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Improving mixture of grains by using bi-dimensional Galton boards

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ABSTRACT

This paper presents an experimental and numerical study that deals with the problem of mixing grains falling down through a bi-dimensional Galton board (BGB). The special issue addressed here is the influence of the presence of lateral walls in the BGB. Disks of equal diameters but different species are launched from the top of the device. During the fall, disks collide with obstacles (arranged to form a triangular lattice) and with the lateral walls. The exit distribution of particles at the bottom of the board is determined and the incidence of the presence of walls in the mixing quality is studied as a function of W the relative separation between lateral walls. Two types of indexes are evaluated to characterize the efficiency in the obtained mixture. The presence of walls has proven to be crucial to enhance the quality of the mixture of particles.

1. Introduction

Mixing particles of different species has been a common and hard-to-solve problem through all ages and widely in many different situations of human life [1–3] and it is still a matter of intensive research due to its strong industrial applications. The problem of segregation of particles due to different mechanisms is always present when any mixing technique is to be applied [4].

In particular, the use of Galton boards to disperse particles has proved to be very efficient in this task. Based on the wide problem of diffusion of particles through periodic arrays of obstacles, these devices have been used to study transport and diffusion in many phenomena ranging from crystals to the present one of mixing grains. The BGB is a two-dimensional system of small pins forming a regular triangular lattice. It has been used intensively for many purposes. A typical sketch of the board is shown in Fig. 1.

There are plenty of examples in the literature of much research done using this setup due to the interesting dynamics present in it. Those examples range from charged particles scattering to dispersive flow of glass beads [5–12].

Usually, the board is arranged in a vertical or nearly vertical position and experiments on it consist in launching small balls or small disks (compared to the spacing between pins of the lattice) from the top row of the array and collecting them at the bottom part of the system by a set of collectors. For the particular case of mixing particles, two parallel containers are placed on top of the device and they are respectively filled with the two types of particles to mix.

As pointed out above, the implementation of a BGB like a particle mixer has been developed and successfully proved in Refs. [9,10,13]. In those works, Bruno et al. studied experimentally and numerically the dynamics of gravity driven particles

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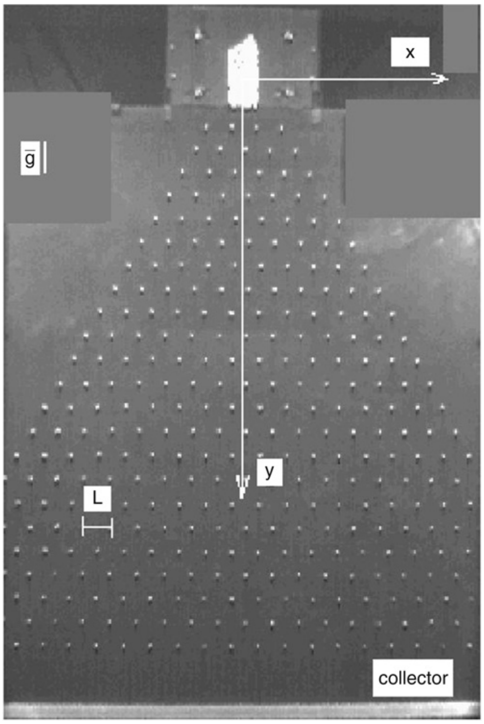

Fig. 1. A typical bi-dimensional Galton Board device. At the top, there is a container with particles to be released. When a mixture experiment is prepared, this cell is replaced by a two compartment one containing respectively in each compartment the species to be mixed. The spacing between pins, L, is indicated as well as the coordinate assignation and gravity direction. This setup is to illustrate the real experimental situation depicted in the text.

going through an ordered array of obstacles. Collisions with obstacles make particles to diffuse. The diffusive regime is attained after a certain number of collisions. Due to this process, particles are forced to mix. They also studied the collective motion of particles and found that the diffusive regime is rapidly attained after few rows in the BGB are crossed. As a second stage, they analyzed the feasibility and efficiency of the mixture of particles with different radii based on a diffusion driven process through the BGB. They developed an analytical method to predict the relative composition of a mixture as a function of the radii ratio of the particles in the mixture. All these results were obtained for boards without the presence of lateral walls, i.e., particles never collide with any wall during their down stream.

Recently, Benito et al. [14,15] have studied and characterized the behavior of the exit distributions of same species of particles falling down through a BGB. They found a change in the behavior of these distributions as the spacing W between walls was reduced. They observed a crossover in the behavior of the exit distributions changing from a Gaussian-like to a uniform-like shape. They determined a simple parabolic relationship that holds when searching for the necessary separation between lateral walls in order to obtain a uniform exit distribution of percolating particles. This work inspires the following question: how can walls in the BGB device help to mix particles? For this reason, the present paper will focus on the problem of mixing particles through the board with the presence of lateral walls.

Small discs of two different species will percolate through a vertical BGB where obstacles are very small pins arranged in a regular triangular lattice and where lateral walls are set using appropriate PVC rods, as will be explained below. Mixing of particles by diffusion through obstacles occurs inevitably. The exit distribution of particles at the bottom of the board is measured by classifying separately the two kinds of particles in different density functions. Adequate indexes are calculated in order to characterize the mixture quality as a function of walls separation.

The present study will contain both experiments and simulations. Simulations will specially be helpful to inspect a large range of parameters that are not easily attainable in experiments. They will also offer better statistics to describe the behavior of mixture indexes.

The following section will explain the experimental setup used here. The next one will explain the simulation algorithms employed. Then, the paper will show the results obtained both in experiments and simulations, discussing the main outstanding features. Finally, conclusions will be drawn.

2. Experimental setup and results

The device used for all the experiments here consists of two parallel plates, a smooth wood one and a glass one of 120 cm wide and 125 cm of height, separated by a small gap of 1.5 mm in which particles fall down. Fig. 2 gives a schematic

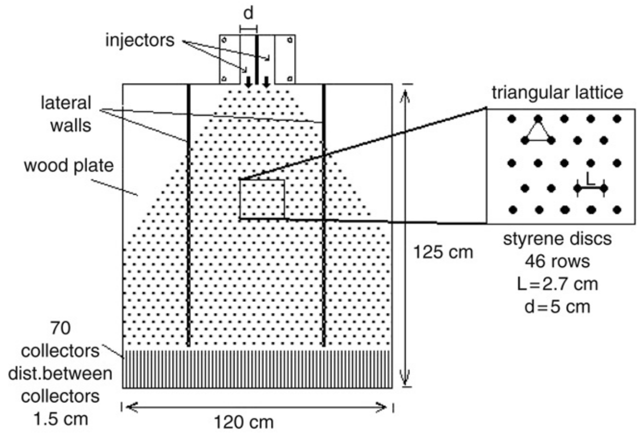

Fig. 2. Schematic draw of a BGB with its main dimensions. On the top, two injectors with a width of 5 cm each one, are placed as indicated. Lateral walls implemented with PVC rods are indicated. Collectors at the bottom will serve to build the final exit distribution function of the particles launched from the top. This sketch serves to illustrate the real experimental setup depicted in the text. The pins observed here are not in the real setup location.

representation of the device. The obstacles, styrene discs, (8 mm diameter, 1.2 mm thick) are glued onto the wooden wall forming a hexagonal network with lattice spacing $L = 2.7$cm, as indicated in Fig. 2. This distance was chosen to be larger than the diameter of the bigger flowing discs in order to avoid the formation of arcs. The number of rows of obstacles N is fixed and equal to 46. The horizontal central coordinate, $x = 0$, is taken at the middle point of the board. Two boxes are symmetrically located around this point and at the top of the BGB. These two boxes have a width of $d = 5$ cm each one, and they are separated by a 1mm PVC road. They can be filled with a large number of discs (each box with different species) that can be released simultaneously or one by one. At the bottom of the board, disks are collected into bins of 1.5 cm width. The maximum number of bins is 70. This sets the maximum possible number of columns present in the BGB and, consequently, is the maximum possible separation between walls. When particles exit the BGB crossing the last row, they are collected in these "bins collectors" so that they are counted and classified in two exit distribution functions, one for each species.

In order to study the influence of the presence of walls two lateral PVC rods of 1 m height and 1 cm width have been placed. The distance W between them can be varied. The following values for W were employed: 70, 60, 50, 40, 31, 19 and 12 measured in number of collected cells, as explained above. The disks are released all simultaneously and a great number of them is used in order to get a good statistic (of the order of 500 disks each run).

To check the behavior for different conditions, two mixing cases were tried: (1) styrene disks, half of them painted black and the other half painted white; (2) styrene disks and rubber disks in the same proportion, painted differently to distinguish among the two materials. All the disks had the same radius of 8 mm and 1.2 mm thick. Each of the two top boxes was filled with one kind of disks (styrene black, styrene white or rubber whose densities are $\rho = 0.9$ and 1.6 and g/cm^3 respectively). In case (2), rubber disks filled the left box. Both boxes contained the same amount of disks. Disks were suddenly released and collected by the bottom collectors. After that, they were counted to build the corresponding histograms. W was then changed and the above process was repeated. From the collected data, the dispersion σ for each histogram and the corresponding mean final horizontal position $\langle x \rangle$ were calculated.

Fig. 3 shows most of the exit histograms obtained for different values of the wall separation W. The selection of the curves shown allows following the crossover obtained in the shape of the exit distributions. Lines indicate experimental values for the two species of disks. In this case they were styrene black (curves at the left) and white (curves at the right). Symbols indicate the two fits performed separately over the histograms corresponding to each set of particles. These fits were drawn with the corresponding statistical information extracted from the data points, i.e., dispersion and mean value. As clearly seen, as long as W diminishes, the exit distributions become uniform in shape [14] and the fitting Gaussians do not represent the distributions any longer. It seems that at the lowest values of W the final distributions of both types of particles completely overlap, giving a better mixture performance. This will be proved later by measured Rose indexes.

Fig. 4 illustrates the results obtained for the exit distributions belonging to the system styrene–rubber. Styrene disks were poured from the right cell of the injector and rubber ones from the left; thus, curves correlate with these initial conditions. Here, again, symbols indicate the Gaussians fits performed to demonstrate the crossover undergone by the distributions as W decreases. As before, the Gaussian behavior is reasonable down to $W \approx 50$ bins, but walls begin to dominate the shape of the distributions for lower walls separations. There is also some nonsymmetric aspect of the curves with respect to a central vertical line that is not present in the preceding figure. This feature is due to the different elastic properties of the material of the disks. Due to their higher restitution coefficient, rubber disks have more likelihood to explore further regions of the BGB than styrene ones.

But after all these results, it is necessary to characterize the quality of the obtained mixture in such a way to be able to predict mixing capability for a BGB with given characteristics (size, number of rows, etc.). For this reason, the Rose index M

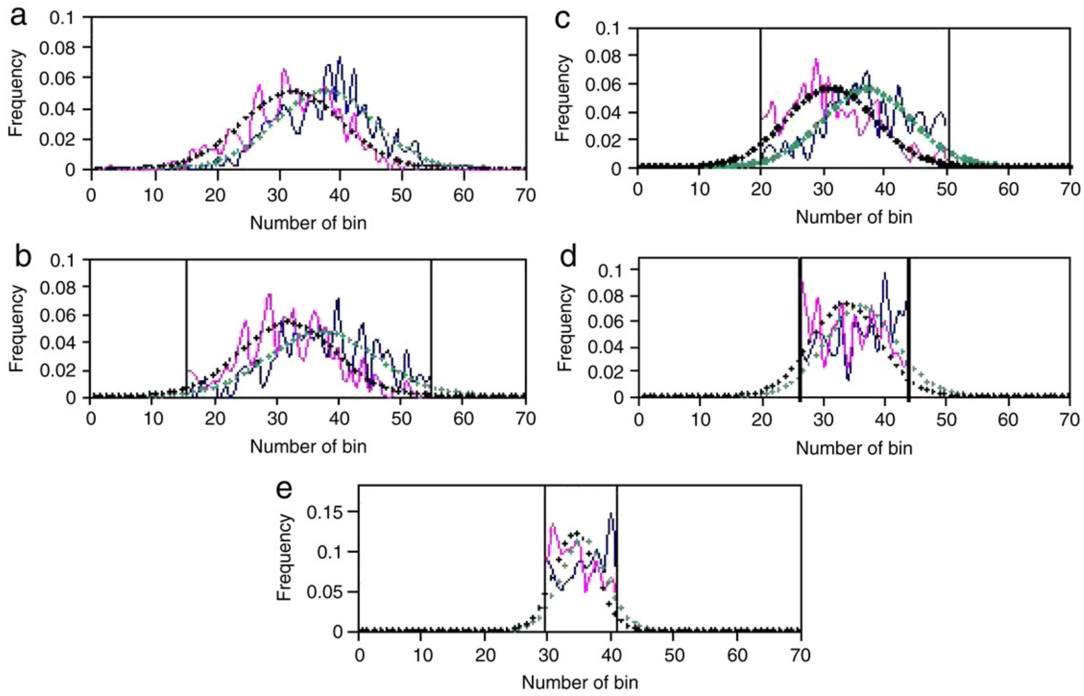

Fig. 3. Distribution functions for the mixture of "white" and "black" styrene disks for some of the cases studied. W attains the following values: (a) 70; (b) 40; (c) 31; (d) 19 and (e) 12, all measured in bins. The other values for W are not shown for shortness. The curves at the right are the one corresponding to "white disks" and the left ones to the "black disks". The corresponding Gaussian fits have been superimposed. Vertical lines about the wall position were drawn to guide the eye.

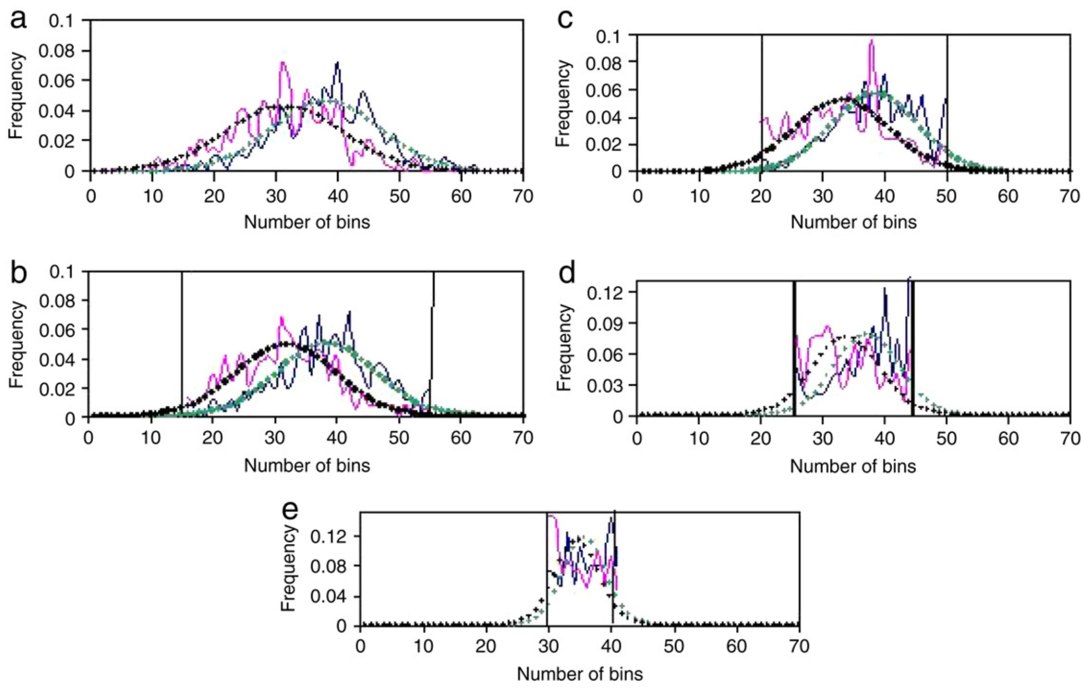

Fig. 4. Distribution functions for the mixture of rubber and styrene disks for some of the cases studied. W attains the following values: (a) 70; (b) 40; (c) 31; (d) 19 and (e) 12, all measured in bins. The other values for W are not shown for shortness. The curves at the right correspond to styrene and the left ones to rubber. The corresponding Gaussian fits have been superimposed. Vertical lines about the wall position were drawn to guide the eye.

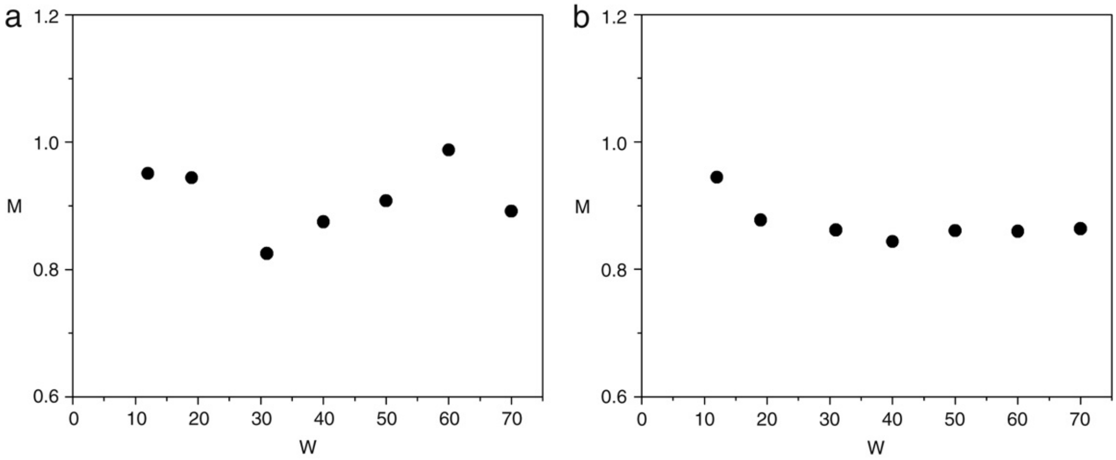

Fig. 5. Global Rose indexes M for the two cases studied: (a) styrene disks "black" and "white", (b) styrene and rubber disks.

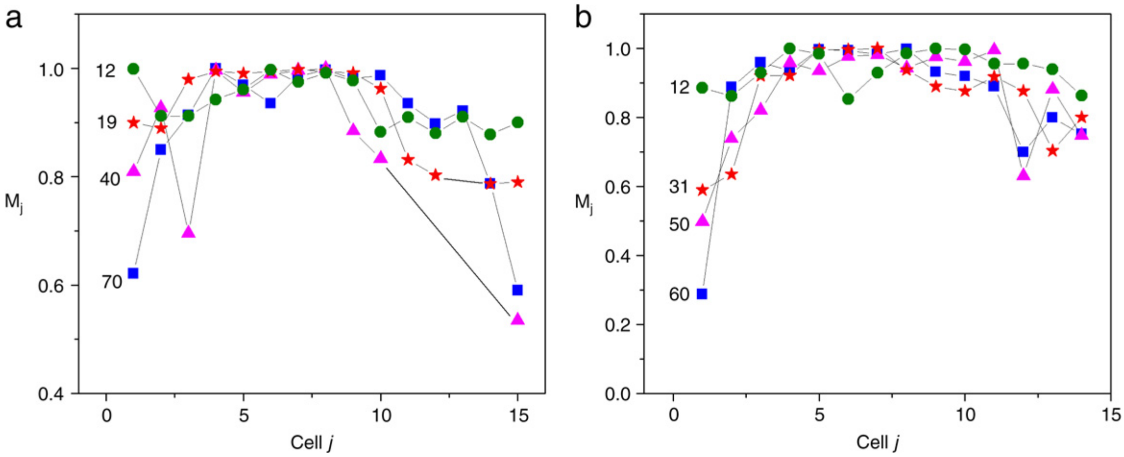

Fig. 6. Behavior of the Rose index M_j per cell j for different W and for the two systems: (a) styrene disks "black" and "white", where diamonds correspond to $W = 12$, stars to $W = 19$, up triangles to $W = 40$ and squares to $W = 70$, as indicated; (b) styrene and rubber disks, where diamonds correspond to $W = 12$, stars to $W = 31$, up triangles to $W = 50$ and squares to $W = 60$, as indicated. The horizontal axis indicates the different values of j labeling each of the cells employed for the calculation of M_j. For all values of W, we used the same number of cells.

of the mixture of particles at the exit of the board was measured. This index is defined as:

$$M = 1 - \frac{V}{V_o} \qquad V = \frac{1}{k}\sum_{j=1}^{k}\left(C_a(x_j) - \frac{1}{2}\right)^2 \tag{1}$$

where V_0 is the variance at the completely segregated state, i.e., $\frac{1}{4}$, and C_a is the concentration for species a, calculated in terms of volume percentage.

To analyze the mixture degree for each W samples, cells of the same mass on the histogram were taken. Inside each cell j, the local concentration of each species is calculated. Then, $V_j = (C_a(j) - \frac{1}{2})^2$ and $M_j = 1 - \frac{V_j}{1/4}$, the Rose index per cell j, were calculated. Finally, M was calculated using (1). Figs. 5 and 6 show $M$and M_j, respectively, for different values of W and for the two mixing cases studied. In the case of Fig. 6, the horizontal axis indicates the different values of j labeling each of the cells employed for the calculation of M_j. For all values of W and a given mixture, we used the same number of cells to span the whole width of the BGB. This number was $j = 15$ and $j = 14$, for the two systems, respectively. Thus, to compare the plots for different W in Fig. 6 (a) (or (b)), one has to keep in mind that, for instance, cell $j = 15$ ($j = 14$) represents always the last cell to the right of a BGB of width W, as well as the cell $j = 0$ represents the first cell to the left of the BGB.

As seen in the cited figures, there is a tendency of the global Rose index to increase as W decreases. As previously discussed in Ref. [10] for open BGB (without walls), only the increment of the number of rows can improve a mixture degree. Implementation of lateral walls allows us to obtain a better mixture using the same number of rows, details of practical importance at an industrial level. This increment found here is based on the fact that the presence of walls produces a gaussian-to-uniform exit distribution crossover that is of crucial effect in the mixing problem. This crossover has been studied and described elsewhere [14,15] and it has been demonstrated that there is an important commitment between the number of rows and the number of columns needed in a BGB to get the so-mentioned crossover. These results, and the ones showed here, demonstrate that a careful election of the BGB dimensions is crucial for mixture improvement. Moreover, the effect of W is especially important for the cell indexes M_j. As W diminishes, the mixture quality per cell mainly increases at the lateral bins and it becomes more even for all the cells in the system. This effect is indeed a very important one that assures that the improvement in the global Rose index is not only showed by the central part of the exit distribution function, but also present all over it. In practical words, one can find a good quality mixture in most of the exit cells of a BGB that feeds several packing devices.

3. Simulation procedure and results

Simulations were driven to reproduce the geometrical features of experiments. We define sites (representing the pins of the board) arranged in a triangular lattice and keeping experimental distances. Disks of the same size like those of the experiment are launched from above and their horizontal positions were taken in such a way to represent experiments, i.e., two virtual boxes were assumed in the simulation, and disks belonging to different species were released, respectively, from each box (cell). Trajectories followed by the disks were calculated in this way: a disk falls down by gravity until it encounters a pin in its trajectory. At this moment, and depending on the relative positions of the centres, the small disk will roll over the pin to the right (the x-coordinate of its centre is to the right of that of the pin) or to the left (the x-coordinate of its centre is to the left of that of the pin). At this stage, we have to decide which will be the new x-position of the particle, i.e., we have to introduce the effect of bouncing without calculating any force (we are not interested here in the dynamics of the problem). Thus, we just choose a random number uniformly distributed between 0 and 1 (ran(0, 1)) and put the particle at the corresponding x-position given by:

$$x_p = x_{\text{pin}} + aR_{\text{pin}} + aR_p + a\delta\, d\, \text{ran}(0, 1). \tag{2}$$

Here, x_{pin} is the x-coordinate of the centre of the pin; R_{pin} is the radius of the pin (the same for all of them in the lattice), R_p is the particle radius, d is the distance between two neighboring pins minus two times their radius, δ is a factor to account for the hardness of the disks and, finally, a is a factor equal to $+1(-1)$ if the x-coordinate of the centre of the particle is to the right (left) of that of the pin. A value of $\delta = 0.7$ was used to simulate styrene hardness and $\delta = 1.0$ for rubber. This selection is totally arbitrary and this parameter will just represent some real bouncing parameter: the smaller the value of δ, the smaller the hardness of the percolating particle and thus, the smaller the lateral displacement.

After a new position for the particle is attained, a new calculation for the trajectory of the particle is performed, following the procedure explained above, and so on. The elastic properties of the lateral walls of the BGB are the same as those of the pins of the board. Thus, in the case a particle collides with one of the walls, the simulated bouncing effect will be similar to that implemented in Eq. (2), i.e.:

$$x_p = bW + aR_p + a\delta\, d\, \text{ran}(0, 1) \qquad \begin{array}{rl} b = 0;\, a = 1 & \text{collision with the left wall} \\ b = 1;\, a = -1 & \text{collision with the right wall} \end{array} \tag{3}$$

When the particle left the last row of the BGB, its final x-position is recorded to build the exit distribution histogram like in real experiments. In simulations, we have the advantage to know exactly the final position of each particle and thus one can classify these data to build a histogram as desired. Thus, in present simulations, we chose as a collector bin unit a width equal to 1.35 cm. This width is precisely one-half the lattice spacing L. A series of 10^5 equal experiments were performed in order to build the exit distribution of percolating particles and a similar analysis like that for experiments was done.

At this stage, it is worth mentioning that the effect of throwing disks one at a time in simulations is not relevant for the final results obtained. In fact, this has been demonstrated experimentally in Refs. [9,10]. In a typical BGB experiment, while the discs are falling, they collide inelastically with the obstacles and other falling discs, losing energy and changing their local velocity direction. Bruno et al [9,10] analyzed the influence of the collective interactions, that is the influence of other falling particles on a tracer disc, by studying the velocity distribution of tracer particle and comparing it to the one obtained when the discs are released one by one and they noticed no significant influence. Even more, they have performed numerical simulations of the spatial frequency of disc–disc and disc–obstacle collisions. They have seen that the spatial frequency of disc–obstacle collisions remains constant in average during all the chute and disc–disc collisions occur mainly close to the entrance where the particles are clustered together (region of high disc–disc interaction density) corresponding to the first four or five rows of obstacles. Afterwards, disc–disc collisions became less probable, when the particles have spread sufficiently due to the presence of obstacles, and the system will behave as a single particle one. For this reason, the results presented and discussed in simulations, will refer to the one particle-at-a-time problem.

Figs. 7 and 8 show some of the exit distributions obtained in simulations for the system of black and white styrene disks and for the case of rubber and styrene, respectively. It should be clear that in these two figures the units for the horizontal axis are set in meters, while those for experimental data in Figs. 3 and 4 are given in number of bins. Nevertheless, this is irrelevant for the main conclusions and comparisons that follow.

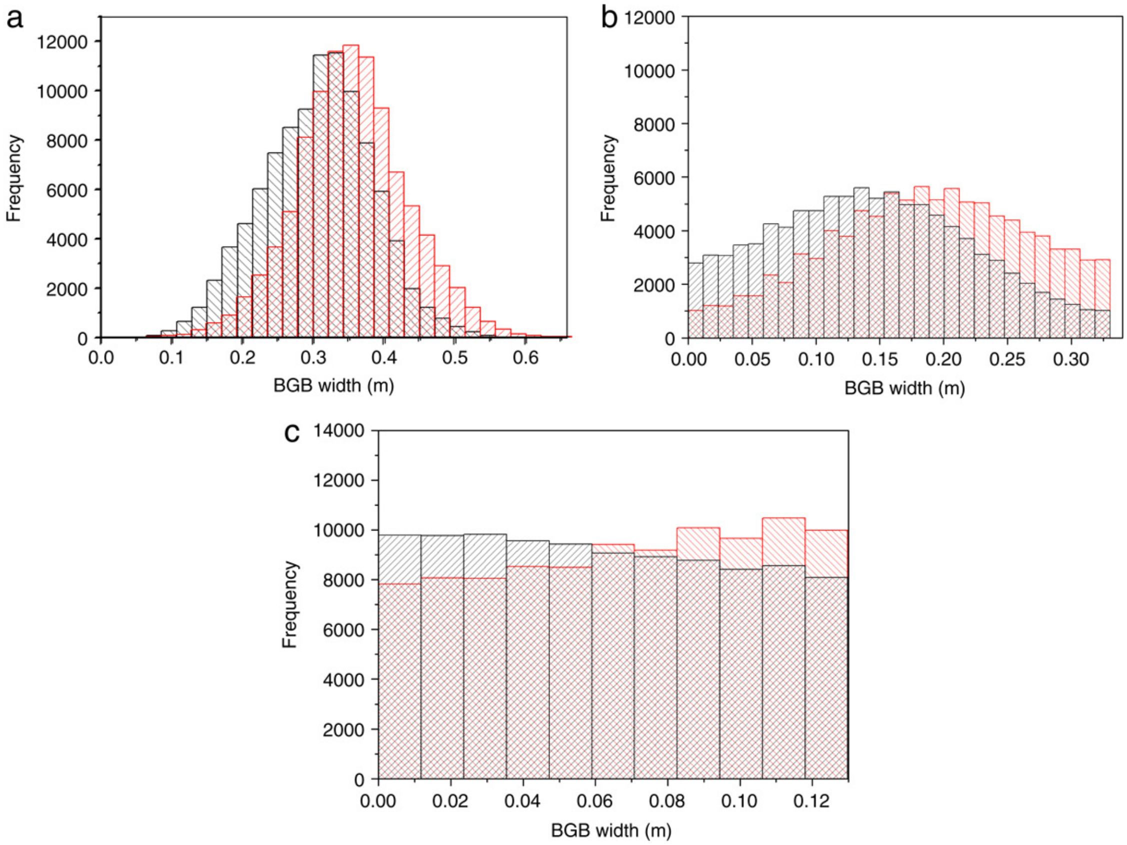

Fig. 7. Some examples of the exit distributions obtained in simulations when mixing disks with the same physical properties (styrene) but distinguished by color (black and white mixture). (a) $W = 57$, (b) $W = 25$ and (c) $W = 11$, measured in number of columns.

As in experiments, a clear crossover is observed in the shape of the final exit distributions of percolating particles. A gaussian description is no longer sustainable to characterize the problem. Another aspect in agreement with experiments is the asymmetric behavior of the two distributions.

In Fig. 7(a), for large W, both distributions are Gaussians, they are not affected by the presence of walls. The mixing region also presents a bell-like shape. Part (b) of the same figure illustrates how the presence of walls begins to affect the distributions. In each distribution, the deformation effect is more pronounced at the side that is closer to the launching point of each one. In part (c), for small W, both distributions tend to uniform behavior, showing just a small slope due to the asymmetry in the launching point, i.e., the effect of each wall is different because they are placed at slightly different distances with respect to the launching point (left launching point is closer to the left wall and vice versa). The mixing region becomes practically uniform and always symmetric.

In Fig. 8(a), for large W, there are no wall effects. The dispersion of the two Gaussians is different, being higher the one corresponding to a greater hardness (rubber in this case). In part (b) of the same figure, the presence of walls is more evident. As seen, each species "notes" the presence of one lateral wall more than the other, i.e., species at the left notes earlier the presence of the left wall, while the other (with less hardness) still has a gaussian shape. Left distribution has still a Gaussian tail at the right and tends to uniform at the left. Finally, for small W, both distributions tend to uniform behavior, but, due to their different hardness, that with greater δ has a lower slope, evidencing that this species is the first one in achieving uniform behavior. The mixing zone is asymmetric due to the different bouncing properties of the particles.

It rests to analyze the behavior of the index of mixture in simulations and compare the results with their experimental counterparts.

Figs. 9 and 10 show, respectively, the global and local (per cell j) Rose indexes for the two mixings. Results qualitatively agree with experiments. Improvement in the mixture quality is evident as W decreases. The advantage of simulations is clear in the figures to follow the trends that the mixing indexes follow as W increases. The great number of samples employed in simulations and the absence of other dynamic effects, allow highlighting the important effect that geometry has in the mixture process. In these figures the units for the horizontal axis are set in number of columns of the BGB.

In Fig. 9(a), the Rose index for the case of mixing discs of equal species is shown (styrene black and white). As seen, the mixture results in a good quality (more than 0.9), even for large W. For $W \approx 25$, the Rose index grows, attaining practically a value equal to 1 for $W \approx 9$.

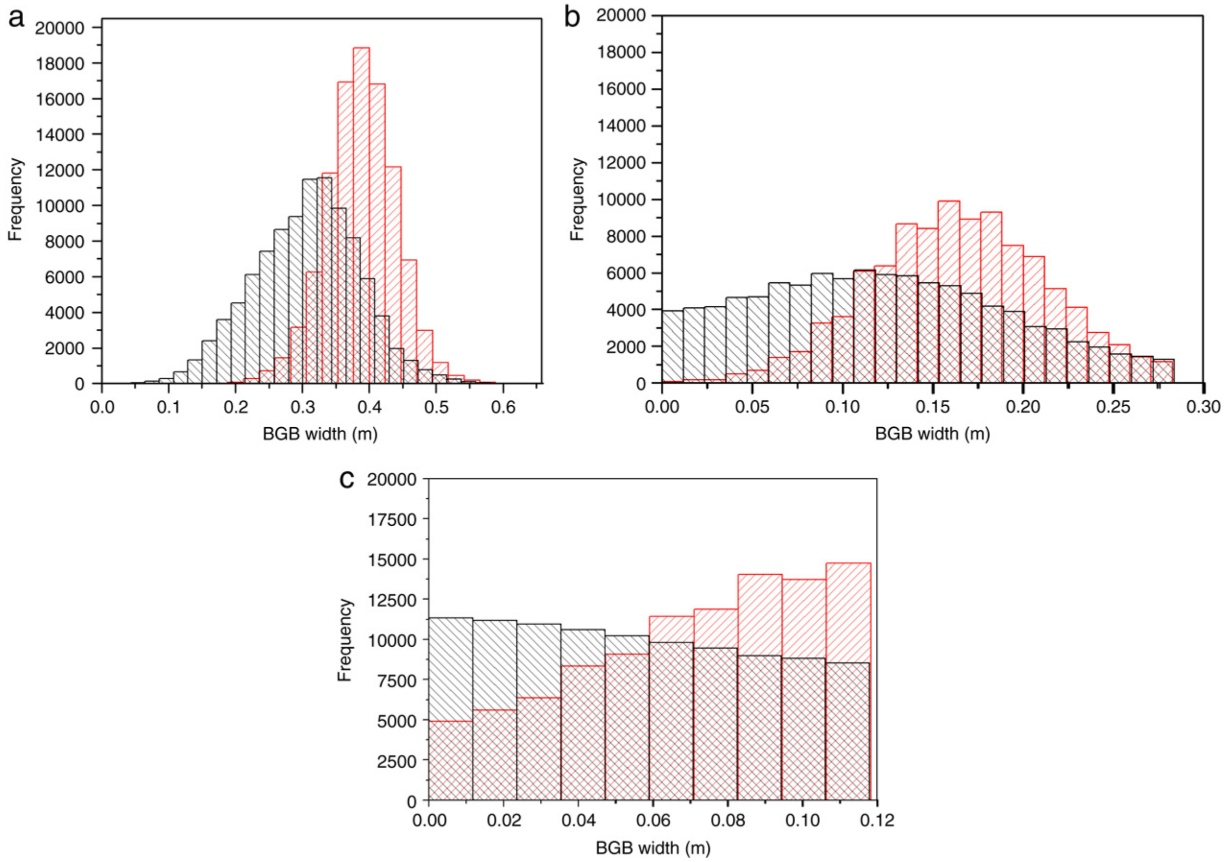

Fig. 8. Some examples of the exit distributions obtained in simulations when mixing disks with different physical properties (styrene–rubber). Histograms to the right correspond to styrene with $\delta = 0.7$ and those falling at the left correspond to rubber with $\delta = 1.0$. (a) $W = 57$, (b) $W = 25$ and (c) $W = 11$, measured in number of columns.

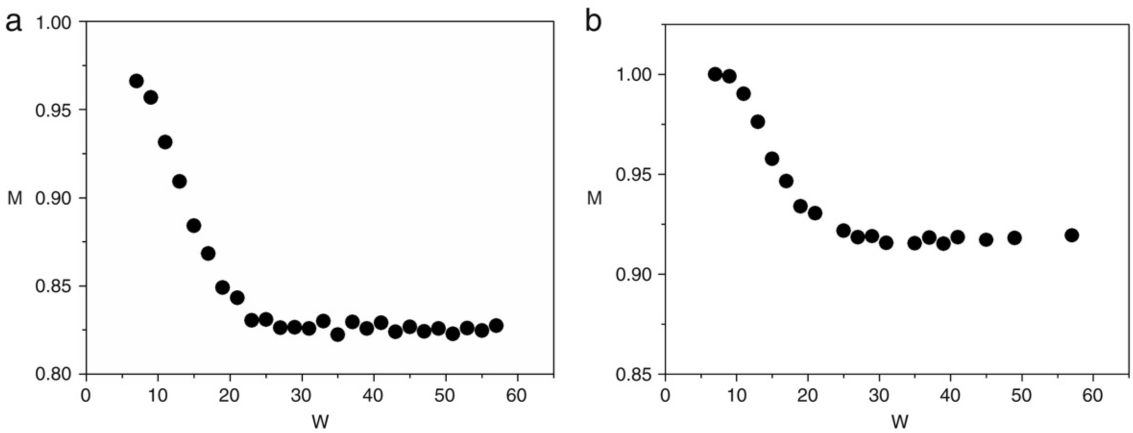

Fig. 9. Simulated global Rose index as a function of W. (a) For styrene black and white particle mixing. (b) For rubber and styrene particle mixing.

In Fig. 9(b), the plot of M as a function of W is presented for the case of styrene–rubber mixture. Here, M presents lower values as compared with the preceding case ($M \approx 0.85$), but they still represent a good degree of mixture. The general trend of the curve is the same as for styrene disks in (a). For $W \approx 25$, the Rose index grows and it reach a value of 0.97 for $W \approx 9$, meaning that the quality of the mixture as been improved with respect to the one for large wall separations.

But the global Rose index does not account for the local mixing quality. This means that, using only M, the homogeneity of the mixture in each collecting cell cannot be ensured. Thus, in Fig. 10, the index M_j for each cell is presented as a function of wall separation, as it was calculated in experiments.

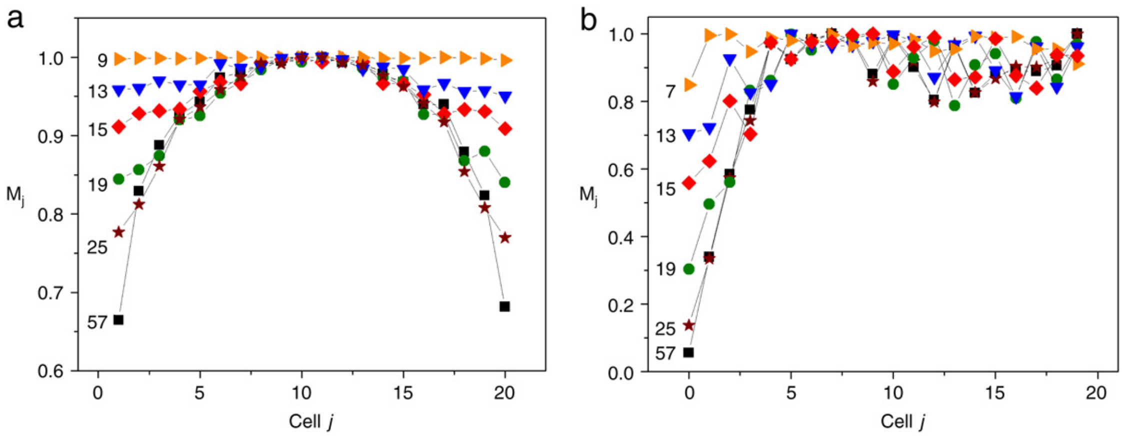

Fig. 10. Rose index M_j for each sample cell j, obtained in simulations for different W, as indicated. (a) For styrene black and white particle mixing. Symbols are as follows: squares for $W = 57$, stars for $W = 25$, circles for $W = 19$, diamonds for $W = 15$, down triangles for $W = 13$ and right triangles for $W = 9$; (b) For rubber and styrene particle mixing, where symbols are: squares for $W = 57$, stars for $W = 25$, circles for $W = 19$, diamonds for $W = 15$, down triangles for $W = 13$ and right triangles for $W = 7$.

In Fig. 10(a) the results for styrene "black" and "white" disks are shown. Here, it is evident that all curves are symmetric with respect to the centre. This behavior was already pointed out when the exit distribution functions were analyzed: the mixing zone was symmetric there. For $W > 25$, M_j presents larger values at the central cells, compared to the lateral cells, the difference being of the order of 45%. Only the central cells present a good mixing quality. This difference is reduced to approximately 20% when $W \approx 25$. From this value on, the difference is still smaller. When $W \approx 9$, the curve is practically constant and all the cells have the same mixing index and homogenization degree.

In Fig. 10(b), it is clear that there is an asymmetric behavior of the indexes M_j due to the difference in the bouncing properties of the particles (styrene–rubber mixing). This fact was also observed at the exit distributions, where the mixing zone offered an asymmetric feature. For large W the mixing indexes at the lateral cells are much lower than the ones at the central cells by around 95%. This difference is mainly due to the different bouncing properties. Disks with less elastic coefficient can hardly reach the walls, specially the farthest one with respect to their launching point. As a result, the concentration of the styrene disks is low at the left cells, giving even worst local mixing indexes than the right part of the BGB. The mixing quality is improved for $W \leq 25$. For small W there still remains a difference of the order of 15% between the last left cell and the rest of them.

Thus, the separation between walls becomes crucial for mixing homogenization in both systems studied.

It is worth remarking that the results shown above correspond to a fixed number of rows in the BGB. Do not lose sight to the fact that the optimum width obtained here for an homogeneous mixture all over the BGB will depend on the number of rows employed in it [12]. Thus, it is important to remark that the present results are scale dependant and a point to highlight is that it is always possible to find the dimensions for the BGB (by just taking into account the ratio between number of rows and number of columns) to obtain an optimum homogenization degree according to the size constraints in real circumstances.

4. Conclusions

This work has presented a detailed experimental and numerical study concerning the influence of lateral walls on the mixing capability of a bi-dimensional Galton Board.

Experiments with particles with two different bouncing properties were performed. The shape of the exit distributions functions and the mixing indexes were measured. By experimentally determining the mean position and variance of each size distribution, we were able to predict the quality of the resulting mixing. The behavior of these quantities was described as the separation between lateral walls was varied.

The mixing region changes as walls become closer. It goes from a bell-like shape to a uniform shape. This uniform shape is better achieved when particles with similar hardness properties are mixed. For particles with different bouncing properties, the final result is not as good as for the first case.

Separation W between walls is crucial in the mixing process through a BGB. As W decreases, the global mixture is more efficient in both cases studied. The enhancement in mixture quality is especially important when one measures the local index M_j. This fact is important at the industrial level. In all cases the global index M is quite higher compared to common powder mixers and the mixing obtained is irreversible. The great number of samples employed in simulations and the absence of other effects, allow highlighting the important effect that geometry has in the mixture process.

Future efforts will be driven to characterize theoretically the evolution of the mixture quality as a function of wall separation and to look over the influence of the arrangement geometry (random lattice) in BGB devices [16].

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