# MULTI-STAGE OPTIMAL CONTROL OF LIGNOCELLULOSIC **BIOETHANOL PRODUCTION**

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Abstract — The dynamics of production and purification of ethanol from sorghum lignocellulosic materials by a three stages process was modeled and optimized in this work. The process involves a first stage for hydrolyzing sweet sorghum bagasse; a second stage for fermenting the generated sugars and a third stage for the separation of ethanol. Kinetic and distillation equations were embedded into macroscopic balances in order to derive a mathematical model used to solve a three-stage optimal control problem. The aim was to maximize the process productivity by optimally managing the controlled flows between units and by optimally fixing switching times between the process stages.

Keywords- multi-stage optimal control; sorghum bagasse; hydrolysis; fermentation; distillation.

# I. INTRODUCTION

Bioethanol is the most widely used biofuel and the most promising alternative to fossil fuels. It is also considered clean because of its inherent characteristics of low pollutant (Andrianantenaina and Ramamonjisoa, 2016). Most bioethanol is produced from sucrose containing or starchbased feedstocks. Crop-based bioethanol imposes an adverse effect on global food supply and a sustainable alternative feedstock which can be used for non-crop bioethanol is lignocellulosic biomass such as rice straw, wheat straw, corn stover, switchgrass, sugarcane bagasse and sorghum bagasse. Lignocellulose mainly consists of cellulose, hemicellulose and lignocellulose. Lignocellulosic bioethanol has not yet been produced on a commercial scale due to lack of cost-effectiveness. Nevertheless, the use of lignocellulosic feedstock is considered renewable, since the carbon released to the environment is captured again by the growth of new crops. Comprehensive efforts are required to reduce costs and maximize the profit throughout the whole process. In general, the productivity of growth-associated products in a chemostat is higher than in a batch reactor but this is not always the case with ethanol production (Shuler and Kargi, 2002). Chemostats outperform batch fermenters in ethanol production from glucose alone as cells grow relatively fast but ethanol productivity from mixed sugars in batch cultures is about two to three times higher than in continuous cultures (Song et al., 2012). The productivity of batch fermenters can be further improved by feeding media leading to the use of fed batch fermenters. Also, the separation of lignocellulosic bioethanol from the culture may strongly affect the process productivity. Increasing the productivity of the whole process should be a preferred target over optimizing individual stages. Modelling is a nontrivial part of optimization aimed at increasing the productivity of a process. This is because a representation adequately predicting the response of the process to all admissible discrete decisions and continuous controls is necessary to optimize it. The optimization of a whole biochemical process would include a multistage optimal control problem involving change of dynamics and control variables on several stages because batch and semi-continuous process may involve nosmooth, switched optimal control problems. These problems were mainly researched for aerospace applications but examples from chemical engineering started to appear during the last decade. See for example De Prada et al. (2009) and Ni et al. (2015). They can be described as follows: given a set of P stages  $p \in [1, ..., P]$ , minimize the cost functional defined by:

$$J = \sum_{p=1}^{P} G(x_p(t_f), x_p(t_0)) + \sum_{p=1}^{P} \int_0^{t_f} F(x_p(t), x_p(t)) dt, \qquad (1)$$

subject to the dynamic constraints:

 $\frac{dx}{dt} = f_p\left(x_p(t), u_p(t)\right), \forall p = 1, \cdots, P$ (2) inequality path constraints:  $C_P^{Min}(t) \le C\left(x_p(t), u_p(t)\right) \le C_P^{Max}(t), \forall p = 1, .., P$ (3)

boundary conditions:

 $\phi_P^{Min}(t) \le \phi\left(x_p(t), u_p(t)\right) \le \phi_P^{Max}(t), \forall p = 1, P (4)$ and the linkage constraints:

 $L_s^{Min} \le L_s(x_p^{ls}(t), p_p^{ls}, t_p^{ls}; x_p^{rs}(t), p_p^{rs}, t_p^{rs}) \le L_s^{Max}$ (5) where  $x_p(t), u_p(t)$  and t are respectively the state variables, the control variables and the time in stages p = 1, ..., P; L is the number of pair of stages to link and s = 1, ..., L are the "left" and "right" stage-numbers respectively (Betts, 2001).

This work is concerned with the numerical derivation of optimal time-profiles for the control variables of a sequential production and separation process for ethanol production from sorghum bagasse and the optimal switching times between hydrolysis, fermentation and distillation stages.

#### **II. MODELS OF THE PROCESS' STAGES**

Several ways to generate ethanol from lignocellulosic residues of sweet sorghum are possible but we restrict this numerical study to the enzymatic hydrolysis of grinded sorghum bagasse with cellulase enzyme; the posterior fermentation of generated sugars by *Saccharomyces cerevisiae* and a final semi-batch distillation stage. The sequential process involves a hydrolysis reactor linked to a separated fermentor as shown in Fig. 1. The feed added to the later reactor contains glucose and some other fermentable sugars arising from the hydrolysis of sorghum bagasse in the former reactor. Afterwards, filtering is performed to feed the aqueous ethanol-solution to the boiler of a distillation column. Since this work presents the numerical application of multi-stage optimal control, materials and methods used to derive experimental information are just referenced.

## A. Hydrolysis kinetics

Lignin is a recalcitrant source of carbon compounds that may be decomposed via pretreatment and hydrolysis into a spectrum of sugars in which glucose and xylose are the first and second most dominant (Song *et al.*, 2012). Joris (2015) and Duarte (2018) researched alternatives for identifying high yield and high conversion-rates hydrolytic enzymes to hydrolyze sweet sorghum bagasse. From these studies we took kinetic data for hydrolyzing the bagasse by cellulase enzyme (FibreZyme® G4, Dyadic's, USA) and performed a least-squares regression in order to determine parameter values listed in Table 1. Kinetic equations used to characterize this hydrolysis are the following:

$$r_M^h = -Y_{S/M} \gamma \frac{M}{M+k_1} M \tag{6}$$

$$r_S^h = \gamma \frac{M}{M+k_1} M \tag{7}$$

where  $r_M^h$  is the rate of depletion of lignocellulosic materials,  $r_M^h$  is the glucose production rate, M is a variable representing an non-dimensional concentration of hydrolysable sugars into lignocellulosic solids,  $\gamma$  the maximum specific hydrolysis rate,  $k_1$  is the saturation constant and  $Y_{S/M}$  is the observable yield of glucose on the lignocellulosic material.

# **B.** Fermentation kinetics

Sugars generated in the above stage are converted to bioethanol by fermentation. An industrial strain of *Saccharomyces cerevisiae* LFF-S04 available on *Laboratorio de Fermentaciones, Facultad de Bioquímica y Ciencias Biológica, Universidad Nacional del Litoral (FBCB -UNL)*, was used by Duarte (2018) to characterize the fermentation of the hydrolysate generated during the previous stage. Although several fermentable sugars are present in the hydrolysate, glucose is the main product and also the limiting substrate for the biomass growth rate. Kinetic rates are characterized by the following equations:

$$r_X = \mu_{max} \frac{S_f}{k_S + S_f} \left( 1 - \frac{P}{k_P} \right) X \tag{8}$$

$$r_P = a\mu_{max} \frac{S_f}{k_S + S_f} X \tag{9}$$

$$r_S = -\frac{1}{\gamma} \mu_{max} \frac{S_f}{k_S + S_f} X \tag{10}$$

Rates  $r_X$ ,  $r_P$  and  $r_S$  are respectively the biomass growth rate; the ethanol production rate and the glucose con-



#### Fig. 1: Schematics of th train

sumption rate. In kinetic Eqs. (8)-(10), *S* is the glucose concentration, *X* is the biomass concentration and *P* is the ethanol concentration;  $\mu_{max}$  is the maximum specific biomass growth rate;  $k_s$  is the Monod constant on glucose;  $k_i$  is an inhibition constant considering the braking effect of ethanol on the biomass growth rate; *a* is the growth-associated Luedeking-Piret specific production rate and *Y* is an observed lumped yield of products (biomass and ethanol) on glucose. Also, a least squares regression was performed to determine parameter values listed in Table 1.

#### C. Distillation dynamics

Although other options are feasible, ethanol from aqueous solutions, like fermentation cultures, is usually separated by standard techniques like filtering and distillation. There are quite standardized mathematical representations for these separation techniques. We utilize the model by Logsdon and Biegler (1993) for a trays column which considers the following assumptions: feeding an aqueous mixture at saturation temperature to the boiler; non-ideal vapor-liquid relationships; negligible vapor holdup in each tray and in the boiler; constant vapor flow and constant liquid holdup in trays and in the condenser; theoretical trays; constant operation pressure; adiabatic (i.e. energy balances neglected) column with n stages of equilibrium; and total condensation of the distillate.

# **III. ASSEMBLING MODELS**

The train illustrated on Fig. 1 comprises the hydrolysis reactor, the fermentor and the distillation column and has three main stages. The first stage begins on the hydrolysis reactor and its aim is to generate glucose and several other fermentable sugars. When the hydrolizable solids are depleted at an unknown time  $t_f^{(1)}$ , the culture is feed to the fermentor starting the biomass growth and the ethanol production. The fermentation finalizes at an unknown time  $t_f^{(2)}$  when fermentable sugars have been practically exhausted. Then, the feeding of the distillation column starts and the separation proceeds until an un-known time  $t_f^{(3)}$ . Filtering of depleted solids and filtering of biomass are respectively performed to feed the fermentor and boiler. To model the train, kinetic equations linked by yield parameters must be embedded into macroscopic balances equations for both reactors. Furthermore, the dynamics of the distillation column considers that the boiler is feed with the filtered culture. The problem involves three stages; a batch hydrolysis  $(p_1)$ ; fedbatch fermentation  $(p_2)$ ; and a semi-continuous distillation  $(p_3)$ ; and three control variables: the flow of filtered hydrolysate solution toward the fermentor  $(u_1)$ ; the flow of the filtered culture toward the column boiler  $(u_2)$ ; and the distillate flow  $(u_3)$  out of the condenser.

# A. Dynamic equations

The dynamics of the whole train is represented by Eqs. (11) to (22). Equations (11)-(12) respectively state the dynamics of sugars-depletion from solids and the glucose production in the hydrolysis rector. Equations (13) and (14) state the dynamics of the reaction volumes,  $V_h$  and  $V_f$ , in the hydrolysis reactor and the fermentor respectively. The dynamics of the biomass concentration, X, ethanol, P, and glucose,  $S_f$ , in the fermentor are given by Eqs. (15), (16) and (17), respectively. Equation (18) defines the dynamics of the molar fraction of ethanol,  $x_i$ , in each tray *i*. The dynamics of the volume of solution available in the boiler, B, is given by Eq. (19). The molar fractions of ethanol in the condenser,  $x_c$ , and in the boiler,  $x_b$ , are respectively defined by Eqs. (20) and (21). Equation (22) gives the dynamics of the condensed distillate, D. In this model, binary parameters  $p_1$ ,  $p_2$  and  $p_3$  take value 1 whenever their respective stages are active and 0 otherwise; i.e. when hydrolysable solids are depleted then  $p_1=0$ ,  $p_2=1$ ,  $p_3=0$  and when fermentation ends then  $p_1=0$ ,  $p_2=0$ ,  $p_3=1$ . Switch-times are determined by the problem solution. Variables  $y_i$ ,  $y_c$  and  $y_b$  respectively state the molar fraction of ethanol in the vapor phase on tray *i*, on the condenser and on the boiler. The liquid flow L in the column is computed as the difference between the vapor flow V and the distillate flow  $u_3$ .

$$\frac{dM_h}{dt} = r_M^h p_1 \tag{11}$$

$$\frac{ds_h}{dt} = r_S^h p_1 \tag{12}$$

$$\frac{dv_n}{dt} = -u_1 p_2 \tag{13}$$

$$\frac{dx_{1}}{dt} = u_{1}^{T} p_{2} - u_{2} p_{3} \tag{14}$$

$$\frac{dx}{dt} = \left(r_x - u_1^J \frac{x}{V_f}\right) p_2 \tag{15}$$

$$\frac{1}{dt} = \left(r_P - u_1' \frac{1}{v_f}\right) p_2 \tag{16}$$

$$\frac{dS_f}{dt} = \left(r_S^f - u_1^f \frac{(S_h^{-3}f)}{V_f}\right) p_2 \tag{17}$$

$$\frac{dx_i}{dt} = \left(\frac{L(x_{i-1} - x_i) + V(y_{i+1} - y_i)}{M_h}\right) p_3, i = 1, \dots, n_{trays} (18)$$

$$\frac{du}{dt} = (u_2' + L - u_1')p_3 \tag{19}$$

$$\frac{dx_c}{dt} = \left(\frac{V}{M_{hc}}(y_1 - y_c)\right)p_3\tag{20}$$

$$\frac{dx_b}{dt} = \frac{1}{B} \left( u_2^f (x_f - x_b) + L(x_h - x_b) - V(y_b - x_b) \right) p_3 (21)$$
$$\frac{dD}{dt} = u_3 p_3 (22)$$

Feeding flows  $u_1^f$  and  $u_2^f$  are respectively related to  $u_1$  and  $u_2$  by algebraic equations to be next defined.

#### **B.** Algebraic equations

Vapor-liquid equilibrium data are computed by interpolation between points provided by a table expressed as:

$$y_i = y(x_i) \tag{23}$$

The filtering factor  $c_1$  given by the volume of filtered solids-free solution feed to the fermentor per liter of non-filtered hydrolysate is computed by:

$$c_1 = \frac{1000 - 0.79M^h(t_0^{(1)})}{1000} y(x_i)$$
(24)

where  $M^h(t_0^{(1)})$  is the initial mass of hydrolizable material per liter in the hydrolysis reactor. The filtering of solids from the hydrolysate implies that the glucose concentration on the solids-free solution  $G_1^f$  and the flow to the fermenter  $u_1^f$  are respectively given by:

$$\dot{S_h} = S_h / c_1 \tag{25}$$

$$u_1^f = c_1 u_1$$
 (26)

The second filtering operation is performed to separate biomass from the culture fed to the boiler. So, the filtering factor  $c_2$  given by the volume of biomass-free solution per liter of non-filtered culture is computed by:

$$c_2 = \frac{1000 - X(t_f^{(2)})}{1000} \tag{27}$$

This implies that ethanol concentration in the biomass-free solution  $P_f$  would be given by:

$$P_f = P/c_2. \tag{28}$$

Since vapor-liquid equilibrium data are expressed in molar fractions, the following expressions must be computed to calculate the molar fraction of ethanol and the molar feeding flow to the boiler:

$$x_f = \frac{\frac{P_{f/46}}{P_{f/46}+(1000-X-P_f)/18}}{(29)}$$

$$u_{2}^{f} = 1e^{-3} \left( \frac{P_{f}}{46} + \frac{(1000 - X - P_{f})}{18} \right) u_{2}.$$
 (30)

Sugars concentration in this flow is assumed negligible because they have been exhausted in the fermentor.

#### C. Path, state control and end constraints

Optimal control of batch distillations usually involves the maximization of the quantity of distillate subject to purity constraints. The inequality imposing a minimum molar fraction of ethanol in the distillate is:

$$x_c^{\min}(t^{(3)}) \ge x_c. \tag{31}$$

Also, a depletion constraint is imposed to the molar fraction of ethanol in the solution on the boiler at the final distillation time:

$$x_b(t_f^{(3)}) \le x_b^{\min}. \tag{32}$$

Both constraints are applied just to the stage  $p_3$  of the problem. Also, limits to the achievable values of some states must be imposed. Here the following constraints must be considered:

$$0 \le V_h(t^{(2)}) \tag{33}$$

$$0 \le V_f(t^{(2)}) \le V_f^{max} \tag{34}$$

$$0 \le B(t^{(3)}) \le B^{max} \tag{35}$$

Equation (33) set the lower bound to the hydrolysis reactor. Equation (34) states that the culture volume in the fermentor is a nonnegative variable which must not exceed the reactor-vessel capacity and Eq. (35) imposes the dome capacity as the upper bound to the volume of filtered culture in the boiler. Flows are constrained by Eqs. (36) to (38). Bounds  $u_1^{max}$  and  $u_2^{max}$  are technical

constraints stating the maximum flow between units. Since the vapor flow in the column is considered constant, it imposes an upper bound to the distillate flow. The liquid flow is the difference between the vapor flow and control variable  $u_2(t^{(3)})$ .

$$0 \le u_1(t^{(2)}) \le u_1^{max}$$
 (36)

$$0 \le u_2(t^{(3)}) \le u_2^{max}$$
 (37)

$$0 \le u_3(t^{(3)}) = V_f - L_f(t^{(3)}) \le V_f$$
(38)

#### **D.** Objective function

As the hydrolysis is an autonomous process, no objective function is considered for this stage and as increasing the productivity of the whole process is the target, the following objectives are stated for the following stages:

1. Stage 2: Maximum ethanol productivity in the fermentor:

$$\max J^{(2)} = \frac{P(t_f^{(2)})V(t_f^{(2)})}{t_f^{(2)}}$$
(39)

2. Stage 3: Maximum distillate productivity:

$$\max J^{(3)} = \frac{D(t_f^{(3)})}{t_f^{(3)}} \tag{40}$$

The problem is a free final-time one defined by the sum of three free-time subproblems. The hydrolytic reaction autonomously evolves until time  $t_f^{(1)}$  when the feeding of the fermentor starts. The fermentor is then feed with the hydrolysate in optimal fashion until time  $t_f^{(2)}$ . Then, the feeding of the boiler starts and the distillation proceed until the final process time  $t_f^{(3)}$ . The global productivity is defined as the sum of  $J^{(2)}$  and  $J^{(3)}$  (in gP/h).

# IV. MULTI-STAGE OPTIMAL CONTROL OF THE TRAIN

GPOPS was developed in response to a demand for software able to solve complex multi-stage optimal control problems. Its freeware 5.2 version implementing the Radau pseudospectral collocation method (Rao et al., 2012) was employed in this work. Pseudo-spectral methods are a special class of orthogonal collocation methods discretizing both control and states variables. A detailed description of the algorithm implemented by GPOPS can be found in Rao et al. (2014). A nominal problem defined by parameters summarized in Table 1 was solved in a 2.0 GHz 16 GB RAM PC. Kinetic and yield parameters for the hydrolytic production of glucose and isomers from Sorghum bagasse with cellulase enzyme and for ethanol production on this hydrolysate with S. cerevisiae were derived from experimental information obtained in the Laboratorio de Fermentaciones (FBCB – UNL). Summarized details about materials and methods are reported in Joris et al. (2017). Non-ideal vapor-liquid water-ethanol equilibrium data were gentle provided by Dr. José Espinoza and coworkers from INGAR (Universidad Tecnológica Nacional – CONICET). The value  $x_c^{min}(t^{(3)})$  was considered constant along the last stage. A higher purity value was not considered because of the energetic inefficiency inducted by high reflux ratios necessary to reach such values. Usually, the obtained distillate is subject to a subsequent purification stage in another smaller distillation column. The output generated for this nominal problem states that the optimal quantity of distillate is  $D(t_f^{(3)}) = 2.256 \text{ kmol} (104.7 \text{ kg})$ . Optimal switch time between the hydrolysis stage and the fermentation stage is  $t_f^{(1)} = 10.02$  h while the optimal switch time between the fermentation stage and the distillation stage is

| Table 1. Parameter values for the nominal prob | lem. |
|--|------|
|--|------|

| Para                        | meter   | type         | Parameter              |                          |                    |                | Value |      |
|-----------------------------|---|--------------|------------------------|--------------------------|--------------------|----------------|-------|------|
| Yields                      |   |              | $Y_{S/M}$ (g S/g M)    |                          |                    |                | 6.550 |      |
|                             |   |              |                        | Y (g X                   | /g S)              |                | 1.126 |      |
| ŀ                           | Kinetic   | s            |                        | γ (h                     | -1)                |                | 0.039 |      |
|                             |   |              |                        | <i>k</i> <sub>1</sub> (g | M)                 |                | 0.010 |      |
|                             |   |              |                        | $\mu_{max}$              | (h <sup>-1</sup> ) |                | 0.259 |      |
|                             |   |              |                        | <i>k<sub>s</sub></i> (g  | ( <b>S</b> )       |                | 10.00 |      |
|                             |   |              | $k_P$ (g P)            |                          |                    |                | 51.37 |      |
|                             |   |              | $k_I (g M)$            |                          |                    |                | 5.000 |      |
|                             |   |              | $a (g P/10^6 cells X)$ |                          |                    |                | 0.655 |      |
| Di                          | stillati  | on           |                        | $V_f$ (km                | ol/h)              |                | 0.600 |      |
|                             |   |              |                        | $M_h$ (k                 | mol)               |                | 0.30  |      |
|                             |   |              |                        | $M_{hc}$ (k              | mol)               |                | 0.90  |      |
|                             |   |              |                        | n                        |                    |                | 4     |      |
|                             |   |              |                        | $x_c^m$                  | in                 |                | 0.50  |      |
| Sta                         | te bou  | nds          |                        | V <sub>Max</sub>         | (1)                |                | 1000  |      |
|                             |   |              | B <sub>Max</sub>       |                          |                    |                | 00    |      |
| Con                         | trol bo   | unds         | $u_2^{Max}$ (l/h)      |                          |                    |                | 250   |      |
|                             |   |              | $u_2^{Max}$ (l/h)      |                          |                    |                | 100   |      |
|                             |   |              | $u_3^{Max}$ (kmol/h)   |                          |                    |                | 0.6   |      |
| End constraint              |   | $X_b^{\min}$ |                        |                          |                    | 0.002          |       |      |
| Initial                     | conditi   | ons          |                        |                          |                    |                |       |      |
| (p = 1)                     |   | (p = 2)      |                        | (p =                     | = 3)               |                |       |      |
| <i>M</i> (g/l)              | 280   |              | X (g/l)                | 0.4                      | x                  | c <sub>b</sub> | 0.012 | 2    |
| $S_h$ (g/l)                 | 0   |              | P(g/l)                 | 0.0                      | $x_i(1,$           | 4)             | equil | ib   |
|                             |   |              | $S_{F}(g/l)$           | 10                       | 2                  | (c             | w.bo  | iler |
|                             |   |              | V (I)                  | 100                      | B (kn              | nol)           | 0.807 | 7    |
|                             |   |              | . (-)                  |                          | D (kn              | nol)           | 0.0   |      |
|                             |   |              |                        |                          | 2 (11              |                | 0.0   |      |
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Fig 3.a: Dynamics of  $M_h(t)$  and  $S_h(t)$ .



Fig. 3.b: Dynamics of reaction volumes  $V_h(t)$  and  $V_f(t)$ .



Fig. 3.c: Dynamics of  $S_f(t)$ , X(t) and P(t).



Fig. 3.d: Dynamics of the distillate D(t).





 $t_f^{(2)} = 53.28$  h and the final process time is  $t_f^{(3)} = 72.54$ .

Optimal trajectories for control variables are summarized in Figs. 2. Evolution of states according the optimal



Fig. 3.f: Dynamics of  $x_i(t)$  and,  $x_c(t)$ .



Fig. 3.g: Dynamics of molar fraction  $x_h(t)$ .

 Table 2 Switching times, objective functions and distillate volume for the nominal and perturbed problems.

| Nominal problem ( $\gamma = 0.039 h^{-1}$ ; $\mu_{max} = 0.259 h^{-1}$ ; $a = 0.655$ ) |               |               |               |           |           |                |  |  |
|--|---------------|---------------|---------------|-----------|-----------|----------------|--|--|
|  | $t_{f}^{(1)}$ | $t_{f}^{(2)}$ | $t_{f}^{(3)}$ | $J^{(2)}$ | $J^{(3)}$ | $D(t_f^{(3)})$ |  |  |
|  | 10.02         | 53.28         | 71.67         | 551.44    | 1.458     | 104.7          |  |  |
| Sensibility to variations on $\gamma$  |               |               |               |           |           |                |  |  |
| γ  | $t_{f}^{(1)}$ | $t_{f}^{(2)}$ | $t_{f}^{(3)}$ | $J^{(2)}$ | $J^{(3)}$ | $D(t_f^{(3)})$ |  |  |
| 0.03   | 11.89         | 55.22         | 74.15         | 524.96    | 1.610     | 104.0          |  |  |
| 0.05   | 8.28          | 51.49         | 73.91         | 577.65    | 1.440     | 106.4          |  |  |
| Sensibility to variations on $\mu_{max}$   |               |               |               |           |           |                |  |  |
| $\mu_{max}$  | $t_{f}^{(1)}$ | $t_{f}^{(2)}$ | $t_{f}^{(3)}$ | $J^{(2)}$ | $J^{(3)}$ | $D(t_f^{(3)})$ |  |  |
| 0.20   | 10.79         | 66.75         | 85.81         | 445.48    | 1.224     | 105.1          |  |  |
| 0.30   | 9.59          | 46.98         | 66.64         | 620.60    | 1.555     | 103.5          |  |  |
| Sensibility to variations on a   |               |               |               |           |           |                |  |  |
| а  | $t_{f}^{(1)}$ | $t_{f}^{(2)}$ | $t_{f}^{(3)}$ | $J^{(2)}$ | $J^{(3)}$ | $D(t_f^{(3)})$ |  |  |
| 0.50   | 10.34         | 51.67         | 70.27         | 445.30    | 1.316     | 92.3           |  |  |
| 0.80   | 9.37          | 54.31         | 76.68         | 628.75    | 1.518     | 116.3          |  |  |

switching times and control trajectories are depicted in Figs. 3.

Note that some numerical instability in the optimal distillate flow can be observed. This is a quite common phenomena observed in optimal control of batch distillations, as noted by Logsdon and Biegler (1993), which seems not to considerably distort the evolution of the quantity of distillate depicted in Fig. 3.d.

Optimal control has been used mostly in fields where process models are well known and, but it has had fewer acceptances in biotechnology, where model uncertainties can be significant. In order to study the effect of variations on parameters  $\gamma$ ,  $\mu_{max}$  and a, we performed a brief sensitivity study by varying these parameters. Results are summarized in Table 2.

From data summarized in Table 2, the following conclusions can be stated: (i) Although stage 1 evolves spontaneously, the final stage time  $t_f^{(1)}$  may change according the dynamics of subsequent stages. (ii) Stage 2 timelength  $(t_f^{(2)}-t_f^{(1)})$  depends mainly the on specific biomass growth rate but its impact on the distillate value  $D(t_f^{(3)})$ is almost negligible. (iii) The distillate value  $D(t_f^{(3)})$  is strongly impacted by variations in the Luedeking-Piret specific production rate but its impact on the final process time is rather minor. (iv) Variation on the specific hydrolysis rate  $\gamma$  affect in the duration of the hydrolysis stage but the impact in remaining performance parameters is almost negligible.

#### V. CONCLUSIONS

A three-stage model for optimizing a train of a hydrolysis reactor, a fermentor and a distillation column was used to optimize units' control variables and switching times between stages. The model involves experimental information for the hydrolytic production of glucose and isomers from Sorghum bagasse with cellulase and the kinetics of ethanol production from generated sugars by S. cerevisiae; and bibliographic information for distillation parameters. Kinetic equations and distillation dynamics equations were introduced into macroscopic balances for modeling a train involving three control variables: the flow of hydrolysate toward the fermentor, the flow of the filtered culture from the fermentor toward the boiler of the distillation column and the distillate flow. Since this multi-stage (bio)chemical process involve switching dynamics and change of control variables along the time, stages time-lengths should not be independently fixed. So, control variables were optimally profiled and timescheduled by using GPOPS 5.2. As a consequence, optimal flow profiles and switching times were computed. A brief sensitivity research on the effect of the variation of main kinetic parameters was also performed. A more compressive sensitivity research based on stochastic programming should be performed but this is out of the scope of this work. Multi-stage process involving switching dynamics and change of control variables are common in (bio)chemical engineering but optimization of such processes arise recently. In this regard, the optimization of the whole train aimed at maximizing the overall productivity shows that separately optimizing independent units is not a good option. The results show that nowadays there are no big obstacles in optimizing mathematical representations of multi-stages process because modern optimal control tools can handle multi-stage mathematical models.

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