Relativity of simultaneity in secondary school: an analysis based on the theory of the conceptual fields

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ABSTRACT

In this paper, we present results about the conceptualization of the relativity of simultaneity and the operational invariants used by high school students. The Theory of Conceptual Fields is adopted to design, analyse and evaluate a didactic sequence to teach the basic aspects of the Theory of Special Relativity in secondary school. Only the situations related to the relativity of simultaneity are analysed here. An inductive categorization is constructed from 256 protocols generated in the implementations carried out in four 11th grade courses (15-16 years) (N = 128) in a public school of Colombia. The categorization identifies the resolutions of the students and the operative invariants that they use in each situation. The results indicate that the proposed situations would produce the emergence and awareness of the operational invariants linked to the relativity of Galileo, which would open the way to the conceptualization of the relativity of simultaneity in secondary school.

KEY-WORDS

Relativity of simultaneity, theory of conceptual fields, secondary school, Special Theory of Relativity

RÉSUMÉ

Dans ce travail on présente des résultats sur la conceptualisation de la relativité de la simultanéité aussi que les invariants opératoires utilisés par les élèves de l'école secondaire. On utilise la Théorie des Champs Conceptuels pour dessiner, analyser et évaluer une séquence didactique dédiée à enseigner des éléments de la théorie

de la relativité restreinte dans l'école secondaire. Ici, on analyse seulement les situations correspondantes à la relativité de la simultanéité. On a développé une catégorisation inductive basée sur 256 protocoles obtenus dans les implémentations réalisées dans quatre cours de 11éme degré (15-16 ans) (N=128) dans une école publique en Colombie. La catégorisation a identifié les résolutions des étudiants et les invariants opératoires qu'ils utilisent dans chaque situation. Les résultats suggèrent que les situations proposées produiraient l'émergence et la prise de conscience des invariants opératoires liés à la relativité de Galileo, et ils ouvriraient la porte à la conceptualisation de la relativité de la simultanéité dans l'école secondaire.

MOTS-CLÉS

Relativité de la simultanéité, Théorie des Champs Conceptuels, école secondaire, Relativité restreinte.

Introduction

The importance and benefits of introducing modern physics in secondary school have been repeatedly mentioned, since it would allow a better understanding of the fundamental concepts of classical physics and could also lead to modify the physics contents proposed for teaching in the High School (HS), studying classical physics with a contemporary vision. However, it is a known that the Special Theory of Relativity (STR) is not generally taught in HS. The reasons for this are as diverse as complex, such as the lack of appropriate didactic devices for teachers to teach STR in HS, and the way in which school physics is taught, which does not allow conceptualizing the basic concepts of neither, classical nor relativistic kinematics.

On the other hand, among the few available investigations on the subject, most focus on the errors of students at all levels, disregarding the genetic and pragmatic aspects of the conceptualization of complex notions such as those involved in both classical and relativistic kinematics.

This work is part of a research project that adopts a developmental and pragmatic conception of conceptualization based on the Theory of Conceptual Fields (TCF) (Vergnaud, 1990). In general terms, this project intends to design, implement and test a didactic sequence to teach the fundamental notions of the STR in HS and to study its conceptualization (Otero, Arlego & Prodanoff, 2015, 2016; Otero & Arlego, 2016; Arlego & Otero, 2017). We focus here on the relativity of simultaneity.

Previous Research

The investigations on the teaching of the STR have been mainly oriented to the university (Angotti et al., 1978; Saltiel & Malgrange, 1980; Panse, Ramadas & Kumar, 1994; Pietrocola & Zylbersztajn, 1999; Scherr, Shaffer & Vokos, 2001, 2002; Pérez & Solbes, 2003; Dimitriadi, Halkia & Skordoulis, 2010; Hosson, Kermen & Parizzot, 2010; Dimitriadi & Halkia, 2012). Regarding HS it is worth mentioning Pérez and Solbes (2006) and Villani and Pacca (1987).

Previous studies report that students have difficulty defining and using the concept of the event (Hewson, 1982) and which confuse the instant of occurrence of an event with the instant it is received (Scherr et al., 2001). It is also mentioned that students reason spontaneously about motion (which they consider absolute), distance and velocity, when they have to explain mechanical phenomena in classical and special relativity (Saltiel & Malgrange 1980; Villani & Pacca, 1987).

The study on10th grade students in Greece shows that for the students the simultaneity is absolute and independent of relative motion (Dimitriadi & Halkia, 2012). According to Scherr et al. (2001), university students consider that each observer is a different reference system, and that if the observers are in the same place, they remain in the same reference system. The HS students would have a pre-Galilean view of the motion, which consider to be absolute (Posner, Strike, Hewson & Gertzog, 1982; Dimitriadi & Halkia, 2012). These ideas survive the school teaching of classical kinematics, and when attempts are made to teach relativistic kinematics, students are expected to become aware of the conceptual break between Galilean and modern physics (Villani & Arruda, 1998). But how can they do so without the prior genesis of the relevant classical concepts?

There are no precedents in the literature of a didactic sequence for HS designed within the reference of the TCF, nor of the analysis of the conceptualization of basic concepts of the STR using this reference. To teach the STR and describe the process of conceptualizing its basic notions in secondary school, we have designed a didactic sequence based on what students know and not what they should know (Otero & Arlego, 2016). The successive versions of the sequence that we have developed are based on the implementations carried out in eight HS courses in Argentina and Colombia (Otero, Arlego & Prodanoff, 2015, 2016; Otero & Arlego, 2016; Arlego & Otero, 2017).

In this work, we present results obtained in two situations of the previously mentioned sequence, which would allow students to arrive at the relativity of simultaneity. An inductive categorization is constructed from 256 protocols obtained during the exploratory implementation of the didactic sequence in four 11th grade courses (15-16 years old) in Colombia. The categorization attempts to identify, describe and analyse the diverse ways of solving situations of the students and the underlying operative invariants, with the aim of testing whether the situations designed produce the emergence and awareness of operative invariants linked to the Galilean relativity that allow arrive at the relativity of simultaneity in secondary school.

The theory of conceptual fields (TCF)

The TCF proposed by Vergnaud (1990, 2013) considers that the conceptualization is the cornerstone of cognitive development. According to Vergnaud, scientific knowledge has as substrate the organizing schemes of behavior. In the TCF, Vergnaud replaces the subject-object relation proposed by Piaget, by the scheme-situation. A scheme contains: 1) anticipations of the goal to be achieved, the effects of achieving it and the intermediate stages. 2) Rules of action of the type "if-then" that allow to generate the sequence of actions of the subject. 3) Inferences and reasoning that allow to calculate the rules and the anticipations from the information and the system of invariants of the subject. 4) Operational invariants that lead to the identification by part of the subject of the relevant elements of the situation and the information assimilation of the situation to be treated.

The Operational invariants (OI) can be of two types: concepts-in-action and theorems-in-action. These invariants organize the action of the subject and make it operational. A concept-in-action is a category, a property, a predicate that is considered relevant to the situation. A theorem-in-action is a proposition that the subject considers true about that situation.

The operational invariants are implicit, and are not comparable with concepts or scientific principles, because the latter are explicit, and their relevance and validity can be discussed. The TCF does not distinguish between everyday concepts and scientific concepts (Otero et al., 2014), the process of conceptualization has the same characteristics in all cases: it consists in identifying objects, their properties and their interrelations (Vergnaud, 2013, p. 41).

According to Vergnaud (1990, 2013) there are two forms of knowledge in interaction. The operational form, which allows the subject to act in a situation, and the predicative form, which allows the subject to enunciate and designate objects, as well as communicate their knowledge.

In the operational form, action does not only refer to its external manifestations - behavior - but also includes the operational and implicit aspects of the action: thought, decision making, anticipation and inferences. When facing a given situation, it is very complex to know what to do and it is not necessarily possible to put into words what is done.

Consequently, teaching-learning cannot be reduced to the predicative form, nor can a concept be reduced to its definition. In this line, our research analyses the activity of the subject in situation, identifying the OI, since these are the most decisive elements of the scheme. The analysis in terms of OI makes it possible to distinguish what characteristics of situations are considered by the subject, what OI they put into play and what are the goals they are trying to achieve and the rules and inferences they use to do it (Vergnaud, 2013).

Methodology

In this paper, we report results from four implementations of the sequence (Otero & Arlego, 2016), carried out in courses of 11th grade in the same public institution of the Colombian basic secondary school (N = 128).

In all courses the teacher was the same and the students had studied the first part of the sequence mentioned before. During the classes, the students met in fixed groups of four or five members chosen by themselves. The situations considered here began by asking for an individual anticipation without calculations, which was immediately given to the teacher. Then, the students met in fixed groups of four or five members chosen by themselves, where they shared and discussed their individual results without the intervention of the teacher. A consensus response was prepared and presented by a member of the group, which was recorded in audio. Finally, the students wrote and individually responded to other proposed tasks and delivered their work to the teacher at the end of each class. The teacher digitized the protocols and gave back them in the next class.

An inductive categorization was constructed from the registers, generating categories and subcategories, which are incorporated into a data matrix containing each student in rows and variables or the nominal categories in columns. The subcategories are associated to the operational invariants that allow to describe the conceptualization in the two situations that treat the relativity of the simultaneity.

Next, we briefly present the sequence implemented detailing the two situations considered in this work. Finally, we describe each category and the subcategories generated and perform a descriptive statistical analysis for qualitative data.

Brief description of the didactic sequence

The didactic sequence (Otero & Arlego, 2016) proposes situations grouped in three parts, with a total of ten situations (S1-S10), which represent the evolution of relativistic ideas in Physics from Galileo to Einstein, so it does not presuppose the basic classical notions. The first part (S1-S5) refers to the classical kinematics and the principle of relativity of Galileo. Here we propose to analyse the motion of objects from different reference systems, introducing the relativity of the motion from the beginning. The situations presented require using of the concept of relative velocity, for instance, a car that comes in the route by the opposite lane approaches us at a higher velocity than the one indicated by its speedometer. These kinds of situations call for the use of the Galileo velocity addition law. This part is completed with situations that use the principle of inertia and Galilean relativity: students experiment with a pendulum hanging from their hand, at rest and with uniform motion (relative to the floor), and when braking, accelerating or moving in a circle. Finally, we try to put in evidence the indistinguishability between rest and uniform translation, which is the basis of the principle of relativity, by considering a situation where it is necessary to decide the state of motion of a car - ideally isolated - only with the help of a pendulum hanging from the ceiling.

The second part (S6-S7) consists of a transition to the STR, where the principle of relativity is retaken and generalized, and the invariance of the velocity of light c is formulated. Both postulates are used to analyse and model situations that allow to treat the relativity of the simultaneity of events from different reference systems, first with low velocity projectiles and then with light. The present work is centred in this part.

The third part (S8-S10) considers the more characteristic kinematic aspects of the STR. We discuss the time dilation, the lengths contraction, and the relativistic addition of velocities. After that, the problem of projectiles is taken up in a completely relativistic context, giving the phenomenon of lack of simultaneity a general character.

Next, we present the two situations whose results we will analyse in this work and we made some didactic considerations regarding their school treatment.

The analysed situations

The two situations considered allow the emergence and development of complex concepts such as: the equivalence between rest and uniform motion, the addition of velocities, the invariance of c, and the consequences of the joint application of the two relativistic postulates, such as the relativity of simultaneity. The S6, considers two rubber balls that are shot at the same time and in opposite directions from the middle of an isolated wagon, which moves uniformly respect to the platform. The question is whether the balls come simultaneously or not to the walls at both ends of the wagon, for a fixed observer inside (middle) the wagon and another fixed on the platform.

SITUATION 6:
An observer is sitting in the middle of an empty wagon. Another observer standing outside the wagon determines that the wagon moves at a constant velocity. The observer in the wagon has a device that can throw rubber balls back and forth, at the same instant.

a) Analyse for each observer, without calculations, whether the projectiles arrive at the same time or not at each wall of the car.
b) Complete the following table for each observer and propose different velocities for the car and the projectiles.

IN Observer			OUT Observer		
$v'_w(m/s)$	$v'_{br}(m/s)$	$v'_{bl}(m/s)$	$v_w(m/s)$	$v_{br}(m/s)$	$v_{bl}(m/s)$

c) Calculate the meeting point (position and time) between the projectiles and the walls of the car, for each observer, considering the different values of velocities proposed.

In classical kinematics, two events occurring in separate places and at the same time in one reference system, are also simultaneous in another reference system. This is common sense in our low velocity world, although we are aware of it. So, why we should expect students value the loss of something, who are unaware of possessing? This is one of the reasons for S6: to use classical kinematic concepts and to become aware of its consequences, as part of the genesis of relativistic concepts.

After predicting without calculations, students should respond using the relative velocity and relativity principle of Galileo, as well as modelling and solving the corresponding meeting problems between projectiles and wagon walls, for both observers. To exemplify, a possible row of the Table 1 has been completed.

TABLE 1

Possible relative velocities for bullets according to IN and OUT observers

IN Observer			OUT Observer		
v'_w(m/s)	v'_{br}(m/s)	v'_{bl}(m/s)	v_w(m/s)	v_{br}(m/s)	v_{bl}(m/s)
0	60	-60	20	80	-40

When students place for inside of wagon (IN) observer, they assume the relativity principle *in action*. The repetition of the meeting point calculation will allow to dominate the procedure. For the first row of Table 1 and if the car has a length of 10 m, in the IN-reference system the calculation would be:

$$x'_{wl} = -5\ \mathrm{m} \qquad x'_{bl} = -60\frac{\mathrm{m}}{\mathrm{s}}t' \qquad -5 = -60\ \frac{1}{\mathrm{s}}t'_l \qquad t'_l = \frac{5}{60}\mathrm{s} \approx 0{,}083\ \mathrm{s}$$

$$x'_{wr} = 5\ \mathrm{m} \qquad x'_{br} = 60\frac{\mathrm{m}}{\mathrm{s}}t' \qquad 5 = 60\frac{1}{\mathrm{s}}t'_r \qquad t'_r = \frac{5}{60}\mathrm{s} \approx 0{,}083\ \mathrm{s}$$

The meeting time on the right and on the left is

$$t'_l = t'_r = \left(\frac{1}{12}\right)\mathrm{s} \approx 0{,}083\ \mathrm{s}.$$

Considering the outside of wagon (OUT)-reference system and the values of the first row of Table 1, for the left wall and the projectile to the left:

$$x_{wl} = -5\ \mathrm{m} + 20\frac{\mathrm{m}}{\mathrm{s}}t\,,\, x_{bl} = -\,40\frac{\mathrm{m}}{\mathrm{s}}t\,, -5\ \mathrm{m} + 20\ \frac{\mathrm{m}}{\mathrm{s}}\ t_{\,l} = -40\ \frac{\mathrm{m}}{\mathrm{s}}\ t_{\,l},\ t_{\,l} = \frac{5}{60}\mathrm{s} \approx 0{,}083\ \mathrm{s}$$

Similarly, for the right wall and the projectile to the right:

$$x_{wr} = 5\ \mathrm{m} + 20\frac{\mathrm{m}}{\mathrm{s}}t\,,\ x_{br} = 80\ \frac{\mathrm{m}}{\mathrm{s}}\ t\,, \qquad 5\ \mathrm{m} + 20\ \frac{\mathrm{m}}{\mathrm{s}}\ t_{\,r} = 80\ \frac{\mathrm{m}}{\mathrm{s}}\ t_{\,r},\ t_{\,r} = \frac{5}{60}\mathrm{s} \approx 0{,}083\ \mathrm{s}$$

$$t_{\,l} = t_{\,r}$$

As expected from classical Galilean relativity, for both observers, the events are simultaneous. Although in the field of STR it is incorrect to state that for OUT observer the projectiles arrive simultaneously to the walls of the wagon, the S6 is necessary because the students do not know the physics of their daily life at low velocities, compared with c. It is necessary to know the Galileo addition of velocities and the results it produces, to later understand the consequences of not using this rule. This is an essential rupture to enter the field of the STR. In Table 2 we synthesize the concepts, questions and main OI necessary to solve S6 according to the classical kinematics.

Table 2

Concepts, questions and OI necessary to address the S6

Concepts	Questions	OI
Reference System (SR) Relativity principle Galileo addition of velocities Simultaneity	How are the velocities of the bullets for each observer determined? How to determine the meeting point? How long do the bullets take in each reference system?	For IN, $v'_v = 0$ For IN the walls are at rest. For IN the projectiles arrive at equal times to the walls of the wagon. Equal velocities and distances, imply equal times. For OUT $v_v \neq 0$ For OUT the bullets have different velocities. For OUT $\mathrm{v_{br}} = v_c + \mathrm{v'_{br}}$ $\mathrm{v_{bl}} = v_v + \mathrm{v'_{bl}}$ The meeting point is calculated with the equations of motion of the bullets and the walls for each observer. For OUT the walls move, one approaches the bullet that goes out back, and the other moves away from the bullet that goes forward. For OUT distances change proportionally with the velocity, leaving the same arrival time to both walls: "Compensation" For OUT the projectiles arrive at equal times to the walls of the wagon.

On the other hand, contrarily to material objects like projectiles, it is an experimental fact that the velocity of light c is independent of the source motion. The interplay between c-invariance and the relativity principle results in the loss of simultaneity, a cornerstone of Einstein relativity. This is reason to introduce the S7, where bullets are replaced by light beams.

SITUATION 7

An observer is sitting in the middle of an empty wagon. Another observer standing outside the wagon determines that the wagon moves at a constant velocity. The observer in the wagon has a device that can throw light beams back and forth, at the same instant.

a) Analyze for each observer, without calculations, whether the light arrive at the same time or not at each wall of the car.
b) Complete the following table for each observer and propose different velocities for the car and the light.

IN observer			OUT observer		
$\mathrm{v'_c}(\mathrm{m/s})$	$\mathrm{v'_{Lr}}(\mathrm{m/s})$	$\mathrm{v'_c}(\mathrm{m/s})$	$\mathrm{v'_{Lr}}(\mathrm{m/s})$	$\mathrm{v'_c}(\mathrm{m/s})$	$\mathrm{v_{Li}}(\mathrm{m/s})$

c) Calculate the meeting point (position and time) between the light beams and the walls of the wagon, for each observer, considering different values of velocities.

Before the presentation of S7 to the students, the class must discuss the c-invariance i.e. that c does not compose velocities with its source. The calculation of the meeting between the light beams of light and the walls for both observers, leads to the result of the loss of simultaneity for OUT observer.

Here, by placing the students assume in action the first postulate and if they do not add values to c, assume the second postulate.

TABLE 3

Possible relative velocities for light beams according to IN and OUT

IN observer			OUT observer		
$v'_c(m/s)$	$v'_{Lr}(m/s)$	$v'_c(m/s)$	$v'_{Lr}(m/s)$	$v'_c(m/s)$	$v_{Li}(m/s)$
0	c	$-c$	20	c	$-c$

For IN:

$$x'_{wl} = -L \quad x'_{Ll} = -c.t \qquad -L = -c.t \qquad t = \frac{L}{c}$$

$$x'_{wr} = L \quad x'_{Lr} = c.t \quad L = c.t \qquad t = \frac{L}{c}$$

For OUT:

$$x_{wl} = -L + 20.t \qquad x_{Ll} = -c.t$$

$$-L + 20.t = -c.t \qquad (c+20).t = L$$

$$t_l = \frac{L}{c+20}$$

$$x_{wr} = L + 20.t \qquad x_{Lr} = c.t$$

$$L + 20.t = c.t \qquad L = (c-20).t$$

$$t_r = \frac{L}{c-20}$$

Light takes longer to reach the wall on the right than the one on the left. In Table 4 we synthesize the concepts, questions and main OIs needed to solve S7.

Table 4

Concepts, questions and OI necessary to address the S7

Concepts	Questions	OI
Reference System (RS) Relativity principle c invariance Relativity of simultaneity	How are the velocities of the light beams for each observer determined? How to determine the meeting point? How long do the light beams take in each reference system? What are the consequences of the relativity of simultaneity?	For IN, $v'_v = 0$ For IN the walls are at rest. For IN the light beams arrive at equal times to the walls of the wagon. Equal distances, imply equal times. For OUT $v_v \neq 0$ For OUT beams are equally fast $v_{Ld} = c$ $v_{Li} = -c$ For OUT the walls move, the left one approaches the beam that goes back and the right one moves away from the beam that goes forward. For OUT the beam that goes backwards arrives before the wall that the one that goes forward. The meeting point is calculated with the equations of motion of the light and the walls for each observer. For IN the times are the same and for OUT they are different. The simultaneity depends on the RS.

Categories of analysis of situation 6

$C1_6$: Simultaneity according to IN in the individual answer to item (a)

$C1M1_6$: In this subcategory are grouped the subjects considering that the bullets arrive together to the walls of the wagon. For example, they express: "*For IN the balls arrive at the same time because their velocities are equal*" or "*For IN, the wagon is in constant motion, we could say that the wagon's velocity is 0; only the velocities of the bullets could be considered, since the velocities are the same and the distances are equal, they must arrive at the same time to the walls*".

$C1M2_6$: not answered

The Table 5 shows that for IN, almost all students predicted that bullets arrive simultaneously at the walls.

Table 5

$C1_6$: Simultaneity according to IN in the individual answer to Item (a)

Subcategory	Number of students
$C1M1_6$ The bullets arrive together	118
$C1M2_6$ Not answered	10
Total	128

C2$_6$: Simultaneity according to OUT in the individual answer to Item (a)

C2M1$_6$: This subcategory groups the students who consider that for OUT, the bullets arrive at different times to the walls of the car, arriving before the shots to the left.

"*OUT sees the bullets arrive at different times because the left bullet goes faster*".

C2M2$_6$: This subcategory groups the students considering that according to OUT, the bullets arrive together to the walls of the wagon.

"*OUT sees the balls arrive at the same time*".

The Table 6 shows that initially more than two-thirds of students predicted that the projectiles arrive at different times to the walls, and first to the left wall. This anticipation would be driven by the IO: if two mobiles are moving in opposite directions, their velocities must be added.

TABLE 6

C2$_6$: Simultaneity according to OUT in the individual answer to Item (a)

Subcategory	Number of students
C2M16 the bullets arrive at different times	85
C2M26 the bullets arrive at the same time	43
Total	128

C3$_6$: Simultaneity according to OUT in the individual answer to Item (a), after group discussion

C3M1$_6$: According to OUT the bullets arrive at the same time. After the group interaction, the students wield arguments like "*OUT notice the movement of the wagon that affects the velocity of the balls. Although the velocity of the balls is different, they will arrive at equal times*".

"*The ball fired in the same direction as the wagon will have a higher velocity, however the wall of the wagon on that side will move away from that ball. In the opposite, that is fired towards the opposite direction of the movement of the wagon will have a lower velocity but the wall on that side is getting closer to that ball, then they arrive at equal times, it does not matter if it is for inside or outside*".

"*For OUT, the balls also arrive at the same time, but for him the velocities are different because we have to add the velocity of the wagon. We must take it into account, and it is something that the groups that say that they arrive at different time, do not consider, and is that the wagon is moving. They have different velocities, yes, but as the wagon is moving, a wall is moving away, and to that wall the ball goes with a higher velocity. That is, it must travel more distance, but it has more velocity. The other ball is slower, but the wall is approaching. This compensates the two factors and the balls arrive at the same time. This is what must be considered by groups* *that think that the balls arrive at separate times that when wagon is observed in motion, the wagon is not static, and therefore arrive at the same time and compensate*".

C3M2$_6$: According to OUT the bullet that goes backwards arrives first.

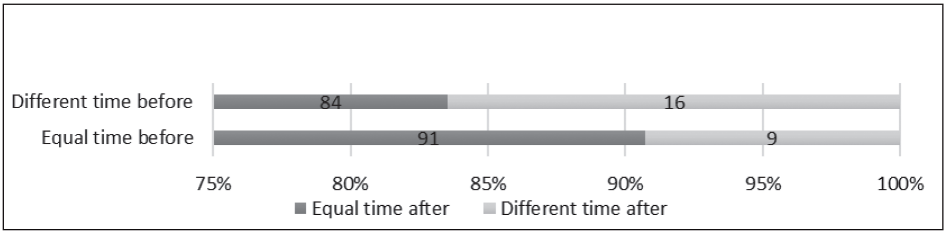
The Table 7 shows that students changed their prediction for item (a). Now, they would consider that according to OUT, the bullets have different velocities and nevertheless, they arrive at the same time to the walls because the velocity of the wagon affects the bullets and because the walls are moving.

TABLE 7

C3$_6$: Simultaneity according to OUT. Item (a), after group discussion

Subcategory	Number of students
C3M1$_6$ *the bullets arrive at the same time*	110
C3M2$_6$ *the bullets arrive at different times*	18
Total	128

GRAPH 1



Prediction of the arrival to the walls according with OUT before and after of the group discussion

The Graph 1 shows the changes of prediction of the students before and after the group discussion (without teacher intervention). Among the 110 students who initially anticipated that the bullets would arrive at different times, 84% said later that they would do it together. The 91% of the 18 students who made the correct prediction kept it. Apparently, because of the group discussion, some operational invariants emerge driving the selection of information about the velocities of the bullets and the wagon in this reference system. On the other hand, 16% of the adherents to the non-simultaneity in the anticipation, maintained their position and 9% of those who had correctly predicted the simultaneity changed their minds.

C4$_6$: Completing the velocities Table according to IN

C4M1$_6$: only the absolute value of the bullets velocity is written.

$C4M2_6$: correct answer: the bullets velocities have the same absolute value and opposite sign.

The Table 8 shows that more than two thirds of the students correctly fill the table with the bullets velocities for the IN-reference system. It is remarkable that students use the first postulate appropriately, writing, that is, they consider the wagon walls at rest in the reference system where IN is located.

TABLE 8

$C4_6$: Filling the Table 1 according to IN

Subcategory	Students
$C4M1_6$ only the absolute value of the bullets velocities is written	28
$C4M2_6$ the bullets velocities have the same absolute value and opposite sign	100
Total	128

$C5_6$: Calculation of the velocities according to OUT

Five ways of completing the table are identified, ranging from incorrect forms to the most correct ones.

$C5M1_6$: Students consider that for OUT "*the left bullet arrives earlier because it goes faster*". They fill *ad hoc* the Table 1 to comply with this IO. They do not compose velocities.

$C5M2_6$: Students who use the operational invariants: "*for OUT, the bullet going to backward arrives before*" and "*when they go in the same direction, you must subtract and in the opposite direction you must add*". Then the velocity of the wagon is subtracted from the bullet fired forward and added if it goes backwards, then they impose the sign. They do not add velocities.

$C5M3_6$: Students who only add the velocity of the car for the bullet fired to the right and use this value with opposite sign for the velocity of the bullet fired to the left.

$C5M4_6$: Students who add to the velocity of the forward bullets the wagon velocity and subtract it to the backward bullets, and then put the sign indicating the sense. They do not use the IO "*in the opposite direction, you must add the velocities and in the same direction you must subtract them*". For example:

"*That is due to the direction of the ball. If it is the left one, that goes to the contrary direction, you must subtract, and if it goes in the same direction, you must add*".

$C5M5_6$: Students who add the velocity of the wagon and solve the algebraic sum: "*for IN the balls had velocities of 10 m/s, for each direction. For OUT, we consider that the velocity of the wagon was 80 m / s. Then the right bullet goes at 90 m / s, and the other one at 70 m / s*".

"Ah well, in fact, for IN we should have also a sum of velocities, but as the wagon goes at 0, that sum of velocities does not affect anything. Outside we also do the sum of the velocities, but it is because the wagon goes at 80 which makes it change the velocity for the balls. I would also like to emphasize that for OA, all the balls move in the same direction, in the direction of the wagon even though they have different velocities".

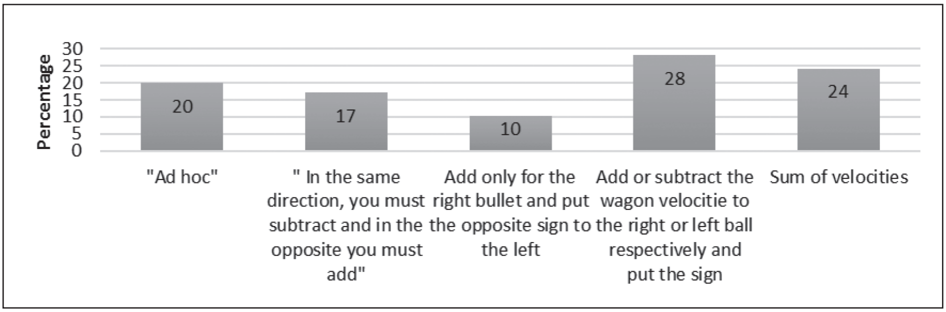
The Table 9 shows the number of students identified in each subcategory.

Table 9

$C5_6$: Sum of velocities filling the Table 1 according to OUT

Subcategory	Students
$C5M1_6$ *"Ad hoc"* according to the IO *"the velocity of the left ball is bigger"*	26
$C5M2_6$ *" in the same direction, you must subtract, and in the opposite, you must add"*	22
$C5M3_6$ add only for the right bullet and put the opposite sign to the left	13
$C5M4_6$ add or subtract the velocity of the wagon to the right or left bullet respectively and put the sign	36
$C5M5_6$ sum of velocities	31
Total	128

Graph 2

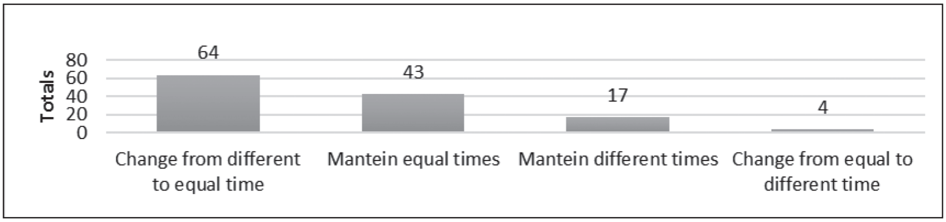


Sum of velocitys in the OUT system of reference

The Graph 2 shows that regarding the subcategories $C5M4_6$ and $C5M5_6$, 52% of students have appropriate IOs in the sum of velocities in this situation.

The Graph 3 shows how many students have changed or not its prediction on the simultaneity of bullets according to OUT, before and after the group interaction (C2 and C3). The first two bars correspond to a change in the correct direction. More than 50% of the students changed to the simultaneity of the bullets according to the reference system of OUT and the 34% that had responded correctly retained their answer. The last two bars represent 16% of students that maintain or change to different time.

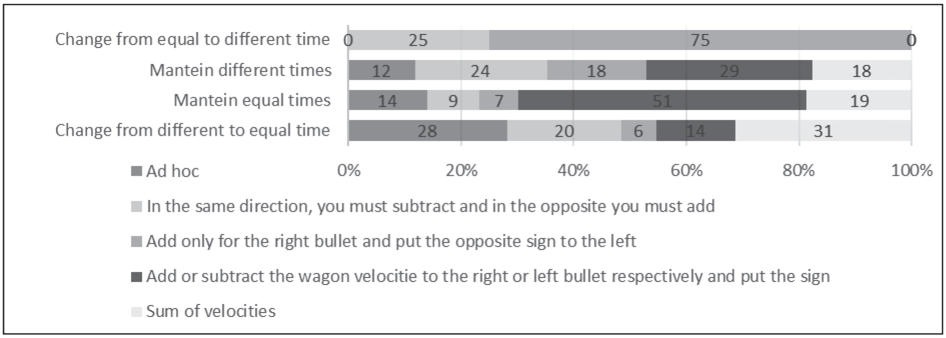
Graph 3



Changes of predictions of the simultaneity of bullets according to OUT, before and after group discussion (N=128)

The Graph 4 shows the distribution of the different forms of composing velocities considering OUT reference system according to whether the prediction about the simultaneity of the bullets has been changed or not.

Graph 4



Distribution of the different forms of composing velocities

Among the 64 students who switched to simultaneity, 45% sum velocities, while 48% did not manage to do so. Among the 43 students who agree to the simultaneity from the beginning, 70% added velocities. Among the 17 students who preserve non-simultaneity, 47% add velocities correctly and 18% do so only to the right. Finally, only 4 students go back changing from equal to different time, and among them the categories of addition are not present.

Regarding to item C, the students calculated without problems the arrival times according to IN, but they couldn't do it according to OUT. The teacher calculated the meeting point with the class, and the students repeated the calculation for several rows of the velocities table.

CATEGORIES OF ANALYSIS AND RESULTS FOR SITUATION 7

Before proposing the S7 the class discussed about the light velocity c. It was identified that the students knew that c is a limiting velocity, its value in the vacuum and they also offered appropriate examples about the finite time (no instantaneous) propagation of c. The teacher dealt with the class the second postulate and the invariance of c regarding the movement of the source and then proposed to the students the new situation.

$C1_7$: Simultaneity of light according to IN, individual anticipation of the item (a)

$C1M1_7$: "*according to IN the beams of light arrive at the same time*".
$C1M2_7$: not answered.

The Table 10 shows the results of this category. Most of the students agree on the simultaneity according to IN.

TABLE 10

$C1_7$: Simultaneity of light according to IN

Subcategory	Students
$C1M1_7$ *"According to IN the beams of light arrive at the same time"*	118
$C1M2_7$ not answered	10
Total	128

$C2_7$: Simultaneity of light according to OUT, individual anticipation of the item (a)

$C2M1_7$: "*According to OUT the beams of light arrive at the same time*".

"*For both IN and OUT observers, they see the light rays coming at the same time, since the velocity of the light is constant and does not change. The velocity of the wagon does not affect the order of arrival because the velocity of light is constant*".

"*For both observers, the light arrives at the same time, since the train has constant velocity and the velocity of light is also constant. That is why the rays arrive at the same time. We can also say that the rays travel the same distance between the observer and the wall, for both observers*".

$C2M2_7$: "*According to OUT the beams of light arrive at different time*".

The Table 11 shows that according to OUT, more than two thirds of students anticipate the simultaneity of light beams.

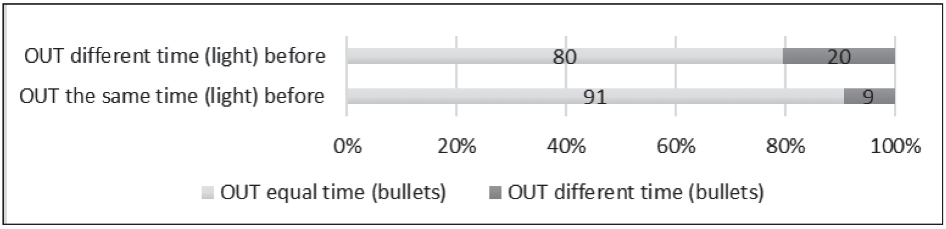
TABLE 11

$C2_7$: Simultaneity of light according to OUT, individual anticipation

Subcategory	Students
$C1M1_7$ *"According to OUT the beams of light arrive at the same time"*	74
$C1M2_7$ *"According to OUT the beams of light arrive at different time"*	54
Total	128

The Graph 5 shows that 80% of the students who had concluded the simultaneity of the bullets at the end of S6, affirms the simultaneity of light in S7, before the group discussion.

GRAPH 5



Distribution of the subcategories of $C3_6$ in those of $C2_7$

$C3_7$: Simultaneity of light according to OUT, after the group discussion

$C3M1_7$:"*according to OUT, the beams of light arrive together*".
$C3M2_7$:"*according to OUT, the beam fired to the left arrives before, but the difference is very small*".

"*They will not arrive at equal times, but they will not delay 5 seconds, but a very small time, because one distance will be larger than another. But as the velocity of light is immense, and I put that the distance from the centre to the wall will be 10 meters, it will take a very small time, it would be 0.00001s out there, then the times will be different but very tiny*".

"*I had said that the rays of light for the OUT observer arrived at separate times, because it was no longer compensated by the velocities. One wall was moving away and the other was approaching and the velocities of light beams always are constant. Then one ray comes first, and the other one comes later*".

"*For the OUT observer, we all agree that the light rays will make contact at separate times. However, the difference is minimal and imperceptible. For OUT observer, the velocity of light is constant, cannot vary, neither decrease nor increase. But OUT observes that the walls of* *the car are moving, one moves away and the other approaches the rays of light. And as the velocity of light does not change the times will be different*".

C3M3$_7$: Students' problems to decide if according to OUT the difference of times really exist, or it is a measurement problem.

Table 12

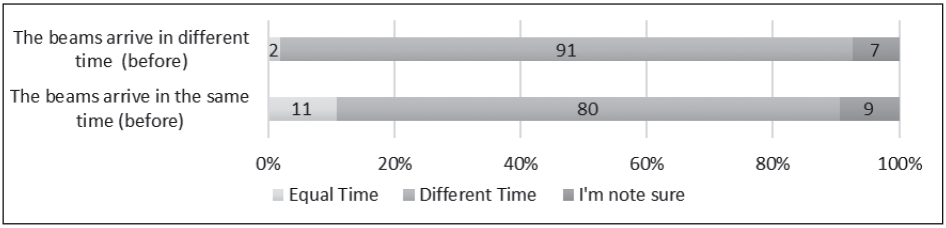
C3$_7$: Results for the item (a) according to OUT, after the group discussion

Subcategory	Students
C3M1$_7$ *"according to OUT, the beams of light arrive simultaneously"*	9
C3M2$_7$ *"according to OUT, the beam fired to the left arrives before, but the difference is very small"*	108
C3M3$_7$ *"the difference of times really exists, or it is a measure problem?"*	11
Total	128

The Table 12 shows that after the group discussion the students anticipated the non-simultaneity of light in the reference system of OUT. The operational invariants linked to the non-addition of c seems to drive the selection of information. As according to OUT the walls move and are stable, then the light arrives before to the back wall.

The Graph 6 represents the disaggregation of the individual anticipation about simultaneity of light, in those corresponding to the group discussion. Among the 74 students who initially held simultaneity of light, 80% changed their position, and among the 54 students who anticipated non- simultaneity, 91% retained their idea Therefore, at the end of the group discussion, almost all students consider that in the OUT-reference system, light does not come simultaneously to the walls of the wagon.

Graph 6



Loss of simultaneity according to OUT before and after of group discussion

C4$_7$: Invariance of c

C4M1$_7$: the velocity of the wagon is not added to c. Students use the IO of c-invariance and the relativity principle.

"*The wagon velocity is zero for the IN observer, the velocity of the right beam is c, for the*

left it is the same, but with the opposite sign. For the OUT, the wagon velocity is 20 and the velocity of light is the same as that of the IN because the velocity of the light never changes".
"*We made two examples with different velocity for the wagon, 23 m / s and 25 m / s, to show that no matter what velocity it is, the velocity of light will never change because it is always constant*".
$C4M1_7$: Not answered.

The Table13 shows that most students use the IO "*you must not add or subtract anything to c*".

TABLE 13

$C4_7$: Invariance of c

Subcategory	Students
$C4M1_7$ *"you must not add or subtract anything to c"*	109
$C4M2_7$ Not answered	19
Total	128

$C5_7$: Calculation of the arrival times of light

$C5M1_7$: The calculation is made correctly, and separate times are obtained.

"*For the OUT observer, only the velocity of the car is considered, and that is what makes the difference of arrival times, although it is a very minimal difference, almost imperceptible, but there is a difference!*".
"*From the equations, it is concluded that for IN observer the rays of light arrive at the same time. For the OUT observer, a light beam comes first than the other one. This is because the velocity of the car is considered. Despite being a small difference, one ray of light will always arrive first than the other one*".
"*The difference for OUT reference system is that it considers the velocity of the car. In addition, unlike the previous situation, the velocity of light is not added or subtracted from the velocity of the car, as this is insignificant compared to the velocity of light*".

$C5M2_7$: Calculation is made correctly, but a "-" sign is added to indicate the time of the beam traveling to the left.

$C5M3_7$: the meeting problem cannot be solved, or it is carried out incorrectly.

The Table 14 shows that 75% of the students perform the calculation correctly, including those who incorporate a negative sign to indicate the time to the left.

TABLE 14

$C5_7$: Calculation of the arrival times of light

Subcategory	Students
$C5M1_7$ The calculation is performed correctly and different times are obtained	81
$C5M2_7$ Calculation is made correctly, but a "-" sign is included to the left time	18
$C5M3_7$ The meeting problem cannot be made or it is carried out incorrectly	29
Total	128

DISCUSSION

The results presented in the previous sections show that in both situations, the responses obtained for the observer inside the wagon are mostly correct. This reference system would not be problematic for students. It is remarkable that they use the First postulate of relativity principle appropriately and consider that according to the IN-reference system the train is at rest. This would be attributed to the first phase of the sequence that has been described in other works (Otero & Arlego, 2016; Arlego & Otero 2017).

On the contrary, in both situations, there are more difficulties and differences in the OUT-reference system respect to the IN case. The S6 seems to offer more inconvenience to the students than the S7. This could be due to the knowledge achieved by the students during S6, especially in relation to the item c.

Particularly for the OUT case, it is interesting to analyse the changes occurred with the predictions about the arrival of the bullets to the walls of a considerable number of students, together with the possible reasons for such changes. When most of the students anticipate: "*the bullet fired backwards will arrive first*", they would be guided by the operational invariants "*the bullet and the back wall go towards the meeting*" and "*on the opposite direction you must add and on the same direction you must subtract velocities*". This would also direct the selection of information and its inferences According to the students the left bullet would be faster than the bullet fired forward, which chases the wall. In the group discussion, the interaction between these students and those who believe that the bullets will arrive together, and the sharing of the whole class, produce the emergence of other invariants. When finally, students accept that for the OUT observer the bullets arrive simultaneously, the students consider the information of the bullets velocities and its relationship with the wagon velocity. This would eventually lead to the inference that times are equal because there is a 'compensation' (see transcriptions for category C3 in S6).

Even in the same situation, each item reinitializes the IOs that the students will use.

For example, most students completed the table with the velocities for IN in a relatively correct way but used several alternatives to do it for the OUT case. These alternative forms of calculating the velocities of the bullets, would indicate different progresses in the conceptualization of the Galilean composition of velocities. It is important to note that the IO "on the opposite direction you must add and on the same you must subtract velocities" is a common case of the addition of velocities in everyday life, on which students have a lot of experience, when they drive their bicycle or travel by car or in the metro and wonder about the velocity of other mobiles. In the S6, this case must be set aside, since it is asked about the velocity of an object launched from a moving system. The third modality of the category $C5_6$ (see Table 9 and Graph 2) identifies a first step in the conceptualization of velocity addition when the velocities are calculated, being the most frequent case. The calculation would correspond to the theorem in act "*on the opposite direction you must subtract and in the same you must add velocities, and later then put the sign*". Velocities composition is fundamental to physically understand why the bullets in the OUT-reference system would arrive simultaneously in a Galilean perspective. In this process, the task of filling the table would promote the analysis on how to calculate the velocity of each bullet. Since the categories are constructed from the individual protocols that each student elaborates to deliver to the teacher at the end of the class, it is possible to infer that now the majority would not be using the theorem "*on the opposite direction you must add and on the same you must subtract velocities*".

To analyse if there would be a relationship between the change in the prediction of the arrival of the bullets and the velocities addition required in item b, we grouped the responses of the students in the four existing possibilities, whose results are presented in the Graph 3. After that, we crossed these categories with those corresponding to the velocities addition. The analysis shows that there is a statistically significant association between both variables. The Graph 4 shows that the velocities composition, from its most primitive variant to the correct one, is related to the change of opinion from non-simultaneity to simultaneity, or the prevalence of the original prediction in the case of projectiles. But even students who persist in the non-simultaneity of projectiles in item a), start to compose velocities in an appropriate way when completing Table 1. This reflects the instability, the contingent and opportunistic nature of conceptualization.

Regarding the last item of S6, the students could not calculate for themselves the meeting times for the OUT case, because they did not know how present the meeting problem in mathematical terms. This procedure was carried out with the assistance of the teacher and repeated by the students several times with different car and projectile velocities.

Regarding S7, in the individual instance most students predicted the simultaneity in the case of light, because they generalize the obtained results with the projectiles (see Graph 5). For the OUT observer, also in this situation, the group discussion and the sharing changed some of the responses to them item a). Many of the students that originally affirmed the simultaneity of the arrival of light on the walls, latter affirmed the loss of simultaneity, as can be observed in the Graph 6. The reason could be that now, the students consider that c is the same in both reference systems, and that according to OUT observer the walls of the wagon are in motion.

The results indicate that almost all students use of the second postulate of c-invariance in the item b of S7, as it is showed in the Table 13. Regarding the calculation of the meeting times, most students solve the problem by themselves, noting the difference in time and its small value, as shown in the Table 14. This ability to solve correctly the meeting problem would be due to the c-invariance that makes the equations of motion mathematically simpler and on the other hand, to the experience gained in the resolution of the equivalent problem with projectiles in the S6.

Conclusion

The results presented in this work show the relevance of the operational invariants in the selection of information, the inferences made by the students and the complexity of the conceptualization process of the simultaneity in a broad context, from Galileo to Einstein relativity. The analysis of S6 shows some promising results about how students can use the velocities composition appropriately. It would be useful for this type of task to be carried out when teaching classical kinematics, helping students to deal with cases that are not usual in their daily life, proposing propitious situations. The S6 would contribute to the awareness that the second postulate of c-invariance eliminates any possibility of "compensation" to "save" the absolute character of simultaneity. This in turn naturally enables the irruption of the concept of relative simultaneity. In summary, our results suggest that the use of situations aimed at explaining and becoming aware of IOs related to the addition of Galilean velocities, would be a previous step that greatly facilitates the understanding of the relativity of the simultaneity.

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