



## Diffusive Drying Kinetics in Wheat, Part 1: Potential for a Simplified Analytical Solution

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Mathematical models for kernel drying predict grain–air mass and heat transfer rates, and the faster and more accurately they can do this the better, especially in simulation-based design and control. Few publications used the simplified analytical solution of the diffusion equation for ‘short times’ based on semi-infinite medium concepts, which is simpler and as accurate as the exact infinite series though it still awaits its place in determining diffusion coefficients. The drying background of wheat and similarly sized grains was reanalysed because assumptions that could not apply to larger grains may be valid for smaller ones to justify analytical solutions. Reliable Biot numbers were calculated to be some 0.5 for heat transfer and 50–3000 (internal control) for mass transfer. Despite heat transfer which individually may or may not be considered to be controlled externally, the grain thermal-to-moisture diffusivity ratio was about 6000, making heat transfer instantaneous compared with mass transfer. To corroborate these indicators, the complete heat conduction–moisture diffusion problem was numerically solved, and the predictions showed, precisely, that analytical solutions may be used to estimate diffusion coefficients in wheat. Then, the steps leading to this overlooked simplified diffusive equation were revised in the light of current knowledge to show how computing time can be saved without loss of accuracy.

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### 1. Introduction

The main practical purpose of mathematical descriptions for fixed bed and continuous dryers (Brooker *et al.*, 1992; Giner *et al.*, 1998) is to predict the residence time or the grain throughput as accurately as possible. Other uses, following proper validation of the model, are to explore the effect of operating variables on dryer performance (Nellist, 1987) and to forecast the effect of design variants (Nellist & Bruce, 1995). Another practical use of simulation models is in their dynamic form for developing model-based, dryer control algorithms (Bruce & McFarlane, 1993; Mujumdar & Passos, 1998). All these uses require models that are accurate yet interactive, and fast to run on the computer.

Dryer models comprise the product model to represent the drying characteristics of a single particle, the

‘deep-bed’ model, dealing with mass and energy balances in the bed, sometimes with airflow prediction, and the ‘equipment model’, which covers the air–grain relative directions, plus features such as air/grain recirculation, reversed airflow and variable bed depth.

Concerning the product model, or thin-layer drying model, the unsteady-state partial differential diffusion equation (Crank, 1975) can be used with simplifying assumptions to allow an analytical solution to be obtained to relate the average grain moisture content  $W$  with time  $t$ . The solution is normally an involved infinite convergent series (Becker & Sallans, 1955; Suárez *et al.*, 1981; Tolaba & Suárez, 1988; Sun & Woods, 1994). Assumptions used to this end are a regular shape and a diffusion coefficient independent of moisture content *during drying*. As observed in drying reviews (Parry, 1985; Sokhansanj & Cenkowski, 1988; Jayas *et al.*, 1991; Nellist



& Bruce, 1995), many previous contributions avoided the infinite series solution in the past, possibly because of difficulties in fitting the data with tedious calculations, resorting to semi-empirical equations such as the Lewis single exponential or to an empirical variant of it with two fitting parameters, the Page equation (Jayas *et al.*, 1991) or else to a phenomenological second-grade polynomial, the expression developed by Thompson *et al.* (1968). The last two reproduce the experimental drying curve better, but none has been as accurate as the diffusive series solution, at least in its form for spheres (Sun & Woods, 1994).

Other authors considered a potential dependency of the effective diffusion coefficient  $D$  with moisture content (m.c.), so they have not used that analytical solution. For barley drying, Bruce (1985) used an exponential function of  $D$  with m.c. in an isotropic sphere, fitting it directly from a numerical solution of the differential diffusion equation, for which purpose the author measured many thin-layer drying runs. While the reviews on diffusion coefficients in foods offered by Zogzas *et al.* (1994, 1996) and Zogzas and Maroulis (1996) have confirmed the Arrhenius-type temperature dependency of  $D$ , this was not the case with water concentration, since the plots of  $D$  versus m.c. presented considerable scatter. Similar conclusions can be drawn from the thin layer drying reviews mentioned above, where scarce attempts to model the dependency of kinetic parameters on m.c. were observed, particularly in wheat.

Ketelaars *et al.* (1995) used scanning neutron radiography to determine moisture gradients inside a non-shrinking kaolin clay sample adapted to have moisture flux in one dimension and found a curious dependency where  $D$  first decreases with m.c. and then increases. Although the authors suggested that the experimental drying data are not suitable to properly assess the effect of moisture on  $D$ , the applicability of scanning neutron radiography to grains is not guaranteed since the method is suitable for materials that do not have much hydrogen.

Another problem for m.c.-dependent diffusion coefficients derived from finite difference schemes is geometry: most models have assumed a spherical shape for rather ellipsoidal grains, such as wheat, barley, rice, so the simplifying assumption is incorporated within the fitting diffusion coefficient and may superimpose on the possible functionality of  $D$  with m.c. Numerical solutions of the diffusion equation by finite elements allow better modelling of grain shape (Haghighi *et al.*, 1990; Irudayaraj & Wu, 1994) but, from the dryer design point of view, they are complex and still very slow to run. Tolaba *et al.* (1997) have solved the diffusion equation for spheres with the Crank–Nicolson finite difference scheme to directly propose a sorptional dependency of  $D$  on m.c. in isothermal drying. However, Krishna and Wesselingh

(1997) have indicated that the possible relationship between  $D$  and m.c. is better based on transport characteristics of the material (pore size, cell wall permeability), than on physicochemical/thermodynamical grounds better suited for explaining the heat of sorption (Gely & Giner, 2000). However, to date, no experimental drying data has been presented for grains as evidence of a dependency between  $D$  and m.c.

Another aspect that inhibits the use of analytical solutions in grain drying is the need for modelling heat conduction inside grains. Sokhansanj and Bruce (1987) have done a valuable experimental and theoretical work to predict intragranular heat transfer. The Biot number they obtained is around the threshold between external and combined internal–external control, so it could well be used to accept a flat internal profile assumption. In turn, the profile was shown to heat so fast during drying that it could even justify the isothermal drying assumption, in view of the difference of rates of relative changes in moisture and temperature. The isothermal drying assumption had been proposed in the well-known work by Pabis and Henderson (1962) for a large grain, such as maize, where the supposition may not be realistic.

The use of analytical solutions can also be justified in terms of the Biot number for mass transfer. Walton *et al.* (1988) defined the Biot number for mass transfer  $Bi_m$  using a water content conversion from grain to air phases based on a slope of an equilibrium m.c. (e.m.c.) versus vapour density isotherm, though the true isotherm is e.m.c. versus water activity  $a_w$ . In turn, Parti (1993) studied drying models based on Biot numbers for heat and mass transfer but, for the latter, no sorption isotherm conversion was included. Even when Biot and Lewis numbers (Foust *et al.*, 1983) may indicate that ‘isothermal drying’ is not valid for maize, they may justify that assumption for wheat and similarly sized kernels, so perhaps an analytical solution can be used in its own right to estimate diffusion coefficients.

Among the analytical solutions of the diffusion equation, that developed by Becker (1959) is equivalent to the infinite series solution for short times; it has only three terms and was used on a few occasions, almost always within drying simulation models (Becker & Isaacson, 1970; Giner *et al.*, 1996). It was seldom used to fit diffusion coefficients despite being easy to implement in statistical software. Surprisingly enough, it was omitted in the thin-layer drying and diffusion coefficient reviews mentioned above. Therefore, as the Becker equation seems to comply with the accuracy and calculation speed required in simulation models for wheat dryer design, the purposes of this work are: (1) to reanalyse the heat and mass transfer fundamentals for wheat drying based on more reliable calculations of Biot numbers, so that the possibilities of using diffusive analytical solutions in the

drying of wheat and similarly sized kernels can be better ascertained; and (2) to highlight the development of the diffusive short time equation (thus termed because it is based on semi-infinite medium concepts) so as to compare it with the analytical infinite series solution for a sphere. Finally, the extent of the possible effects of geometry assumption on the diffusion coefficient values is discussed.

## 2. Theoretical considerations

### 2.1. Microscopic mass balance in the grain with transport by liquid diffusion

By taking the grain volume as a system and assuming internal water transport by liquid diffusion, the microscopic mass balance can be expressed in the following way (Crank, 1975):

$$\frac{\partial W_l}{\partial t} = \nabla(D\nabla W_l) \quad (1)$$

This partial differential equation holds in each internal point of the solid, and gives the local m.c. of the diffusing component  $W_l$  as a function of time  $t$  and the radial spherical coordinate  $r$ , whose axis is always normal to the surface and whose origin is placed at the centre of symmetry (Becker, 1959). The symbol  $D$  is the transport parameter, *i.e.* the effective diffusion coefficient of water relative to the dry matter. If shrinkage is neglected in view of the low moisture ranges normally covered in wheat drying, the diffusive flux with respect to dry matter coincides with that taken with reference to a fixed system of coordinates (Bird *et al.*, 1960). For radial water flux in spherical geometry, Eqn (1) can be developed to give (Bruce, 1985)

$$\frac{\partial W_l}{\partial t} = D \left( \frac{\partial^2 W_l}{\partial r^2} + \frac{2}{r} \frac{\partial W_l}{\partial r} \right) + \left( \frac{\partial W_l}{\partial r} \right)^2 \frac{\partial D}{\partial W_l} \quad (2)$$

#### 2.1.1. Simplified form of the differential equation

If  $D$  is taken to be independent of the local m.c. during drying, then

$$\frac{\partial W_l}{\partial t} = D \left( \frac{\partial^2 W_l}{\partial r^2} + \frac{2}{r} \frac{\partial W_l}{\partial r} \right) \quad (3)$$

For either form, the appropriate initial and boundary conditions must be defined.

#### 2.1.2. Initial and boundary conditions in mass transfer

The initial and boundary conditions typical to solve the differential equations, Eqns (2) or (3), are the following. Initial condition:

$$t = 0, \quad W_l = W_0, \quad 0 \leq r \leq R \quad (4)$$

where:  $W_0$  is the initial grain moisture content and  $R$  is the grain radius. Boundary condition at the centre:

$$r = 0, \quad \frac{\partial W_l}{\partial r} = 0, \quad t > 0 \quad (5)$$

Boundary condition at the surface, where two alternatives, Eqns (6a) and (6b) are possible:

$$r = R, \quad -\rho_d D \frac{\partial W_s}{\partial r} = k_p(p_{vs} - p_{va}) \\ \equiv k_w(W_s - W_e), \quad t > 0 \quad (6a)$$

$$r = R, \quad W_s = W_e, \quad t > 0 \quad (6b)$$

where:  $W_s$  is the particular value of  $W_l$  at the surface;  $\rho_d$  is the density of the dry matter;  $k_p$  is the air-grain mass transfer coefficient based on vapour pressure; and  $k_w$  is a transformed version of  $k_p$ , based on grain moisture units [see Eqn (16)]. The surface condition of Eqn (6a) is general and assumes a gradual decrease of the surface m.c. towards the equilibrium value given by  $W_e$  (for air temperature  $T_a$  and air humidity  $h_{ra}$ ) or, in equivalent terms, a gradual decrease of the surface water vapour pressure in the grain side  $p_{vs}$  towards the vapour partial pressure in the bulk air  $p_{va}$ .

### 2.2. Microscopic energy balance inside the kernel

By considering heat conduction inside the grain, the microscopic energy balance for spheres becomes (Sokhansanj & Bruce, 1987)

$$\rho_d C_p \frac{\partial T_l}{\partial t} = k_T \left( \frac{\partial^2 T_l}{\partial r^2} + \frac{2}{r} \frac{\partial T_l}{\partial r} \right) \quad (7)$$

where the specific heat  $C_p$  and thermal conductivity of the moist grain  $k_T$  both depend on m.c., and  $T_l$  is the local temperature.

#### 2.2.1. Initial and boundary conditions in heat transfer with surface water evaporation

Initial and boundary conditions for solving the microscopic energy balance, where the contribution of radiation heat transfer is neglected, are as follows:

$$t = 0, \quad T_l = T_0, \quad 0 \leq r \leq R \quad (8)$$

$$r = 0, \quad \frac{\partial T_l}{\partial r} = 0, \quad t > 0 \quad (9)$$

$$r = R, \quad k_T \frac{\partial T_s}{\partial r} = h_T(T_a - T_s) - L_g k_w(W_s - W_e), \quad t > 0 \quad (10)$$

where:  $L_g$  is the heat of sorption;  $T_a$  is the air temperature;  $T_s$  being the particular value of  $T_l$  at the grain

surface;  $T_0$  is the initial temperature; and  $h_T$  is the air-grain heat transfer coefficient.

The surface condition ( $r = R$ ) indicates that the heat flux entering the grain—which is used to increase grain temperature—results from the difference of the heat received from the air by convection and that used to evaporate the moisture at the surface. The heat of sorption should be evaluated at the surface m.c. and temperature for being an implicit assumption of the liquid diffusion theory with surface evaporation (Giner, 1999).

### 2.3. Controlling mechanisms of heat and mass transfer rates for calculating Biot numbers

It has been reported that cereal grains, and especially wheat, dry during a definite falling drying rate period (Becker, 1959; Sun & Woods, 1994). However, when the radial coordinate is at the grain surface, precise calculations of the Biot number for mass transfer are required to decide whether boundary conditions [Eqn (6a) or Eqn (6b)] apply. Moreover, the evaluation of the Biot number for heat transfer  $Bi_c$  is also useful to find out whether or not the microscopic energy balance [Eqn (5)] should be resolved.

Sokhansanj and Bruce (1987) have calculated a value for  $Bi_c$  of 0.13 in a typical wheat drying situation using  $R/3$  as characteristic dimension for spherical geometry, so they had obtained about 0.4 if based on  $R$ . In turn, Parti (1993) used  $Bi_c$  and  $Bi_m$  to study simulations of several cases. Though his calculations could have been correct, his values of  $Bi_m$ , which were assigned, were not defined by the conversion of vapour pressure into moisture needed for uniformity of the the grain-air driving force.

### 2.4. Definitions of Biot numbers

#### 2.4.1. Heat transfer

The Biot number for heat transfer is defined here using  $R$  as characteristic dimension (Foust *et al.*, 1983):

$$Bi_c = \frac{h_T R}{k_T} \quad (11)$$

The calculation of  $Bi_m$  and  $Bi_c$  require thermophysical and transport properties.

#### 2.4.2. Procedure for deducing the Biot number for mass transfer

By writing the surface boundary condition [Eqn (6a)] in dimensionless form, the Biot number for mass transfer can be found to identify the controlling mechanism for

the mass flux. To this end, the following equivalence can be proposed:

$$\frac{\partial W_s}{\partial r} \equiv \frac{\partial W_s}{\partial p_{vs}} \frac{\partial p_{vs}}{\partial r} \quad (12)$$

where  $\partial W_s / \partial p_{vs}$  can be obtained from the sorption isotherm equation while  $\partial p_{vs} / \partial r$  is the gradient of sorbed water vapour pressure across the interface viewed from the grain side. Replacing this into the boundary condition [Eqn (6a)], yields

$$r = R, \quad -\rho_a D \frac{\partial p_{vs}}{\partial r} = \frac{k_p}{(\partial W_s / \partial p_{vs})} (p_{vs} - p_{va}), \quad t > 0 \quad (13)$$

$p_{vs}$  is expressed in dimensionless form  $p_{vs,dim}$  as follows:

$$p_{vs,dim} \equiv \frac{p_{vs} - p_{va}}{p_{vs0} - p_{va}} \quad (14)$$

where  $p_{vs0}$  is the vapour pressure at the grain surface evaluated at  $W_0$ . Substituting Eqn (14) into Eqn (13), the dimensionless general boundary condition for the surface ( $r = R$ ) is obtained

$$-\frac{1}{p_{vs,dim}} \frac{\partial p_{vs,dim}}{\partial r_{dim}} = \frac{k_p}{(\partial W_s / \partial p_{vs})} \frac{R}{\rho_a D} \equiv Bi_m \quad (15)$$

where  $r_{dim}$  is given by the ratio  $r/R$ . This definition of  $Bi_m$  was presented for a general geometry and material by Crapiste and Rotstein (1997) and constitutes the right form of expressing this important dimensionless number for mass transfer. To follow the notation of the surface boundary condition written in Eqn (6a),  $Bi_m$  can be expressed with two equivalent notations:

$$Bi_m = \frac{k_p}{(\partial W_s / \partial p_{vs})} \frac{R}{\rho_a D} \equiv \frac{k_w R}{\rho_a D} \quad (16)$$

It must be said that the calculation of  $k_w$  given by  $k_p / (\partial W_s / \partial p_{vs})$  must be updated continuously during a drying run. The conversion factor  $\partial W_s / \partial p_{vs}$  is required because only the vapour pressure is a uniform field across the grain-air interface unlike moisture content (grain) or humidity (air). As anticipated, this definition does not coincide with those given by the previous contributions, so values could be different.

### 2.5. Methods for calculating properties and coefficients

#### 2.5.1. Thermal conductivity of the wet grain particle

For the particle thermal conductivity, a correlation reported by Sokhansanj and Bruce (1987) was used:

$$k_T = 0.14 + 0.68W \quad (17)$$

For typical postharvest moisture,  $k_T$  varies between 0.20 and 0.25 W m<sup>-1</sup> K, as in asbestos (Wakao & Kaguei, 1982). This shows that the grain itself conducts heat rather poorly.

### 2.5.2. Air-grain heat and mass transfer coefficients

The study carried out by Sokhansanj and Bruce (1987) on barley, where geometry and drying behaviour are similar to those of wheat, led to the following general expression relating the Colburn factor  $J_H$  with the particle Reynolds number  $Re_p$ :

$$J_H = 3.27 Re_p^{-0.65} \quad (18)$$

the Reynolds number was obtained from the expression  $G_a d_p / \mu_a$ , and the correlation holds for values of  $Re_p$  between 20 and 1000. For a typical air mass flow rate  $G_a$  of 0.3 kg m<sup>-2</sup> s (Woods & Favier, 1993), grain particle diameter  $d_p$  of  $3.6 \times 10^{-3}$  m (Giner, 1999), and an air viscosity  $\mu_a$  of  $1.85 \times 10^{-5}$  kg m<sup>-1</sup> s, the value for  $Re_p$  is about 50, *i.e.* transitional regime (laminar flow holds for  $Re_p < 1$  while fully turbulent flow begins at  $Re_p$  of 10<sup>3</sup>). To obtain heat and mass transfer coefficients, the above correlation of  $J_H$  in Eqn (18) has to be equated with the original definition of the Colburn factor. By the heat and mass transfer analogy, the Colburn factor for mass transfer  $J_M$  is equal to that for heat transfer  $J_H$ , which also allows the mass transfer coefficient to be estimated (Green & Maloney, 1984). These original definitions are

$$J_H = Nu Re_p^{-1} Pr^{-1/3} \quad (19)$$

$$J_M = Sh Re_p^{-1} Sc^{-1/3} \quad (20)$$

The Nusselt number  $Nu$  is given by  $h_T d_p / k_{Ta}$ , while the Prandtl number  $Pr$  is given by  $\nu_a / \alpha_a$ , a ratio of momentum (kinematic viscosity) to thermal diffusivities. The Prandtl number is also equivalent to  $(\mu_a / \rho_a) / (k_{Ta} / \rho_a C_{pa})$  where  $\mu_a$  is the viscosity of the air, and the relationship becomes  $\mu_a C_{pa} / k_{Ta}$ . In turn, the Sherwood number  $Sh$  is given by  $k_c d_p / D_{AB}$  where  $k_c$  is the air-grain mass transfer coefficient based on the vapour concentration in the air, and  $D_{AB}$  is the binary diffusion coefficient of water vapour in air, with the Schmidt number  $Sc$  given by  $\nu_a / D_{AB}$ , the ratio of momentum to mass diffusivities in air. Air viscosity and thermal conductivity were predicted with straight lines fitted to data published by Heldman and Singh (1981) with the coefficient of determination  $r^2$  above 0.999 in both cases

$$\mu_a = 1.735 \times 10^{-5} + 4.318 \times 10^{-8} T_a \quad (21)$$

$$k_{Ta} = 0.0238 + 6.838 \times 10^{-5} T_a \quad (22)$$

The water vapour-air binary diffusivity is also required to determine the complete variation of air-grain heat and mass transfer coefficients.

### 2.5.3. Water vapour-air diffusivity

Reid *et al.* (1977) have presented the following correlation for gas mixtures, among them the vapour-air system:

$$D_{AB} = \frac{10^{-7} T_k^{1.75} [(M_A + M_B) / M_A M_B]^{1/2}}{p / 1.013 \times 10^5 (\sum v_A)^{1/3} + (\sum v_B)^{1/3}} \quad (23)$$

where  $D_{AB}$  is expressed in m<sup>2</sup> s<sup>-1</sup> for absolute pressure  $p$  in Pa and temperature  $T_k$  in K. Subscripts  $A$  and  $B$  may indistinctly denote water or air since  $D_{AB}$  is equivalent to  $D_{BA}$  (Treybal, 1980). The symbol  $M$  indicate molecular mass, while the sums included in the denominator are molecular parameters whose values are 20.1 for air and 12.7 in water.

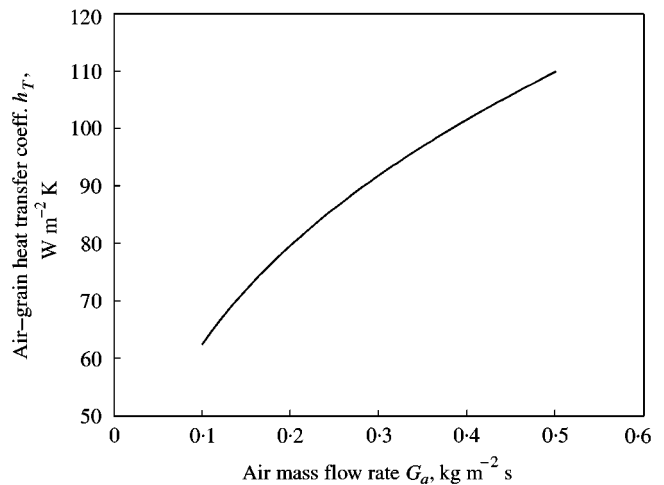
Therefore, after using the procedure mentioned above [Eqns (18) and (19)] to solve for  $Nu$  or  $Sh$  or, dimensionally for  $h_T$  or  $k_c$ , it was found that they depend mostly on the system fluid dynamics, being  $Nu \propto Re_p^{0.35}$ , or  $h_T \propto G_a^{0.35}$ , the powers being the same also for the proportionalities of  $Sh$  to  $Re_p$  and  $k_c$  to  $G_a$ . The relationship of  $k_c$  and its more useful version,  $k_p$  is the following:

$$k_p = k_c \frac{M_v}{(R_g T_k)} \quad (24)$$

where  $R_g$  is the gas constant and  $M_v$  the molecular mass of vapour. *Figure 1* shows an almost two-fold increase of the heat transfer coefficient in the normal range of airflows ascribed to wheat drying.

### 2.5.4. Conversion factor in the Biot number for mass transfer

The  $Bi_m$  definition [Eqn (16)] requires  $\partial W / \partial p_{vs}$  to be obtained from a sorption isotherm equation. The



*Fig. 1.* Air-grain heat transfer coefficient for wheat as a function of the air mass flow rate, at 60°C (after Sokhansanj & Bruce, 1987)

Modified Henderson equation (ASAE, 1991) was used here, written for the surface moisture:

$$W_s = 0.01 \left[ -\frac{\ln(1 - a_w)}{k_{MH}(T + C_{MH})} \right]^{1/N} \quad (25)$$

For hard wheat, the value for  $k_{MH}$  is  $2.31 \times 10^{-5}$ , that for  $C_{MH}$  of 55.815, while the value for the constant  $N$  is 2.2857 (ASAE, 1991). The water activity  $a_w$  is equal to  $p_{vs}/p_{sat}$ , the saturation vapour pressure  $p_{sat}$  in Pa being given by the following correlation:

$$p_{sat} = \exp \left( A_{sat} - \frac{B_{sat}}{T_k} - C_{sat} \ln T_k \right) \quad (26)$$

with values for the coefficients  $A_{sat}$ ,  $B_{sat}$ , and  $C_{sat}$  of 54.119, 6547.1 and 4.230, respectively (Giner *et al.*, 1996). In this work,  $T_k$  is the temperature in K, while  $T$  is expressed in °C. Therefore, the derivative needed is

$$\left( \frac{\partial W_s}{\partial p_{vs}} \right)_T = \frac{0.01}{[k_{MH}(T_a + C_{MH})]^{1/N} N} \left[ -\ln \left( 1 - \frac{p_{vs}}{p_{sat}} \right) \right]^{(1-N)/N} \frac{p_{sat}}{(p_{sat} - p_{vs})} \quad (27)$$

2.5.5. Heat of sorption estimated by the Clapeyron-sorption isotherm model

As indicated in the previous literature (Brooker *et al.*, 1992), the latent heat of vaporization of water  $L_w$  can be calculated with the Clapeyron expression together with a correlation for the saturation vapour pressure:

$$L_w = \frac{RT_k^2}{M_v} \frac{d \ln p_{sat}}{dT_k} \quad (28)$$

which predicts that  $L_w$  decreases by 10% between 0 and 100°C. Gely and Giner (2000) used a similar expression for the heat of sorption:

$$L_g = \frac{RT_k^2}{M_v} \left( \frac{\partial \ln p_{vs}}{\partial T_k} \right)_w \quad (29)$$

where, by replacing  $p_{vs}$  by  $a_w p_{sat}$ , the expression becomes

$$L_g = \frac{RT_k^2}{M_v} \left[ \frac{d \ln p_{sat}}{dT_k} + \left( \frac{\partial \ln a_w}{\partial T_k} \right)_w \right] \quad (30)$$

which can be rewritten in simpler form as

$$L_g = L_w + \frac{RT_k^2}{M_v} \left( \frac{\partial \ln a_w}{\partial T_k} \right)_w \quad (31)$$

to find that  $L_g$  has a pure water term [ $L_w$ , Eqn (29)], which is only temperature-dependent, plus a sorptional term owing to the additional binding energy between

water and grain components, a function of both m.c. and temperature. The derivative  $(\partial \ln a_w / \partial T)_w$  is developed from Eqn (25):

$$\left( \frac{\partial \ln a_w}{\partial T} \right)_w = \frac{k_{MH}(100W)^N \exp[-k_{MH}(T + C_{MH})(100W)^N]}{1 - \exp\{-k_{MH}(T + C_{MH})(100W)^N\}} \quad (32)$$

2.5.6. Correlation used for the effective diffusion coefficient of water in wheat

Another property required to evaluate  $Bi_m$  is  $D$ . An Arrhenius-type relationship developed by Sun and Woods (1994), independent of moisture content, was used.

$$D = 7.507 \times 10^{-8} \exp \left( \frac{-2806.5}{T_a + 273.16} \right) \quad (33)$$

where  $T_a + 273.16$  is the absolute temperature of the drying air, since it is customary to correlate diffusion coefficients with that temperature instead of that of the grain,  $T_k$  (Nellist & Bruce, 1995).

3. Results and discussion

3.1. Biot number for heat transfer

Values of  $Bi_c$  were calculated for  $a_w$  from 0.7 to 0.95, a common interval in wheat postharvest. Results were plotted in Fig. 2 for three air mass flow rates, corresponding

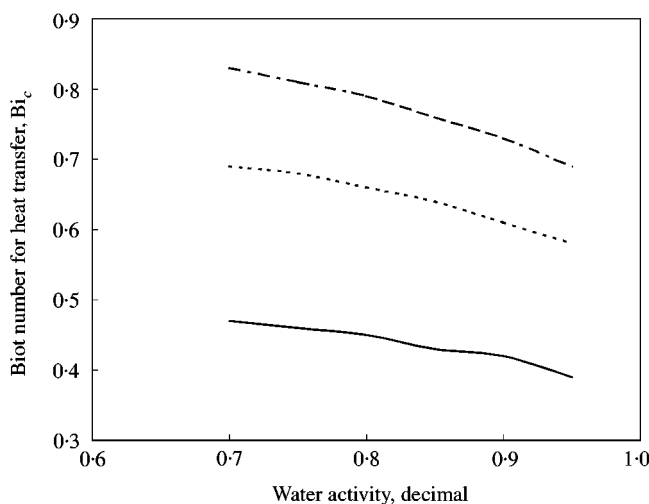


Fig. 2. Biot number (evaluated at 60°C) for heat transfer as a function of wheat water activity for three air mass flow rates: —, 0.1 kg m<sup>-2</sup> s; ----, 0.3 kg m<sup>-2</sup> s; - · - ·, 0.5 kg m<sup>-2</sup> s

to particle Reynolds numbers between 20 and 100. Figure 2 indicates that values are rather low, around 0.5, and close to the zone of external control. As the Biot number [Eqn (11)] is defined as the ratio between the internal resistance to the heat flux  $\varphi_{int}$  given by  $R/k_T$  and the external one  $\varphi_{ext}$  given by  $1/h_T$ , and as the value of  $h_T$  is high ( $\approx 80 \text{ W m}^{-2} \text{ K}$ , see Fig. 1), the external resistance cannot be large despite being greater than the internal. Concerning the internal resistance of wheat, which conducts heat rather poorly, the reciprocal of the conductivity,  $1/k_T$  would make  $\varphi_{int}$  high rather than low. Therefore, it is found that  $\varphi_{int}$  is less than  $\varphi_{ext}$  solely because wheat is a small kernel.

Foust *et al.* (1983) have presented graphs for heat conduction in plane plates, infinite cylinders and spheres, where the analytical series solutions for constant  $k_T$  are represented as dimensionless temperature *versus* dimensionless time for various dimensionless positions inside the body and several Biot numbers. For Biot numbers below 0.17, the curves of all the points within the body coincide, so there are no internal temperature gradients (flat profile). The values for  $Bi_c$  predicted in Fig. 2 are in the same order as that required for no gradients but somewhat higher than 0.17. In turn, Parti (1993) has presented a wider range for using external control-based models, indicating that they are valid for  $Bi_c \leq 1.5$ . Based on Parti's range, the values for  $Bi_c$  in Fig. 2 largely comply with this condition. Villermaux (1993) has suggested that a model aimed at design should be as simple (fast) and accurate as possible, so if the temperature profile is considered flat, this would eliminate the need of solving the partial differential equation inside the grain. However, the Biot number for mass transfer must also be analysed comparatively, since one Biot number alone should not be considered separately in a coupled transfer process.

### 3.2. Biot number for mass transfer

By taking a grain density value of  $1300 \text{ kg m}^{-3}$  (Giner & Denisienia, 1996), the dry mass referred to the wet volume  $\rho_{d0}$  is  $1300/(1 + W_0)$  and was used here to calculate  $Bi_m$  [Eqn (16)], which is plotted in Fig. 3 against  $a_w$  for different airflow rates. A strict internal control for mass transfer ( $Bi_m > 50$ ) is established as soon as drying starts ( $a_w = 0.95$ ), even at the lower airflow. Therefore, the fixed boundary condition [Eqn (6b)], can be used from the beginning and the problem becomes simpler to solve. Another feature observed is that the increase of airflow leads to an increase in the value of  $Bi_m$  because the external film resistance is decreased. The results of Fig. 3 are also useful to note that there is no need of using airflows above those of practical dryers ( $\approx 0.3 \text{ kg m}^{-2} \text{ s}$ )

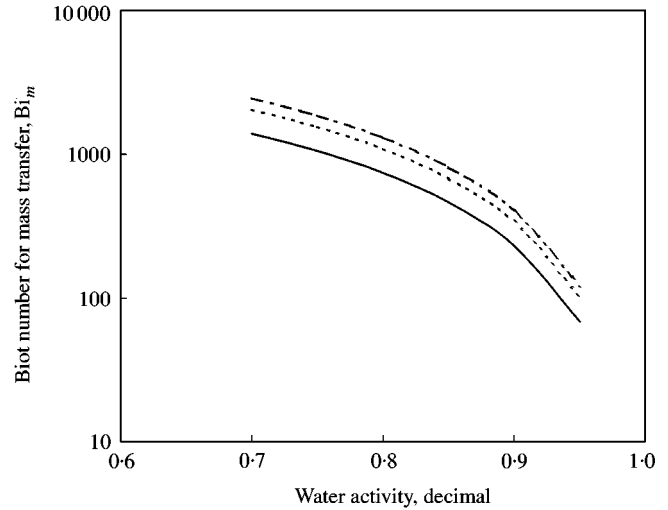


Fig. 3. Biot number for mass transfer (evaluated at  $60^\circ\text{C}$ ) as a function of wheat water activity for different air mass flow rates: —,  $0.1 \text{ kg m}^{-2} \text{ s}$ ; ---,  $0.3 \text{ kg m}^{-2} \text{ s}$ ; - · - ·,  $0.5 \text{ kg m}^{-2} \text{ s}$

to obtain internal control when determining diffusion coefficients with fixed surface values. Whilst the value for  $Bi_c$  does not vary considerably during a drying run, that for  $Bi_m$  increases at least one order of magnitude *during the course of drying*. This is caused by the pronounced m.c. gradients that develop inside the grain.

### 3.3. First resulting form of the model

#### 3.3.1. Mass transfer

The model to be used is the differential diffusion equation [Eqn (3)] with fixed surface boundary condition [Eqn (6b)]. If numerically solved, the average m.c.  $W$  must be calculated at each time:

$$W = \frac{3}{R^3} \int_0^R W_r r^2 dr \quad (34)$$

#### 3.3.2. Heat transfer

A macroscopic balance which as such considers flat internal temperature profile in the grain can represent the energy exchange of grain and air

$$\rho_d C_p \frac{dT}{dt} = h_T a_v (T_a - T) - \rho_d \left( -\frac{dW}{dt} \right) L_g \quad (35)$$

where  $a_v$  is the specific grain surface (ratio of grain surface area to grain kernel volume), while  $(-dW/dt)$  is the drying rate based on the average m.c. at time  $t$ . A nonlinear, ordinary differential equation is obtained for heat transfer in an initial value problem.



3.4. *Applicability of the assumption of isothermal conditions in the thin-layer drying of wheat*

The Pabis and Henderson (1962) recommendation of considering isothermal drying for maize, a grain whose value for  $Bi_c$  must be around 1 may not be as realistic as for wheat and similarly sized kernels. Sokhansanj and Bruce (1987) found that a barley kernel—similar in size to wheat—takes some 4–5 min to reach the air temperature asymptote. Fortes *et al.* (1981) have found comparable results for wheat, and the interval is small compared to the total drying times needed for wheat in dryers, typically 60–120 min (Nellist, 1987). To corroborate the validity of the ‘isothermal drying’ assumption, authors such as Parry (1985) have evaluated the ratio of thermal  $\alpha$  to moisture  $D$  diffusivities in the grain, the former being defined as

$$\alpha = \frac{k_T}{\rho_d C_p} \tag{36}$$

using the thermal conductivity by Eqn (17) and a value of  $C_p$  given by

$$C_p = C_{pd} + C_{pw} W \tag{37}$$

the value for the specific heat of the dry matter  $C_{pd}$  is 1300 J kg<sup>-1</sup>°C and that for water  $C_{pw}$  is 4187 J kg<sup>-1</sup>°C (Bruce & Giner, 1993). For the value of  $D$ , Eqn (33) was used. The  $\alpha/D$  ratios were plotted as a function of  $a_w$  at 60°C in Fig. 4 and it is observed that the relaxation rate for heat transfer is some 6000 times as fast as that for mass transfer. According to Parry (1985), heat transfer could be assumed ‘instantaneous’ compared with the ‘slow’ internal moisture transport. The order of magni-

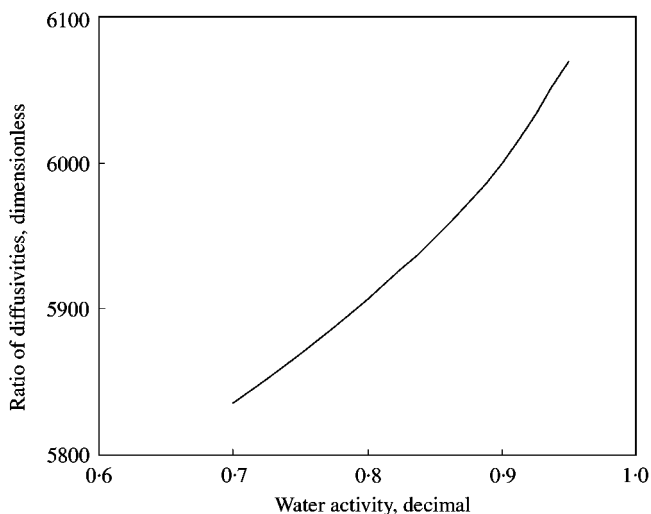


Fig. 4. Values of the ratio between thermal and moisture diffusivities ( $\alpha/D$ ) as a function of water activity (evaluated at 60°C)

tude of  $\alpha$  is about 10<sup>-7</sup> m<sup>2</sup> s<sup>-1</sup> while that of  $D$  is about 10<sup>-11</sup> m<sup>2</sup> s<sup>-1</sup>. Consequently, despite the value for  $Bi_c$  alone not being as low as ideally acceptable for assuming external control for heat transfer, the strong internal control for mass transfer makes the precise limit of the  $Bi_c$  less relevant, ‘dragging’ it towards a more externally controlled situation. This was tested in a sensitivity analysis of a deep bed simulation model, where the change of  $h_T$  from 10 to 100 W m<sup>-2</sup> K had almost no effect on performance parameters such as residence time, maximum grain temperature and specific energy consumption (Giner, 1999).

3.4.1. *Further verification by solving the complete system by finite differences*

As the  $\alpha/D$  ratio does not consider the size of the body being analysed, one interpretation is that, although heat transfer is much faster than mass transfer, it does not necessarily imply that heat transfer has no gradients. Therefore, to confirm the applicability of isothermal drying, an additional verification was carried out: the differential diffusion equation [Eqn (3)] with the general surface condition Eqn (6a) was solved numerically together with the heat conduction equation [Eqn (7)] plus its general initial and boundary conditions [Eqns (8)–(10)]. The numerical method used was an explicit finite difference scheme using 20 concentric, equally spaced shells to divide a sphere with a value of  $R$  of 2 × 10<sup>-3</sup> m. The transport and thermal properties used were already described. Operating conditions used for these predictions were the following: air temperature, 70°C; relative humidity, 0.06 (decimal); air mass flow rate, 0.3 kg m<sup>-2</sup> s; initial grain m.c., 0.20 dec., d.b.; and initial grain temperature, 20°C. Moisture content and temperature gradients are represented in Fig. 5 [(a) and (b), respectively], for several times from 0 up to 6 min.

Given that by using the general surface boundary condition for mass transfer, the predictions of Fig. 5(a) show a very fast convergence to a fixed surface m.c., it is verified that such a Dirichlet case [surface condition Eqn (6b)] could be assumed from the beginning. In turn, for each time, Fig. 5(b) shows a definitely flat temperature gradient and a sharp rise towards the drying air temperature after only 1–2 min of drying. Therefore, this situation approaches isothermal conditions and allows analytical solutions to be used in wheat.

3.5. *Standard analytical solution of the diffusion equation*

Analytical solutions also need to assume that  $D$  is independent of m.c. during drying, and it was shown in the introduction that, in principle, this is possible in view of the lack of a definite law of variation stated in various

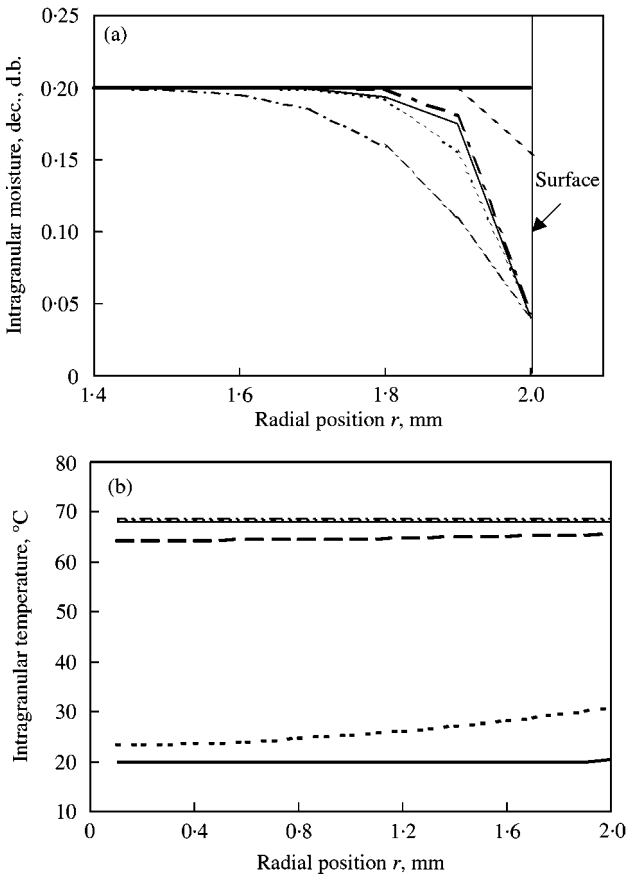


Fig. 5. (a) Moisture gradients inside a wheat grain dried at 70°C and plotted from an initial moisture content of 0.2 for several times: —, 0.01 s; - - -, 0.1 min; - · - ·, 1 min; — · — ·, 2 min; · · · · ·, 4 min; - · - ·, 6 min; (b) corresponding temperature gradients for grain starting at a temperature  $T_0$  of 20°C for the same times

reviews (Zogzas *et al.*, 1994, 1996; Zogzas & Maroulis, 1996). In fact, direct evidence is needed to demonstrate a reliable dependency of the diffusion coefficient with m.c. Meanwhile, it can be assumed that an average value of  $D$  remains constant during drying. The analytical series solution obtained after integrating the local m.c. as a dimensionless ratio  $W_{dim}$  in the sphere volume (Becker & Sallans, 1955; Crank, 1975; Suárez *et al.*, 1981; Parry, 1985; Tolaba & Suárez, 1988; Sun & Woods, 1994) presented according to Becker (1959) is

$$W_{dim} = \frac{W - W_e}{W_0 - W_e} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left[ -\frac{n^2 \pi^2}{9} X^2 \right] \quad (38)$$

where the dimensionless time  $X^2$  is equal to  $a_v^2 Dt$  (related to the better-known Fourier number  $Fo$  by  $Fo = Dt/R^2 = X^2/9$ ). The value of the kernel-specific surface area  $a_v$  is  $3/R$  for a sphere. In spite of its tested accuracy in wheat (Sun & Woods, 1994), this infinite series requires

a considerable number of terms  $n$  to reach convergence in each time, especially at short times.

On the other hand, for long times,  $X > 0.8$  ( $W_{dim} < 0.3$ ), the series converges with only one term:

$$W_{dim} = \frac{W - W_e}{W_0 - W_e} = \frac{6}{\pi^2} \exp \left[ -\frac{\pi^2}{9} X^2 \right] \quad (39)$$

which can be plotted as a straight line on a semi-logarithmic graph.

To fit the analytical solution to the observed drying data, using  $D$  as fitting parameter, the experimental m.c. must be expressed in dimensionless form. The value of  $W_{dim}$  is initially 1 at  $t = 0$  and rarely fall below 0.3 in grain drying (Bruce, 1985). Thus, the complete infinite series [Eqn (38)] is needed for fitting purposes, since the long times form [Eqn (39)] is not valid in this practical range. This subject is treated in more detail below.

### 3.6. Limitations of the complete series form

Equation (38) presents several drawbacks.

- Expressions having numerous terms cannot be written in commercial non-linear fitting software.
- Often, the number of terms required to ensure convergence is unknown and varies with time (Gupta & Woods, 1992).
- It may be necessary to write a computer program for the least-squares method that includes two convergence criteria: one at each time of a drying curve to calculate the series with the necessary number of terms, the other in each entire drying curve, to find the optimizing  $D$ .
- Once the parameters were fitted, the use of the series to predict thin layer drying curves require normally another program, *e.g.* a modified version of (c).
- The use of an infinite series as a thin layer equation in fixed-bed drying models increases the calculation time considerably, given that a bed is divided normally into 50 or 100 'thin layers'.

#### 3.6.1. Limitations of long times series convergence

Giving a straight line in a semi-logarithmic graph, Eqn (39) was used in numerous previous works, as observed from Zogzas *et al.* (1994). However, its application requires the curve of  $W$  versus  $t$  to be determined for  $W_{dim} < 0.3$ , outside the range of practical interest, where moisture varies too slowly and is more exposed to measuring errors. Besides, should  $D$  have a dependency on m.c., the value of  $D$  obtained would be an average of a 'low moisture zone', not representative of the short times (high moisture) zone, the practical region, where

that  $D$  value must be used subsequently in the complete infinite series.

3.7. The simplified diffusive analytical solution for 'short times'

Becker (1959) has presented an original study on grain drying kinetics, proposing a special solution of the differential diffusion equation which, with a relatively simple and compact form, predicts the drying curve in the range of  $0.2 < W_{dim} < 1$  or  $X < 1$  with the same accuracy as the infinite series.

The basis is that for 'short times', changes are limited to the vicinity of the surface, so, in principle, the body could be seen as a semi-infinite plane medium. On these grounds, Becker used the Gaussian error integral (instead of the infinite series) as a solution for the plane plate form of Eqn (3) (i.e. the  $2/r$  term disappears). As it is believed that Becker's development was not discussed in agri/food publications, it was adapted and included here.

3.7.1. Development of the short times diffusive analytical solution

Given that, in the first moments ( $t \approx 0$ ), the changes are restricted to the neighbourhood of the surface, the problem can be represented as that of a plane semi-infinite solid. The diffusion equation [Eqn (3)] simplifies for plane geometry to

$$\frac{\partial W_l}{\partial t} = D \left( \frac{\partial^2 W_l}{\partial r^2} \right) \tag{40}$$

where, by defining a transformed coordinate  $s$  as  $R - r$ , the initial and boundary conditions result:

$$t = 0 \rightarrow W_l = W_0 \tag{41}$$

$$s = 0; t > 0 \rightarrow W_l = W_s \tag{42}$$

(where  $s = 0$  indicates that  $r = R$ , the full characteristic length).

For this situation, the solution of the differential equation, using the initial and boundary conditions take the form of the Gaussian error integral:

$$1 - W_{l\ dim} = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-Z^2) dz \tag{43}$$

Symbols  $W_{l\ dim}$  and  $Z$  are defined as follows:

$$W_{l\ dim} = \frac{W_l - W_s}{W_0 - W_s} \tag{44}$$

$$Z = \frac{s}{2\sqrt{Dt}} \tag{45}$$

To obtain the average concentration, e.g. the mean grain m.c.,  $W$  in a finite solid as a function of time close to zero, a mass balance is written for the transfer through the surface:

$$(W_0 - W)\rho_d V = \int_0^t J S dt \tag{46}$$

where  $V$  is the kernel volume in  $m^3$ ,  $S$  the kernel surface area in  $m^2$ . By Fick's first law, the water flux is given by

$$J = -\rho_d D \left( \frac{\partial W_l}{\partial s} \right)_{s=0} \tag{47}$$

Then, by replacing the flux  $J$ , the basic equation to solve becomes

$$(W_0 - W) = \frac{S}{V} \int_0^t -D \left( \frac{\partial W_l}{\partial s} \right)_{s=0} dt \tag{48}$$

The partial derivative  $(\partial W_l / \partial s)_{s=0}$  is evaluated from the Gaussian solution [Eqn (43)]. To do this, the definition of  $W_l$  [Eqn (44)] must be used in Eqn (43), where  $dZ$  must be replaced by  $ds/2\sqrt{Dt}$  [see definition of  $Z$  in Eqn (45)]. Advantage was taken of the fact that  $\exp(-Z^2)$  approaches 1 as  $s \rightarrow 0$ .

By carrying out such steps, the following is obtained:

$$-\left( \frac{\partial W_l}{\partial s} \right)_{s=0} = \frac{W_0 - W_s}{\sqrt{\pi D t}} \tag{49}$$

Now, by replacing Eqn (49) into Eqn (48), an intermediate expression is obtained for the average moisture content  $W$  (note that  $a_v$  is being given by  $S/V$ )

$$\frac{W_0 - W}{W_0 - W_s} = a_v \frac{\sqrt{D}}{\sqrt{\pi}} \int_0^t \frac{1}{\sqrt{t}} dt \tag{50}$$

which, on integrating, gives rise to the 'short time expression' for the average m.c. as a function of time of a slab

$$W_{dim} = \frac{W - W_s}{W_0 - W_s} = 1 - \frac{2}{\sqrt{\pi}} X \tag{51}$$

To extend the validity of Eqn (51) to other geometries, such as the spherical, the solution must be expanded in McLaurin series (Taylor series around  $X = 0$ ). As  $W_{dim}$  is a function of  $X$ , the expansion can be written as

$$f(X) = f(0) + f'(0)X + \frac{f''(0)}{2} X^2 + \dots + \frac{f^n(0)}{n!} X^n \tag{52}$$

where  $f(0) = 1$  and  $f'(0) = -2/\sqrt{\pi}$ , being the higher order derivatives dependent on solid body shape. However, in view of the requirement that the series should converge rapidly around  $X = 0$ , Becker (1959) neglected the terms

whose order was higher than  $f''(0)$ . Therefore, the following equation was determined:

$$W_{dim} = 1 - \frac{2}{\sqrt{\pi}} X + \frac{f''(0)}{2} X^2 \tag{53}$$

whose validity range can be ascertained by comparing it with some infinite series solutions of the diffusion differential equation [e.g. Eqn (38)]. To evaluate  $f''(0)/2$ , Eqn (53) was reordered as a straight line by expressing the ordinate as  $Y = (1 - W_{dim})/X$ . The slope of the resulting equation is  $f''(0)/2$  as shown below

$$Y = \frac{1 - W_{dim}}{X} = \frac{2}{\sqrt{\pi}} - \frac{f''(0)}{2} X \tag{54}$$

Becker (1959) plotted the complete infinite series solutions for different shapes as  $Y$  versus  $X$ . For spheres, the graph of the reordered Eqn (38) is plotted here in Fig. 6. Hence, in view of the fact that the true analytical solution  $Y$  versus  $X$  presents a perfectly linear behaviour (intercept  $2/\sqrt{\pi}$ ) with a slope of  $-0.331$  for  $X \leq 1$ , the second derivative of Maclaurin expansion is

$$\frac{f''(0)}{2} = 0.331 \tag{55}$$

Then by including Eqn (55) in Eqn (53), and replacing  $X$  by its definition, the thin layer equation for short times in spheres becomes

$$W_{dim} = 1 - \frac{2}{\sqrt{\pi}} a_v \sqrt{Dt} + 0.331 a_v^2 Dt \tag{56}$$

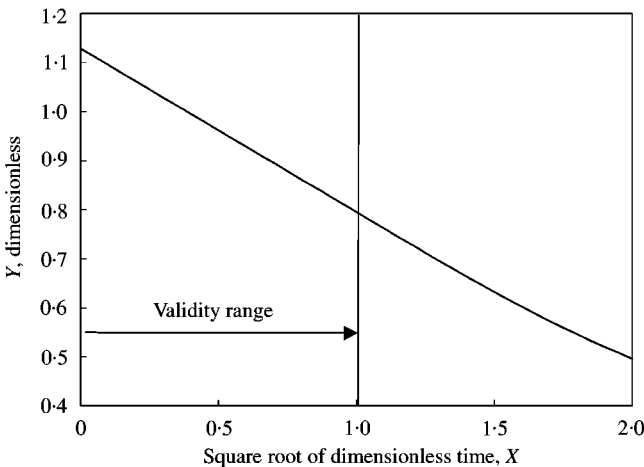


Fig. 6. Infinite series solution for spheres represented as  $Y = [(1 - W_{dim})/X]$  versus  $X$ ; the moisture ratio is defined as  $W_{dim} = (W - W_e)/(W_0 - W_e)$ , where  $W$  is the mean grain moisture, dec., d.b., while  $W_0$  and  $W_e$  are the initial and equilibrium moisture contents, respectively; the straight line behaviour holds for  $X$  greater than 1; the intercept is  $2/\sqrt{\pi}$ , while the slope is  $0.331$ ; therefore, the Maclaurin expansion around  $X = 0$  yields  $f''(0)/2 = 0.331$

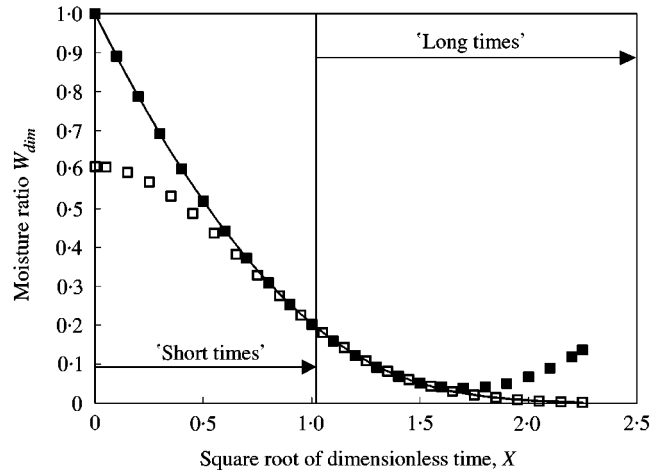


Fig. 7. Complete analytical infinite series for spheres (—), Becker's solution for short times (■) and convergence of the infinite series in one term for long times (□)

which is recommended as the constitutive expression for grain-air mass transfer in wheat and similarly sized kernels.

### 3.7.2. Usefulness of the simplified diffusive short time equation

Figure 7 shows plots of the complete infinite series solution [Eqn (38)], together with the short times equation reanalysed here [Eqn (53)] and the one term convergence of the series solution [Eqn (39)] for long times.

In effect, the short times equation coincides with the complete series for  $X$  ranging from 0 to 1, and even to higher values, while the long times convergence tends to coincide with the series solution for  $X$  values above 1. Outside its validity range, the m.c. predicted with the short time equation begins to increase because of the influence of the positive term in  $t$ . Therefore, according to the graph, the switch from the short time to the long time equation can be done in the range of  $0.2 < W_{dim} < 0.3$  or a value of  $X$  around 1. This confirms that the time-consuming infinite series can be avoided. The practical significance of this equation can be demonstrated upon a value of  $D$  of  $1.6 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}$ , estimated using Eqn (33) for an air temperature of  $60^\circ\text{C}$ . From the definition of  $X^2$  given by  $a_v Dt$ , fixing the value for  $X$  as 1, and using a typical wheat grain equivalent radius  $R$  of  $2 \times 10^{-3} \text{ m}$ , so  $a_v$  given by  $3/R$  takes the value of  $1500 \text{ m}^{-1}$ , a maximum allowable time of 7.5 h from the start of drying is found within which the 'short time' equation is valid. The time would be longer for lower drying temperatures but it would still be about 3 h at  $90^\circ\text{C}$ , giving sufficient time for studying grain drying kinetics in practical situations. This corroborates the value of the simplified diffusive short time expression for

applying it in 'thin layer drying' studies on wheat, rice, barley, oats, in smaller grains such as rapeseed and, possibly even in a tiny grain such as amaranth (as long as it keeps the internal control for mass transfer). The usefulness of the short time equation reviewed here comes not only from its simplicity and accuracy in estimating diffusion coefficients in the practical time range, but also from the benefit of using it in dryer simulation models aiming at design or model-based control. Concerning diffusion coefficient estimation, the use of the equivalent spherical shape—of less surface area for the same true particle volume—would make the values for  $D$  slightly above, but proportional to, those obtained by finite elements in axisymmetric ellipsoidal shape, according to preliminary results obtained in the laboratory. This means that results obtained with analytical solutions for different conditions could be used in intraparticle simulation after being corrected by a kind of shape factor.

#### 4. Conclusions

The Biot number for mass transfer in wheat was found to change by more than an order of magnitude, *e.g.* from 100 to 3000 during the course of drying. The strong internal control holds from the beginning of drying.

The Biot number for heat transfer in wheat is about 0.5, slightly above the maximum value for strict external control. However, as the internal transport of heat was found to be some 6000 times faster than that of moisture, the heat flux control operates as if its Biot number were shifted to a more externally controlled situation, giving the possibility of assuming a flat temperature profile inside the grain.

To corroborate this, the complete problem of moisture diffusion plus heat conduction in a wheat kernel was numerically solved with the general boundary conditions. A strict internal control for mass transfer was verified, with fixed surface moisture while the temperature profile was, in effect, predicted as flat, reaching the air temperature very rapidly.

These findings give the possibility of using analytical solutions in wheat for isothermal systems, especially if used for diffusion coefficient fitting purposes plus subsequent utilization for the prediction of dryer performance.

The analytical solution of the diffusion equation for spheres is a convergent, infinite series neither easy to use for parameter fitting purposes nor when included in simulation models because it makes them slow to run, less interactive for design and of slower response for drying control situations.

The diffusive analytical solution for short times due to Becker was reviewed here and proved to be as accurate as

the standard analytical series solution without losing physical meaning, but much faster to run. Besides, unlike the convergence in one term of the infinite series, it can be used to estimate diffusion coefficients in the practical drying range, giving sufficient time (some 7 h at 60°C, some 3 h at 90°C) to collect kinetic data.

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