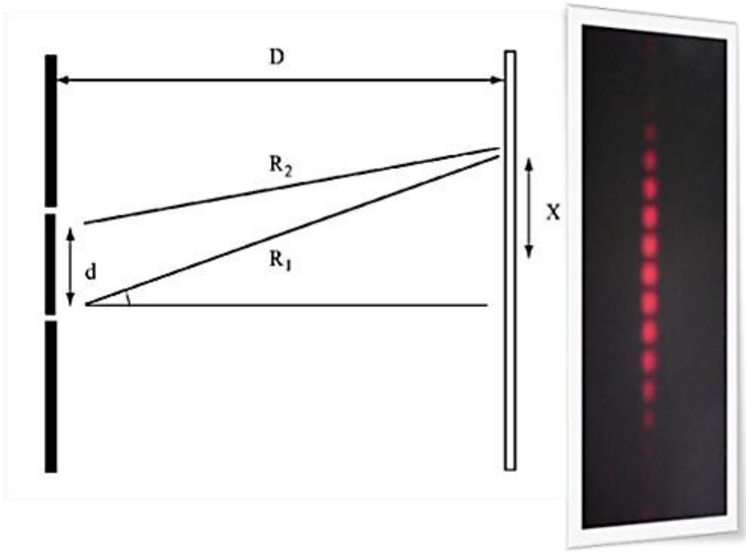

Figure 2. Scheme of the DSE and the corresponding results obtained in the screen

If we use these results at the DSE (Fanaro, Arlego and Otero, 2014), we must consider two main classical paths (due to the cancellation of the paths commented above). Applying basic mathematics we obtain an expression describing the probability of the light distribution on the screen (Eq. 1)

$$P(x) \approx cos^2(\frac{\omega.d}{c.D}.x) \quad (1)$$

where (D) is the distance from the slits to the light detection screen, (d) is the separation of the slits and ω is a parameter that corresponds to the "type" of light.

This result indicates that the probability function has maxima and minima, in agreement with the alternated pattern, as it was observed in the experiment. The students had to relate the minima and maxima of the P(x) with the results of the DSE, and answer the following questions:

$Q_{4.1}$) What characteristics has the P(x) graph got? How does the probability change with the distance to the center of the screen? Consider D (the distance from the slits to the wall) and d (the distance between slits) and the value of the proportionality constant k, which corresponds to the red laser, and draw it.
$Q_{4.2}$) Which is the relation between the minima and maxima of P(x) drawn above with the results of the DSE?

Then, the students had to run the simulation of the DSE created with the simulation software Modellus (**Figure 3**). The students had to relate the resulting vector with the probability function.

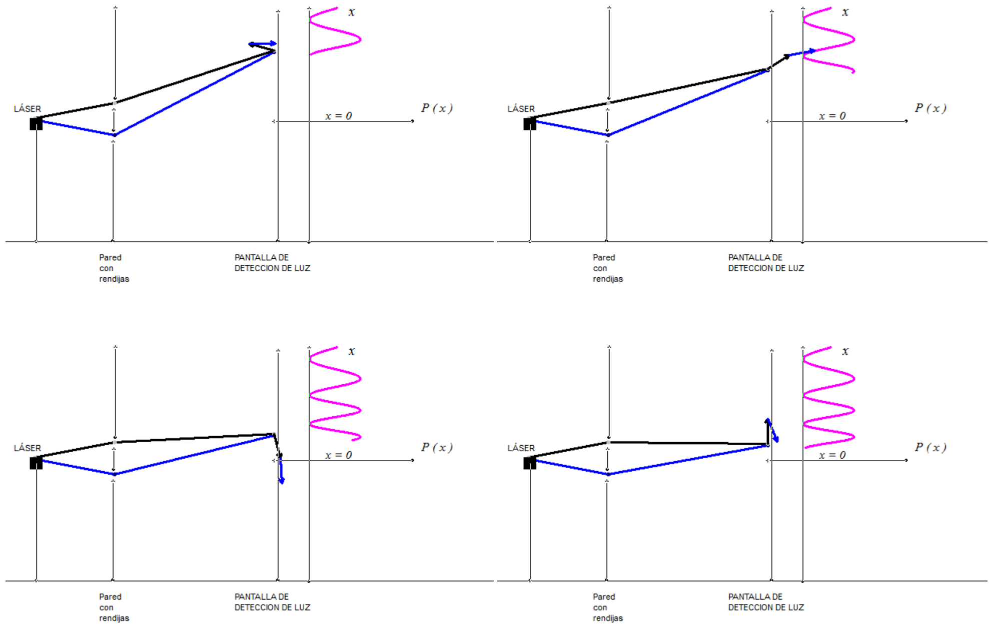

Figure 3. Selected snapshots of the Modellus simulation relative to the DSE and the corresponding P(x)

The question raised was:

$Q_{4.3}$) Which is the relation between the sum of the associated vectors to each main path and the P(x) function drawn by the software? In this paper we are focused on the the students´ conceptualization process about quantum interpretation of DSE, therefore we consider the answers $Q_{4.2}$ and $Q_{4.3}$

a) Regarding the relation between the characteristics of the graph of P(x) and the results of the DSE (Q4.2)

Most students (n=70) could interpret the graph of P(x) previously obtained. They identified the maximum value of the probability function with the light fringes, and the minimum values of the function with the places where light was not observed when they performed the experiment. Approximately half of them (34/70) used the probability concept in the relationship, whereas the other half (36/70) did not use this concept but expressed "where there is a minimum of probability, there is darkness". It is important to highlight that many students of this group represented graphically the modulations (due to single slit diffraction) observed in the classes, although it wasn´t considered in the Feynman´s "Sum of all Paths" (**Figure 4**).

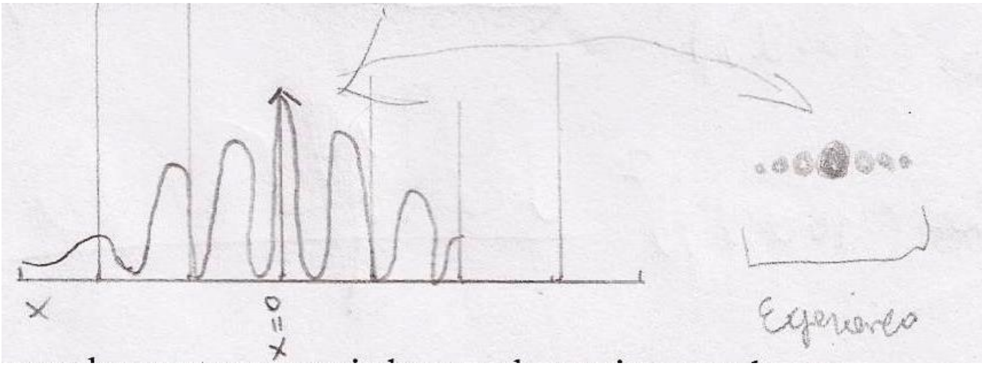

Figure 4. Drawing done by a student representing P(x) and the correspondence with the DSE.

Note the complexity of both the graphical representation and the appropriate identification done by the student, which was represented by the curved arrow between both graphs. Finally, a few students (n=13) either did not answer Q4.2 or gave a confused answer, e.g. considering that light copies the shape of the slits as it goes through, i.e. using ray optics. Some students of this group did not use the proper concepts of maxima and minima of probability but they established the relationship inadequately considering the maxima as the absence of light and the minima as the presence of light. Both issues present a big obstacle for the conceptualization of the quantum concept of probability. This denotes the complexity of the situation for the students, since they were working with situation four and they still did not consider the probability as the central concept of the situation.

b) Regarding the relation between the sum of the vectors associated to each main path and the function P(x) plotted by the software (Q4.3), more than half of the students (n=47) established the adequate relationship, using the probability concept. They pointed out that the sum vector represents the probability of detecting light at the screen, and the larger the module of the sum vector, larger the probability of detecting light. Additionally they reinforced the concept by pointing maxima and minima on the P(x) graph. **Figure 5** shows a student´s answer diagram overlapping the real probability function (not simply the modeled by Feynman´s "Sum of all Paths"), the DSE image obtained in the class and the sum of the two associated vectors, all in spatial correspondence. It is outstanding that this diagram was designed by the student himself denoting the high degree of association achieved between the experiment and the proposed model.

A group of n= 21 students established that when the sum of the vectors is maximum, there would be light on the screen, and when it is minimal there would be darkness, but without referring to the probability concept. In this sense, the conceptualization was incomplete from the point of view of the association with the Feynman´s Multiple Path model.

Figure 5. Diagram developed by a student to answer Q4,3. It is remarkable the high level of association achieved between the model and the experiment in this case

Finally, the rest of the students (n=5) neither answered nor established a clear relationship between model and experiment. For example, the answer in **Figure 6** shows that this student did not consider any periodicity, drawing the sum of vectors practically with the same length. These results indicate the complexity that supposes for the students to use more than two systems of graphical representations, together with the algebraic representations. For these students it was difficult to identify the physical concepts with the three representation systems that this situation demanded: algebraic and graphical representation of P (x) and the representation of experiment carried out in class. This difficulty is partly due to the complexity of the quantum concepts involved, and the proper conceptualization process that is only achieved in the long term.

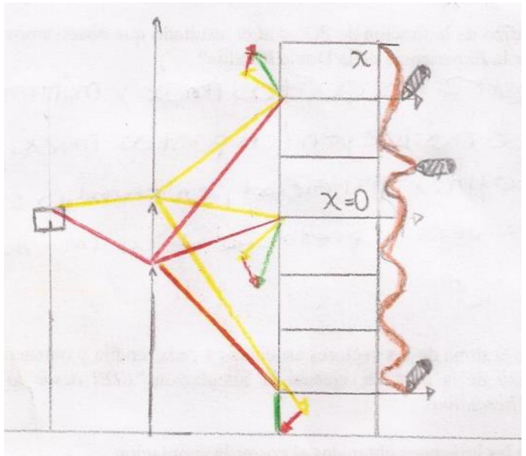

Figure 6. A student´s plot of DSE and its corresponding function P(x). Note that in this case there is no association between the maxima and the resulting vectors (see the text for more details).

Conclusions

The results of the implementation are promising, given that a good part of the students conceptualized the probability in the DSE, from the Feynman´s "Sum of all Paths" formulation. It is of great encouragement that many students tried to identify the shape of the probability function graph with the presence or absence of light in terms of probability. That is, they conceptualized that the expression of P (x) describes the results obtained when performing the experiment. It is remarkable that the students, without handling the algebraic expression of probability, represented the modulation in the graph of the function, so that it can be more appropriate to the results obtained in the experiment.

The students had faced situations prior to those analysed in this paper, where the concept of probability and its relation to possible paths were raised. However, the DSE context, requires a more complex association of representation systems. Therefore, we believe that these results support the conclusion that the conceptualization is complex, and slow, due to both the concepts involved and the representation systems demanded by the situations. To help the conceptualization, we propose to design a new set of situations that allows students to work more in depth with the concept of probability from the first instances of the sequence. For example, situations that allow students to calculate probabilities of different events, since in this sequence the probability concept was approached in a qualitative way. This is not difficult to incorporate with simple tools such as spreadsheets, and we are working in this direction.

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