Oxford Development Studies
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713439972

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Maria Emma Santos[ab]
[a] Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET), Universidad Nacional del Sur, Argentina [b] Oxford Poverty and Human Development Initiative (OPHI), Departamento de Economía, Universidad Nacional del Sur, Bahía Blanca, Argentina

Online publication date: 17 March 2011

To cite this Article Santos, Maria Emma(2011) 'Human Capital and the Quality of Education in a Poverty Trap Model', Oxford Development Studies, 39: 1, 25 — 47
To link to this Article: DOI: 10.1080/13600818.2010.551003
URL: http://dx.doi.org/10.1080/13600818.2010.551003

Oxford Development Studies,
Vol. 39, No. 1, March 2011

Human Capital and the Quality of Education in a Poverty Trap Model

MARIA EMMA SANTOS

ABSTRACT *This paper presents a model of a poverty trap that is caused by an unequal initial income and human capital distribution and differences in the quality of education between children from more and less advantaged social sectors. Under certain conditions, the economy converges to a situation with three stable and simultaneous equilibria, two of which constitute poverty traps, lowering the economy's current and steady-state aggregate output level as well as its growth rate. The model suggests that a policy oriented towards equalizing the quality of education would, in the long run, have the potential to reduce initial inequalities.*

JEL Classification: D31, O12, I21

1. Introduction

Being educated is broadly recognized as an end of human development in itself as well as a human right. It is also recognized as a tool with high instrumental value for other development outcomes, especially for moving out of poverty, if not within an individual's own lifespan, at least intergenerationally. Much international and national effort has so far been focused on making education available to all human beings. In fact, the second Millennium Development Goal is precisely to "Achieve Universal Primary Education".

Although such efforts have been of tremendous importance for nations' progress towards universal education, in the past 15 years there has been increasing concern regarding the quality of education. In particular, there is evidence that in many countries, both developing and developed, variability in the quality of education is highly associated with the socio-economic intake of schools. In other words, children from low socio-economic backgrounds access poor-quality education whereas children from favourable socio-economic backgrounds access high-quality education. Under these circumstances, education cannot work effectively as a tool to break down the intergenerational transmission of poverty because the cognitive skills acquired will not suffice for obtaining an income that makes it possible to leave poverty.

Maria Emma Santos, Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET), Universidad Nacional del Sur, Argentina, and Oxford Poverty and Human Development Initiative (OPHI), Departamento de Economía, Universidad Nacional del Sur, 12 de Octubre y San Juan, 7mo Piso, 8000 Bahía Blanca, Argentina. Email: msantos@uns.edu.ar; maria.santos@qeh.ox.ac.uk

I am grateful to two anonymous referees and to Emma Samman and Sabina Alkire for very useful comments. Any errors are my responsibility.

ISSN 1360-0818 print/ISSN 1469-9966 online/11/010025-23

DOI: 10.1080/13600818.2010.551003

The contribution of this paper is to present a model that formalizes this idea, namely, that segmentation in educational quality reinforces inequalities and can lead to poverty traps. While this has been argued conceptually, to the author's knowledge, it has not yet been formalized. The model is based on Berti Ceroni (2001), with the important difference that this approach incorporates the quality of education.

The policy implications derived from the model are related to Roemer's (1998) theory of equality of opportunity, according to which unequal results cannot be justified if they are due to differences in circumstances rather than differences in efforts. This is also in line with the 2006 *World Development Report*: "the equalizing promise of education can be realized only if children from different backgrounds have equal opportunities to benefit from quality education" (World Bank, 2005, p. 140).

Section 2 briefly reviews the literature on poverty traps. Section 3 describes the model. Section 4 derives the policy implications and their motivation and provides some specific alternatives based on empirical evidence. Finally, Section 5 concludes. More technical issues are presented in two appendices.

2. Poverty Trap Models in the Literature

"A poverty trap is any reinforcing mechanism that causes poverty to persist" (Azariadis & Stachurski, 2005, p. 33). From the beginning of development theory, this concept has been useful in explaining the observed differences in per capita income between countries. Poverty trap models are associated with some type of departure from the neoclassical assumptions, such as non-convexities (scale economies, positive externalities, and other increasing returns), the existence of imperfect competition, some market failures (especially capital markets), and acknowledgement of the importance of institutional frameworks. These allow the emergence of multiple equilibria, at high and low income levels, so that if an economy starts below a certain threshold it remains trapped in a "bad equilibrium".

The concept of poverty traps was implicit in the early development theories of Rosenstein-Rodan (1943), Nurske (1952), Leibenstein (1957) and Myrdal (1957). These ideas were abandoned for some decades, possibly because of the lack of formalization (Ros, 2001). However, given the difficulties of neoclassical growth theory, in particular of Solow's (1956) model, in explaining the big differences in per capita income between countries, they re-emerged in a more formalized framework. In this way, Murphy *et al.* (1989) and Matsuyama (1995) proposed models in which the complementarities in investment decisions lead to the existence of a *good* and a *bad* equilibrium. An analogous reasoning was developed for investment in human capital, as proposed by Kremer (1993), Redding (1996) and Acemoglu (1997).[1]

The concept of poverty traps helps explain cross-country difference, but can also be applied to explain situations of within-country *economic duality*, in which a fraction of the population reaches a good equilibrium, with a high income level, while another fraction remains trapped in a bad equilibrium, with a low income level. This situation is described as fractal poverty traps by Easterly (2001) and Barrett & Swallow (2003).

The papers of Galor & Zeira (1993), Galor & Tsiddon (1997) and Berti Ceroni (2001) present models of this type of poverty trap, and in all of them the trap operates on the human capital accumulation function. In Galor & Zeira's (1993) model, the poverty trap is driven

by credit market imperfections, with a borrower's interest rate increasing with lenders' monitoring costs, which in turn increase in the amount lent (because the borrower has a higher incentive to default). In this way, the initial distribution of wealth determines each dynasty's human capital accumulation path and steady state. The economy becomes segmented into two groups: the skilled and wealthy workers, for whom investing in education is an optimal decision; and the unskilled and unwealthy workers, who derive a higher utility from not investing in education. Matsuyama (2000) presents a similar model, with the difference that the threshold that divides the rich and the poor is endogenously determined.

Rather than focusing on credit market imperfections as the source of a trap, Galor & Tsiddon (1997) proposed a model in which multiple equilibria emerge as a consequence of external effects and are a (temporary) stage of development. Specifically, the model presents two types of positive externality: a home environment externality, according to which an individual's level of human capital is an increasing function of the parental level of human capital; and a global technological externality, according to which technological progress is positively related to the average level of human capital in society. In the first phase of development, the home environment externality is the dominating factor, creating strong inequalities in human capital distribution. However, as investment in human capital of the highly educated segments of society increases, the global technological externality starts to dominate, leading to income convergence. The model suggests that equality-enhancing policies implemented prematurely may lead to a low output trap.

Berti Ceroni (2001) criticizes both models: Galor & Zeira's (1993) on the credit market as the source of the trap, arguing that credit market imperfections are more likely to take the extreme form of self-financing constraints, as expected human capital is not generally accepted as collateral; and Galor & Tsiddon's (1997) on the non-convexities assumed in individual human capital accumulation, arguing that empirical evidence on that is inconclusive. The author offers a model in which a poverty trap is generated by non-homothetic preferences so that the poor require higher returns to education than the rich in order to invest in education.

In two recent linked empirical studies, Knight *et al.* (2009, 2010) argue the existence of a poverty trap involving education and income in rural China. In the first paper they found support for poverty having an adverse effect on both the quantity and quality of education. They also found evidence of poor quality contributing to low quantity of education. In the second paper, they found that both low quantity and low quality of education reduce the income benefits of education through different channels, closing the vicious circle.

The model developed in this paper is based on Berti Ceroni (2001), and it is conceptually connected to the evidence found in Knight *et al.* (2009, 2010). The main difference is that here the quality of education is incorporated as a key element reinforcing the segmentation of the economy into social sectors or networks determined by education.[2] Instead of reaching a situation with two stable equilibria (one good and one bad), the economy converges to a situation with three stable equilibria, two bad and one good. Similarities and differences with Berti Ceroni (2001) are pointed out in the presentation of the model in Section 3.

3. The Model

The model is based on Berti Ceroni (2001) in that it is a model of overlapping generations. Each family is composed of two individuals, parent and child. Each individual is born with the same ability, lives two periods and is endowed with one unit of time in each period. Individuals can make decisions only in the second period of their lives. When young, individuals receive an education if their parents so decide, in which case they assign their unit of time to school. Children who do not go to school acquire a fixed level of human capital as a consequence of the passage of time. In the second period of their lives all individuals offer their time unit on the labour market, earn an income proportional to their level of human capital, and decide how to allocate it between consumption and spending on their children's education.

Following Berti Ceroni (2001), the utility function of parent i in time t depends on consumption in period t and on the stock of human capital of the ith child in period $t + 1$. It takes the form:

$$u^i(c_t^i, h_{t+1}^i) = \ln(c_t^i) + \delta h_{t+1}^i, \tag{1}$$

where δ is a parameter that measures the altruistic motive, with $0 \leq \delta \leq 1$. The human capital production function presents the first departure from Berti Ceroni's model. It is assumed that there is segmentation in the economy between the families of more educated parents and families of less educated parents. Such segmentation is usually observed, especially in developing countries. It can be seen in terms of social circles, networks, class or even neighbourhoods. Superscript j denotes the social circle.

$$h_{t+1}^{ij} = \begin{cases} \mu^j & e_t^{ij} \leq b^j \\ \ln[q^j(e_t^{ij} - b^j) + v^j] & e_t^{ij} > b^j \end{cases} \tag{2}$$

$$\text{with } \mu^j = \ln(v^j).$$

h_{t+1}^{ij} is the level of human capital acquired by the child of parent i, from social circle j. μ^j is the level of human capital achieved by the child if he or she does not receive any formal education. This level varies with the social sector to which the parent belongs. It is assumed that given two parents, one with a higher educational level than the other, $h_t^{i1} < h_t^{i2}$, if none of the parents decides to provide formal education to their children, the child of the better educated parent (h_t^{i2}) will enjoy a level of human capital equal to or greater than the level of human capital of the child of the less educated parent (h_t^{i1}), that is: $\mu^1 \leq \mu^2$. Parameter μ^j depends on v^j, which represents the knowledge and basic abilities provided at home and it is assumed that for $h_t^{i1} < h_t^{i2}$, $v^1 \leq v^2$.

Education here is considered to be public; this is the second difference from Berti Ceroni's model. However, there exists a private cost of education e_t^{ij}, given by the cost of complementary goods, such as books and transportation to school, and by the opportunity cost of non-working. A priori, this cost is assumed to be independent of the parent's level of education. However, when solving the maximization problem it will be seen that the optimal level of private spending in education will be different for different social circles. Apart from parameters μ^j and v^j, a third key parameter is b^j, which also depends on the social circle to which the child belongs. It is the education spending threshold level that is necessary so that the child's human capital starts to increase. In other words, it is the

spending level at which spending in education starts to be *effective*. As before, for $h_t^{i1} < h_t^{i2}$, $b^1 \leq b^2$. A simple example might help to clarify this assumption. It is very common that children whose parents are highly educated start the first grade of primary school already knowing how to read and write and perform some simple mathematical operations. This, on the other hand, is very rare among children whose parents have a low level of education. Therefore, the minimum level of education (and so the minimum educational spending) needed by the children of the better educated parents in order to exhibit an improvement in their skills is higher than the minimum required by children of the less educated parents.

Finally, q^j represents the quality of education received by the child belonging to social sector j, which is of particular interest in the model. The quality of education that the child receives is not a decision variable for the parents. It is determined by the allocation of public resources to each school and the characteristics of the school system. Here it is taken as exogenously given.

In particular, it is assumed that there is segmentation between social circles, which is expressed in the assumption that schools with students coming from better educated families have a better quality of education than schools with students coming from less educated families, that is, for $h_t^{i1} < h_t^{i2}$, $q^1 \leq q^2$. Such segmentation is closely related to the concept of *horizontal inequalities*, that is, inequalities between groups encompassing common cultural identities, which can be driven by ethnicity, religion, race, region or even class (Stewart, 2001).

The assumption of educational quality segmentation is key, and it is supported by recent empirical evidence from international evaluations. A sign of quality segmentation is when between-schools variance accounts for a high proportion of overall variance in students' performance. As a reference, in the OECD countries evaluated in the Programme for International Students Assessment (PISA) it was found that—on average—the between-school variance accounts for 36% of overall variance in students' performance in reading (OECD, 2005, on PISA 2000), 33.6% in maths (OECD, 2004, on PISA 2003) and 33% in science (OECD, 2007, on PISA 2006). While many of the OECD countries as well as the non-OECD countries participating in the study exhibit much lower between-school variance than the OECD average, many others exhibit much higher variance. In particular, Italy, the Czech Republic, Hungary, Austria, Germany, Belgium, Bulgaria, Chile, Argentina, Netherlands, Japan, Brazil, Mexico, Poland, Peru, Greece and Slovenia are some of the countries that have shown a between-school variance that accounts for 50% or more of overall variance in performance of at least one of the three evaluated skills (reading, maths or science). Most frequently, a high proportion of the between-school variance is associated with the socio-economic background of students, indicating significant inequalities in the quality of education by social group, as suggested by the model.[3]

Applying an innovative methodology on data from PISA, Barros *et al.* (2008) found that the median Latin American country is more opportunity-unequal than the typical industrial nation. In the first case, the child's circumstances, notably mother's education and father's occupation, as well as location in some cases, accounted for about 20% of total inequality, whereas in the second the same circumstances accounted for about 15% of total inequality.

The empirical evidence from Knight *et al.* (2009) also supports the model's assumption. They found that education quality (as measured by self-rated school performance, time

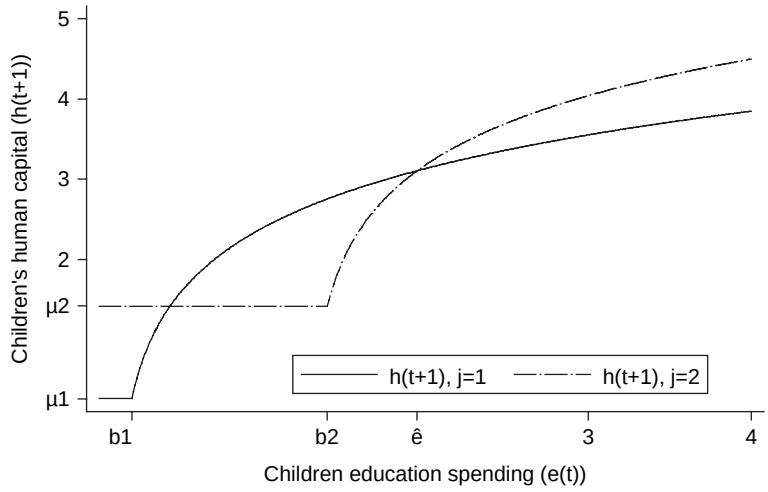

Figure 1. Human capital accumulation function.

spent learning and household education expenditure per child) is positively associated with households' socio-economic status.

Figure 1 presents the human capital production function for a possible set of values of the parameters (v^j, b^j, q^j) and for two circles or social sectors $j = 1, 2$. Sector $j = 2$ parents' human capital level is higher than sector $j = 1$ parents' human capital level $(h_t^{i1} < h_t^{i2})$.[4] It can be noted that for both social sectors, when education spending is lower than the minimum required for it to be effective (b^1, b^2, respectively), children remain at the fixed human capital level μ^1 and μ^2, respectively, which is higher in the case of children of parents with higher education. As soon as education spending exceeds the threshold, children's human capital level starts to increase, and it does so at a decreasing rate. This diminishing returns behaviour is a natural way of thinking of human capital accumulation, reflecting the limits imposed by the capacity of any human being, which restricts the conversion of increasing education spending into ever-increasing cognitive skills.

The figure clearly indicates the effect of segmentation and initial disadvantages. Children of the less educated parents have a lower education level at home, which results in a lower education spending threshold. At the same time, they benefit from lower external effects generated by the interaction with other people within their social circle, and they attend schools with lower-quality education.

As in Berti Ceroni's model, the economy produces a final good only through a linear technology that uses human capital as the only production factor:

$$Y_t = H_t = \int_I h_t^i g_t(h_t^i) dh_t^i, \tag{3}$$

where H_t is the aggregate stock of human capital in period t and $g_t(h_t^i)$ is the density function that characterizes the distribution of human capital among parents in period t, so that $g_t(h_t^i) \geq 0$ and $\int g_t(h_t^i) dh_t^i = 1$. The distribution of human capital in the initial generation of parents is exogenously given: $g_0(h_0^i)$, with $h_0^i \in (\alpha, \beta)$ and $\mu_1 = \alpha < \beta$.

3.1 The Solution to the Microeconomic Problem

The individual maximization programme that parent i has to solve in time t is given by:

$$\begin{aligned} &\max_{e_t^i} u^i(c_t^{ij}, h_{t+1}^{ij}) = \ln(c_t^{ij}) + \delta h_{t+1}^{ij} \\ &\text{s.t} \\ &\qquad c_t^{ij} = h_t^{ij} - e_t^{ij} \\ &h_{t+1}^{ij} = \begin{cases} \mu^j & e_t^{ij} \le b^j \\ \ln\left[q^j(e_t^{ij} - b^j) + v^j\right] & e_t^{ij} > b^j \end{cases} \\ &\qquad (e_t^{ij}, c_t^{ij}) \ge (0,0). \end{aligned} \tag{4}$$

Following Behrman & Birdsall (1983), it is assumed that the quality of education is determined by public resource allocation to schooling out of general overall revenues so there is no direct relation between the quality in a particular area and the tax burden of a particular household located in that area. Therefore, the budget constraint does not consider taxes.

As in Berti Ceroni's model, the utility function is non-homothetic. The marginal rate of substitution between the parent's consumption in period t and the child's human capital for a given ratio between the two is decreasing in the parent's human capital stock, so that the poor require relatively higher returns from education to start investing.

Replacing the budget constraint and the human capital production function in the utility function and maximizing with respect to e_t^{ij}, the expression of optimal spending in education is obtained:

$$e_t^{*i}(h_t^{ij}) = \begin{cases} b^j & h_t^{ij} \le \bar{h}^j \\ \frac{\delta h_t^{ij} + b^j}{(1+\delta)} - \frac{v^j}{q^j(1+\delta)} & h_t^{ij} > \bar{h}^j \end{cases}, \tag{5}$$

$$\text{where } \bar{h}^j = b^j + \frac{v^j}{\delta q^j}. \tag{6}$$

Note that for human capital levels equal to or lower than the threshold $\bar{h}^j$, education spending is constant at the minimum required level of spending (b^j). For human capital levels above the threshold $\bar{h}_j$, the proportion of income assigned to education increases with the educational level of the parent. The spending functions of the different social circles differ in two main points. First, they differ in the threshold level ($\bar{h}^j$) at which spending in education starts to increase in the parents' human capital level. As equation (6) shows, this threshold is increasing in b^j and in v^j, and decreasing in the quality of education q^j and parents' altruism δ. It has already been assumed that parameters b^j, v^j and q^j are higher for more educated social circles. It is now also assumed that the threshold $\bar{h}^j$ is also higher for higher social circles. Second, for each human capital value h, the level of spending will be higher for higher values of b^j and q^j, this last effect suggesting a positive feedback effect by which higher education quality fosters higher investment by parents. In other words, it is assumed that the parameters' values are such that the spending function of the more advantaged social circle runs above that of the less advantaged circle. However, note that the rate at which one additional unit of parents' human capital

translates into higher spending is the same for both social circles, as both types of parent have the same degree of altruism (δ). That is, in its increasing part, the slope of each spending function is the same.

Replacing equation (5) in equation (2), the transition equation that describes the evolution of dynasty i's human capital is obtained:

$$h_{t+1}^{ij} = \phi^j(h_t^{ij}) = \begin{cases} \mu^j & h_t^{ij} \leq \bar{h}^j \\ \ln\left[\frac{q^j\delta(h_t^{ij}-b^j)+\delta v^j}{(1+\delta)}\right] & h_t^{ij} > \bar{h}^j \end{cases} . \tag{7}$$

Under the assumptions mentioned regarding the parameters, the dynamics of human capital accumulation of each dynasty is independent of the aggregate dynamic, but is dependent on the social circle j to which the dynasty belongs. This transition function $\phi^j(h_t^{ij})$ has a positive slope and is concave for $h_t^{ij} > \bar{h}_j$.

In what follows, it is assumed that there are only two sectors or social circles with initial human capital levels clearly differentiated: $j = 1,2$, with $h_t^{i1} < h_t^{i2}$. Each social sector has its transition function.

The emergence of poverty traps requires the individual transition function to exhibit multiple steady states. Appendix 1 specifies the conditions under which three stable equilibria emerge. Some of these conditions have already been stated. In the first place, parameters v, b and q and the human capital level of children who do not receive formal education (μ) are required to be non-negative (conditions C1 and C2); the parameters v, b and q for the social sector of higher human capital are assumed to be higher than those for the social sector of lower human capital (condition C3); the human capital threshold at which the spending function starts to increase in the parents' human capital is assumed to be higher for dynasties with a higher initial education level ($\bar{h}^1 < \bar{h}^2$, condition C4); the human capital threshold at which the education spending function starts to increase in the parent's human capital level is required to be higher than the child's human capital level if he or she does not receive education ($\bar{h}^j > \ln(v^j)$, condition C5); and, finally, it is assumed that the derivative of each transition function j in its concave part (when $h_t^{ij} > \bar{h}^j$) has a slope greater than unity when it is evaluated at the point where $h_t^{ij} = h_{t+1}^{ij}$ (condition C6). This point is called h_u^j. Under such conditions, Figure 2 depicts how the functions of the two social sectors can produce multiple equilibria for each j-transition function.[5] Each individual transition function $\phi^1(h_t^{i1})$ (labelled in the figure as Transition Fc. $j = 1$) and $\phi^2(h_t^{i2})$ (labelled in the figure as Transition Fc. $j = 2$) presents three steady states, at the human capital levels μ_1, h_u^1, h_L^*, μ_2, h_u^2, h_H^* correspondingly. Note that $\mu_1, h_L^*, \mu_2, h_H^*$ are stable equilibria, whereas h_u^1 and h_u^2 are unstable ones.

3.2 Equilibria at the Aggregate Level: The Poverty Traps

While each of the two j-transition functions presents three equilibria, the equilibria that prevail at the aggregate level depend on the interval of human capital levels in which each human capital accumulation function h_{t+1}^{i1} and h_{t+1}^{i2} operates. In other words, the number and type of equilibria that are determined in the economy depend on the human capital level that distinguishes the more and the less educated social circles. This threshold will be called $\hat{h}$. As an example, $\hat{h}$ could correspond to tertiary (university or other post-secondary) education.

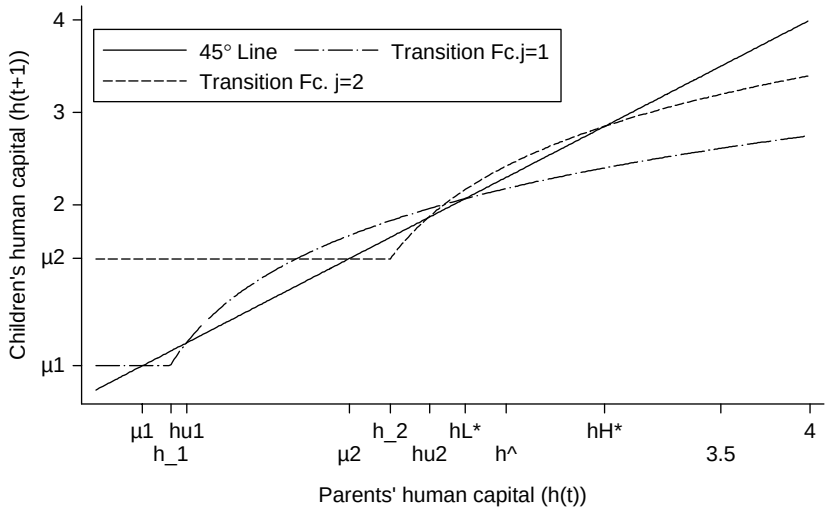

Figure 2. Transition functions.

To define this level, two additional conditions will be defined, which allow the configuration of equilibria of interest in this paper. In the first place, it is assumed that:

$$(\text{C7})\ \mu^2 < h_L^*.$$

This condition requires the human capital level achieved by the children of the more educated parents when they do not receive formal education to be lower than the human capital level achieved by the children of the less educated parents when they receive formal education. If this condition is not satisfied, the children of the rich, educated parents who receive no formal education will end up having a steadystate human capital (μ^2) higher than the human capital of the children of the uneducated and poor parents who do receive formal education (h_L^*), which is counter-intuitive. Formally, this condition guarantees the intersection of the curves of human capital accumulation of the two social sectors, $\phi^1(h_t^{i1})$ and $\phi^2(h_t^{i2})$.

Second, it is required that:

$$(\text{C8})\ h_u^2 < h_L^* < \hat{h}.$$

The first part of this condition requires the human capital level that works as a threshold in sector $j = 2$, below which dynasties end up being non-educated, to be lower than the maximum human capital level that dynasties from sector $j = 1$ can achieve. Although this sounds arbitrary, it may be argued that the opposite case would be extreme in the assumptions regarding social segmentation. The second part of this condition requires the human capital level that distinguishes the two sectors ($\hat{h}$) to be higher than the human capital level to which the initially less educated dynasties that invest in education converge (h_L^*). Again, although it seems an arbitrary condition, together with the previous ones it guarantees an intuitive result: that at least some of the poor decide to invest in education.

Considering all the conditions mentioned, the aggregate transition function is obtained. For parents' human capital levels lower than the threshold ($h_t < \hat{h}$), the human capital accumulation function that prevails is the one corresponding to $j = 1$; for parents' human

capital levels above the threshold ($h_t \geq \hat{h}$), the prevailing human capital accumulation is the one that corresponds to $j = 2$. The expression for this aggregate function is given by:

$$\phi(h_t^i) = \begin{cases} \ln(v^1) & h_t^i \leq \bar{h}^1 \\ \ln\left[\frac{q^1\delta(h_t^i - b^1) + \delta v^1}{1+\delta}\right] & \bar{h}^1 < h_t^i < \hat{h} \\ \ln\left[\frac{q^2\delta(h_t^i - b^2) + \delta v^2}{1+\delta}\right] & h_t^i \geq \hat{h} \end{cases} . \tag{8}$$

Figure 3 presents the aggregate transition function under the stated conditions. The figure shows that the aggregate transition function has a discontinuity at the threshold level $\hat{h}$, and that three stable equilibria are defined at levels μ^1, h_L^*, h_H^* (and an unstable one at h_u^1).[6] Then, in the long run, dynasties are concentrated in three groups. The *very poor* are dynasties with an initial human capital level below h_u^1. The parents of these dynasties may initially invest in educating their children (with a spending higher than the required threshold $e_t^{i1} > b^1$), but eventually they will stop doing so because the human capital stock decreases from one generation to the next, and they will converge to the steadystate level of human capital given by μ^1, remaining forever below the level h_u^1, that is, poor and uneducated. This result is analogous to the first equilibrium in Berti Ceroni's model.

The second equilibrium h_L^* corresponds to the *poor*. Dynasties that converge to this equilibrium are those whose initial human capital is above h_u^1 but below $\hat{h}$.[7] The parents of these dynasties invest enough in their children's education ($e_t^{i1} > b^1$). However, given that they move in a low-educated social circle and receive low-quality education, the human capital level to which they eventually converge is considerably lower than the one corresponding to the third possible equilibrium h_H^*. This third human capital level corresponds to the *non-poor*, and is reached only by those dynasties with an initial human capital above $\hat{h}$, not only because of their favourable initial conditions, but also because they receive high-quality education.

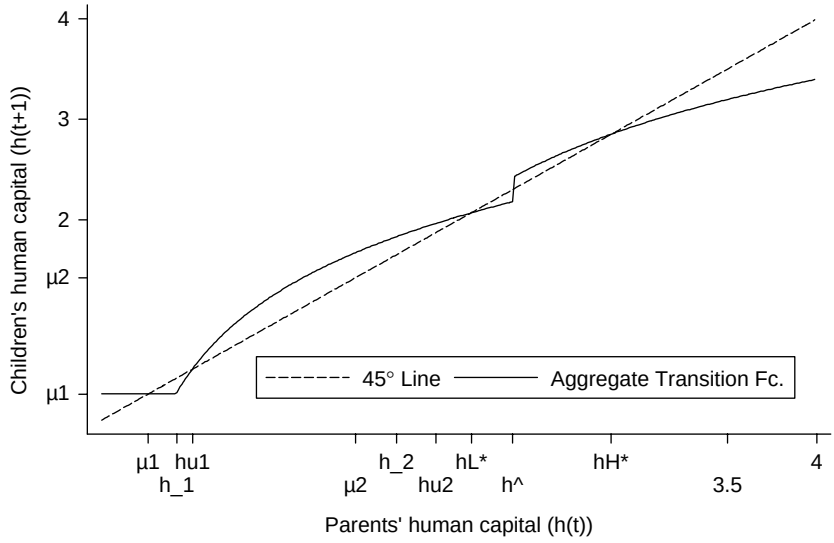

Figure 3. Aggregate transition function.

The two poverty traps constituted by equilibriums μ^1 and h_L^* can be seen not only in relative terms but also in absolute terms if one thinks of two poverty lines, one for extreme poverty (z_1) and one for poverty (z_2), such that $\mu^1 < z_1 < h_L^*$ and $h_L^* < z_2 < h_H^*$.

3.3 Income Distribution and Macroeconomic Equilibrium

Once the three long-run equilibria to which different fractions of the society converge are identified, it is possible to obtain the aggregate level of education spending, the output in each period t and the long-run output. As in Berti Ceroni (2001), it is assumed that the current income distribution determines the future one:

$$g_t(h_t^{ij}) = g_{t-1}[\phi^{-1}(h_t^{ij})] \quad h_t^{ij} \in [\alpha, \beta] \tag{9}$$

and that at any point in time, income distribution determines current aggregate investment in education and aggregate human capital and income of the next period. In what follows, it is useful to define $G_t(\bar{h}^1) = \int_{\mu^1}^{\bar{h}^1} g(h_t^i)dh_t$ and $G_t(\hat{h}) = \int_{\mu^1}^{\hat{h}} g(h_t^i)dh_t$. Considering equation (5), the threshold localization given by conditions C7 and C8 ($\mu^2 < h_L^* < \hat{h}$), and that $\int_I g_t(h_t^i)dh_t^i = 1$, the aggregate education spending E_t is given by:

$$\begin{aligned} E_t = b^1 G(\bar{h}^1) + \frac{\delta}{1+\delta}\int_{\bar{h}^1}^{\beta} h_t g_t(h_t^i)dh_t + \left(\frac{q^1 b^1 - v^1}{q^1(1+\delta)}\right)[G(\hat{h}) - G(\bar{h}^1)] \\ + \left(\frac{q^2 b^2 - v^2}{q^2(1+\delta)}\right)[1 - G(\hat{h})]. \end{aligned} \tag{10}$$

Considering equations (3) and (8), the aggregate output is given by:

$$\begin{aligned} Y_t = \mu^1 G_{t-1}(\bar{h}^1) + \int_{\bar{h}^1}^{\hat{h}} \ln[q^1(h_{t-1} - b^1) + v^1] g_{t-1}(h_{t-1})dh_{t-1} \\ + \int_{\hat{h}}^{\beta} \ln[q^2(h_{t-1} - b^2) + v^2] g_{t-1}(h_{t-1})dh_{t-1} + \ln\left(\frac{\delta}{1+\delta}\right)[1 - G_{t-1}(\bar{h}^1)]. \end{aligned} \tag{11}$$

As in Berti Ceroni's model, the current—and therefore the initial—income distribution affects the aggregate accumulation of human capital and the output growth along the transition path to the steady state. The more unequal the initial income distribution is, the slower the human capital accumulation. Dynasties with an initial human capital below h_u^1 have (zero or) negative growth rates, as do dynasties with an initial human capital level $h_L^* < h < \hat{h}$. Only dynasties with an initial human capital level $h_u^1 < h < h_L^*$ or $h > \hat{h}$ have positive growth rates.

The negative effect of inequality in the initial human capital distribution on aggregate output persists in the long run because of the existence of poverty traps. The steadystate aggregate output is given by:

$$\begin{aligned} Y_\infty &= \mu^1 G_0(h_u^1) + h_L^*[G_0(\hat{h}) - G_0(h_u^1)] + h_H^*[1 - G_0(\hat{h})] \\ &= h_H^* - (h_H^* - h_L^*)G_0(\hat{h}) - (h_L^* - \mu_1)G_0(h_u^1), \end{aligned} \tag{12}$$

where one can see that the highest potential output level h_H^* is reduced because fraction $[G_0(\hat{h}) - G_0(h_u^1)]$ of the population can reach only output level h_L^*, and fraction $G_0(h_u^1)$ can reach only output level μ^1.

4. Policy Implications

4.1 The Prediction of an Equal-Quality Education Policy

Among the set of parameters v^j, b^j, q^j, δ that affect the human capital transition equation, only parameter q^j, the quality of education, is a potential policy instrument. Parameters v^j, b^j refer to intrinsic characteristics of the two social sectors, the more and the less educated, whereas parameter δ measures the degree of altruism of parents to children and is common to both social sectors. None of these can be influenced by public policy.

A natural question that arises in this setting is what would be the effect of a policy that guaranteed the same quality of education to all children, independent of the social sector from which they came? In other words, what would happen if there was no education quality segmentation by social groups? Figure 4 depicts one possible solution of the effect of equating the quality of education in the two sectors ($q^1 = q^2$) and holding everything else constant. Two equilibria were obtained.[8] The new solution was obtained by replacing q^1 with q^2 in the second line of equation (8); because of the increase in the education quality of the less advantaged social group, the slope of the transition equation for $j = 1$ dynasties increased substantially. Thus, all dynasties with an initial human capital level above h_u^1 converged to the steady-state human capital level h_H^*, independent of whether their human capital was below or above $\hat{h}$, that is, independent of whether they belonged to the more or the less educated sector.[9]

A fraction of the population, however, remained uneducated, with the human capital level μ^1; they are the dynasties that started with a human capital level lower than h_u^1. Yet, this fraction $G(h_u^1)$ is smaller than before because the increase in the quality of education

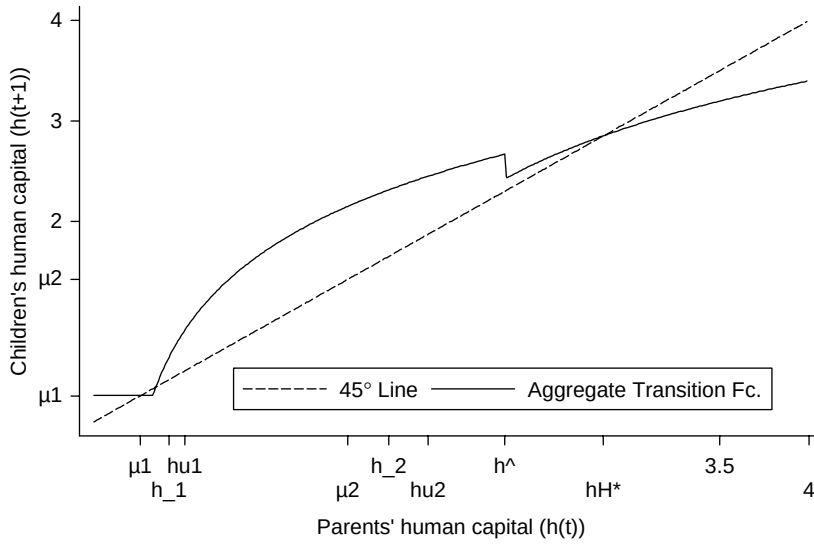

Figure 4. Aggregate transition function with equal quality of education.

for all $j = 1$ makes the threshold level $\bar{h}^1$ decrease (recall equation (6)). This decrease makes the value h_u^1 also lower (the whole curve $\phi^1(h_t^{i1})$ is now to the left of the original one). It can be verified that both the aggregate education spending as well as the current output level and the steady-state output level are higher.

This policy exercise exemplifies the crucial role that the quality of education can play in the economy. If the education received by a child from an advantaged social circle is higher than that received by a child from a disadvantaged one, social inequalities will be perpetuated, causing *poverty traps*. Even when the years of schooling are the same for these two children, the acquired cognitive skills, that is, the acquired human capital h_{t+1}^{ij}, will not be the same, affecting their future employability in the labour market, and therefore their future incomes.

The model suggests that even with marked initial disadvantages represented by lower values of parameters v^j and b^j, if children from the disadvantaged sectors receive the same quality of education as children from the advantaged sectors, this can significantly reduce the initial inequalities in the long run. In fact, for a sufficiently high quality of education, the model allows for the possibility of a complete elimination of initial inequalities, with everyone converging to the same human capital level.[10] Clearly, this is an excessively optimistic perspective on the effect of such a policy, which arises because this is a schematic model.

Although the prediction of overall convergence to the same human capital level is unrealistic, a milder version of it, namely, the eradication of a fraction of the population with no education and *some* degree of convergence of cognitive skills, may not be. In fact, this milder prediction finds support in recommendations made by international organizations. The sixth goal stated by the 2000 World Education Forum, aimed at reaffirming the 1990 *World Declaration on Education For All*, consists of "improving all aspects of the quality of education . . . " (UNESCO, 2000, pp. 8, 15) and it is emphasised that " . . . quality must not suffer as access expands and that improvements in quality should not benefit the economically well-off at the expense of the poor" (UNESCO, 2000, p. 13).[11] Based on the empirical evidence from PISA 2003, OECD has made similar claims (OECD, 2004, p. 267), as well as the 2006 *World Development Report* (e.g. World Bank, 2005, p. 2).

Evidence suggests that it is possible to make progress towards both guaranteeing equal-quality education and reducing the lack of education in each country. In 2007, while 15% of children of basic education age (primary and lower secondary combined) in developing countries were out of school, the figure was 6% in transition countries and 4% in developed ones (UNESCO, 2010, p. 74). Primary school net enrolment in Sub-Saharan Africa increased by 17 percentage points (from 56 to 73%) between 1999 and 2007; in South and West Asia net enrolment increased by 12 percentage points in the same period (UNESCO, 2010, p. 62). These figures still imply that about 72 million children and 71 million adolescents are out of school. Moreover, school attendance is no guarantee of successful learning outcomes. However, the progress made even among very poor areas of the world constitutes a sign that it is feasible to eradicate the lack of education.

Even the convergence to a similar human capital level suggested by the model has some plausibility. Clearly, in reality there would always be some heterogeneity. Notwithstanding that, there are countries that exhibit low socio-economic segregation (as measured by the impact of the school's socio-economic background on students' performance) and high overall performance in international tests simultaneously. Canada, Finland, Sweden, Iceland, Ireland, New Zealand, Denmark, Estonia, Hong Kong-China and Macao-China are some examples obtained from the PISA evaluations (OECD/UNESCO-UIS, 2003,

p. 219; OECD, 2004, p. 188, 2007, p. 194). This suggests that not only is it possible to reduce education quality segmentation and the dispersion in students' performance, but also that this can be done without risking average performance.

The key question, however, is which policies could promote equal education quality? Although this requires a country-specific understanding of the mechanisms that contribute to between-school socio-economic segregation, empirical evidence on different types of policy offers some guidelines. This is addressed in the next section.

4.2 Policies towards Equal-Quality Education

There is a vast literature on the determinants of education quality and the impact of different types of policy intervention on educational outcomes and their distribution. However, evidence is very often mixed. A first type of policy consists of *investing in school inputs* in schools in marginalized areas, as these schools are usually deprived in these aspects. School inputs include physical resources, from educational material such as textbooks to the infrastructure of the school (e.g. roofs and heating), as well as human resources—the quantity and quality of teachers.

A systematic review of the empirical evidence indicates that there is little to suggest that investing more in physical resources will significantly improve student achievement. Hanushek (1995) reviewed evidence on estimates of educational production function from developing countries and Hanushek (1997) compiled evidence from the USA. Glewwe (2002) and Glewwe & Kremer (2006) also reviewed evidence from natural experiments and randomized trials, methods they advocate as the best for policy evaluation. Results on the impact of the different variables vary significantly from one study to the other, and include intuitive and counter-intuitive results, or simply non-significant effects. A few randomized evaluations found some positive impact of certain interventions in certain contexts.[12] However, there is no consistent pattern across developing countries regarding this type of investment.

In terms of human resources, however, there seems to be a more salient and consistent result. Value-added models, as estimated by Rivkin *et al.* (2005) using panel data from the UTD Texas School Project, measure teacher quality on the basis of student achievement gains, and find a very strong effect of better teachers. Analogous estimates using different data sets (from Tennessee, Chicago and New York City, cited in Hanushek & Rivkin (2010)) have similar results. These results suggest that high-quality instruction throughout primary school could substantially offset disadvantages associated with low socio-economic background (Rivkin *et al.*, 2005, p. 419). Yet, one striking result, also consistent across these studies, is that measured characteristics of teachers, such as their education, do very little to explain the overall variance in teacher quality. In particular, experience is not significantly related to achievement following the initial years in the profession. This is consistent with a lack of systematic evidence on the effect of these observable teacher characteristics in other types of study (Hanushek, 1995). However, Rivkin *et al.* (2005) mention that research shows that principals can, when asked, separate teachers on the basis of quality.[13] Thus, they argue that rather than tightening standards for teachers, policies should be oriented towards effective practices on hiring, firing, mentoring and promotion practised by school principals. Providing principals with more autonomy for these issues then constitutes a policy option to be explored.

Rivkin *et al.* (2005) also emphasize that economically disadvantaged students face higher teacher turnover and tend to be taught more frequently by new teachers. Given the finding that initial years of experience do affect teacher quality, policies should be developed both to keep more senior teachers in the classrooms of disadvantaged students and to mitigate the impact of inexperience (Rivkin *et al.*, 2005, p. 450). This is in line with one of the policy recommendations of the 2010 EFA Report (UNESCO, 2010), which states the need for policies that attract the best teachers to marginalized schools.[14] Clearly, much more research is needed on the best policies to retain and reward good teachers.

In terms of the quantity of teachers, the study of Rivkin *et al.* (2005) found that students do benefit from smaller classes. This is in line with results elsewhere (Hanushek & Luque (2003), in a number of developed and developing countries; Drèze & Kingdon (2001), in India; Case & Deaton (1999), in South Africa; and Angrist & Lavy (1999), in Israel). However, the authors also found that the costly policy of reducing class size by 10 students produces smaller benefits than a one standard deviation improvement in teacher quality.

A second type of policy consists of *investing in complementary goods, particularly health*. In fact, the ability to enjoy the benefits of education is highly conditioned by the child's health status. Although not abundant, the available evidence on school-based health programmes such as administrating de-worming drugs, iron supplements and school meals is favourable (see Kremer, 2003, and references therein).

A third type of policies consists of educational *reforms* that are meant to introduce better incentives to foster quality. Options include decentralization, the introduction of test score-based accountability systems (sometimes with rewards and sanctions), teachers' incentives and voucher systems. Unfortunately, evidence on these policies is still scarce and not conclusive, but suggests that design aspects play a key role in all the options outlined.

On *decentralization*, Jimenez & Sawada (1999) found a positive impact on expanding education in poor rural areas of El Salvador but no effect on student achievement. In the case of Argentina, Galiani & Schargrodsky (2002) found a positive impact on students' achievement when schools were transferred to fiscally ordered provinces, but a negative one when provinces were in fiscal deficits. There is also evidence on some negative impacts of Kenya's mix of centralized and decentralized control over different aspects of education (Glewwe & Kremer, 2006). Cross-country evidence from PISA suggests that after accounting for demographic and socio-economic background, education systems where schools have a higher degree of autonomy in budgeting tend to perform better (OECD, 2007).

Regarding *accountability systems*, evidence from PISA suggests that, other things being equal, students in schools that post their achievements publicly perform—on average—better than students in schools that do not post their achievement data publicly (OECD, 2007). However, many argue against these accountability systems, especially when they entail rewards and sanctions (Rothstein, 2000; Hanushek & Raymond, 2001; Taut *et al.*, 2009, among others). In the USA, Hanushek & Raymond (2004) found evidence that states that introduced incentives related to performance with the No Child Left Behind Act had a positive impact on average achievement but widened the racial achievement gap.

Proposals on introducing *teachers' incentives* are motivated by the fact that in most countries "good and bad teachers can expect about the same career progressions, pay and other outcomes" (Hanushek, 2002, p. 2089). Moreover, in some countries such as Kenya and India, teachers' absenteeism and lack of engagement in teaching are very serious and

widespread problems (Kremer, 2003; Kremer *et al.*, 2005; Kingdon, 2007). However, limited information is available about the design and impact of alternative incentive schemes in schools (Hanushek, 2002). Lavy (2002) found some positive evidence of an incentives programme in Israel, but Glewwe *et al.* (2004) found no long-term impact of a teachers' prize programme in Kenya.

Finally, *voucher programmes* are proposed on the assumption that creating a competitive market between schools will raise quality. Such programmes can essentially be of two types: a large-scale, *universal* voucher system; or a means-tested voucher system targeted at low-income families.[15] Empirical evidence on universal programmes is not conclusive as to whether they increase overall achievement (Ladd, 2002; Neal, 2002). Most importantly, a universal voucher programme is likely to generate greater socio-economic and racial polarization of students between schools as students seek to improve the quality of their peers (Epple & Romano, 1998; Ladd, 2002). There is some evidence of this in the case of Chile (Hsieh & Urquiola, 2002).

In contrast to a universal programme, however, a means-tested voucher programme may be useful for reducing segregation and increasing overall quality by generating greater competition between schools. Neal (2002) and Ladd (2002) advocate this option but emphasize the importance of its design. In particular, schools must be required not to charge any additional tuition or other fees and oversubscribed schools must be obliged to select students randomly. The programme must also offer transportation to any selected school. Angrist *et al.* (2002) found a positive impact of this type of voucher programme in Colombia (the Paces programme) on school attendance, completion and academic achievement.

One further potential reform is worth mentioning. Evidence from PISA (OECD, 2004, 2005, 2007) indicates that school systems characterized by tracking and streaming (separating children into different types of school according to academic ability and/or ability grouping in *all* subjects within schools) tend to have wider disparities in performance associated with the school's socio-economic composition, reinforcing the socio-economic segregation between schools. Such school systems also appear to have lower average performance. Thus, *eliminating tracking procedures*, or at least delaying the age at which this is done, can prove effective at reducing inequalities and improving overall achievement simultaneously. Poland constitutes a case study of the benefits of removing tracking (OECD, 2007, chapter 5).

5. Conclusions

The contribution of this paper has been to formalize the idea that when the poor can access only poor-quality education, this can lead to poverty traps. The model is based on Berti Ceroni (2001), with the difference that it incorporates the effect of segregation in education quality. Starting with initial inequality in the distribution of human capital and income, the human capital accumulation dynamics lead, under certain conditions, to a situation with three simultaneous steady-state equilibria. For those individuals with an initial, low human capital endowment, it is not profitable to invest in education, and these people remain in that situation forever. Those exceeding a certain human capital threshold will invest in education, but as they receive low-quality education they remain trapped at low levels of human capital and income. Only those dynasties that initially have high levels of human capital reach the steady state at a high human capital and income level.

The existence of poverty traps lowers the economy's current and steady-state aggregate output level as well as the growth rate.

The model constitutes only the first step in formalizing the concept that differences in the quality of education may lead to poverty traps. It has some restrictive assumptions, such as the linear production function and non-consideration of physical capital. At the same time, the equilibria configuration analysed here is not the only possible one and depends on the position of the threshold level $\hat{h}$ that distinguishes the two social sectors. Appendix 2 details other possible equilibria configurations. Although differences in the quality of education received by children from different social sectors do not necessarily lead to poverty traps, it is possible that they do so.

Despite its limitations, the policy recommendation of aiming to equalize the quality of education for all social sectors finds support in recommendations made by international organizations such as UNESCO, the OECD and the World Bank. Empirical evidence suggests that policies towards retaining and attracting more senior teachers to schools with disadvantaged students could increase education quality in marginalized schools. Providing school principals with greater autonomy and proper incentives for effective hiring, firing and promotion of teaching staff can also be effective. School-based health programmes to treat diseases with high prevalence and/or school meal programmes can be important complements of the above. Avoiding and suppressing (if they exist) tracking and streaming procedures seems to reduce segmentation. Finally, a means-tested voucher programme for disadvantaged students is a further option. Interestingly, empirical evidence suggests that it is possible to reduce segregation without hampering overall student performance. In this case, equity and efficiency do not seem incompatible goals; on the contrary, they may reinforce each other.

Notes

[1] For a thorough recent review of the literature on poverty traps, see Azariadis & Stachurski (2005).

[2] The effect of social segmentation on inequality is also explored by Bowles *et al.* (2007).

[3] Note that PISA has been performed mainly in upper-middle and high-income countries. Lower-middle and low-income countries where there is high inequality are likely to present the same or an even more pronounced segmentation pattern than that observed in some of the richer countries.

[4] The figure uses the following values of the parameters: $b^1 = 0.2$, $b^2 = 1.4$, $v^1 = 1.3$, $v^2 = 4.12$, $q^1 = 12$, $q^2 = 33$.

[5] Figure 2 uses the same values of the parameters as in Figure 1 and a value of $\delta = 0.5$.

[6] If all conditions of Section 3.2 are satisfied (so that the transition equation of each sector has three stable equilibria) but conditions C7 or C8 are not satisfied, other equilibria configurations can occur at the aggregate level. Although these are not of interest for this paper, they are detailed in Appendix 2.

[7] It is worth noting that there are dynasties that, although they start with a human capital level higher than h_L^* (but lower than $\hat{h}$), they converge to the steady-state human capital level h_L^*, lower than the initial state.

[8] All the values of Figure 4 are the same as those in Figure 3, except that now $q^1 = q^2 = 33$.

[9] Part of condition C8 no longer holds, because now $\hat{h} < h_L^*$.

[10] If the quality of education were set equal for the two sectors at a higher level than that used for Figure 4, the $j = 1$ transition function would not intersect the 45° line at low human capital levels, eliminating the two first equilibria (μ^1, h_u^1), and all dynasties would converge to the high human capital and income equilibrium h_H^*. In that case, condition C5 would not be satisfied.

[11] The *World Declaration on Education for All* was agreed by delegates from 155 countries together with representatives from about 150 governmental and non-governmental organizations.

[12] For example, a radio mathematics instruction in Nicaragua, a programme on remedial education, and a computer-assisted learning programme in India (see references in Glewwe & Kremer, 2006).

[13] This is consistent with evidence from PISA on the correlation between principals' perceptions of a lack of qualified teachers hindering instruction and students' performance (OECD, 2007).

[14] The report also argues that wherever segmentation is linked to ethnicity, this can also be complemented by recruiting teachers from ethnic minorities so that children are taught in their home-language as well as in the vernacular language (UNESCO, 2010, p. 197). Moreover, the report suggests that teachers should be trained to address marginalization in classrooms, which is in tune with some evidence of the pervasive effect of stigma on school performance.

[15] There is actually a third option: a voucher system targeted at residents in low-income areas (school districts). Neal (2002) provides good arguments as to why this is not a good option.

[16] In general terms, given an expression $Y = Xe^X$, the W function (also called product log) provides a solution to it given by: $X = W(Y)$. The W Lambert function can be expanded in series: $W(Y) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^{n-2}}{(n-1)!} Y^n$.

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Appendix 1. Conditions for the Emergence of Multiple Equilibria

It is assumed that there are only two sectors or social circles with initial human capital levels clearly differentiated: $j = 1,2$, with $h_t^{i1} < h_t^{i2}$. The following conditions for the emergence of multiple equilibria are required.

(C1) $(v^j, b^j, q^j) \geq (0, 0, 0) \quad \forall j = 1, 2.$

(C2) It is understood that the human capital level of a child who does not receive formal education is non-negative, so that $\mu^j = \ln(v^j) \quad \forall j = 1, 2.$

(C3) As explained in Section 3.1, it is assumed that $v^1 \leq v^2; b^1 \leq b^2; q^1 \leq q^2$.

(C4) It is also assumed that the human capital level at which the spending function starts to be increasing in the parent's human capital is higher for dynasties with a higher initial education level. This is consistent with the assumption that the threshold at which education spending starts to be effective is higher for children with more educated parents than for children with less educated parents ($b^1 \leq b^2$). Formally, the condition is: $\bar{h}^1 < \bar{h}^2$. Replacing equation (6) in both sides of the inequality and rearranging, the condition is re-expressed as: $[(v^1/\delta q^1) - (v^2/\delta q^2)] < [b^2 - b^1]$.

(C5) It is required that for each j, the human capital threshold at which the education spending function starts to be increasing in the parent's human capital level is higher than the child's human capital level if he or she does not receive education: $\bar{h}^j > \ln(v^j) \quad \forall j = 1, 2$. This condition guarantees that the human capital accumulation curve for each j intersects the 45° line, so that corner solutions appear at low income levels, constituting a stable equilibrium: once the dynasty reaches the human capital level μ^j, it remains there forever. Replacing equation (6) at the left of the inequality and

rearranging, the condition can be re-expressed as: $(v^j/\delta q^j) > [\ln(v^j) - b^j]$ $\forall j = 1, 2$.

For the existence of other equilibria at higher income levels and for the emergence of poverty traps, it is required that the derivative of each transition function j in its concave part (when $h_t^{ij} > \bar{h}^j$) has a slope greater than unity when it is evaluated at the point where $h_t^{ij} = h_{t+1}^{ij}$. This point will be called h_u^j. Formally, it is required that:

$$\frac{\partial \phi^j(h_t^{ij})}{\partial h_t^{ij}}\Big|_{h_{t+1}^{ij}=h_t^{ij}=h_u^j} > 1.$$

To find an expression of this condition in terms of the parameters, it is first necessary to find an expression for h_u^j for which the transition equation (7) needs to be solved, evaluated at the point in which $h_{t+1}^{ij} = h_t^{ij} = h_u^j$:

$$\ln\left[\frac{q^j\delta(h_u^j - b^j) + \delta v^j}{1+\delta}\right] = h_u^j. \tag{A1}$$

For simplicity, the following notation will be used: $m^j = [\delta/(1+\delta)]q^j$, $p^j = [\delta/(1+\delta)](v^j - q^j b^j)$. Applying the exponential to both sides of equation (A1), it can be rewritten as:

$$(m^j h_u^j + p^j) = e^{h_u^j}. \tag{A2}$$

Multiplying both sides of equation (A2) by $-e^{(-p^j/m^j)}$ and rearranging the terms, equation (A3) is obtained:

$$-\frac{e^{(-p^j/m^j)}}{m^j} = -\frac{(m^j h_u^j + p^j)}{m^j} e^{\left\{-\frac{(m^j h_u^j + p^j)}{m^j}\right\}}. \tag{A3}$$

Using the W Lambert function (cited in Euler, 1783), the solution to equation (A3) is given by:[16]

$$h_u^j = -\left\{W\left(-\frac{e^{(-p^j/m^j)}}{m^j}\right) + p^j/m^j\right\}. \tag{A4}$$

Replacing equation (A4) in the condition for the existence of multiple equilibria, which requires that in h_u^j the transition function has a slope higher than unity, and using the definitions of m^j and p^j above, the condition can be stated as:

$$\frac{\partial \phi^j(h_t^{ij})}{\partial h_t^{ij}}\Big|_{h_t^{ij}=h_u^j} = -\frac{1}{W\left(-\frac{e^{(-p^j/m^j)}}{m^j}\right)} > 1. \tag{A5}$$

For condition (A5) to be satisfied, it must hold that:

$$-1 < W\left(-\frac{e^{(-p^j/m^j)}}{m^j}\right) < 0. \tag{A6}$$

Given that $m^j, e^{(-p^j/m^j)} > 0$, the argument of the W function is negative. At the same time, for real values between $[-1/e, 0]$, function W takes values between [−1,0], with

$W(-1/e) = -1$ and $W(0) = 0$. Then for equation (A6) to be satisfied, it is required that:

$$-\frac{1}{e} < -\frac{e^{(-p^j/m^j)}}{m^j}.$$

(C6) Replacing m^j, p^j by their original expressions and rearranging the inequality, it is found that the condition for the existence of multiple equilibria is:

$$\frac{\delta}{1+\delta} > \frac{e^{\left[1-\frac{v^j}{q^j}+b^j\right]}}{q^j}.$$

If condition C6 is satisfied, the transition function $\phi^j(h_t^{ij})$ has a slope higher than unity in h_u^j, and so h_u^j is an unstable point. Given that the function is concave, the slope decreases tending to zero and it intersects the 45° line one more time, at a point that is called h_L^* for $j = 1$ and in h_H^* for $j = 2$.

Appendix 2

This Appendix analyses the other possible equilibria that arise when the two additional conditions C7 and C8 are not satisfied. The case of the three stable equilibria ($\mu^1 < h_L^* < h_H^*$) described in Section 3.2 will be called scenario 1.

(1) In the case condition C7 is not satisfied, that is, if $\mu^2 \geq h_L^*$, the following may occur: that the curves $\phi^1(h_t^{i1})$ and $\phi^2(h_t^{i2})$ do not intersect (curve $\phi^2(h_t^{i2})$ would be above curve $\phi^1(h_t^{i1})$), that they intersect for the first time at $\mu^2 = h_L^*$, or that they intersect at some point to the right of μ^2. In any of these cases, because condition C7 is not satisfied, the first part of condition C8 will not be satisfied either, so that $h_u^2 > h_L^*$. Under these conditions, the following possibilities can occur regarding condition C8.

(1a) One possibility is that $\hat{h} < h_L^* < h_u^2$. In this case, there will be three equilibria at $\mu^1 < \mu^2 < h_H^*$. This will be called scenario 2. In it, all the poor (who start with a human capital level lower than $\hat{h}$) end with a steadystate human capital level equal to μ^1, that is, they are trapped in a very low poverty trap forever. On the other hand, the rich (those with a human capital level higher than $\hat{h}$) may converge to two possible steady states, depending on their initial human capital level. If $h_t < h_u^2$, they will converge to the steadystate level μ^2, whereas if $h_t > h_u^2$, they will converge to the steadystate level h_H^*.

(1b) Another possibility is that $h_L^* < \hat{h} < h_u^2$, in which case, to avoid indeterminacies, it is required that $\hat{h} < \mu^2$. Under these conditions, four equilibria are produced at $\mu^1 < h_L^* < \mu^2 < h_h^*$. This will be called scenario 3. In it, the poor with an initial human capital level $h_t < h_u^1$ will end up being not educated, converging to equilibrium μ^1; the poor with an initial human capital level $h_u^1 < h_t < \hat{h}$ invest in education and converge to equilibrium h_L^*. On the other hand, the rich with an initial human capital level $\hat{h} < h_t < h_u^2$ do not invest in education and converge to equilibrium μ^2. Only the rich with an initial human capital level $h_u^2 < h_t$ converge to the highest equilibrium h_H^*. In this case there are three poverty traps at different human capital and income levels: μ^1, h_L^*, μ^2.

(1c) Finally, it is also possible that $h_L^* < h_u^2 < \hat{h}$, in which case the resulting equilibria are the same as with scenario 1.

(2) It is possible that condition C7 is satisfied ($\mu^2 < h_L^*$, so that the two curves $\phi^1(h_t^{i1})$ and $\phi^2(h_t^{i2})$ intersect), but that condition C8 is not satisfied. In that case, the following options are possible.

(2a) It could happen that $h_u^2 < \hat{h} < h_L^*$, in which event two equilibria are produced at human capital levels $\mu^1 < h_H^*$. This will be called scenario 4. In this setting all the poor ($h_t < \hat{h}$) remain poor and with low human capital, as they converge to μ^1. All the rich $h_t > \hat{h}$ stay rich and with high human capital, as they converge to h_H^*.

(2b) It may also happen that $h_L^* < h_u^2 < \hat{h}$, in which case the equilibria of scenario 1 arise.

(2c) If $h_L^* < \hat{h} < h_u^2$, there is an indeterminacy for dynasties that start with a human capital level $\hat{h} < h_t < h_u^2$, as μ^2 is not an equilibrium in this case.

(2d) Another possibility is that $\hat{h} < h_L^* < h_u^2$, in which case the equilibria of scenario 2 arise.

(2e) Finally, it is also possible that $\hat{h} < h_u^2 < h_L^*$, in which case it is required that $\hat{h} < \mu^2$ to avoid indeterminacies. Under these conditions the equilibria of scenario 2 arise.