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Bread baking: Technological considerations based on process modelling and simulation

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Abstract

This paper presents a study of bread baking, mainly from a technological point of view, i.e. focused on transport phenomena and major quality changes occurring during the process. Such study was carried out by numerical simulation of a previously developed and validated mathematical model, which describes the simultaneous heat and mass transfer (with phase change in a moving boundary) taking place in bread during baking. Kinetic models for starch gelatinization and browning development were coupled to the transport model. Input variables to the model were oven temperature, heat transfer coefficient, and bread radius. A total of 105 operating conditions were simulated using the finite element method, and the end point of baking was established for three values of surface lightness. It is shown that an intense heating strategy can produce a browned but unbaked product, besides nutritional quality is negatively affected. Furthermore, minimization of baking time is restricted by internal resistance to heat transfer.

Keywords: Bakery products; Optimization; Control; Baking strategy; Acrylamide; Maillard reaction.

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Nomenclature

a_w	water activity
C_p	specific heat (J kg^{-1} K^{-1})
D	water (liquid or vapour) diffusion coefficient of product (m^2 s^{-1})
D_{va}	water vapour diffusion coefficient in air (m^2 s^{-1})
E_a	activation energy of starch gelatinization (J mol^{-1})
h	heat transfer coefficient (W m^{-2} K^{-1})
K	rate constant of starch gelatinization (s^{-1})
K_0	pre-exponential factor in Eq. (19) (s^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
k_b	rate constant of browning (min^{-1})
k_g	corrected mass transfer coefficient (kg Pa^{-1} m^{-2} s^{-1})
k_g^*	mass transfer coefficient from Eq. (16) (kg Pa^{-1} m^{-2} s^{-1})
L^*	lightness
M	molecular mass (g mol^{-1})
P	water vapour pressure (Pa)
Pr	Prandlt number
Q	heat uptake in starch gelatinization (J)
R, r	radius (m)
R_g	universal gas constant (8.314 J K^{-1} mol^{-1})
RH	relative humidity (%)
Sc	Schmidt number
T	temperature (K)
t	time (s)

W	water (liquid or vapour) content (kg kg^{-1})

Greek symbols

α	degree of starch gelatinization
δ	Delta-type function
ΔT	temperature range of phase change (K)
ε	emissivity
λ_v	latent heat of evaporation (J kg^{-1})
ρ	density (kg m^{-3})
σ	Stefan-Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4})

Subscripts

∞	ambient
air	air
atm	atmospheric
c	centre
f	phase change
s	solid or surface
sat	saturated
w	water

1. Introduction

Baking is the final and most important step in bread making. During the baking process, simultaneous heat and mass transfer occurs within the product producing several physical and chemical changes, which are responsible for the typical features of bread. Basically, dough is transformed into crumb due to starch gelatinization and protein denaturation, and thermal expansion of carbon dioxide (produced by leavening agents) and water vapour; crust is subsequently formed as a result of water evaporation, cross-linking reactions and browning development, which is associated with flavour and harmful compounds formation (Mondal and Datta, 2008; Purlis, 2010; Sablani et al., 1998; Scanlon and Zghal, 2001; Vanin et al., 2009; Yin and Walker, 1995).

Despite of technological advances and process automation, bread making is a traditional food process that still largely depends on experience of skilled technologists (Fahloul et al., 1994; Hadiyanto et al., 2007). Since no microbiological risk is involved a priori, as in other food processes such as pasteurization or sterilization, the end point of baking mainly depends on quality aspects (sensorial attributes) which are critical in the acceptance of the product by consumers, i.e. the surface colour together with texture and flavour (Ahrné et al., 2007; Purlis and Salvadori, 2007). On the other hand, knowledge about the process time as a function of material properties and operating conditions is one of the main interests of design engineers and equipment users (Goñi et al., 2008). So, to better understand and therefore to predict, optimize and control baking, it is essential to consider both transport phenomena and quality changes taking place in bread during the process.

Some efforts have been made to integrate all changes occurring during baking in the context of process optimization, where different approaches were applied. On the one hand, experimental based studies have been performed. In this sense, empirical models (e.g. polynomial functions) are proposed to describe the variation of state variables or quality attributes as a function of operating conditions. Afterwards, optimization can be performed using different methods. For instance, response surface methodology (RSM) has been applied to develop and improve new baking technologies for bread and cake (Demirekler et al., 2004; Sevimli et al., 2005). Another possibility is to perform process optimization using nonlinear programming (Dingstad et al., 2004; Therdthai et al., 2002). On the other hand, transport models describing transformations of the product (e.g. heat and mass transfer model coupled with quality kinetic models) have been used as starting point for baking optimization. On this concept, Hadiyanto et al. (2007, 2008a,b, 2009) developed and applied a series of optimization algorithms for a quality driven process design to improve bakery production.

In bread baking, it is clear that either for optimization or direct technological application, it is necessary to define parameters based on empirical information. For instance, even multi-objective optimization based on sophisticated algorithms uses weight factors and setting values (e.g. end point of the process) to establish the global objective function, which are based on previous experience since sensorial attributes are involved. In addition, there exist a variety of products or specifications according to different cultures and regulations. Therefore, it is difficult to develop an objective methodology to optimize the process or to determine a general heating strategy. In this context, the objective of this paper was to carry out a study of bread baking analyzing simultaneously quality and process aspects. For this purpose, numerical simulation of baking using (previously) validated transport and quality kinetic models was performed for a wide range of operating conditions. In this way, this work seeks to contribute to a better understanding of bread baking, mainly from a technological point of view, and it is expected to be considered as a reference guide for food engineers in bakery industry; final parameters and decision would depend on each product and equipment.

2. Methodology

The presented study was performed by simulation of a previously developed and validated simultaneous heat and mass transfer (SHMT) model for bread baking (Purlis and Salvadori, 2009a,b, 2010). In addition, kinetic models for describing product quality changes, i.e. starch gelatinization (Zanoni et al., 1995a,b) and surface browning (Purlis and Salvadori, 2009c), during the process were coupled to the transport model. Numerical simulation instead of performing experimental tests to analyze the process, allows working under standardized operating conditions, thus minimizing the uncertainties associated with such a complex process as bread baking.

2.1. Mathematical model for heat and mass transfer

The SHMT model includes the main distinguishing features of bread baking, i.e. the rapid heating of bread core and the development of a dry crust. The former has been explained by the evaporation-condensation mechanism (de Vries et al., 1989; Sluimer and Krist-Spit, 1987; Wagner et al., 2007), while the later is due to the formation and advancing of an evaporation front towards the bread core (Zanoni et al., 1993, 1994). In this way, bread baking is considered as a moving boundary problem (MBP) where SHMT with phase change occurs in a porous medium. Then, bread is modelled as a system containing three different regions: (1) crumb: wet inner zone, where temperature does not exceed 100 ºC and dehydration does not occur; (2) crust: dry outer zone, where temperature increases above 100 ºC and dehydration takes place; (3) evaporation front: between the crumb and crust, where temperature is ca. 100 ºC and water evaporates (liquid-vapour transition).

Mathematically, the MBP is formulated using a physical approach, where the enthalpy jump corresponding to phase change is incorporated in the model by defining equivalent thermophysical properties (Bonacina et al., 1973). Such definition means that evaporation takes place within a temperature range rather than at a fixed temperature. Other major assumptions are the following: (1) bread is homogeneous and continuous; the porous medium concept is included through effective or apparent thermophysical properties; (2) heat is transported by conduction inside bread according to Fourier's law, but an effective thermal conductivity is used to incorporate the evaporation-condensation mechanism in heat transfer; (3) only liquid diffusion in the crumb and only vapour diffusion in the crust are assumed to occur (Luikov, 1975); (4) volume change is neglected. For a detailed description of the SHMT model, including thermophysical properties, the reader is referred to Purlis and Salvadori (2009a,b, 2010).

2.1.1. Governing equations

In this study, bread (French type) is considered as an infinite cylinder of radius *R*, so a one dimensional problem can be obtained from the axial symmetry assumption. For initial conditions, uniform temperature and water content are assumed.

Heat balance equation:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) \quad (1)$$

Mass balance equation:

$$\frac{\partial W}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial W}{\partial r}\right) \tag{2}$$

2.1.2. Boundary conditions

The heat arrives to the bread surface by convection and radiation, and is balanced by conduction inside the bread:

$$-k\frac{\partial T}{\partial r} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_\infty^4) \tag{3}$$

The water migrating towards the bread surface is balanced by convective flux:

$$-D\rho_s\frac{\partial W}{\partial r} = k_g(P_s(T_s) - P_\infty(T_\infty)) \tag{4}$$

where $P_s = a_w P_{sat}(T_s)$ and $P_\infty = (RH/100)\, P_{sat}(T_\infty)$.

At the centre, i.e. $r = 0$:

$$\frac{\partial T}{\partial r} = 0 \tag{5}$$

$$\frac{\partial W}{\partial r} = 0 \tag{6}$$

2.1.3. Thermophysical properties

According to the MBP formulation, equivalent thermophysical properties are defined including the phase transition occurring during the process, i.e. an equivalent property is valid for dough/crumb and crust.

Specific heat:

$$C_p(T,W) = C_p^*(T,W) + \lambda_v W\delta(T - T_f, \Delta T) \tag{7}$$

$$C_p^*(T,W) = C_{p,s}(T) + WC_{p,w}(T) \tag{8}$$

$$C_{p,s} = 5T + 25 \tag{9}$$

$$C_{p,w} = (5.207 - 73.17\times10^{-4}\,T + 1.35\times10^{-5}\,T^{2})\,1000 \tag{10}$$

Thermal conductivity:

$$k(T) = \begin{cases} \dfrac{0.9}{1+\exp(-0.1(T-353.16))} + 0.2 & if \quad T \le T_f - \Delta T \\ 0.2 & if \quad T > T_f + \Delta T \end{cases} \tag{11}$$

Density:

$$\rho(T) = \begin{cases} 180.61 & if \quad T \le T_f - \Delta T \\ 321.31 & if \quad T > T_f + \Delta T \end{cases} \tag{12}$$

Density for solid (ρ_s) that appears in Eq. (4) is equal to 241.76 kg m^{-3}.

Mass diffusivity:

$$D(T) = \begin{cases} 1\times10^{-10} & if \quad T \le T_f - \Delta T \\ 1.32\times10^{-3}\,D_{va}(T) & if \quad T > T_f + \Delta T \end{cases} \tag{13}$$

$$D_{va}(T) = \left(2.302\,\frac{p_0}{p}\left(\frac{T}{T_0}\right)^{1.81} \right) 1\times10^{-5} \tag{14}$$

where p_0 = 0.98×10^5 Pa and T_0 = 256 K (Eckert and Drake, 1959); p = P_{atm} = 101325 Pa.

A smoothed Heaviside function with continuous derivative is used to incorporate the phase transition into thermophysical properties, with parameters T_f = 100 ºC and ΔT = 0.5 ºC. In addition, the delta-type function $\delta(T - T_f, \Delta T)$ describing the enthalpy jump (Eq. (7)) is defined by the sum of two smoothed Heaviside functions with different sign.

Water activity:

$$a_w(T,W) = \left[\left(\frac{100\,W}{\exp(-0.0056\,T + 5.5)} \right)^{-1/0.38} + 1 \right]^{-1} \tag{15}$$

The heat transfer coefficient (h) is a model input for process simulation (see Section 2.4), and the mass transfer coefficient (k_g) is determined by using the Chilton-Colburn (or heat-mass) analogy and a correction factor (Purlis and Salvadori, 2009b):

$$\frac{h}{k_g^*} = \frac{M_{air}}{M_w} P_{atm} C_{p,air} \left(\frac{Sc}{Pr}\right)^{2/3} \tag{16}$$

$$k_g = 7.83 \times 10^{-2} k_g^* \tag{17}$$

Regarding heat transfer by radiation, the emissivity of bread surface is considered equal to 0.9 (Hamdami et al., 2004).

2.2. Kinetic model for starch gelatinization extent

In bread baking, the extent of starch gelatinization in dough should be used to determine the minimum process time, since the sensory acceptability of the product will not be guaranteed if a complete starch gelatinization is not achieved (Zanoni et al., 1995a). Starch gelatinization (together with protein denaturation) is responsible for the dough/bread transition and starts at about 50 °C (Zanoni et al., 1995b). The standard procedure for evaluating the degree of starch gelatinization is differential scanning calorimetry (DSC), which measures the temperature and enthalpy of this endothermic process (Fennema, 1996). On the other hand, carrying out a DSC test during baking is not possible in a practical sense. To solve this technological issue, Zanoni et al. (1995a,b) developed and validated a kinetic model of starch gelatinization for bread, which is temperature dependent. In such model, the extent of starch gelatinization follows first-order kinetics and the reaction rate constant is temperature dependent according to the Arrhenius equation:

$$\frac{d(1-\alpha)}{dt} = -K(1-\alpha) \tag{18}$$

$$K = K_0 \exp\left(\frac{-E_a}{R_g T}\right) \tag{19}$$

where $K_0 = 2.8\times10^{18}$ s^{-1} and $E_a = 139$ kJ mol^{-1}. The gelatinization degree (α) is defined as:

$$\alpha(t) = 1 - \frac{Q(t)}{Q_{max}} \tag{20}$$

where $Q(t)$ and Q_{max} are the heat uptakes for partially baked and raw dough, respectively (Zanoni et al., 1995a,b). At initial condition, $\alpha = 0$, i.e. $Q = Q_{max}$ (raw dough).

A complete starch gelatinization in the product can be assumed when the coldest point of bread achieves a value of $\alpha \geq 0.98$; only after reaching this point, bread can be considered as properly baked. This limit value has been established according to data previously published (Therdthai et al., 2002; Zanoni et al., 1995a,b). It is worth to note that the bread recipe used to validate the SMHT model is similar to the one reported by Zanoni et al. (1995b) for the set up of the kinetic model of starch gelatinization.

2.3. Kinetic model for browning development

For bakery products, surface colour is one of the main quality features considering preference of consumers, and therefore it is often used to judge the completion of baking (Abdullah, 2008; Ahrné et al., 2007). The formation of colour, i.e. browning, is the result of non-enzymatic chemical reactions (Maillard reaction and caramelization of sugars) that produce coloured compounds, which are accumulated during baking. The development of browning in bread during baking is a dynamic process which depends on local temperature and water activity, so it should not be decoupled from transport phenomena occurring in the product (Purlis, 2010). In this sense, Purlis and Salvadori (2009c) proposed a kinetic model for browning development based on a non-isothermal kinetic approach, and assuming a general mechanism of browning, which can be described by lightness variation (L^* parameter of the CIE $L^*a^*b^*$ colour space). In such model, browning is described by first-order kinetics, and the rate constant is dependent on temperature and water activity:

$$\frac{dL^*}{dt} = -k_b L^* \tag{21}$$

$$k_b = \left(7.9233\times10^6 + \frac{2.7397\times10^6}{a_w}\right)\exp\left(-\frac{\left(8.7015\times10^3 + \dfrac{49.4738}{a_w}\right)}{T}\right) \tag{22}$$

Kinetic parameters of Eq. (22) were estimated from non-isothermal experiments using real bread samples, instead of isothermal tests and/or ideal systems, in order to better represent actual industrial baking conditions. The kinetic model was validated (mean absolute percentage error = 3.61%) using experimental data obtained at 180, 200, and 220 ºC oven temperature, for natural (h = 7-8 W m^{-2} K^{-1}) and forced convection (h = 12 W m^{-2} K^{-1}) baking modes. Finally, it has been established that colour formation is initiated when temperature surpasses 120 ºC, while raw dough (standard recipe for French bread: 100% wheat flour, 54.1% water, 1.6% salt, 1.6% sugar, 1.6% margarine, 1.2% dry yeast) has an initial value of L^* = 85 (Purlis and Salvadori, 2009c).

2.4. Numerical simulation

Bread baking was simulated for several operating conditions. For this aim, input variables to the SHMT model were oven temperature (180, 190, 200, 210, 220, 230, and 240 ºC), heat transfer coefficient (5, 10, 15, 20, and 25 W m^{-2} K^{-1}), and product radius

(0.025, 0.03, and 0.035 m). These values were selected according to reported data for conventional baking ovens and common industrial practice (Baik et al., 1999, 2000; Carson et al., 2006; Li and Walker, 1996; Sakin et al., 2009; Therdthai et al., 2002; Zareifard et al., 2009). Initial temperature and water content were assumed to be uniform and equal to 25 ºC and 0.65 kg kg^{-1} (dry basis), respectively. Relative humidity (or water vapour pressure) in oven ambient was assumed to be negligible (conventional baking).

The system of nonlinear partial differential equations describing the MBP stated in section 2.1 was solved using the finite element method (Zienkiewicz, 1989). The numerical procedure was implemented in COMSOL Multiphysics 3.2 (COMSOL AB, Sweden) and MATLAB 7.0 (The MathWorks Inc, USA). The method of lines is used in COMSOL Multiphysics for discretization of the partial differential equations, so a differential algebraic equation system is obtained. This new system is solved using an implicit time-stepping scheme (backward differentiation), i.e. a Newton’s method together with a COMSOL Multiphysics linear system solver (UMFPACK). The time step taken by the algorithm is variable (COMSOL AB, 2005), but it was ensured to be small enough (< 5 s) to do not miss the latent heat peak corresponding to phase transition. The finite element mesh consisted in 240 elements in all cases. Finally, a medium order Runge-Kutta routine (function *ode45* from MATLAB) was used to solve (numerically) the quality kinetic models from temperature and moisture content profiles obtained through transport model simulation, using the same criterion for time step as before.

3. Results and discussion

In this work, bread baking was simulated for 105 different operating conditions, according to selected values of input variables to the SHMT model, i.e. oven temperature, heat transfer coefficient, and characteristic length of bread (radius). Note that mass transfer coefficient also changed due to heat-mass transfer analogy (Eq. (16)). Then, both natural and forced convection baking modes were analyzed (Purlis and Salvadori, 2009b). Numerical simulation of the SHMT model allowed obtaining high amount of data, especially, because kinetic models describing quality changes were coupled to transport phenomena (see Appendix). Since the aim of this work was to present a technological perspective of bread baking, results and discussion are focused on practical implications rather than on a detailed description of transport phenomena taking place during the process. This last aspect has been extensively covered in previous papers (Purlis and Salvadori, 2009a,b, 2010). Therefore, temperature and water content profiles were condensed into core and surface temperatures, weight loss, and surface lightness and starch gelatinization extent of the coldest point (bread centre).

Results for two different operating conditions (but for the same bread radius) are shown in Figures 1 and 2; typical variation of temperature at core and surface, and weight loss of bread can be seen in Figures 1a and 2a. At the centre, temperature rises (after a lag phase where thermal gradient is established) until reaching 100 ºC asymptotically, in a sigmoid way, while surface temperature increases continuously towards the oven air temperature. Consequently (and simultaneously), inner zone of bread does not suffer dehydration, which is characteristic of the crumb; on the other hand, a dry crust is formed at outer zone of the product. As a matter of fact, the continuous dehydration of bread, characterized by the advance of the established evaporation front (ca. 100 ºC), is translated into a continuous weight loss of the product, which is responsible for the enlargement of the crust. Note that quantitative differences observed between Figures 1a and 2a are due to the magnitude of heat and mass fluxes in each case, i.e. at higher oven temperature and heat transfer coefficient, more rapid heating and drying may occur.

Regarding the quality aspects of the process, variation of surface lightness and starch gelatinization extent of the centre of bread during baking are presented in Figures 1b and 2b. The development of browning and transformation of dough into crumb are proportional to heat and mass fluxes established by operating conditions, because kinetic models for these quality indices are based on temperature and water activity, and temperature of the product, respectively. Therefore, it is essential to understand transport phenomena in order to design, optimize and control a given process.

To study bread baking from a technological point of view, it is necessary to consider the process time. In this sense, a criterion to determine the end point of baking is required. In this work, surface colour by means of the L^* value (see section 2.3) was used for this aim, and to provide a reference as general as possible, three values of surface lightness were considered, i.e. 80, 75, and 70 (lighter to darker, Figure 3). These values were chosen according to previous experience and with the aim of covering a wide range of baking conditions; the ultimate decision will depend on each particular case. For instance, a sensorial evaluation would be very useful to identify preference of consumers, and afterwards, to establish target values or operating limits.

In Figures 1 and 2, it was indicated (with dashed lines) the end point of the process according to different final values of L^*. From transport phenomena theory, it is expected that for increasing heat and mass transfer fluxes, and longer baking times, darker products will be obtained since higher temperature and lower water activity are reached at surface. On the other hand, the evolution of starch gelatinization extent is not straightforward, in the analyzed context. Assuming that a value of $\alpha \geq 0.98$ ensures a complete transition of dough into crumb, which should be considered as a minimum requirement for baking, all situations for condition shown in Figure 1 accomplished this constraint. However, there exist some cases where this critical requirement could not be achieved, e.g. case 1 in Figure 2; Table 1 summarizes such situations for the range of operating conditions simulated in this work. So, a control variable should be established to overcome this problem, i.e. achieving the target value of surface lightness without a complete baking. One possibility would be measuring the core temperature at the end of the process and verifying a value greater than 95-96 ºC (Tables A.1-A.3). Other authors established a minimum or shortest baking time as the time needed for the bread centre to reach a temperature of 98 ºC (Ahrné et al., 2007; Therdthai et al., 2002). An alternative (or additionally) solution could be establishing an empirical correlation between starch gelatinization degree and weight loss of the product: from the analysis of obtained results (see Appendix), it can be seen that all baked samples suffer 8-10% of weight loss, at least. Such correlation must be developed for each particular case, since weight loss depends on product geometry, as well as other factors, e.g. the use of a mould or container. It is worth to note that weight loss is an easy, low-cost and rapid variable to monitor in an industrial process, besides it has been correlated with colour development during bread baking (Purlis and Salvadori, 2007).

Notice that a complete starch gelatinization was not produced when high heat (and mass) flux was established and lighter surface of bread was required. In addition, this situation was favoured with the increase in the characteristic length of bread. This is because browning is a superficial phenomenon mainly (it only occurs when temperature is greater than 120 ºC), and transition of dough into crumb is assessed in the coldest point of the product. Then, if development of browning is accelerated, e.g. increasing *h* and oven temperature, and thermal gradient is diminished, e.g. increasing characteristic length of product, the time required to achieve a low decrease in L^* is not enough to generate a complete starch gelatinization at bread core. Consequently, it is not recommended to establish a high driven force, i.e. $h > 15$ W m^{-2} K^{-1} and $T_\infty > 220$ ºC, in the baking process when slightly browned products are sought.

According to different final values of L^*, the baking time was determined, and then, surface temperature and weight loss of bread were calculated for the end point of the process. In this way, the influence of operating conditions on bread baking could be studied. Following, such study is presented for one condition, i.e. final $L^* = 75$ and $R = 0.03$ m; this is considered as a representative situation of the process, so derived conclusions are valid for the rest of tested situations.

Firstly, baking time decreases when oven temperature and heat transfer coefficient are increased, showing an exponential trend (Figure 4); this is consistent with transport phenomena theory. On the other hand, for $h > 15$ W m^{-2} K^{-1}, i.e. forced convection baking mode, diminution of process time is produced in a slower manner. In this sense, when forced convection is applied, the cost of increasing the value of h (e.g. increasing the oven fan velocity) would not be directly translated into a reduction of baking time, i.e. the strategy of increasing h to diminish the process time loses efficiency for h values greater than 15 W m^{-2} K^{-1}. This can be explained by the relationship between internal ($(k/R)^{-1}$) and external (h^{-1}) resistance to heat transfer (i.e. Biot number, defined as hR/k): as h increases, the external resistance to heat transfer becomes negligible (i.e. boundary condition tending to prescribed temperature) and all resistance is due to (low) thermal conductivity of the product.

The situation described above has a negative impact on the process, mainly from a nutritional point of view: high temperatures at bread surface can be achieved when using high values of heat transfer coefficient and oven temperature, since surface temperature increases almost constantly with these two operating variables (Figure 5). Though browning and gelatinization constraints are achieved for the depicted operating condition, the pathway for accomplishing the target L^* can produce a major detriment to bread quality. This is because the Maillard reaction is associated with the formation of harmful compounds, such as acrylamide and hydroxymethylfurfural (HMF) (Mottram et al., 2002; Stadler et al., 2002). In particular, the production of acrylamide is strongly correlated with baking temperature and time, and apparently starts at 120-130 ºC, so it could be only found in the crust of bakery products (Ahrné et al., 2007; Becalski et al., 2003; Bråthen and Knutsen, 2005; Surdyk et al., 2004). In this way, it would be desirable to reduce surface temperature of bread during baking as much as possible.

Secondly, weight loss of bread decreases, following a linear behaviour approximately, as oven temperature (T_∞) is augmented, for a fixed final value of L^* and product radius (Figure 6). This is because shorter times are required to achieve the final L^* value for increasing baking temperature, as the heat flux is augmented (e.g. Figure 1a). Nevertheless, it can be seen that weight loss is almost independent of heat transfer coefficient. To understand this behaviour, it is helpful to analyze simultaneously the variation of L^* and weight loss with baking time for different values of h, but with equal oven temperature and bread radius (Figure 7). For instance, it can be observed that increasing heat transfer coefficient from 15 to 25 W m^{-2} K^{-1} does not produce any change in weight loss, approximately. Experimental data included in a previous work supported this observation (Purlis and Salvadori, 2007). This behaviour can be explained by the criterion used to establish the end point of baking: browning development depends on temperature and water activity (Eq. (22)), and therefore on the simultaneous heat and mass transfer process taking place at product surface.

4. Conclusions

Bread baking is a very complex process that involves many variables, regarding both quality and operating aspects. In this way, it is essential to understand transport phenomena to design, control and/or optimize the baking process. Then, it is very useful to carry out simulations based on a transport model coupled with (kinetic) models describing sensorial and nutritional changes in the product, as a function of operating conditions and state variables.

The following technological considerations about the bread baking process arise from the present work:

- Though the end point of baking may be determined by colour development of product surface, a control variable should be established in order to ensure the complete baking of food (dough/bread transition). Such variable could be the core temperature with a lower limit value of 95-96 ºC (the development of empirical correlations with other variables such as weight loss could also be a feasible solution).
- Intense heating as a baking strategy should be avoided. For instance, using values of (convective) heat transfer coefficient greater than 15 W m^{-2} K^{-1} and oven temperature above 220 ºC, could produce unbaked foods, besides the baking time is not substantially decreased because of the low thermal conductivity of bread (internal resistance to heat transfer).
- An advantageous strategy would be a low intensity baking process (e.g. $h < 15$ W m^{-2} K^{-1}, $T_\infty < 220$ ºC): high quality products are obtained since lower values of surface temperature are achieved, which avoids the generation of harmful compounds.

- Finally, it will be important to promote the production and consumption of slightly or minimally browned products, since development of browning reactions is associated with accumulation of toxic compounds. Besides high quality food will be obtained, avoiding the advance of such reactions (e.g. slight decrease of initial L^* value) will also reduce the weight loss of bread and energy consumption, generating economical benefits.

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Appendix

Post-processed data obtained from numerical simulation of bread baking (all operating conditions and end points) are given in Tables A.1-A.3. Values are shown in the following units: h in W m^{-2} K^{-1}, temperatures in ºC, time in min, and weight loss in %.

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Figure captions

Figure 1. Variation of (a) core (green) and surface (black) temperature, and weight loss (blue), and (b) surface lightness (red) and degree of starch gelatinization at core (black) of bread during baking. Values for input variables are: oven temperature, 200 ºC; heat transfer coefficient, 15 W m^{-2} K^{-1}; bread radius, 0.03 m. Dashed lines account for different end points of baking (Figure 3). Arrows indicate data corresponding to the secondary axis.

Figure 2. Variation of (a) core (green) and surface (black) temperature, and weight loss (blue), and (b) surface lightness (red) and degree of starch gelatinization at core (black) of bread during baking. Values for input variables are: oven temperature, 240 ºC; heat transfer coefficient, 25 W m^{-2} K^{-1}; bread radius, 0.03 m. Dashed lines account for different end points of baking (Figure 3). Arrows indicate data corresponding to the secondary axis.

Figure 3. Images of bread samples corresponding to different values of lightness considered to establish the end point of baking. Samples were prepared using a standard recipe for French bread with wheat flour; see section 2.3 (Purlis and Salvadori, 2009c).

Figure 4. Baking time for final L^* = 75 and R = 0.03 m, as a function of oven temperature, for different values of heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Figure 5. Surface temperature of bread for final L^* = 75 and R = 0.03 m, as a function of oven temperature, for different values of heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Figure 6. Weight loss of bread for final L^* = 75 and R = 0.03 m, as a function of oven temperature, for different values of heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Figure 7. Variation of (a) lightness and (b) weight loss of bread with for h = 15 W m^{-2} K^{-1} (blue lines) and h = 25 W m^{-2} K^{-1} (black lines). Other values of input variables are: oven temperature, 200 ºC; bread radius, 0.03 m. Dashed lines indicate results for final L^* = 80.

Figure 1 – Purlis

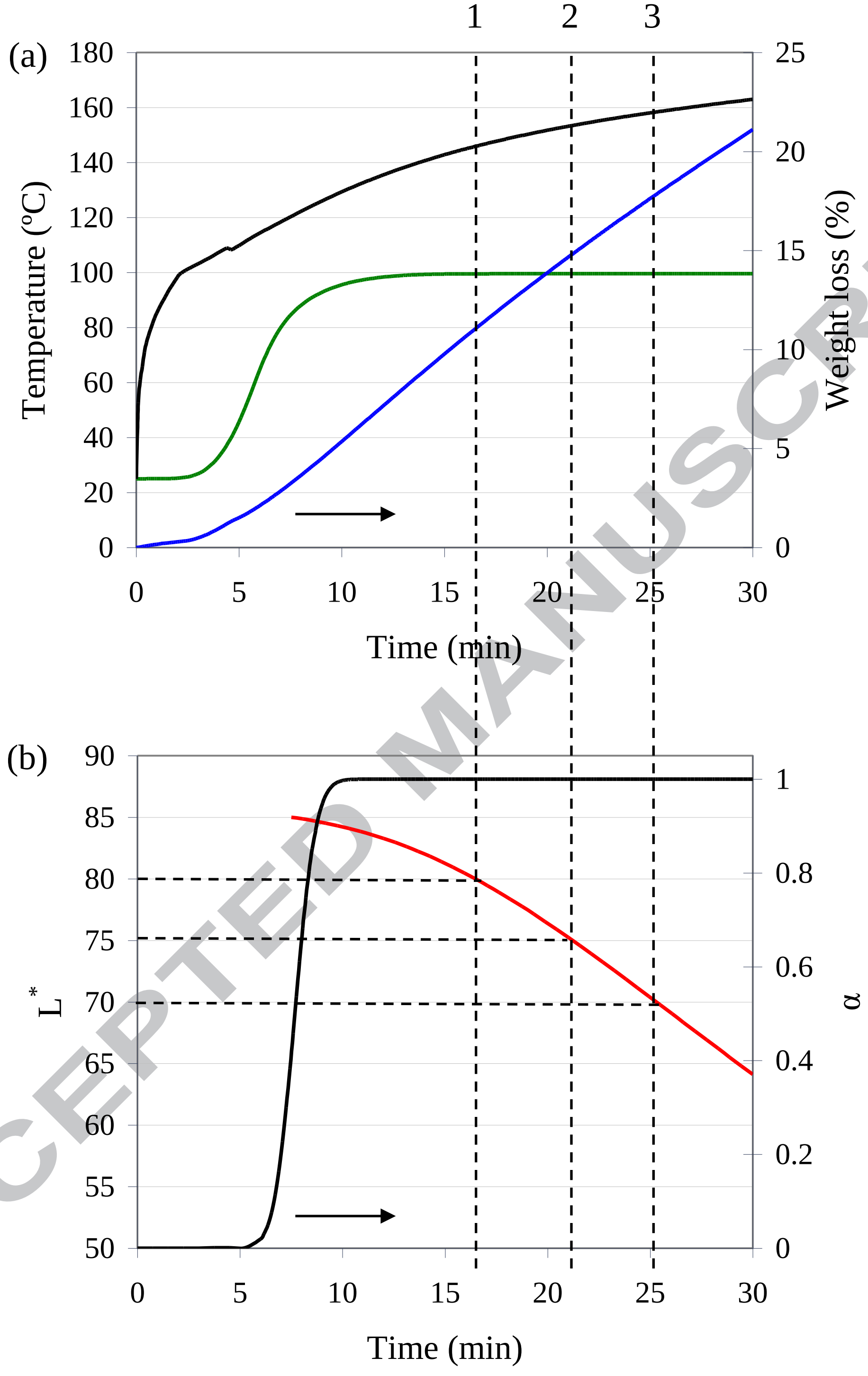

Figure 1. Variation of (a) core (green) and surface (black) temperature, and weight loss (blue), and (b) surface lightness (red) and degree of starch gelatinization at core (black) of bread during baking. Values for input variables are: oven temperature, 200 ºC; heat transfer coefficient, 15 W m^{-2} K^{-1}; bread radius, 0.03 m. Dashed lines account for different end points of baking (Figure 3). Arrows indicate data corresponding to the secondary axis.

Figure 2 – Purlis

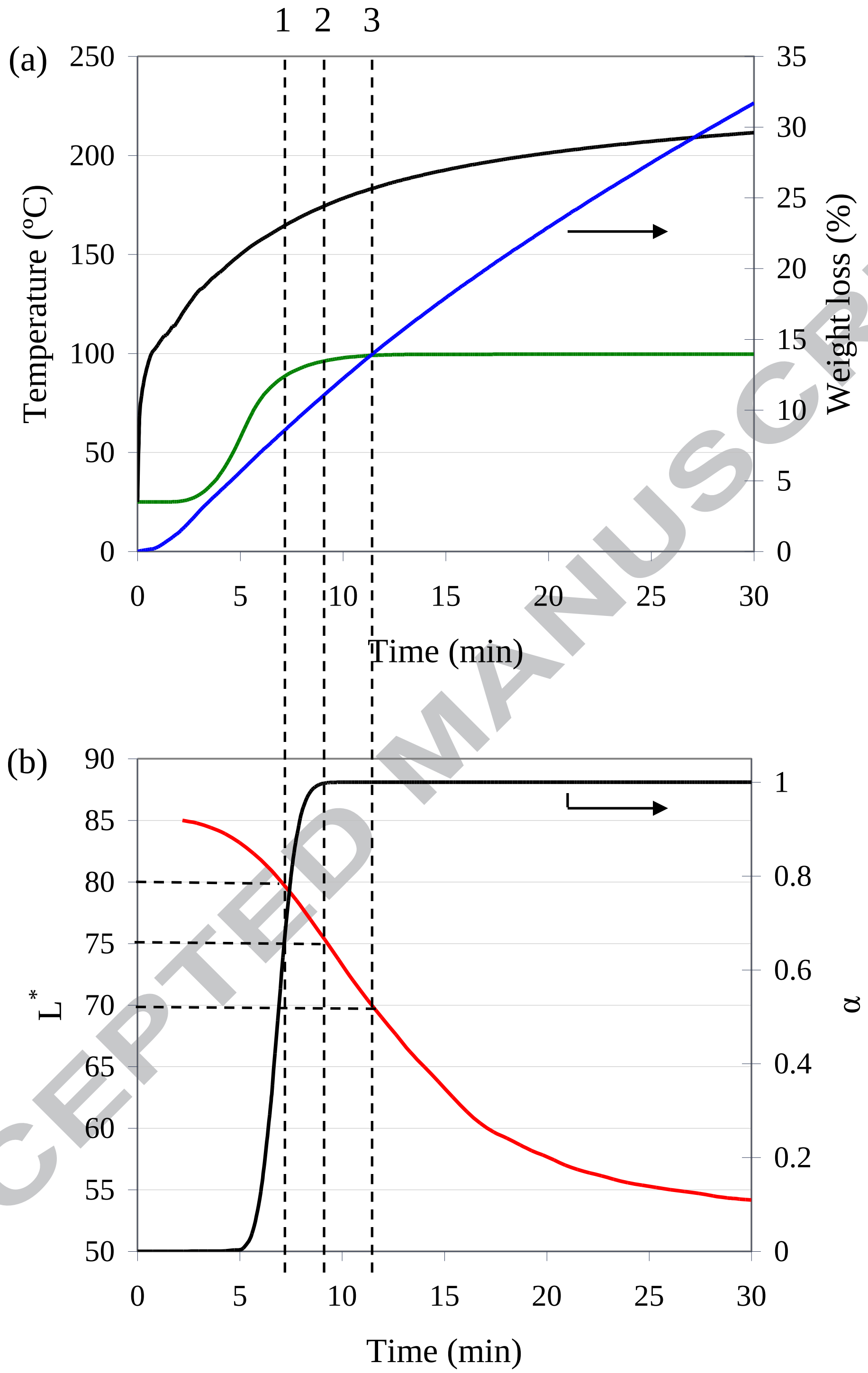

Figure 2. Variation of (a) core (green) and surface (black) temperature, and weight loss (blue), and (b) surface lightness (red) and degree of starch gelatinization at core (black) of bread during baking. Values for input variables are: oven temperature, 240 ºC; heat transfer coefficient, 25 W m^{-2} K^{-1}; bread radius, 0.03 m. Dashed lines account for different end points of baking (Figure 3). Arrows indicate data corresponding to the secondary axis.

Figure 3 – Purlis

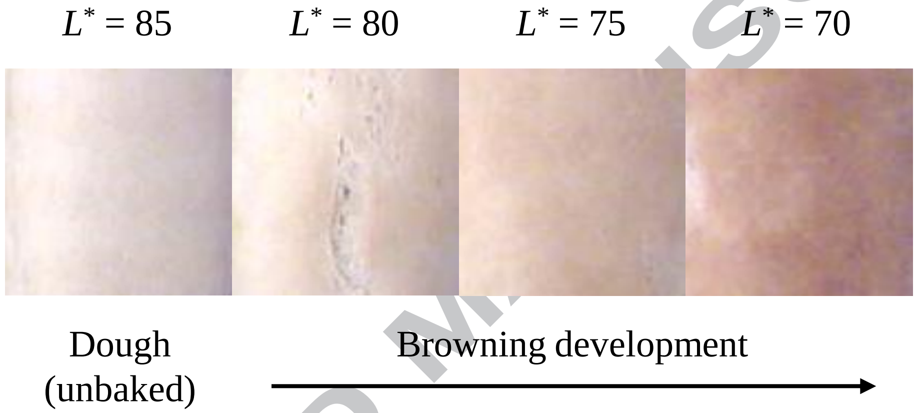

Figure 3. Images of bread samples corresponding to different values of lightness considered to establish the end point of baking. Samples were prepared using a standard recipe for French bread with wheat flour; see section 2.3 (Purlis and Salvadori, 2009c).

Figure 4 – Purlis

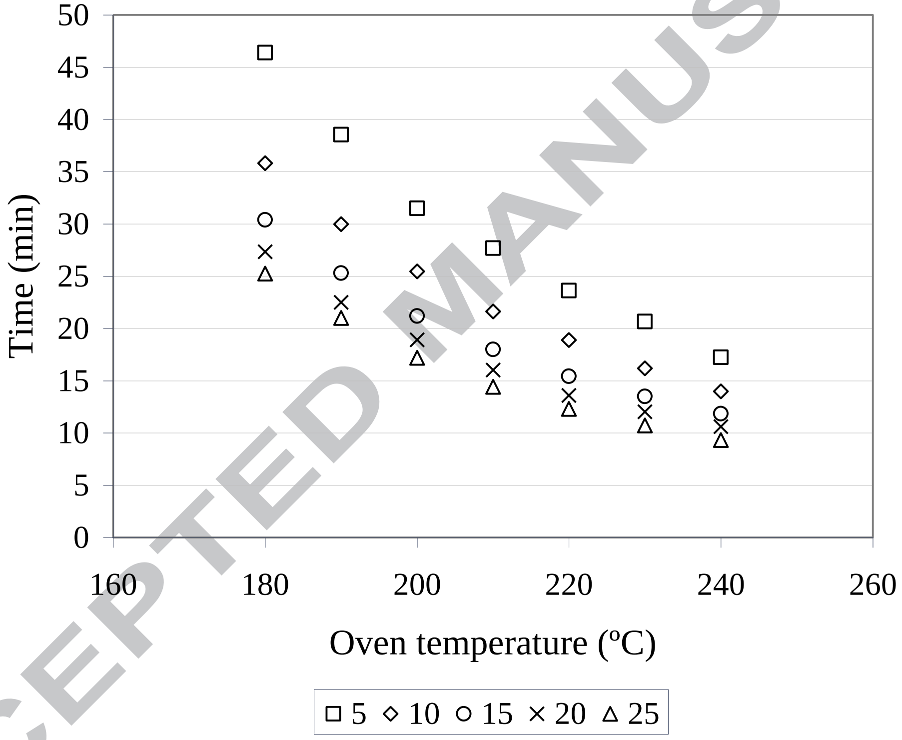

Figure 4. Baking time for final L^* = 75 and R = 0.03 m, as a function of oven temperature, for different values of heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Figure 5 – Purlis

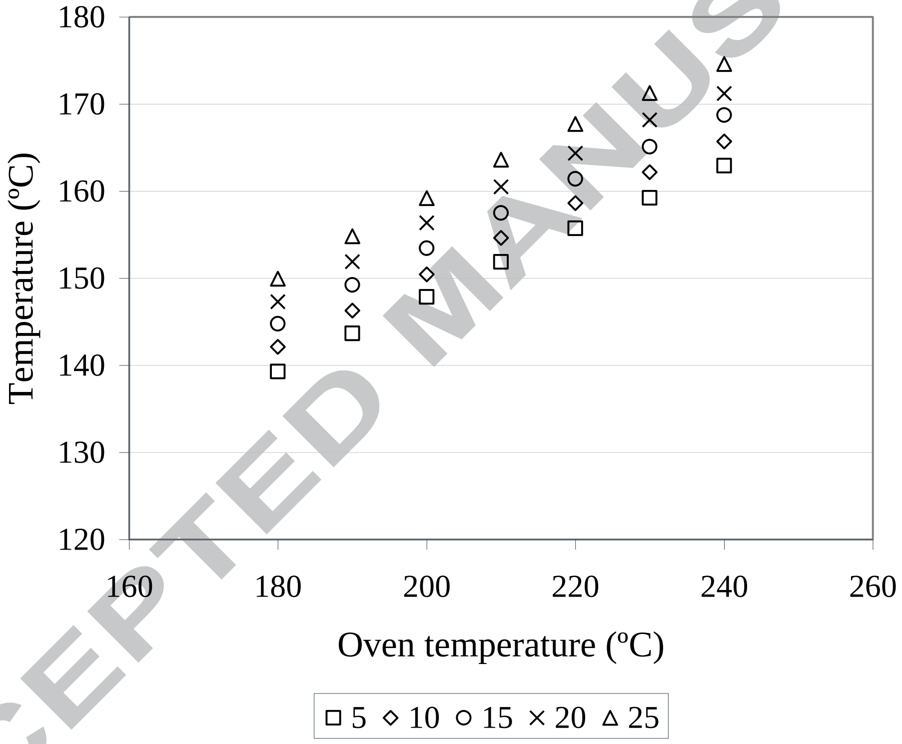

Figure 5. Surface temperature of bread for final L^* = 75 and R = 0.03 m, as a function of oven temperature, for different values of heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Figure 6 – Purlis

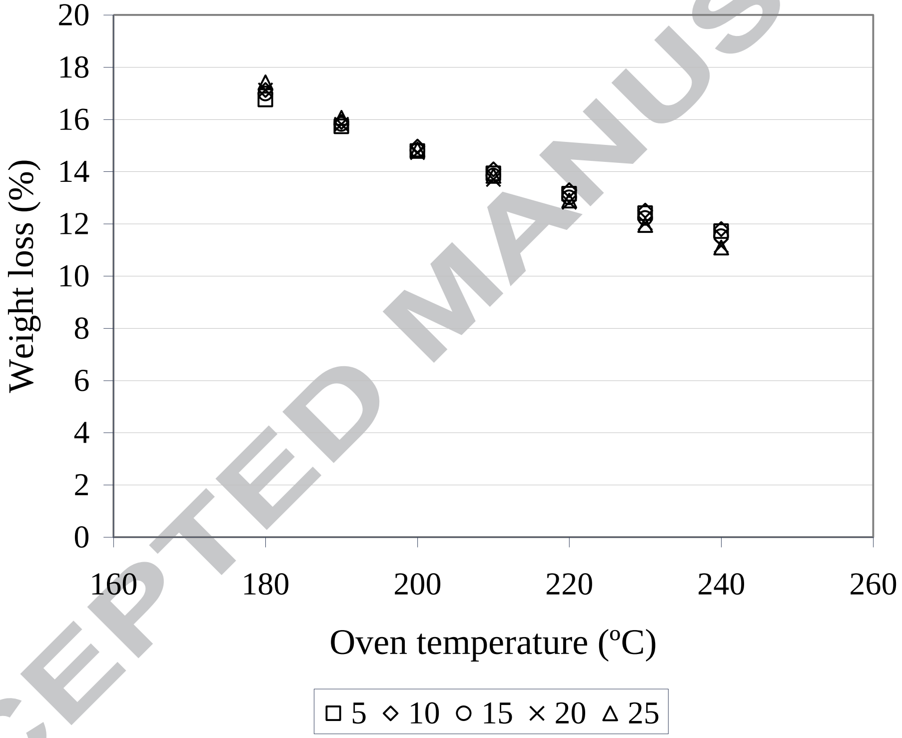

Figure 6. Weight loss of bread for final L^* = 75 and R = 0.03 m, as a function of oven temperature, for different values of heat transfer coefficient (symbols, in W m^{-2} K^{-1}).

Figure 7 – Purlis

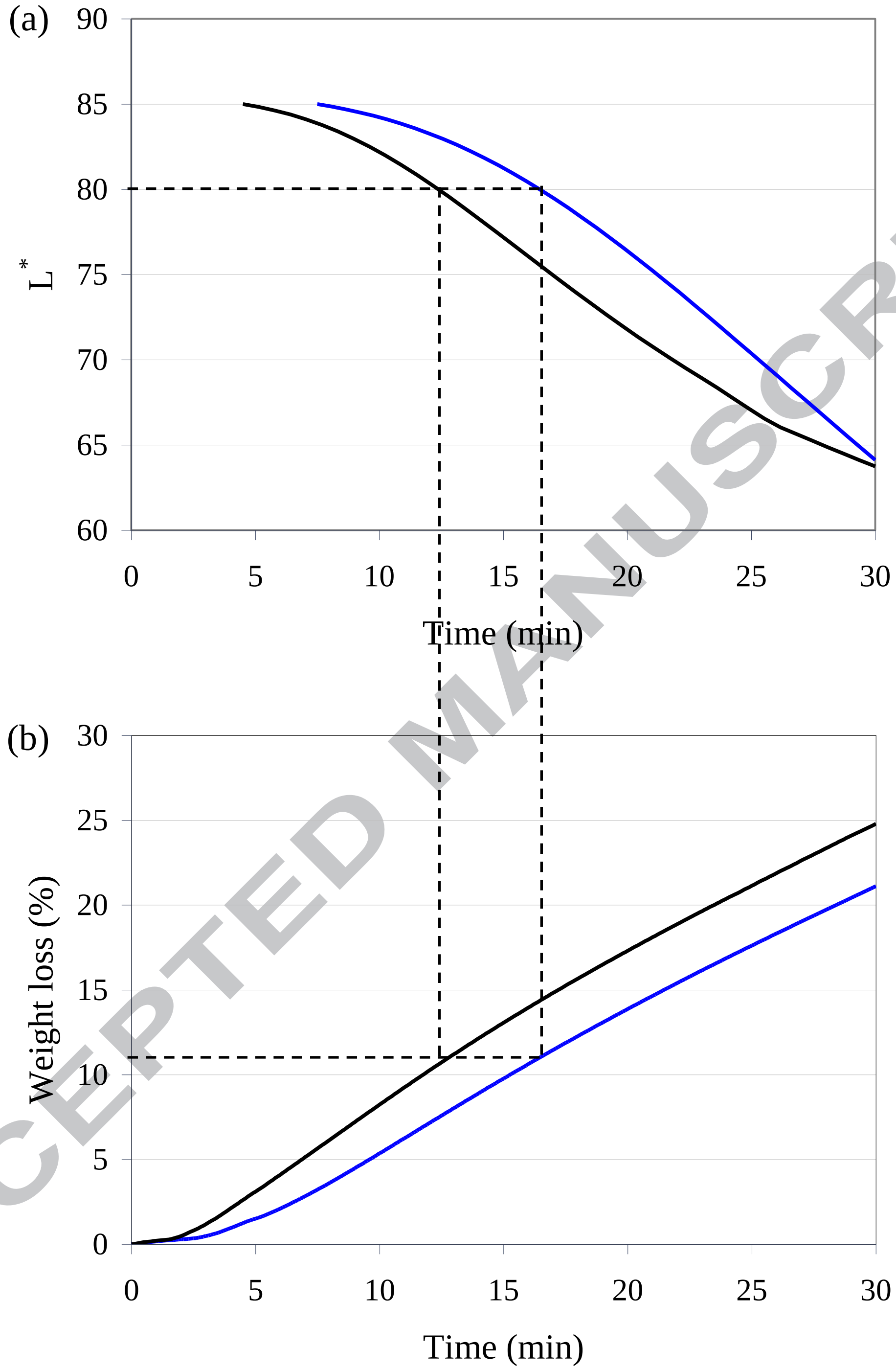

Figure 7. Variation of (a) lightness and (b) weight loss of bread with for *h* = 15 W m^{-2} K^{-1} (blue lines) and *h* = 25 W m^{-2} K^{-1} (black lines). Other values of input variables are: oven temperature, 200 ºC; bread radius, 0.03 m. Dashed lines indicate results for final L^* = 80.

Table 1

Operating conditions (bread radius, heat transfer coefficient, oven temperature) that did not produce the complete gelatinization of bread dough, represented by $\alpha \geq 0.98$.

Final L^*	R (m)	h (W m^{-2} K^{-1})	T_∞ (ºC)	α
80	0.03	20	240	0.94
		25	230	0.92
			240	0.63*
	0.035	15	240	0.86
		20	230	0.64
			240	0.28
		25	220	0.82
			230	0.28
			240	0.11
75	0.035	25	240	0.93

*Case 1 in Figure 2.

Table A.1. Results from bread baking simulation, obtained for bread radius equal to 0.025 m.

		$L^* = 80$					$L^* = 75$					$L^* = 70$				
h	T_∞	t	WL	T_s	T_c	α	t	WL	T_s	T_c	α	t	WL	T_s	T_c	α
5	180	34.86	14.84	134.03	99.66	1.00	43.99	20.33	140.16	99.70	1.00	51.32	24.66	144.22	99.72	1.00
	190	28.83	13.78	137.52	99.65	1.00	36.34	18.99	144.36	99.68	1.00	42.21	22.98	148.80	99.70	1.00
	200	24.34	12.95	141.19	99.63	1.00	30.47	17.88	148.73	99.66	1.00	35.17	21.58	153.52	99.68	1.00
	210	21.02	12.21	144.64	99.61	1.00	26.14	16.85	152.75	99.64	1.00	29.97	20.23	157.81	99.66	1.00
	220	18.07	11.53	147.93	99.60	1.00	22.37	15.85	156.45	99.63	1.00	25.56	18.95	161.73	99.64	1.00
	230	15.84	10.98	151.45	99.58	1.00	19.41	15.01	160.43	99.61	1.00	22.06	17.88	166.01	99.63	1.00
	240	13.80	10.39	154.47	99.57	1.00	16.91	14.23	164.15	99.60	1.00	19.11	16.87	169.92	99.61	1.00
10	180	26.33	15.33	136.61	99.64	1.00	33.96	20.74	142.92	99.67	1.00	40.39	25.07	147.08	99.69	1.00
	190	22.19	14.29	139.99	99.62	1.00	28.48	19.24	146.93	99.65	1.00	33.64	23.13	151.54	99.67	1.00
	200	18.86	13.53	143.85	99.60	1.00	23.93	18.11	151.31	99.63	1.00	28.03	21.63	156.11	99.65	1.00
	210	16.14	12.78	147.45	99.59	1.00	20.31	17.05	155.55	99.62	1.00	23.61	20.26	160.62	99.64	1.00
	220	14.02	12.10	150.87	99.57	1.00	17.49	16.06	159.48	99.60	1.00	20.21	18.99	164.87	99.62	1.00
	230	12.14	11.43	153.84	99.55	1.00	15.11	15.07	162.95	99.59	1.00	17.39	17.77	168.75	99.61	1.00
	240	11.10	10.95	157.42	99.52	1.00	13.58	14.31	166.75	99.57	1.00	15.49	16.77	172.73	99.59	1.00
15	180	21.89	15.07	138.69	99.62	1.00	29.08	20.57	145.35	99.66	1.00	35.52	25.28	149.98	99.68	1.00
	190	18.39	14.17	142.71	99.60	1.00	24.07	19.19	149.93	99.64	1.00	29.10	23.33	154.71	99.66	1.00
	200	15.48	13.36	146.56	99.58	1.00	20.05	17.95	154.32	99.62	1.00	24.01	21.65	159.41	99.64	1.00
	210	13.36	12.56	150.08	99.56	1.00	17.13	16.80	158.44	99.60	1.00	20.33	20.14	163.84	99.62	1.00
	220	11.49	11.84	153.43	99.54	1.00	14.62	15.74	162.43	99.58	1.00	17.22	18.78	168.14	99.61	1.00
	230	10.18	11.23	156.80	99.47	1.00	12.84	14.86	166.26	99.57	1.00	14.99	17.59	172.20	99.59	1.00
	240	8.84	10.56	159.63	99.10	1.00	11.12	13.94	169.70	99.54	1.00	12.94	16.46	176.02	99.57	1.00
20	180	18.89	14.93	141.00	99.61	1.00	25.97	20.77	148.03	99.65	1.00	33.02	26.20	153.13	99.68	1.00
	190	15.74	13.97	145.08	99.59	1.00	21.28	19.24	152.72	99.63	1.00	26.70	23.94	157.98	99.65	1.00
	200	13.47	13.11	148.99	99.57	1.00	17.89	17.89	157.21	99.61	1.00	22.08	22.02	162.77	99.63	1.00
	210	11.56	12.33	152.56	99.54	1.00	15.21	16.68	161.37	99.59	1.00	18.53	20.37	167.45	99.62	1.00
	220	10.07	11.63	156.14	99.46	1.00	13.06	15.62	165.57	99.57	1.00	15.74	18.93	171.81	99.60	1.00
	230	8.65	10.92	159.19	99.06	1.00	11.21	14.63	169.40	99.55	1.00	13.42	17.66	176.14	99.58	1.00
	240	7.69	10.33	162.42	98.27	1.00	9.85	13.78	173.22	99.49	1.00	11.66	16.48	180.06	99.56	1.00
25	180	16.76	14.83	143.15	99.60	1.00	24.09	21.27	150.75	99.64	1.00	32.86	28.19	156.63	99.68	1.00
	190	14.02	13.83	147.28	99.57	1.00	19.74	19.54	155.49	99.62	1.00	26.13	25.33	161.72	99.66	1.00
	200	11.89	12.93	151.19	99.55	1.00	16.40	18.05	160.18	99.60	1.00	21.18	23.02	166.61	99.63	1.00
	210	10.20	12.15	154.93	99.50	1.00	13.80	16.73	164.52	99.58	1.00	17.53	21.08	171.32	99.61	1.00
	220	8.70	11.38	158.50	99.11	1.00	11.70	15.62	168.75	99.56	1.00	14.65	19.40	175.71	99.59	1.00
	230	7.62	10.74	161.82	98.28	1.00	10.14	14.63	172.81	99.52	1.00	12.46	17.93	179.96	99.57	1.00
	240	6.86	10.21	165.32	97.10	1.00	9.14	14.02	177.26	99.40	1.00	11.21	17.16	185.09	99.56	1.00

Table A.2. Results from bread baking simulation, obtained for bread radius equal to 0.03 m.

		L^* = 80					L^* = 75					L^* = 70				
h	T_∞	t	WL	T_s	T_c	α	t	WL	T_s	T_c	α	t	WL	T_s	T_c	α
5	180	36.91	12.29	133.44	99.63	1.00	46.41	16.76	139.30	99.66	1.00	54.12	20.28	143.17	99.68	1.00
	190	30.79	11.45	137.08	99.61	1.00	38.55	15.73	143.66	99.64	1.00	44.68	19.01	147.91	99.66	1.00
	200	25.10	10.73	140.51	99.59	1.00	31.49	14.79	147.86	99.62	1.00	36.43	17.83	152.50	99.64	1.00
	210	22.36	10.11	143.93	99.58	1.00	27.68	13.93	151.87	99.61	1.00	31.71	16.73	156.81	99.63	1.00
	220	19.16	9.57	147.23	99.56	1.00	23.62	13.15	155.72	99.59	1.00	26.94	15.71	160.92	99.61	1.00
	230	16.86	9.07	150.29	99.54	1.00	20.67	12.41	159.23	99.58	1.00	23.45	14.76	164.76	99.59	1.00
	240	14.04	8.61	153.43	99.34	1.00	17.23	11.72	162.91	99.55	1.00	19.57	13.93	168.72	99.57	1.00
10	180	27.92	12.72	136.06	99.61	1.00	35.82	17.12	142.13	99.64	1.00	42.52	20.64	146.10	99.66	1.00
	190	23.49	11.83	139.41	99.59	1.00	29.98	15.91	146.27	99.62	1.00	35.29	19.13	150.77	99.64	1.00
	200	20.21	11.18	143.17	99.57	1.00	25.47	14.96	150.45	99.60	1.00	29.76	17.85	155.11	99.62	1.00
	210	17.29	10.54	146.67	99.54	1.00	21.64	14.09	154.63	99.58	1.00	25.09	16.74	159.62	99.60	1.00
	220	15.29	10.03	150.16	99.48	1.00	18.88	13.29	158.61	99.56	1.00	21.73	15.73	163.90	99.59	1.00
	230	13.06	9.43	152.93	99.15	1.00	16.18	12.51	162.18	99.54	1.00	18.56	14.75	167.87	99.57	1.00
	240	11.41	9.01	156.33	98.34	1.00	13.99	11.80	165.71	99.43	1.00	15.99	13.86	171.63	99.54	1.00
15	180	23.08	12.44	138.13	99.59	1.00	30.39	16.99	144.77	99.62	1.00	36.94	20.79	149.04	99.65	1.00
	190	19.46	11.65	141.97	99.56	1.00	25.30	15.83	149.24	99.60	1.00	30.40	19.21	153.86	99.63	1.00
	200	16.43	11.01	145.75	99.53	1.00	21.19	14.81	153.44	99.58	1.00	25.29	17.84	158.39	99.61	1.00
	210	14.08	10.35	149.18	99.38	1.00	18.00	13.85	157.48	99.56	1.00	21.32	16.59	162.78	99.59	1.00
	220	12.11	9.75	152.47	98.81	1.00	15.42	13.01	161.40	99.53	1.00	18.13	15.49	167.03	99.57	1.00
	230	10.77	9.29	155.87	97.86	1.00	13.49	12.22	165.10	99.38	1.00	15.75	14.50	171.04	99.54	1.00
	240	9.46	8.70	158.55	96.01	1.00	11.84	11.52	168.74	98.88	1.00	13.72	13.61	175.02	99.45	1.00
20	180	20.11	12.36	140.49	99.57	1.00	27.33	17.13	147.31	99.61	1.00	34.46	21.45	152.12	99.64	1.00
	190	16.78	11.49	144.26	99.54	1.00	22.51	15.81	151.90	99.59	1.00	27.88	19.60	157.07	99.62	1.00
	200	14.35	10.80	148.12	99.43	1.00	18.93	14.73	156.36	99.57	1.00	23.12	18.08	161.84	99.60	1.00
	210	12.26	10.06	151.44	98.90	1.00	16.02	13.71	160.50	99.54	1.00	19.39	16.71	166.30	99.58	1.00
	220	10.49	9.57	155.06	97.68	1.00	13.59	12.83	164.35	99.42	1.00	16.36	15.55	170.69	99.55	1.00
	230	9.36	8.93	157.95	95.92	1.00	12.03	12.03	168.19	99.01	1.00	14.33	14.53	174.88	99.51	1.00
	240	8.25	8.24	159.77	92.73	0.94	10.61	11.17	171.22	98.04	1.00	12.58	13.48	178.51	99.26	1.00
25	180	17.83	12.19	142.45	99.56	1.00	25.26	17.42	149.95	99.61	1.00	33.57	22.71	155.37	99.64	1.00
	190	15.29	11.40	146.63	99.51	1.00	20.99	16.06	154.81	99.58	1.00	27.24	20.62	160.53	99.62	1.00
	200	12.53	10.64	150.39	99.07	1.00	17.19	14.85	159.20	99.56	1.00	21.97	18.79	165.41	99.60	1.00
	210	10.60	9.97	154.02	97.88	1.00	14.43	13.84	163.60	99.51	1.00	18.18	17.33	170.20	99.57	1.00
	220	9.15	9.39	157.50	95.75	1.00	12.30	12.89	167.71	99.18	1.00	15.37	16.05	174.76	99.54	1.00
	230	8.06	8.74	160.30	92.58	0.92	10.71	11.96	171.26	98.32	1.00	13.18	14.74	178.64	99.44	1.00
	240	7.15	8.25	163.34	88.15	0.63	9.32	11.10	174.59	96.55	1.00	11.34	13.60	182.36	98.90	1.00

Table A.3. Results from bread baking simulation, obtained for bread radius equal to 0.035 m.

		$L^* = 80$					$L^* = 75$					$L^* = 70$				
h	T_∞	t	WL	T_s	T_c	α	t	WL	T_s	T_c	α	t	WL	T_s	T_c	α
5	180	38.58	10.47	132.96	99.60	1.00	48.37	14.24	138.70	99.63	1.00	56.33	17.24	142.63	99.65	1.00
	190	32.21	9.77	136.58	99.58	1.00	40.20	13.39	143.05	99.61	1.00	46.55	16.16	147.19	99.63	1.00
	200	27.21	9.16	140.07	99.56	1.00	33.80	12.61	147.29	99.60	1.00	38.89	15.18	151.76	99.61	1.00
	210	23.31	8.63	143.35	99.53	1.00	28.79	11.86	151.18	99.58	1.00	32.98	14.24	156.01	99.60	1.00
	220	20.55	8.20	146.69	99.45	1.00	25.13	11.19	154.91	99.56	1.00	28.61	13.39	160.04	99.58	1.00
	230	17.58	7.73	149.51	99.04	1.00	21.53	10.56	158.40	99.53	1.00	24.45	12.58	163.82	99.56	1.00
	240	15.14	7.39	152.86	97.97	1.00	18.40	10.00	162.15	99.31	1.00	20.84	11.89	167.93	99.52	1.00
10	180	29.45	10.81	135.64	99.57	1.00	37.55	14.56	141.59	99.61	1.00	44.43	17.54	145.48	99.63	1.00
	190	24.54	10.13	139.25	99.55	1.00	31.08	13.63	145.95	99.59	1.00	36.51	16.34	150.19	99.61	1.00
	200	20.84	9.46	142.36	99.49	1.00	26.34	12.71	149.85	99.57	1.00	30.71	15.19	154.59	99.59	1.00
	210	18.06	8.95	145.87	99.15	1.00	22.60	11.97	153.80	99.54	1.00	26.19	14.21	158.75	99.57	1.00
	220	15.85	8.52	149.38	98.39	1.00	19.57	11.30	157.82	99.45	1.00	22.50	13.37	163.08	99.55	1.00
	230	13.59	7.99	152.12	96.68	1.00	16.84	10.64	161.38	99.02	1.00	19.28	12.53	166.91	99.47	1.00
	240	12.08	7.58	155.02	94.45	1.00	14.83	10.00	164.61	98.13	1.00	16.92	11.77	170.67	99.12	1.00
15	180	24.24	10.65	137.89	99.55	1.00	31.70	14.48	144.21	99.59	1.00	38.44	17.66	148.34	99.62	1.00
	190	20.21	9.90	141.47	99.45	1.00	26.20	13.46	148.65	99.57	1.00	31.42	16.32	153.18	99.60	1.00
	200	17.18	9.28	144.93	98.98	1.00	22.11	12.56	152.81	99.54	1.00	26.28	15.14	157.73	99.57	1.00
	210	15.17	8.77	148.39	98.09	1.00	19.24	11.75	156.70	99.44	1.00	22.62	14.08	162.08	99.55	1.00
	220	12.68	8.21	151.34	95.65	1.00	16.16	11.01	160.42	98.87	1.00	18.96	13.14	166.21	99.46	1.00
	230	11.34	7.83	154.72	93.00	0.98	14.21	10.36	164.11	97.81	1.00	16.56	12.32	170.18	99.09	1.00
	240	10.32	7.28	156.93	89.82	0.86	12.87	9.72	167.49	96.38	1.00	14.79	11.48	173.81	98.33	1.00
20	180	20.85	10.43	139.96	99.50	1.00	28.15	14.51	146.77	99.58	1.00	35.31	18.10	151.31	99.61	1.00
	190	17.45	9.73	143.76	99.08	1.00	23.25	13.43	151.30	99.55	1.00	28.74	16.60	156.25	99.59	1.00
	200	14.63	9.13	147.33	97.85	1.00	19.35	12.47	155.52	99.48	1.00	23.69	15.31	161.02	99.56	1.00
	210	12.74	8.51	150.52	95.87	1.00	16.67	11.61	159.63	99.06	1.00	20.12	14.16	165.50	99.52	1.00
	220	11.22	8.04	153.90	92.92	0.98	14.55	10.94	163.79	98.14	1.00	17.32	13.20	169.82	99.30	1.00
	230	9.50	7.57	156.55	86.64	0.64	12.36	10.21	167.16	95.89	1.00	14.78	12.34	173.89	98.48	1.00
	240	8.59	7.08	159.12	80.75	0.28	11.05	9.55	170.53	93.17	0.98	13.03	11.46	177.54	97.03	1.00
25	180	18.54	10.37	141.99	99.32	1.00	25.98	14.77	149.39	99.57	1.00	34.24	19.10	154.53	99.61	1.00
	190	15.63	9.63	145.98	98.47	1.00	21.50	13.59	154.06	99.54	1.00	27.73	17.32	159.59	99.59	1.00
	200	13.34	8.97	149.49	96.74	1.00	18.08	12.54	158.49	99.36	1.00	22.89	15.86	164.57	99.56	1.00
	210	11.11	8.45	153.16	92.85	0.98	15.04	11.72	162.82	98.53	1.00	18.83	14.61	169.22	99.50	1.00
	220	10.02	7.99	156.65	89.36	0.82	13.18	10.85	166.67	97.09	1.00	16.11	13.32	173.26	99.08	1.00
	230	8.55	7.46	159.55	80.83	0.28	11.38	10.26	170.70	94.27	0.99	14.08	12.76	178.36	98.19	1.00
	240	7.91	7.06	162.02	74.89	0.11	10.39	9.62	174.24	91.46	0.93	12.57	11.80	182.13	96.65	1.00