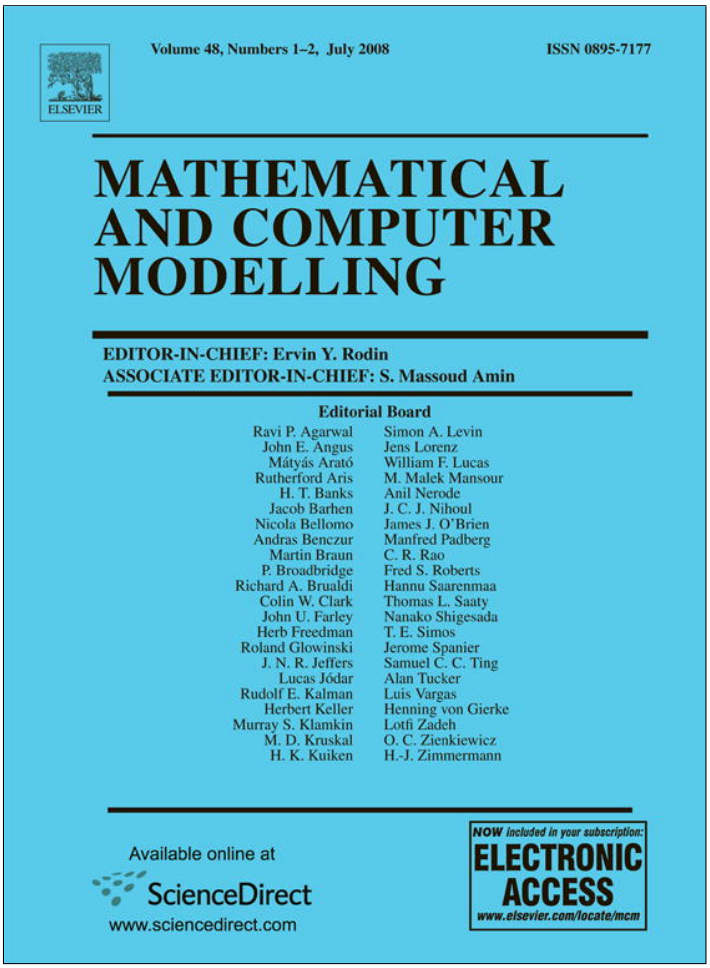

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An approach for identification of uncertain Wiener systems

J.L. Figueroa*,[1], S.I. Biagiola[1], O.E. Agamennoni[2]

Instituto de Investigaciones en Ingeniería Eléctrica - IIIE (UNS-CONICET), Departamento de Ingeniería Eléctrica y de Computadoras, Universidad Nacional del Sur, Avda. Alem 1253; (8000) Bahía Blanca, Argentina

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Abstract

As reported in the literature, Wiener models have arisen as an appealing proposal for nonlinear process representation due to their simplicity and their property of being valid over a larger operating region than a LTI model. These models consist of a cascade connection of a linear time invariant system and a static nonlinearity. In the description of these models, there are several ways to represent the linear and the nonlinear blocks, and several approaches can be found in the literature to perform the identification process.

In this article, we provide a parametric description for the Wiener system. This approach allows us to describe the uncertainty as a set of parameters. The proposed algorithm is illustrated through a pH neutralization process.

Keywords: Wiener models; Nonlinear identification; Uncertainty control

1. Introduction

Nonlinear model-based control has been widely used in the chemical engineering field. The reason for this is that all processes are nonlinear and, in many cases, controllers based on linear models are no longer satisfactory. The use of models based entirely on fundamental process understanding has the advantage of possessing a clear physical interpretation. However, these models tend to be highly complex making impossible their application in popular model-based control strategies [1]. On the other hand, purely empirical models (black-box), based entirely on input/output data, lack physical interpretation. Although black-box models can be established without knowing the detailed underlying physics and dynamics of the system, they have shown to be useful and flexible for many applications.

A third approach is used when some physical insight is available, but several parameters remain to be determined from the observed data. In this category, Pearson and Pottmann [2], included the Wiener model. This model is a kind of block-oriented nonlinear model consisting in a dynamic linear time invariant (LTI) submodel $H(z)$, and a static (i.e. memoryless) nonlinear block $N(.)$.

* Corresponding author. Tel.: +54 291 4595101x3325; fax: +54 291 4595154.
E-mail address: figueroa@uns.edu.ar (J.L. Figueroa).
[1] Also with CONICET.
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The use of these models has been treated in the literature in various contexts such as chemical and biological processes [3–8], among others [9–12]. The advantage of using Wiener models are twofold: the low computational effort associated to the identification and the suitability for control design.

Some representation for the linear and nonlinear blocks of Wiener models have been presented in the literature [2,11,13–21].

In general, the linear dynamics and the static nonlinearity cannot be identified in an independent way due to the cascade structure of the Wiener model. In this sense, several identification algorithms had been presented. One of these approaches is based on correlation techniques [22,23] but it needs the verification of the rather restrictive requirement on the input to be white noise. Another approach is the iterative algorithm proposed by Narendra and Gallman [24]. Originally, this scheme was proposed for the identification of Hammerstein models and it is the most direct approach, and is based on a "successive" procedure where the parameters of the linear and nonlinear blocks are computed recursively to refine the model. The more recent approach is based on the "simultaneous" identification of the complete set of parameters. It was introduced by Kalafakis et al. [3,17] and extended to more general block-oriented models by Bai [25] and Gómez and Baeyens [20].

In this paper, we present a novel characterization approach as well as an identification algorithm for uncertain Wiener models. The goal is to obtain a nominal model of the process plus a parametric description of the uncertainty, which is the main contribution of this work. For this purpose, Laguerre polynomials are used to model the linear dynamic block, and a piecewise linear (PWL) representation of the nonlinear static block is provided. PWL models are often used to represent the behaviour of nonlinear systems, since they constitute the simplest extension of linear systems that approximate nonlinear systems with an arbitrary accuracy. This modeling approach proves to be advantageous due to its simplicity, easy implementation and good application results. Moreover, it happens to be a convenient formulation in order to model the uncertainty which, in this way, can be easily mapped onto the model parameters. From the first canonical expression by Chua and Kang [26], the theory of canonical representations of PWL functions has evolved in a substantial manner. That paper was followed by the improvements and contributions introduced by Güzelis and Göknar [27], Kahlert and Chua [28], and Lin et al.. [29]. Since then, PWL have been used in many applications in a wide variety of fields such as the modeling of electronic devices [30], in the analysis and synthesis of systems with complex dynamics [31,32], as well as in the areas of modeling and control [33–35]. Recently, in [36,34,37], a systematic way of defining the PWL representation for arbitrary (continuous) domains in a more compact and efficient form was introduced. This PWL representation uses the concept of simplicial partitions of the domain of interest. This is the approach followed in the present work.

The paper is organized as follows. In Section 2, general concepts about models and uncertainties are introduced. In Section 3 the description and identification technique of Wiener systems are reviewed. The proposed uncertainty model is presented in Section 4 and an algorithm for parameter uncertainty characterization is introduced. In Section 5, the results are evaluated on the basis of a simulation of a pH neutralization process. The final remarks are made in Section 6.

2. Process information, models and uncertainties

Let us consider that process data are available in the form of two sets of process inputs ($\mathbf{u} = \{u_0, u_1, \ldots, u_N\}$) and outputs ($\mathbf{y} = \{y_0, y_1, \ldots, y_N\}$). Then, we aim at finding an appropriate mathematical model, i.e. a model whose response matches the collected input–output data. This is performed in a two-step procedure. In the first step, a model structure is selected. This structure indicates in which way the input–output data are related. We use the previous knowledge about the process:

$$\hat{y}_{k+1} = f(\hat{y}_k, \ldots, \hat{y}_{k-N_y}, u_k, \ldots, u_{k-N_u}, \theta), \tag{1}$$

where the predicted output at time $k+1$ depends on the present and the N_u previous inputs, as well as on the predicted present and the N_y previous outputs. The set of parameters (θ) is also needed to determine $\hat{y}_{k+1}$. In the second step, the parameters (θ) are computed to minimize the difference between the process and model outputs ($y_k - \hat{y}_k$) to any time. This can be performed using an optimization algorithm. In what follows we denote this set of parameters as *nominal parameters* θ_N.

When the interest aims at obtaining an uncertainty related with this nominal model, a typical approach is to define a set of possible models to represent all the process behaviours. This is performed by considering a set of model

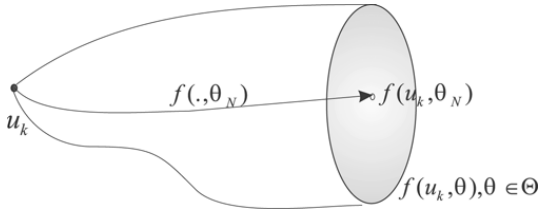

Fig. 1. Model under uncertainties.

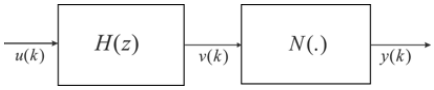

Fig. 2. The Wiener model structure.

parameters Θ such that when these parameters $\theta \in \Theta$ are used, the whole set of exciting inputs $\mathbf{u}$ is "mapped" onto an output set which contains the set of the output data $\mathbf{y}$ (see Fig. 1). In this way, we assume the same format for all the possible models in the uncertain set. These models' family is defined in terms of a set of parameters. Based on this, we can have an idea of the conservatism of the model description, by analyzing the difference between the sets $\{\hat{y} \mid \hat{y} = f(\mathbf{u}, \theta), \theta \in \Theta\}$ and $\mathbf{y}$.

3. Wiener model identification

3.1. Model description

Fig. 2 depicts a Wiener model. It consists of a LTI system $H(z)$ followed by a static nonlinearity $N(.)$. That is, the linear model $H(z)$ maps the input sequence $\{u(k)\}$ into the intermediate sequence $\{v(k)\}$, and the overall model output is $y(k) = N(v(k))$.

In this paper, the linear block of order N_l is described as [38–40].

$$H(z) = \sum_{i=0}^{N_l} h_i L_i(z, a) \tag{2}$$

$$L_i(z, a) = \frac{\sqrt{1-a^2}}{z-a}\left(\frac{1-az}{z-a}\right)^i, \tag{3}$$

where the parameters of the model are the coefficients h_i, the functions $L_i(z, a)$ are the Laguerre basis for LTI models and $a \in \Re$, with $|a| < 1$, is a filter coefficient chosen a priori.

The advantage in using the Laguerre basis is that it needs a lower number of parameters to describe a system with a slow impulse response or a damped system than other representations. Moreover, it allows the use of prior knowledge about the dominant poles (parameter a).

The nonlinear block $N(.)$ is, in general, a real-value function of one variable, i.e. $y = N(v)$. We describe the nonlinear function as

$$y = \sum_{i=0}^{N_n} \tilde{f}_i \tilde{\Lambda}(v, \beta_i), \tag{4}$$

where the basis functions $\tilde{\Lambda}(v, \beta_i)$ are predetermined PWL functions, the values $\tilde{f}_i$ are the parameters that should be computed and N_n will be referred to as "order" of the nonlinearity. The use of fixed basis functions $\tilde{\Lambda}(v, \beta_i)$ makes the output to be a linear function of the parameters. This allows us to use a linear regression to estimate the parameters. The two basic advantages of this approach are the low complexity and the uniqueness of the solution.

The PWL functions have proved to be a very powerful tool in the modeling and analysis of nonlinear systems. The general formulation of PWL functions allows us to represent a nonlinear system through a set of linear expressions, each of them valid in a certain operation region. To make this approximation, the domain of variables $\aleph$ is partitioned

into a set of σ nonempty regions $\aleph^i$, such that $\aleph = \bigcup_{i=1}^{\sigma} \aleph^i$. In each of these regions the nonlinear function is approximated using a linear (affine) representation. These functions allow a systematic and accurate treatment of the approximating functions. It can be proved [41] that any nonlinear continuous function $N(v) : \Re^m \to \Re^1$ can be uniquely represented using PWL functions in the form of Eq. (4). In this description, β_i are given parameters that define the partition of the domain of v, and $\tilde{\Lambda}$ are functions that involve nested absolute values.

3.2. Nominal model identification

As mentioned in the Introduction, different methods for Wiener models identification have been reported in the literature. A simultaneous approach (see for example, [17]) will be herein used for nominal model identification.

Let us assume that an input–output data set is available, noted as u_k and y_k, respectively. To obtain these data sets, several aspects should be taken into account. For example, the process should be persistently excited in the whole domain of the nonlinear block, such that all the relevant dynamics are captured.

From Fig. 2, the signal v_k can be written as the output of the linear block:

$$v_k = H(z) \bullet u_k. \tag{5}$$

On the other hand, v_k is also the input of the nonlinear block and then, it can be obtained from the output y_k and the inverse $N^{-1}(.)$. Therefore:

$$v_k = N^{-1}(y_k) = \sum_{i=0}^{N_n} f_i \Lambda\left(y_k, \beta_i\right). \tag{6}$$

Equating both sides of these equations (with the inclusion of an error function $\epsilon(k)$ to allow for modeling error) the following equation is obtained

$$\sum_{i=0}^{N_n} f_i \Lambda\left(y_k, \beta_i\right) = h_0 l_0(u_k) + \sum_{i=1}^{N_l} h_i l_i(u_k) + \epsilon(k) \tag{7}$$

or, equivalently,

$$\epsilon(k) = \sum_{i=0}^{N_n} f_i \Lambda\left(y_k, \beta_i\right) - h_0 l_0(u_k) - \sum_{i=1}^{N_l} h_i l_i(u_k) \tag{8}$$

which is a linear regression. Define

$$\theta = \left[f_0, f_1, \ldots, f_{N_n}, h_1, h_2, \ldots, h_{N_l}\right]^{\mathrm{T}} \tag{9}$$

$$\phi(k) = \left[\Lambda\left(y_k, \beta_0\right), \Lambda\left(y_k, \beta_1\right), \ldots, \Lambda\left(y_k, \beta_{N_n}\right), -l_1(u_k), -l_2(u_k), \ldots, -l_{N_l}(u_k)\right]^{\mathrm{T}}. \tag{10}$$

Then, Eq. (8) can be written as

$$\epsilon(k) = \theta^{\mathrm{T}} \phi - l_0(u_k). \tag{11}$$

Note that, without loss of generality, the coefficient h_0 is set equal to one. Now, an estimate $\hat{\theta}$ of θ can be computed by minimizing a quadratic criterion on the prediction errors $\epsilon(k)$ (i.e. the least squares estimate). It is well-known that this estimate is given by:

$$\hat{\theta} = \left(\Phi_N \Phi_N^{\mathrm{T}}\right)^{-1} \Phi_N \Gamma, \tag{12}$$

where $\Gamma = [-l_0(u_1), \ldots, -l_0(u_N)]^{\mathrm{T}}$ and $\Phi_N = [\phi(1), \ldots, \phi(N)]$ are formed using the set of the N data available from the process.

Now, estimates of the parameters $\hat{f}_i$ $(i = 0, \ldots, N_n)$, $\hat{h}_0 = 1$ and $\hat{h}_i$ $(i = 1, \ldots, N_l)$ can be computed by partitioning the estimate $\hat{\theta}$, according to the definition of θ in (9). It is important to remark that we are identifying the inverse of the nonlinearity, which is frequently used in many control applications.

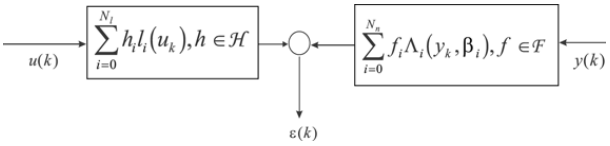

Fig. 3. Uncertainties in the Wiener model.

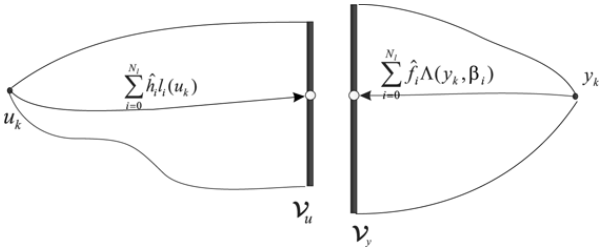

Fig. 4. Uncertainty sets in the Wiener model.

4. Uncertainty characterization

In this section we develop an algorithm, based on the ideas of Section 2, to characterize the uncertainties of the model obtained in Section 3. We introduce a set of parameters $\mathcal{H}$ for the linear dynamic block and a set $\mathcal{F}$ for the parameters of the inverse of the nonlinear block (see Fig. 3):

$$\mathcal{H} = \left\{ h : h = \hat{h} + \delta^h, -h_i^l \leq \delta_i^h \leq h_i^u, 0 \leq i \leq N_l \right\} \tag{13}$$

$$\mathcal{F} = \left\{ f : f = \hat{f} + \delta^f, -f_i^l \leq \delta_i^f \leq f_i^u, 0 \leq i \leq N_n \right\}. \tag{14}$$

To define these bounds, let us define some sets. Given the input data u_k, the linear uncertain system is defined by $\mathcal{H}$ maps at some specific time k over a set

$$\mathcal{V}_u = \left\{ v : v = \sum_{i=0}^{N_l} h_i l_i(u_k), h \in \mathcal{H} \right\}. \tag{15}$$

Given an input u_k, the Laguerre term of order i, i.e. $l_i(u_k)$, is a real number and the set $\mathcal{V}_u$ takes the form of $\mathcal{V}_u = \{v : v_l \leq v \leq v_u\}$.

On the other hand, if we consider the parameter uncertainty description in $\mathcal{F}$, a given output y_k maps at some specific time k over a set

$$\mathcal{V}_y = \left\{ v : v = \sum_{i=0}^{N_n} f_i \Lambda(y_k, \beta_i),\ f \in \mathcal{F} \right\}. \tag{16}$$

This situation is showed in Fig. 4. From this picture it is clear that the parameter set will match the uncertainty description of Section 2 if $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$. In this way, the point u_k is mapped onto $\mathcal{V}_u$ through $\mathcal{H}$. Then, since $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$, this point will be mapped in y_k through the inverse of $\mathcal{F}$. Then, it is only necessary to compute the parameter bounds to satisfy this condition. The nominal linear model parameters $\hat{h}_i$ can be written as a vector, by considering that the Laguerre basis $l_i(u_k)$ is a set of real numbers for each input u_k. Let $l(u_k)$ be the vector whose ith entry is the Laguerre basis $l_i(u_k)$. Then, the expression of the linear model is

$$\hat{v}(k) = \hat{h}^{\mathrm{T}} l(u_k). \tag{17}$$

In a similar way, the PWL basis $\Lambda(y_k, \beta_i)$ is a set of positive real numbers for each output y_k. $\Lambda(y_k)$ is the vector whose ith entry is the PWL basis $\Lambda(y_k, \beta_i)$. Then, the nonlinear model expression is:

$$v(k) = \hat{f}^{\mathrm{T}} \Lambda(y_k). \tag{18}$$

In the following, let us analyze the bounds on the parameters.

4.1. Uncertainty concentrated in the linear block

In this case, let us assume that the uncertainty is concentrated in the linear block.[3] We are looking for the uncertain linear model that maps the set of data $\mathbf{u}$ to the set $\mathbf{v} = \hat{f}^{\mathrm{T}}\Lambda(\mathbf{y})$. To define an uncertain model that allows us to describe the complete set of data, we should compute the set $\{h : h = \hat{h} + \delta^h, -h_i^l \leq \delta_i^h \leq h_i^u,\ i = 0, \ldots, N_l\}$. Now, since that the entries of $l(u_k)$ could be positive or negative, it is possible to split the vector $l(u_k)$ by defining $l^+(u_k) = \max(l(u_k), 0)$ and $l^-(u_k) = \min(l(u_k), 0)$. Then, forming the vector $\gamma = [-(l^-(u_k))^{\mathrm{T}}, (l^+(u_k))^{\mathrm{T}}]^{\mathrm{T}}$, we can compute the uncertainty bounds as

$$\min_{h^l, h^u} \sum_{i=0}^{N_l} \left(h_i^l + h_i^u\right)$$

subject to

$$\begin{aligned}
&\left[(h^l)^{\mathrm{T}}, (h^u)^{\mathrm{T}}\right]\gamma \geq e(k), \quad \text{if } \epsilon(k) \geq 0; k = 1, \ldots, N\\
&-\left[(h^u)^{\mathrm{T}}, (h^l)^{\mathrm{T}}\right]\gamma \leq e(k), \quad \text{if } \epsilon(k) \leq 0; k = 1, \ldots, N\\
&h_i^l, h_i^u \geq 0,
\end{aligned}$$

where

$$\epsilon(k) = \hat{f}^{\mathrm{T}}\Lambda(y_k) - \hat{h}^{\mathrm{T}} l(u_k). \tag{19}$$

4.2. Uncertainty concentrated in the nonlinear block

In this case, let us assume that the uncertainty is concentrated in the nonlinear stationary block [43]. Then, we are looking for the uncertain model that maps the set of data $\mathbf{y}$ to the set $\mathbf{v} = \hat{h}^{\mathrm{T}} l(\mathbf{u})$. Then, to define an uncertain model that allows us to describe the complete set of data, we should compute the set $\{f : f = \hat{f} + \delta^f, -f_i^l \leq \delta_i^f \leq f_i^u,\ i = 0, \ldots, N_n\}$. Now, since the entries of $\Lambda(y_k)$ are positive, we can compute the uncertainty upper bound as

$$\min_{f^u} \sum_{i=0}^{N_n} f_i^u$$

subject to

$$\begin{aligned}
&-(f^u)^{\mathrm{T}}\Lambda(y_k) \leq \epsilon(k), \quad k = 1, \ldots, N\\
&f_i^u \geq 0
\end{aligned}$$

and the lower bound as

$$\min_{f^l} \sum_{i=0}^{N_n} f_i^l$$

subject to

$$\begin{aligned}
&(f^l)^{\mathrm{T}}\Lambda(y_k) \geq \epsilon(k), \quad k = 1, \ldots, N\\
&f_i^l \geq 0.
\end{aligned}$$

4.3. Uncertainty in both the linear and nonlinear blocks

In this case, we consider the most general case, where the uncertainty is split into both the linear and the nonlinear blocks. Note that the intersection of the uncertainties in the linear and nonlinear models should be nonempty. This can be solved as:

[3] Note that in this paper we propose a parametric description for the uncertainty. However, there are other methods to characterize the uncertainty of LTI control systems, such as the frequency response approach [42].

Table 1
Neutralization parameters

Parameter	Value
x_{1i}	0.0012 mol HCL/l
x_{2i}	0.0020 mol NaOH/l
x_{3i}	0.0025 mol $NaHCO_3$/l
K_x	10^{-7} mol/l
K_w	10^{-14}mol^2/l^2
q_A	1 l/m
V	2.5 l

$$\min_{h^l,h^u,f^l,f^u}\left(\alpha\sum_{i=0}^{N_l}(h_i^l+h_i^u)+(1-\alpha)\sum_{i=0}^{N_n}(f_i^l+f_i^u)\right)$$

subject to

$$\left[(h^l)^{\mathrm{T}},(h^u)^{\mathrm{T}},(f^l)^{\mathrm{T}}\right]\begin{bmatrix}\gamma\\ \Lambda(y_k)\end{bmatrix}\geq e(k),\quad \text{if } \epsilon(k)\geq 0;\ k=1,\ldots,N$$

$$\left[-(h^u)^{\mathrm{T}},-(h^l)^{\mathrm{T}},-(f^u)^{\mathrm{T}}\right]\begin{bmatrix}\gamma\\ \Lambda(y_k)\end{bmatrix}\leq e(k),\quad \text{if } \epsilon(k)\leq 0;\ k=1,\ldots,N,$$

where the parameter $\alpha\in(0,1)$ is a selected factor which allows us to weight the uncertainty on the linear or nonlinear block.

5. Process description

To illustrate both the uncertainty characterization and the identification procedures, simulation results were obtained. The example consists of the neutralization reaction between a strong acid (HA) and a strong base (BOH) in the presence of a buffer agent (BX) [44,45]. A Wiener model has often been chosen in the literature for pH process representation. The linear block can be used to describe the mixing dynamics of the reagent streams inside the reactor, while the static nonlinearity stands for the titration curve, which can represent the mathematical relationship between the chemical species and the pH [3].

The neutralization herein considered takes place in a CSTR with a constant volume V. An acidic solution with a time-varying flow $q_A(t)$ of composition $x_{1i}(t)$ is neutralized by using an alkaline solution with flow $q_B(t)$ of known composition made up of base x_{2i} and buffer agent x_{3i}. For this specific case, under some assumptions [44], the dynamic behaviour of the process can be described by considering the state variables: $x_1=[A^-]$, $x_2=[B^+]$ and $x_3=[X^-]$. Then, the mathematical model of the process is:

$$\dot{x}_1=q_A/Vx_{1i}-(q_A+q_B)/Vx_1 \tag{20}$$

$$\dot{x}_2=q_B/Vx_{2i}-(q_A+q_B)/Vx_2 \tag{21}$$

$$\dot{x}_3=q_B/Vx_{3i}-(q_A+q_B)/Vx_3 \tag{22}$$

$$F(x,\xi)\equiv\xi+x_2+x_3-x_1-K_w/\xi-x_3/[1+(K_x\xi/K_w)]=0, \tag{23}$$

where $\xi=10^{-\mathrm{pH}}$. The parameters of the system are addressed in Table 1. Using this model a set of data is generated by simulating 2000 samples with a sample time $T_s=0.5$. A random signal uniformly distributed in [0, 1] is applied to the manipulated variable q_B, this input changes after every set of five samples. Before proceeding with the identification, the steady values are removed from the input ($q_B=0.5$) and output (pH = 7.7182) data, respectively.

In a first step, we compute a nominal Wiener Model as described in Section 3. We consider three Laguerre polynomials (i.e. $N_l=2$) with $a=0.7$ to represent the linear model. It should be remarked that, the present approach does not require prior knowledge of the real linear subsystem structure. As regards the nonlinear block, a PWL model with an 8-section partition was selected to describe the static gain. The identification is performed using a set of 1000 data, and the remaining data are used for validation. Fig. 5 shows a set of these results, restricted to 400 samples (half for identification and half for validation). Two curves are shown: the signal $v(k)$ as the output of the linear block and

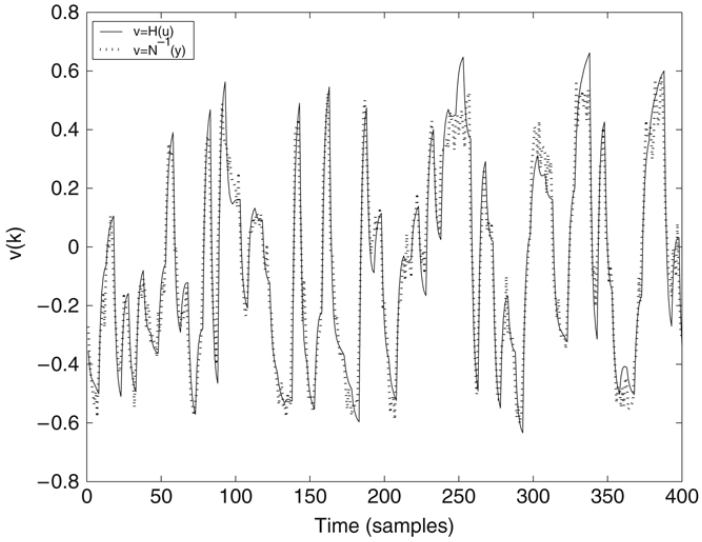

Fig. 5. Simulation for the nominal Wiener model.

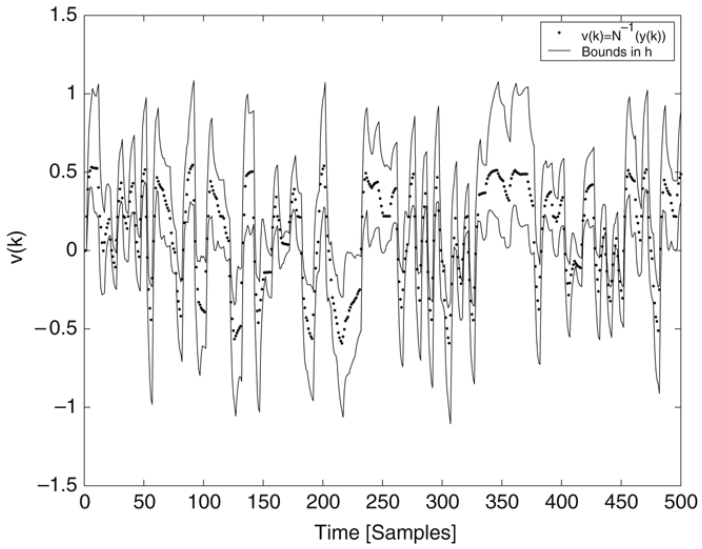

Fig. 6. Uncertainty in the linear parameters.

as the output of the inverse of the nonlinear block $N^{-1}(y(k))$. The parameters are:

$$\hat{h}^{\mathrm{T}} = \begin{bmatrix} 1 & -0.2022 & 0.1386 \end{bmatrix}$$
$$\hat{f}^{\mathrm{T}} = [-0.660 \quad -0.445 \quad -0.416 \quad -0.389 \quad -0.374 \quad -0.303 \quad -0.042 \quad 0.132 \quad 0.204 \quad 0.219 \quad 0.557]$$

for the linear and the nonlinear blocks, respectively.

In a second step, we assume that the uncertainty is concentrated in the linear block. From the physical point of view, this assumption involves the fact that the titration curve is unique. By solving the problem described in Section 4.1, the uncertainty (see Fig. 6) in the parameters is described by:

$$h^u = \begin{bmatrix} 0.9060 & 0.0879 & 0.6786 \end{bmatrix}$$
$$h^l = \begin{bmatrix} 0.8732 & 0.4189 & 0.0226 \end{bmatrix}.$$

A measure for this uncertainty is $error_h = \sum_{i=0}^{N_l} (h_i^u + h_i^u) = 2.9872$.

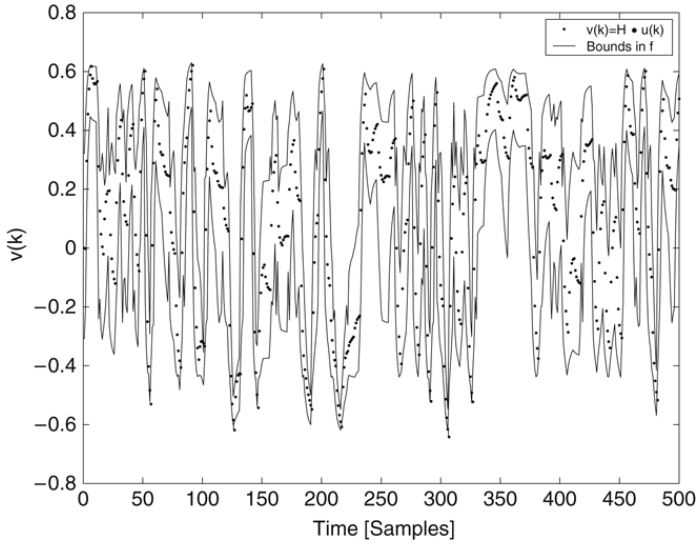

Fig. 7. Uncertainty in the nonlinear parameters.

The case in which the uncertain parameters are all concentrated in the nonlinear block is now considered. This case can be interpreted as if the whole source of uncertainty resides in the titration curve, i.e. the nonlinear gain of the process. Solving the problem of Section 4.2, the parameter bounds (see Fig. 7) are:

$$\begin{aligned} f^u &= [0.0000 \quad 0.1377 \quad 0.2601 \quad 0.0000 \quad 0.2738 \quad 0.3048 \\ &\quad\ 0.4046 \quad 0.0544 \quad 0.2952 \quad 0.2060 \quad 0.0798]^{\mathrm{T}} \\ f^l &= [0.0000 \quad 0.0835 \quad 0.0602 \quad 0.0740 \quad 0.0562 \quad 0.1354 \\ &\quad\ 0.2938 \quad 0.3556 \quad 0.2162 \quad 0.4788 \quad 0.0530]^{\mathrm{T}}. \end{aligned}$$

A measure for this uncertainty is $error_f = \sum_{i=0}^{N_n}(f_i^u + f_i^u) = 3.8232$.

Finally, let us consider the case with uncertainty in both blocks. Fig. 8 shows the functions $error_h$ and $error_f$ as functions of α. For example, for $\alpha = 0.5$ the parameter bounds (see Fig. 9) are:

$$\begin{aligned} f^u &= [0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0428 \\ &\quad\ 0.2192 \quad 0.0036 \quad 0.0000 \quad 0.0000 \quad 0.0000]^{\mathrm{T}} \\ f^l &= [0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \\ &\quad\ 0.0000 \quad 0.2474 \quad 0.0109 \quad 0.1978 \quad 0.0755 \quad 0.0000]^{\mathrm{T}} \\ h^u &= \begin{bmatrix} 0.6080 & 0.000 & 0.1573 \end{bmatrix} \\ h^l &= \begin{bmatrix} 0.3385 & 0.000 & 0.000 \end{bmatrix}. \end{aligned}$$

For this case, the norms of these uncertainties are $error_h = \sum_{i=0}^{N_l}(h_i^u + h_i^u) = 1.1037$ and $error_f = \sum_{i=0}^{N_n}(f_i^u + f_i^u) = 0.7973$. Note that even the sum of both errors ($error_h$ and $error_f$) is lower than the values that were previously calculated for the cases of uncertainty concentrated in only one of the blocks. The interpretation for this result, is that this last case (i.e. the one of distributed uncertainty) admits an additional degree of freedom, which allows the Wiener model to fit better the input–output collected data.

6. Conclusions

In this article, identification and robustness analysis of Wiener systems are considered. The main contribution of this work is a method for robust identification of both the linear and the nonlinear blocks present in an uncertain Wiener model. For this purpose, a parametric representation has been used in order to describe the model. The proposed approach accomplishes the parameter identification by solving an optimization problem. Once the identification is performed, a convex hull of parameters is obtained, with the fact that the overall data used in the identification can be

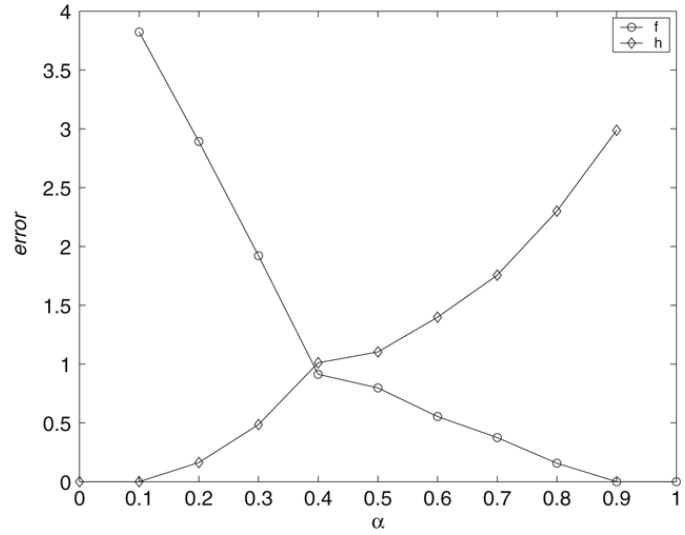

Fig. 8. Uncertainty norm as a function of parameter α.

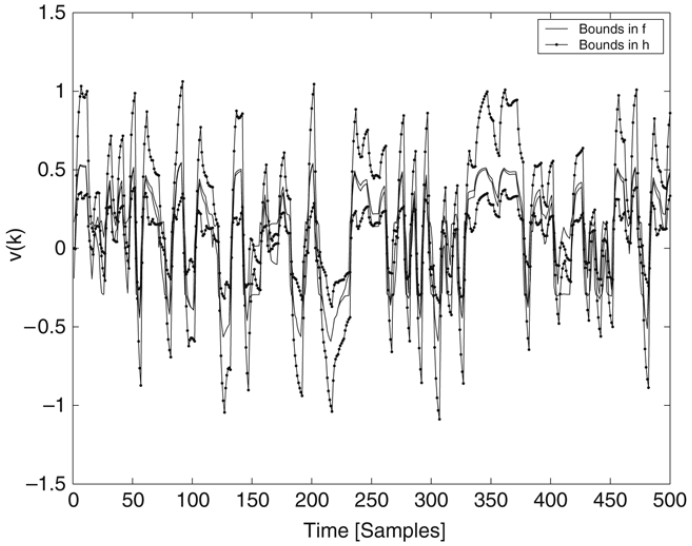

Fig. 9. Uncertainty in the linear and nonlinear parameters.

generated by the model's family associated to the set of parameters. Although the uncertain model herein proposed makes use of the Laguerre and PWL bases for the linear and nonlinear blocks, respectively, it must be remarked that the identification approach is also valid for any other parametric description of the model with uncertainty.

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