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DYNAMIC REGRESSION MODELS WITH STOCHASTIC TRENDS: AN APPLICATION TO THE UNEMPLOYMENT-GROWTH RELATION

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ABSTRACT

The form in which the national product affects the level of unemployment has become one of the most relevant macroeconomic subjects. A simple form of measuring the relation between the increase (decrease) of the national product and the decrease (increase) of the unemplyment rate is explained by the so called ``Okun Law", according to which it is not expected, except under very special conditions, that each point of increase of the product results in a point of increase of the employment and, consequently, a fall of one point in the unemployment rate. The relation between growth and unemployment is much more complex and can be resumed in the following equation

$$u_t - u_{t-1} = -\rho(g_t - c_t),$$

where u_t is the unemployment rate, g_t the percentage growth of the product and c_t represents the percentage growth of the product that it is needed to maintain the unemployment level of the previous period. One of our objetives is to estimate c_t .

In the econometric formulation of this equation, this work takes into consideration that, if we include a deterministic trend within the explicative variables, the resulting model will necessarilly be very restrictive in the sense that, any impulse no matter how intense it was, will not have effects in the long run, because everything will be captured by the given trend. The necessary flexibility is introduced by allowing the trend to evolve in time as a stochastic process, which in turn is equivalent to establishing the “stylized facts” associated with these time series.

Our approach is therefore focused on the intruduction of a stochastic trend in the regression model, seeking to take into account the structural changes that occurred during the most recent years in the Argentinian economy. From the statistical point of view, the key way for modeling a stochastic trend is the state space form. This methodology allows the unknown parameters to be estimated via the prediction error decomposition and the forecastings to be computed extending the Kalman filter. Moreover, a smoothing algorithm based on the Kalman approach can be used to provide an optimal estimator of the trend at each point of the sample period.

KEYWORDS

Growth; Kalman filter; Okun Law; State space; Smoother; Structural time series models; Stylized facts, Unemployment.

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1. INTRODUCTION

Since the product is determined by the action of production factors, every increase of the production requires, in principle, an increase in these factors, particularly the labour. Product movements affects, in principle, the unemployment rate. But the relationship between these two magnitudes is not as simple as to expect that each point of increase in the product implies a point of increase in the employment and, simultaneously, a drop of a point in the unemployment rate. In the United States of America (USA), Arthur Okun showed that when the economy is leaving a recession, the national product grows in a bigger proportion than the employment, and when it is entering a recession, the production decreases in a bigger proportion than the fall of the employment.

The conclusions of this work gave rise to what is actually known as the “Okun Law”. It is, in fact, a simple hypothesis of statistical regularity about the way in which the changes in the national product affect the level of employment and the rate of unemployment. In Okun's original work, the production grew about three per cent for each one per cent in the growing of the employment.

A common practice in many companies consists in maintaining the employees in their posts even when their services are not fully used. This is one of the economic factors that can explain the relation between the fall or growth in the product and the level of unemployment. That underuse of human resources is due to the simple fact that it is costly for the firms to contract new workers and even more to capacitate them for specific jobs. Therefore, when the demand of the goods produced by the firm decreases temporarily, the companies do not dismiss or suspend immediately the workers. This practice, which is usually called “employment stocked”, constitutes a sort of hidden unemployment: even when keeping their jobs, the workers do not contribute with plenitude to the production. Consequently, we can say that in recessions not all the workers really “work”.

Besides the interpretations the economic theory could give to explain the Okun Law, what it really does is basically to show the relation between the growth of the national product and the changes in the unemployment rate. The empirical evidence in different countries showed an inverse relation between these two variables, and in this work our objective is to estimate this relation in Argentina for the period 1980-2002.

2. THE OKUN RELATION

The following equation resumes the relation specified by the Okun Law and is the basis for our empirical work, aimed to use this formulation in the Argentinian case with data starting in 1980

$$u_t - u_{t-1} = -\rho(g_t - c_t), \tag{1}$$

where u_t is the unemployment rate and g_t the percentage growth of the product.

The component c_t in the equation represents the percentage growth of the product that it is needed to maintain the unemployment level of the previous period. This growth, necessary to avoid an increase of unemployment, is basically imputed to the increment in the labour force and to the productivity of this factor. Since the unemployment rate is given as the ratio between the amount of unemployed and the total of the labour force, an increment in this later, of one per cent say, implies that the product should grow at least one per cent if we want to keep the same unemployment rate as that of the former period. Similarly, the rate of growth of the product should be equal to the productivity of the labour in order that the unemployment rate remains stable[(1)].

In the case of the USA, for example, the sum of the growth rate of the active population and the labour productivity is, since 1960, equal to three per cent. In the increment of the active population, the increasing women participation in the labour market plays an important role. The women participation rate was in this country, during the 60's, equal to 40%, increasing up to 60% three decades later. Since the men participation rate had a slight decrease during this period, the observed increment in the global participation rate, which goes from less than 60% to more than 67%, is entirely due to the women advance in the global labour supply. In this country, during the 1950's one out of three women was in the labour force, while actually the participation is two out of three.

Some authors (see, for example, Blanchard, 1996, page 362) call the sum of the growth rates of the productivity and the labour force, "normal" growth rate of the product, introducing it in the formulation of the Okun Law as a constant (three per cent in the case of USA). In the case of Argentina, the great variability of both the labour force and the labour productivity during a period which contains a hyperinflationary crisis, a rapid stabilization and a recession, encourage us to treat this rate as variable, leaving the econometric model to determine its value. In equation (1) this "normal" growth rate is represented by c_t.

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(1) A long run vision of the economy shows without any ambiguity that the increse in the productivity is translated in a process which creates globally more employment than that it destroys it. This is the fundamental reason upon which the economic thought is place itself on the antipodes of the general belief, still well spreds out over the populations, that the growth in productivity as well as the technological advance of any king appear as the cause of unemployment. From the empirical point of view, no stable relation between productivity and unemployment has been probed valid in the short run [see, for example, Krugman (1994)].

Let us analyze the parameter ρ in equation (1). This parameter represents the excess in the growth of the product above the “normal” growth which is transferred to a decrease in the unemployment rate. For example, if its value is 0.5, this means that for each percentage point of economic growth above the normal, the unemployment rate will decreases half per cent. For the USA case mentioned before, if we assume the “normal” growth at about three per cent, a value of 0.5 for ρ means that it is needed an increment of about five per cent in the product to diminish one point the unemployment rate.

It is expected that $0 < \rho < 1$, basically for two reasons. Firstly, the firms do not adjust the employment one-to-one with the deviations of the observed growth in relation with the growth considered as normal. In general, this relation is less than one not only in periods of apogee but also during recession periods. A possible explanation for this is that the training of new workers results costly for the firm, therefore it is sometimes preferable to keep the actual team of workers, paying them for the extra time that they could be working or “stocking up” employment according to the moment of the business cycle and at least in the short run. Moreover, not all the employment depends directly upon the sales of the firm.

Secondly, an increase in the employment does not lead to a one-to-one decrease in the unemployment, because the rate of labour participation normally shows a constant growth. Not every new job is allocated to an unemployed. Some of these new workers come from the populations which was out of the labour force before or out of the economically active population; they are probably people who had abandoned in the past the search for a job but, in front of new and good perspectives in the labour market, they decide to start looking for a job again.

To finish this general comment about Okun Law, let us say that, in the econometric formulation of equation (1), we take into consideration that, if we include a deterministic trend within the explicative variables, the resulting model will necessarily be very restrictive, in the sense that any impulse of any intensity will not have effects in the long run, because everything will return to the given trend. The necessary flexibility is introduced by allowing the trend to evolve in time as a stochastic process, which in turn is equivalent to establishing the “stylized facts” associated with these time series. For such facts to be useful they should (i) be consistent with the stochastic properties of the data, and (ii) present meaningful information. However, many stylized facts reported in the literature do not fulfill these criteria. Particular, if the information is based on mechanically detrended series it can easily give a spurious impression of cyclical behaviour. The analysis based on autoregressive-integrated-moving average (ARIMA) models can also be misleading if such models are chosen primarily on grounds of parsimony (for a treatment of these ideas see Harvey and Jaeger, 1993).

Our approach is therefore focused in the introduction of a stochastic trend in the regression model, taking into account the structural changes that occurred during the most recent years in the Argentinian economy. From the statistical point of view, the key way for modeling a stochastic trend is the use of a structural time series model written in the state space form. This allows the unknown parameters to be estimated via the prediction error decomposition and the forecasting to be computed extending the Kalman filter. Moreover, a smoothing algorithm based on the Kalman approach can be used to provide an optimal estimator of the trend at each point of the sample period.

3. STRUCTURAL TIME SERIES MODELS

Let us consider a model of the form

$$y_t = \mu_t + \gamma_t + \mathbf{z}_t'\ddot{\mathbf{a}}_t + \varepsilon_t, \quad t = 1, \ldots, n, \tag{2}$$

where μ_t is the trend, γ_t is the seasonality, ε_t is an irregular component serially independent, normally distributed with mean zero and constant variance, i. e. $\varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$, $\mathbf{z}_t$ is a $p \times 1$ vector of observed explanatory variables some of them could be lagged values of the dependent variable as well as lagged values of exogenous variables and $\ddot{\mathbf{a}}_t$ es a $p \times 1$ vector of unknown parameters. If the vector $\ddot{\mathbf{a}}_t$ does not depend on the time, then $\ddot{\mathbf{a}}_t = \ddot{\mathbf{a}}_{t-1}$. Models of the form (2) are called structural time series models.

A stochastic formulation of the trend allows the level μ_t and the slope β_t to evolve over time. Then

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, \quad & \eta_t &\sim NID(0, \sigma_\eta^2), \\ \beta_t &= \beta_{t-1} + \zeta_t, & \zeta_t &\sim NID(0, \sigma_\zeta^2), \end{aligned} \tag{3}$$

where the errors ε_t, η_t and ζ_t are mutually independent at all time moments.

A stochastic formulation of the seasonality with period s is given by

$$\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{j,t}, \tag{4}$$

where $[x]$ represents the integer part of x and $\gamma_{j,t}$ is generated by

$$\begin{pmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{pmatrix} = \begin{pmatrix} \cos\lambda_j & \mathrm{sen}\,\lambda_j \\ -\mathrm{sen}\,\lambda_j & \cos\lambda_j \end{pmatrix} \begin{pmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{pmatrix}, \quad j = 1, \ldots, [s/2], \tag{5}$$

where $\lambda_j = 2\pi j / s$ is the frequency in radians, $\omega_{j,t}$ and $\omega_{j,t}^*$ are white noise mutually uncorrelated errors with zero mean and common variance σ_ω^2 for $j = 1, \ldots, [s/2]$ and $t = 1, \ldots, n$. For s even $[s/2] = s/2$, while for s odd $[s/2] = (s-1)/2$. For s even, the component for $j = s/2$ becomes

$$\gamma_{j,t} = \gamma_{j,t-1}\cos\lambda_j + \omega_{j,t}, \quad j = s/2. \tag{6}$$

The extent to which the level, μ_t, the slope, β_t, and the seasonality, γ_t, change over time is governed by the relative hyperparameters $q_\eta = \sigma_\eta^2 / \sigma_\varepsilon^2$, $q_\zeta = \sigma_\zeta^2 / \sigma_\varepsilon^2$ and $q_\omega = \sigma_\omega^2 / \sigma_\varepsilon^2$.

When $q_\zeta = 0$, we have that $\beta_t = \beta_{t-1} = \cdots = \beta$. Now, if β is different from zero, the trend is a random walk plus drift: $\mu_t = \mu_{t-1} + \beta + \eta_t$. In such a case, with $\alpha = \mu_0$ and combining these expressions in the corresponding first equation of (3), we easily verify that

$$\mu_t = \alpha + \beta t + \sum_{i=1}^{t} \eta_i, \quad t = 1, \ldots, n. \tag{7}$$

Here, the behaviour of μ_t is governed by two nonstationary components: a linear deterministic trend and the stochastic trend $\sum \eta_i$.

When $q_\zeta = 0$ and if $\beta = 0$, solving for μ_t with $\alpha = \mu_0$

$$\mu_t = \alpha + \sum_{i=1}^{t} \eta_i, \quad t = 1, \ldots, n. \tag{8}$$

This trend is usually known as “local level”. Notice that the successive η_t shocks have permanent effects on the $\{\mu_t\}$ sequence in that there is no decay factor on past values of η_{t-i}. Hence, μ_t is a stochastic trend.

Returning to the general trend presented in (3) we have firstly, solving for β_t,

$$\beta_t = \beta_0 + \sum_{i=1}^{t} \zeta_i, \quad t = 1, \ldots, n. \tag{9}$$

Then, using this solution we can write μ_t as

$$\mu_t = \mu_0 + \beta_0 t + \sum_{j=1}^{t-1} (t-j)\zeta_j + \sum_{i=1}^{t} \eta_i, \quad t = 1, \ldots, n. \tag{10}$$

Here, each element of the $\{\mu_t\}$ sequence contains a deterministic trend and a stochastic trend. It can be observed that the stochastic part of the coefficients on time depends on the past realizations of the $\{\zeta_t\}$ sequence. These coefficients can be positive for some values of *t* and negative for others.

In the limit case when the hyperparameters q_η and q_ζ are both zero, the trend is deterministic and has the form $\alpha + \beta t$ with $\alpha = \mu_0$.

The statistical treatment of the structural time series models (2) is based on the state space form (SSF), the Kalman filter and the associated smoother.

3.1. THE STATE SPACE FORM

All linear time series models have a state space representation. This representation relates the disturbance vector $\{\text{å}_t\}$ to the observation vector $\{\boldsymbol{y}_t\}$ via a Markov process $\{\text{á}_t\}$. A convenient expression of the state space form is

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{Z}_t \text{á}_t + \text{å}_t, & \text{å}_t &\sim N(\mathbf{0}, \boldsymbol{H}_t), \\ \text{á}_t &= \boldsymbol{T}_t \text{á}_{t-1} + \boldsymbol{R}_t \text{ç}_t^*, & \text{ç}_t^* &\sim N(\mathbf{0}, \boldsymbol{Q}_t), \end{aligned} \tag{11}$$

for $t = 1, ..., n$, where $\boldsymbol{y}_t$ is a $r \times 1$ vector of observations and á_t is the $m \times 1$ unobservable vector called state vector. The matrices $\boldsymbol{Z}_t$, $\boldsymbol{T}_t$, $\boldsymbol{R}_t$, $\boldsymbol{H}_t$, and $\boldsymbol{Q}_t$ have dimensions $r \times m$, $m \times m$, $m \times k$, $r \times r$ and $k \times k$ respectively, and their unknown elements, if any, are place in the hyperparameter vector ø which can be estimated by maximum likelihood. The error terms å_t and ç_t^* are assumed serially independent and independent among them at all time moments. The matrices $\boldsymbol{Z}_t$ and $\boldsymbol{T}_t$ may depend upon $\boldsymbol{y}_1, \boldsymbol{y}_2, ..., \boldsymbol{y}_{t-1}$. The first equation of (11) is usually called the measurement equation and the second one, the transition equation.

The initial state vector á_0 is assumed to be $N(\boldsymbol{a}_0, \boldsymbol{P}_0)$ independently of $\text{å}_1, \ldots, \text{å}_n$ and $\text{ç}_1^*, \ldots, \text{ç}_n^*$, where $\boldsymbol{a}_0$ and $\boldsymbol{P}_0$ are known. If á_t is nonstationary then á_0 is thought of having a diffuse prior, that is the variance matrix $\boldsymbol{P}_0$ is equal to $\theta \boldsymbol{I}$, where θ is a scalar which tends to infinity; see Harvey (1989, subsection 3.3.4) and Abril (1999, section 3.3).

3.2. KALMAN FILTER

The object of the Kalman filter is to update our knowledge of the system each time a new observation $\boldsymbol{y}_t$ is brought in. Once the model has been put in the state space form, the way is opened for the application of an important number of algorithms. In the centre of these is the Kalman filter. This filter is a recursive procedure for computing the optimal estimator of the state vector at time t, based on the information available up to this time t. This information consists of the observations up to and including $\boldsymbol{y}_t$.

The Kalman filter enables the estimate of the state vector to be continually updated as new observations become available. At first sight, the value of such a procedure in economic applications would appear to be limited. New observations tend to appear at rather less frequent intervals and the emphasis is on making predictions of future observations based on a given sample. The state vector dos not always have an economic interpretation and, in cases where it does, it is more appropriate to estimate its value at a particular point in time using all the information in the sample, not just a part of it. These two problems are known as prediction and smoothing respectively. It turns out that the Kalman filter provides the basis for the solution of both of them.

Another reason for the central role of the Kalman filter is that when the disturbances and the initial state vectors are normally distributed, it enables the likelihood function to be calculated via what is known as the prediction error decomposition. This opens the way for the estimation of any unknown parameters in the model. It also provides the basis for statistical testing and model specification.

The derivation of the Kalman filter given below rests on the assumption that the disturbances and the initial state vector are normally distributed. A standard result on the multivariate normal distribution is then used to show how it is possible to calculate recursively the distribution of $\acute{\mathbf{a}}_t$ conditional on the information set at time t, for all t from 1 to n. These conditional distributions are themselves normal and hence are completely specified by their means and variance matrices. It is these quantities which the Kalman filter computes.

In the Gaussian state space model, the Kalman filter evaluates the minimum mean squared error estimator of the state vector $\acute{\mathbf{a}}_{t+1}$ using the set of observations $\boldsymbol{Y}_t = \{\boldsymbol{y}_1, ..., \boldsymbol{y}_t\}$, denoted $\boldsymbol{a}_{t+1} = E(\acute{\mathbf{a}}_{t+1} | \boldsymbol{Y}_t)$, and the corresponding variance matrix $\boldsymbol{P}_{t+1} = Var(\acute{\mathbf{a}}_{t+1} | \boldsymbol{Y}_t)$, for $t = 1, ..., n$. The Kalman filter is given by

$$\begin{aligned} \boldsymbol{v}_t &= \boldsymbol{y}_t - \boldsymbol{Z}_t \boldsymbol{a}_t, \quad & \boldsymbol{F}_t &= \boldsymbol{Z}_t \boldsymbol{P}_t \boldsymbol{Z}_t' + \boldsymbol{H}_t, \\ & & \boldsymbol{K}_t &= \boldsymbol{T}_{t+1} \boldsymbol{P}_t \boldsymbol{Z}_t' \boldsymbol{F}_t^{-1}, \\ \boldsymbol{a}_t &= \boldsymbol{T}_t \boldsymbol{a}_{t-1} + \boldsymbol{K}_{t-1} \boldsymbol{v}_{t-1}, \quad & \boldsymbol{P}_t &= \boldsymbol{T}_t \boldsymbol{P}_{t-1} \boldsymbol{L}_{t-1}' + \boldsymbol{R}_t \boldsymbol{Q}_t \boldsymbol{R}_t', \quad t = 1, \ldots, n, \end{aligned} \tag{12}$$

where $\boldsymbol{L}_{t-1} = \boldsymbol{T}_t - \boldsymbol{K}_{t-1} \boldsymbol{Z}_{t-1}$, with the initialization $\boldsymbol{K}_0 = \mathbf{0}$.

The derivation of the Kalman recursions can be found in Anderson and Moore (1979), Harvey (1989) and Abril (1999). The limiting case $\boldsymbol{P}_0 = \theta \boldsymbol{I}$, where $\theta \to \infty$, can be handled using a relatively straightforward modification of the Kalman filter as proposed by Ansley and Kohn (1985, 1990) and developed further by Koopman (1997). Other treatments of the limiting case are given by de Jong (1991) and Snyder and Saligari (1996). The one-step ahead prediction error of the observation vector is $\boldsymbol{v}_t = \boldsymbol{y}_t - E(\boldsymbol{y}_t | \boldsymbol{Y}_{t-1})$ with variance matrix $\boldsymbol{F}_t = Var(\boldsymbol{y}_t | \boldsymbol{Y}_{t-1}) = Var(\boldsymbol{v}_t)$. The output of the Kalman filter is used to compute the log-likelihood function $\ell(\boldsymbol{y};\, \mathbf{ø})$ in terms of the one-step ahead prediction, conditional on the hyperparameter vector $\mathbf{ø}$, as given, apart from a constant, by

$$\ell(\boldsymbol{y};\, \mathbf{ø}) = -\frac{nr}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{n}\log|\boldsymbol{F}_t| - \frac{1}{2}\sum_{t=1}^{n}\boldsymbol{v}_t' \boldsymbol{F}_t^{-1} \boldsymbol{v}_t. \tag{13}$$

Numerical maximization of $\ell(\boldsymbol{y};\, \mathbf{ø})$ with respect to the hyperparameter vector $\mathbf{ø}$, yields the maximum likelihood estimator $\tilde{\mathbf{ø}}$.

Once the hyperparameters have been estimated, the filter is used for obtaining one-step ahead prediction errors, which enable to calculate diagnostic statistics for normality, serial correlation and goodness-of-fit.

When the normality assumption is dropped, there is no longer any guarantee that the Kalman filter will give the conditional mean of the state vector. However, it is still an optimal estimator in the sense that it minimizes the mean square error within the class of all linear estimators.

3.3. SMOOTHING

The work of de Jong (1988, 1989), Kohn and Ansley (1989) and Koopman (1993) leads to a smoothing algorithm from which different estimators can be computed based on the full sample $\boldsymbol{Y}_n$. Smoothing takes the form of a backwards recursion, giving

$$\begin{aligned} \boldsymbol{u}_t &= \boldsymbol{F}_t^{-1}\boldsymbol{v}_t - \boldsymbol{K}_t'\boldsymbol{r}_t, \qquad \boldsymbol{M}_t = \boldsymbol{F}_t^{-1} + \boldsymbol{K}_t'\boldsymbol{N}_t\boldsymbol{K}_t, \qquad t = n, \ldots, 1, \\ \boldsymbol{r}_{t-1} &= \boldsymbol{Z}_t'\boldsymbol{F}_t^{-1}\boldsymbol{v}_t + \boldsymbol{L}_t'\boldsymbol{r}_t, \quad \boldsymbol{N}_{t-1} = \boldsymbol{Z}_t'\boldsymbol{F}_t^{-1}\boldsymbol{Z}_t + \boldsymbol{L}_t'\boldsymbol{N}_t\boldsymbol{L}_t, \end{aligned} \tag{14}$$

where $\boldsymbol{r}_n = \boldsymbol{0}$ and $\boldsymbol{N}_n = \boldsymbol{0}$. The recursions require memory space for storing the Kalman output $\boldsymbol{v}_t$, $\boldsymbol{F}_t$ and $\boldsymbol{K}_t$, for $t = 1, \ldots, n$. The matrices $\boldsymbol{M}_t$ and $\boldsymbol{N}_{t-1}$ constitute the estimated variances of $\boldsymbol{u}_t$ and $\boldsymbol{r}_{t-1}$, respectively. The series $\{\boldsymbol{u}_t\}$ will be referred to as *smoothing errors*. As we will see later, the smoothing quantities $\boldsymbol{u}_t$ and $\boldsymbol{r}_t$ play a pivotal role in the construction of diagnostic tests for outliers and structural breaks. The smoother can be used to compute the smoothed estimator of the disturbance vector of the transition equation $\hat{\boldsymbol{\varsigma}}_t^* = E(\boldsymbol{\varsigma}_t^* | \boldsymbol{Y}_n)$, that is

$$\boldsymbol{\varsigma}_t^* = \mathrm{Q}_t\mathrm{R}_t'\mathrm{r}_{t-1}, \quad \mathrm{Var}(\boldsymbol{\varsigma}_t^*) = \mathrm{Q}_t\mathrm{R}_t'\mathrm{N}_{t-1}\mathrm{R}_t\mathrm{Q}_t, \quad \mathrm{t} = \mathrm{n}, \ldots, 1, \tag{15}$$

where $\boldsymbol{r}_{t-1}$ satisfies (14); see Koopman (1993) and Abril (1999, section 3.4). The smoothed estimator of the state vector $\hat{\mathbf{a}}_t = E(\acute{\mathbf{a}}_t | \boldsymbol{Y}_n)$ is constructed *via* the simple forward recursion

$$\hat{\mathbf{a}}_t = \mathrm{T}_t\hat{\mathbf{a}}_{t-1} + \mathrm{R}_t\hat{\boldsymbol{\varsigma}}_t^*, \quad \mathrm{t} = 1, \ldots, \mathrm{n}-1, \tag{16}$$

together with $\hat{\mathbf{a}}_n = \boldsymbol{a}_n + \boldsymbol{P}_n\boldsymbol{Z}_n'\boldsymbol{F}_n^{-1}\boldsymbol{v}_n$. The starting value is $\hat{\mathbf{a}}_0 = \boldsymbol{a}_0 + \hat{\boldsymbol{\varsigma}}_0^*$ with $\hat{\boldsymbol{\varsigma}}_0^* = \boldsymbol{P}_0\boldsymbol{r}_0$. A more elaborate algorithm for computing the smoothed state vector, including the evaluation of its covariance matrix, is given by de Jong (1988, 1989) and Kohn and Ansley (1989). Finally, the output of the smoother can also be used to compute the exact score for hyperparameters; see Koopman and Shephard (1992).

Finally, the results of the smoother can also be used to maximize the function $\ell(\boldsymbol{y}; \boldsymbol{ø})$ defined in (13), with respect to the elements of the hyperparameter vector $\boldsymbol{ø}$.

The smoother is used to estimate unobserved components such as the trend and the seasonality, and for calculating diagnostic statistics useful to detect outliers and structural breaks.

3.4. OUTLIERS AND STRUCTURAL BREAKS

An outlier is an observation, which is not consistent with a model which is thought to be appropriate for the overwhelming majority of the observations. It can be captured by a dummy explanatory variable in the measurement equation [equation (2)], known as an impulse intervention variable, which takes the value one at the time of the outlier and zero elsewhere.

A structural break occurs when the level of the series shifts up or down, usually because of some specific event. It is modelled by a step intervention variable in the measurement equation [equation (2)] which is zero before the event and one on the event and after. Alternatively, it can be modelled by a dummy explanatory variable in the corresponding transition equation [equation (3)] which takes the value one at the time of the structural break in the level and zero elsewhere.

A structural break in the slope can be modelled by a staircase intervention in the measurement equation [equation (2)] which is a trend variable taking the values, 1, 2, 3, ..., starting in the period of the break. Alternatively, it can be modelled by a dummy explanatory variable in the corresponding transition equation [equation (3)] which takes the value one at the time of the structural break in the slope and zero elsewhere.

The concept of outliers and structural breaks apply quite generally. However, it is helpful for what follows to note that the level and slope breaks can be viewed in terms of impulse interventions applied to the level and slope equations of the model defined in (2), (3), (4) and (5). The structural framework also suggests that it may sometimes be more natural to think of an outlier as an unusually large value for the irregular disturbance. This leads to the notion of a level shift arising from an unusually large value of the level disturbance while a slope break can be thought of as a large disturbance to the slope component. Thus interventions can be seen as fixed or random effects, however, the random effects approach is more flexible. For example, introducing an outlier intervention at $t = \tau$ is equivalent to regarding the irregular variance at this point as being infinity. By using a large finite variance, we can ensure that the observation y_τ is downweighted without being removed altogether.

Viewing intervention effects as random is consistent with the representation of a stochastic trend in the equations (2), (3), (4) and (5). In this model the level and slope components are subject to random shocks at each point in time. When such movements are abnormally large, increasing the variance of the relevant disturbance or including an intervention variable may be appropriate.

Suppose that we want to test for an outlier at time $t = \tau$. It can be shown (see, for example, Abril, 1997; de Jong, 1989; and de Jong and Penzer, 1997a) that the smoothing errors, $\boldsymbol{u}_t$, and their unconditional variances, $\boldsymbol{M}_t$, at $t = \tau$, constitute the basic elements for the construction of the tests. In fact, the standardization of the $\boldsymbol{u}_t$'s, called the *standardized smoothing errors*, constitute the test statistics for testing for an outlier at any point in the sample. Fortunately, the Kalman algorithm gives the values of these standardized smoothing errors for all time periods so only one pass is needed to produce them. These are valid tests for cases in which the outliers are produced by both a fixed effect model as well as a random effect model. The STAMP (Koopman, Harvey, Doornik and Shephard, 1995) package computes routinely the values of these statistics. They are known there with the name *irregular auxiliary residuals*. This terminology was introduced by Harvey and Koopman (1992).

For the detection of structural breaks, de Jong and Penzer (1997a) showed that all that is required is to have the model set up in state space form in such a way that the level shift can be introduced by a pulse intervention somewhere in the transition equation. As observed earlier, a structural time series model with a level or trend component is of such a form. Given such a setup, the Kalman filter and smoother produces directly the test statistic as the element of $\boldsymbol{r}_t$ in the position corresponding to the pulse intervention. Its variance is automatically available from $\boldsymbol{N}_t$ in (14). The standardization then leads to the *level auxiliary residuals* which constitute the values of the tests used for detecting shifts in the level of a series.

As in the case of the level, the test statistics for changes in the slope are obtained from the appropriate component of $\boldsymbol{r}_t$ computed by the Kalman filter and smoother. Its variance is automatically available from $\boldsymbol{N}_t$. The standardization then leads to the *slope auxiliary residuals* which constitute the values of the tests used for detecting changes in the slope of a series.

The auxiliary residuals are smoothed estimations of the irregular, level and slope disturbances. Although they are serially correlated and correlated among them, they play an important role in the sense that they split up portions of information which usually is mixed up in the innovations residuals. In particular, they are useful for detecting and distinguishing between outliers and structural changes. In order to make the relevant tests, it is possible to show that the statistics follow approximately a normal distribution (see, for example, Abril, 1997; and Koopman, Harvey, Doornik and Shephard, 1995).

3.5. STATE SPACE REPRESENTATION OF THE STRUCTURAL MODEL

The structural model defined in equations (2), (3), (4) and (5) has a state space representation like that given in (11). In this case, and assuming that the vector $\ddot{\mathbf{a}}_t$ does not depend on the time, y_t is 1×1, the $1 \times (2[s/2] + 2 + p)$ order matrix $\boldsymbol{Z}_t$ results to be

$$\boldsymbol{Z}_t = \begin{pmatrix} 1 & 0 & \mathbf{1}' & \mathbf{0}' & \boldsymbol{z}_t' \end{pmatrix}, \qquad (17)$$

where $\boldsymbol{z}_t'$ was defined in (2), $\mathbf{1}$ is a column vector of order $[s/2]$ with all its elements equal to 1 and $\mathbf{0}$ is a column vector of order $[s/2]$ with all its elements equal to 0. The 1×1 matrix $\boldsymbol{H}_t$ turns out to be equal to σ_ε^2. The state vector $\acute{\mathbf{a}}_t$ of order $(2[s/2] + 2 + p)$ is

$$\acute{\mathbf{a}}_t = \begin{pmatrix} \mu_t & \beta_t & \gamma_{1,t} & \cdots & \gamma_{[s/2],t} & \gamma^*_{1,t} & \cdots & \gamma^*_{[s/2],t} & \ddot{\mathbf{a}}_t' \end{pmatrix}', \qquad (18)$$

where $\ddot{\mathbf{a}}_t = \ddot{\mathbf{a}}$ for all t. The $(2[s/2] + 2 + p) \times (2[s/2] + 2 + p)$ order matrix $\boldsymbol{T}_t$ and the $(2[s/2] + 2 + p) \times (2[s/2] + 2)$ order matrix $\boldsymbol{R}_t$ are given by

$$\boldsymbol{T}_t = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{C} & \boldsymbol{B} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_p \end{pmatrix}, \quad \boldsymbol{R}_t = \begin{pmatrix} \boldsymbol{I}_{2[s/2]+2} \\ \boldsymbol{0} \end{pmatrix}, \tag{19}$$

where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \tag{20}$$

the matrix $\boldsymbol{B}$ is diagonal of order $[s/2] \times [s/2]$ with diagonal elements equal to $\cos\lambda_j$, $\lambda_j = 2\pi\, j/s$, $j = 1, ..., [s/2]$, the matrix $\boldsymbol{C}$ is diagonal of order $[s/2] \times [s/2]$ with diagonal elements equal to $\sin\lambda_j$, $\lambda_j = 2\pi\, j/s$, $j = 1, ..., [s/2]$, $\boldsymbol{0}$ represents a vector or a matrix with all its elements equal to zero and $\boldsymbol{I}_x$ represents a x-order identity matrix. Finally, the $(2[s/2] + 2) \times 1$ order column vector $\boldsymbol{\varsigma}_t^*$ and the $(2[s/2] + 2) \times (2[s/2] + 2)$ order diagonal matrix $\boldsymbol{Q}_t$ are

$$\boldsymbol{\varsigma}_t^* = \begin{pmatrix} \eta_t \\ \zeta_t \\ \omega_{1,t} \\ \vdots \\ \omega_{[s/2],t} \\ \omega_1^* \\ \vdots \\ \omega_{[s/2],t}^* \end{pmatrix}, \quad \boldsymbol{Q}_t = \begin{pmatrix} \sigma_\eta^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_\zeta^2 & 0 & \cdots & 0 \\ 0 & 0 & \sigma_\omega^2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \sigma_\omega^2 \end{pmatrix}. \tag{21}$$

Starting values for the Kalman filter are computed from the first $(2[s/2] + 2 + p)$ observations and the likelihood function is then evaluated in terms of the prediction errors from $(2[s/2] + 2 + p) + 1$ to n. By including the parameters $\ddot{\mathbf{a}}$ in the state vector, they are effectively concentrated out of the likelihood function. As a result the likelihood function only needs to be maximized numerically with respect to the relative hyperparameters $q_\eta = \sigma_\eta^2/\sigma_\varepsilon^2$, $q_\zeta = \sigma_\zeta^2/\sigma_\varepsilon^2$ and $q_\omega = \sigma_\omega^2/\sigma_\varepsilon^2$. Estimation of the hyperparameters can be carried out by maximum likelihood either in the time domain or the frequency domain. Once this has been done, estimates of the trend, seasonal and irregular components as well as the coefficients of the explanatory variables are obtained from a smoothing algorithm. These calculations can be carried out very rapidly on a PC using the STAMP package (see Koopman, Harvey, Doornik and Shephard, 1995).

4. UNEMPLOYMENT AND GROWTH

Our basic data are the unemployment rate and the Gross National Product (GNP) of Argentina from 1980 to 2002, inclusive. The data were taken from the “Economic

Report" published by the Ministry of Economy and Public Work and Services of Argentina (2002). It has to be noted that the unemployment rate is estimated twice a year, they usually corresponds to the months of May and October, but for some years the first estimation corresponds to April and the second to November. On the other hand, the GNP is calculated at the end of every quarter of each year. Therefore and in order that the measurement periods of the data coincide, we adjusted the GNP in such a way that it agrees with the measurement period of the unemployment rate. The last data available corresponds to May 2002. Figure 1 shows the unemployment rate for the period 1980-2002 and Figure 2 shows the GNP centred at the moment of measurement of the unemployment rate for the same period.

According to equation (1), the dependent variable, which will be denoted as y_t, is equal to the first difference of the unemployment rate and the explanatory variable is the growth of the economy which will be denoted as g_t. The growth is defined as the percentage change in the GNP. Figure 3 shows the first differences of the unemployment for the considered period and Figure 4 shows the growth of the economy for the same period.

Initially, we estimated the basic structural model given in equations (2), (3), (4) and (5) written in the state space form given in (11) with the dependent variable y_t equals to the first differences of the unemployment rate and the explanatory variable equals to the growth of the economy denoted by g_t.

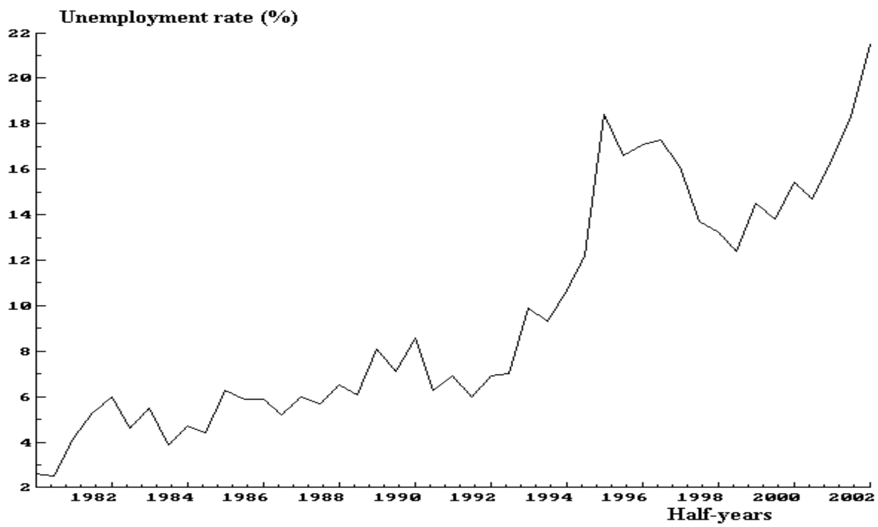

Figure 1: Unemployment rate. Bi-annual values from 1980 to 2002

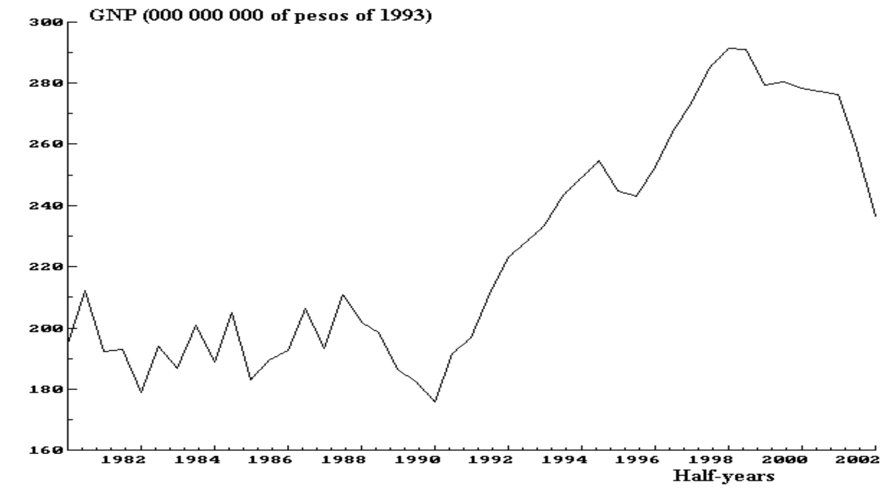

Figure 2: Gross National Product of Argentina. Bi-annual values, at prices of 1993, from 1980 to 2002

As a result of this preliminary estimation we detected that $q_{\zeta} = 0$ and $\beta = 0$, leaving the trend to be equal to the first equation of (3) with $\beta_t = 0$ for all *t*. A trend of this kind is known as local level. We found out as well, based on the tests presented in the previous section, an outliers in the first period of 1995, and that there exist two level shift, the first of them in the second period of 1991 and the second one, in the second period of 1995.

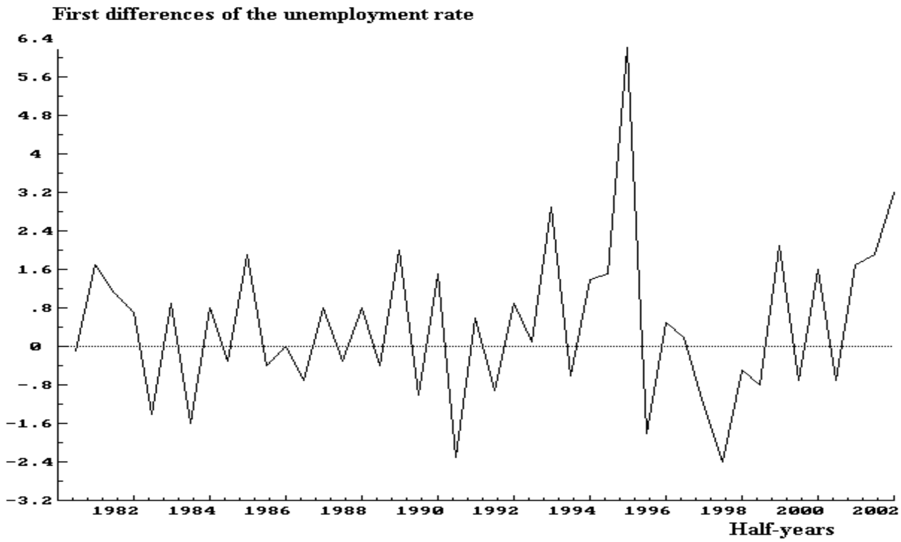

Figure 3: First differences of the unemployment rate. 1980-2002

Introducing interventions for the level, one in the second period of 1991 and another in the second period of 1995, an irregular intervention in the first period of 1995, and deleting the slope of the trend, we made the final estimation with 44 observations, and we obtained

$$y_t = \tilde{\mu}_{t/n} - 0.161449 g_t + \tilde{\gamma}_{t/n} + 3.2925 D_t + \tilde{\varepsilon}_{t/n} \ , \qquad (22)$$

where t = 1980-2, ..., 2002-1, noting that 1980-2 means the second half of 1980 and so on, n = 2002-1; $\tilde{\mu}_{t/n}$, $\tilde{\gamma}_{t/n}$ and $\tilde{\varepsilon}_{t/n}$ are the smoothed estimates of the trend, the seasonality and the irregular component at time t and based on the n observations. Moreover, as we will see later,

$$\tilde{c}_t = \frac{\tilde{\mu}_{t/n}}{0.161449} \ , \qquad (23)$$

where $\tilde{c}_t$ is the estimated "normal" growth rate of the product and their values are given in Table 1. D_t is equal to one when t = 1995-1 and zero otherwise, and

$$\begin{aligned} \tilde{\gamma}_{t/n} &= 0.80375, \quad \text{for the first half of the years,} \\ &= -0.80375, \quad \text{for the second half of the years.} \end{aligned} \qquad (24)$$

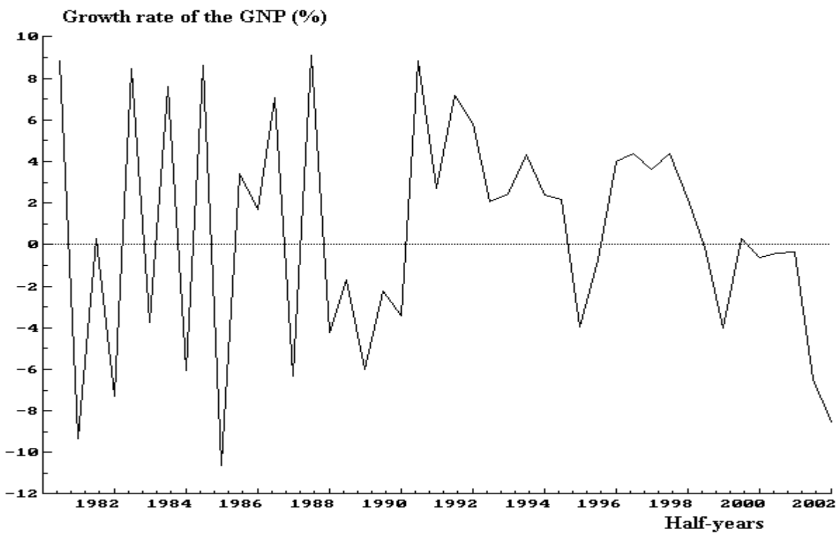

Figure 4: Growth rate of the GNP of Argentina. 1980-2002

It has to be noted that, except the seasonality, all the estimations of (22) are highly significant. Figure 5 shows the components of (22) with the original series of first differences of the unemployment rate for the period under consideration as the dependent variable, i. e. it shows the estimates of the trend, the trend plus the explanatory variables, the seasonality and the irregular component.

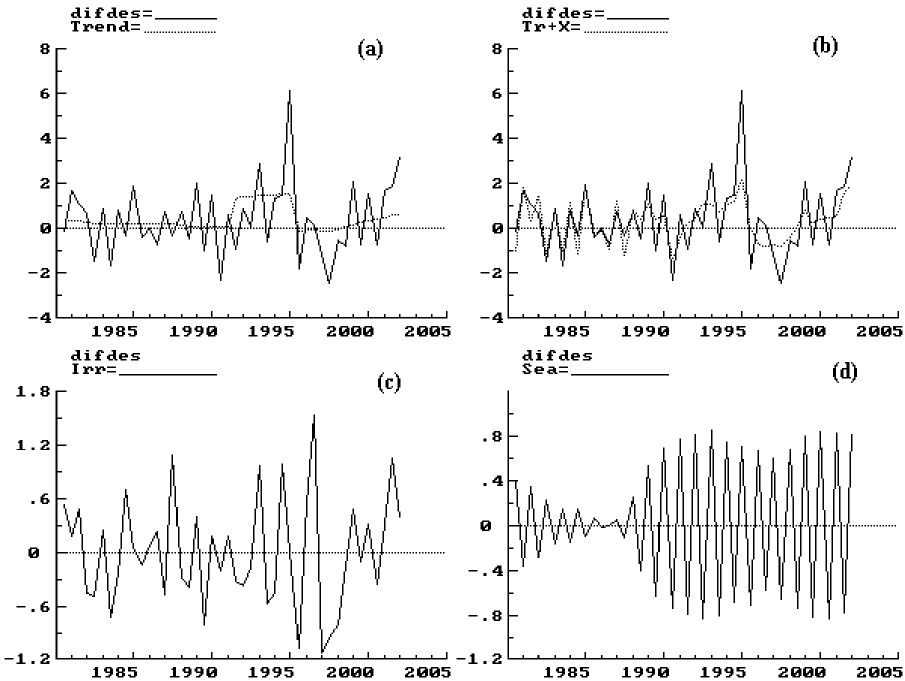

Figure 5: First differences of the unemployment rate. Original series [full line of (a) and (b)], estimated trend [dotted line of (a)], estimated trend plus explanatory variables [dotted line of (b)], estimated irregular component (c) and estimated seasonality (d)

Values of relevant statistics for measuring the goodness of fit are: the coefficient of determination $R_D^2 = 0.8763$; the values of the statistics N_{BS} = 0.4901 of Bowman and Shenton (1975) and N_{DH} = 0.2382 of Doornik and Hansen (1994) for testing the normality of the residuals of the adjusted model; the value of the Durbin-Watson statistic d = 2.196 for testing serial correlation in the residuals of the adjusted model and the value of the Box-Ljung statistic $Q(8, 6)$ = 4.798 for testing the hypothesis that the residuals are white noise.

The interpretation of R_D^2 is similar to that of the coefficient of determination in a usual regression, but its calculation, which can be seen in Koopman, Harvey, Doornik and Shephard (1995), is somehow different from this last one. We observe that the value of R_D^2 is high, which means an adequate fit. The statistics N_{BS} and N_{DH} have a χ^2 with two degrees of freedom and their values lead us to accept the hypothesis of normality of the residuals of the fitted model. The Durbin-Watson d statistic directs us to accept the hypothesis of lack of serial correlation of these residuals. The statistic $Q(8, 6)$ is based on the first eight residuals autocorrelations and has a χ^2 distribution with six degrees of freedom, its value leads us to accept the hypothesis that the residuals are white noise.

In Figure 6 we can see the residuals of the fitted model with its 95% confidence band, the correlogram also with its 95% confidence band, the periodogram, the estimated spectral density and the frequency distribution of these residuals. With the arguments given above and the observation of this figure we can conclude that the fitting is highly satisfactory.

Similar analyses were made with the residual of the irregular component and with the residuals of the trend. In both cases we accept the null hypothesis of normality. These can be seen graphically in Figure 7, where it is observed, on the upper part, the irregular auxiliary residuals with the 95% confidence band and its frequency distribution, and on the lower part, the level (trend) auxiliary residuals with the 95% confidence band and its frequency distribution.

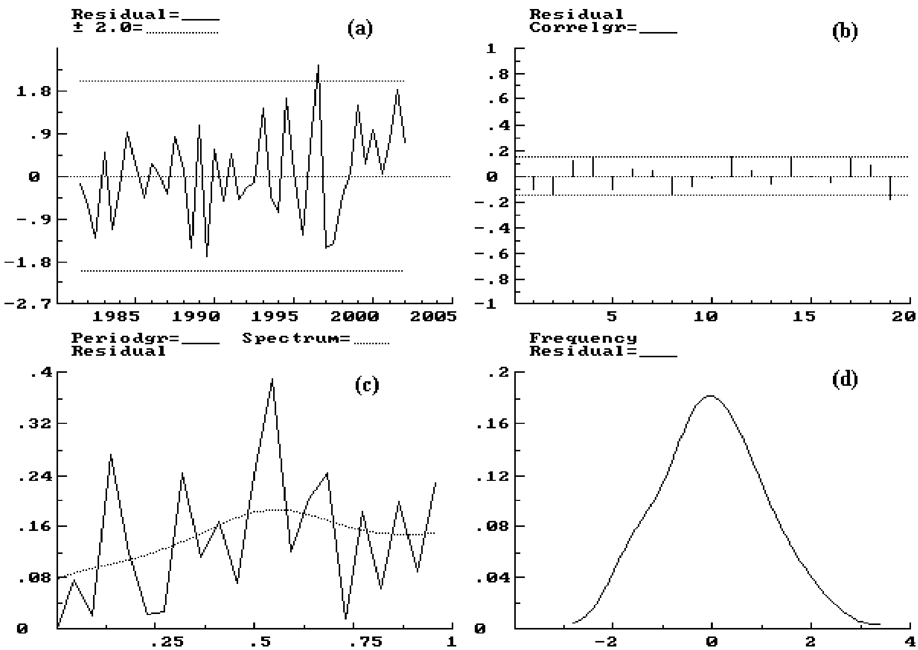

Figure 6: Residuals of the adjusted model in (22). Residuals with 95% confidence band (a), correlogram of the residuals with 95% confidence band (b), periodrogram (full line) and spectral density (dotted line) of the residuals (c) and frequency distribution of the residuals (d)

In order to determine the ability of the fitted model (22) to perform predictions, the last four observations were predicted and some tests were carry out. The value of Chow's statistics within the sample period turn out to be 1.4660, which is significantly lower than the value of the $F(4, 38)$ distribution, pointing out that the model is well specified for prediction purposes. On the other hand, for testing the constancy of the mean of the process y_t over the prediction period, the CUSUM was calculated, giving a value of 1.7773, which is within the acceptance region of the hypothesis of stable mean and it is based on a t distribution with 38 degrees of freedom. All of these can be seen in Figure 8, where it is observed the series y_t, the prediction of its last four values together with the 95% confidence bands for the predictions and for the prediction residuals, and the corresponding CUSUM for the predictions with its 90% confidence band. Clearly, we see that the performance of the model seems to be satisfactory for making predictions.

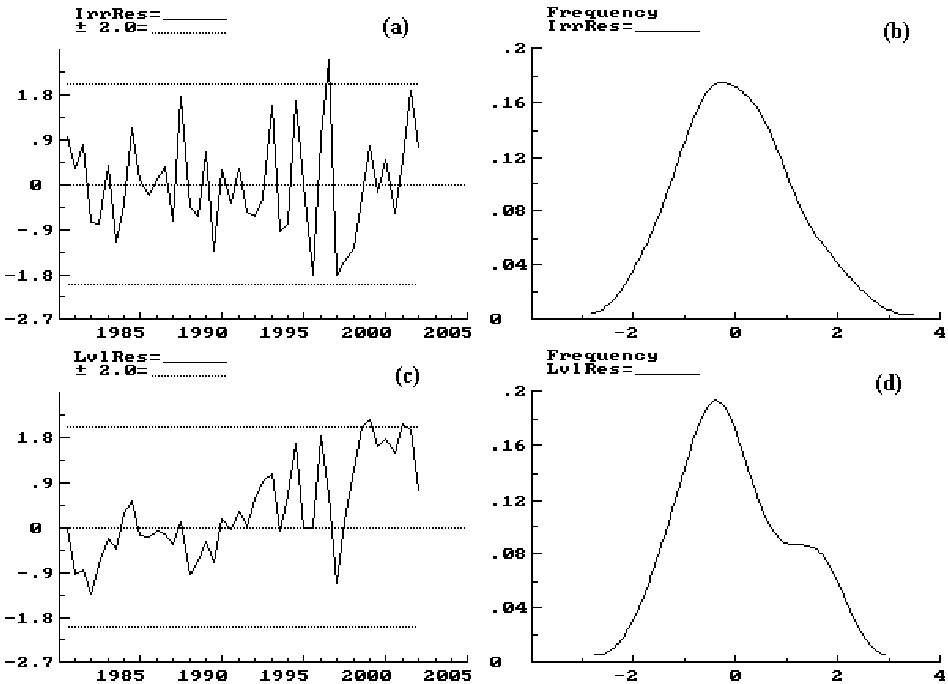

Figure 7: Auxiliary residuals. Irregular auxiliary residuals with 95% confidence band (a), frequency distribution of the irregular auxiliary residuals (b), level auxiliary residuals with 95% confidence band (c) and frequency distribution of the level auxiliary residuals

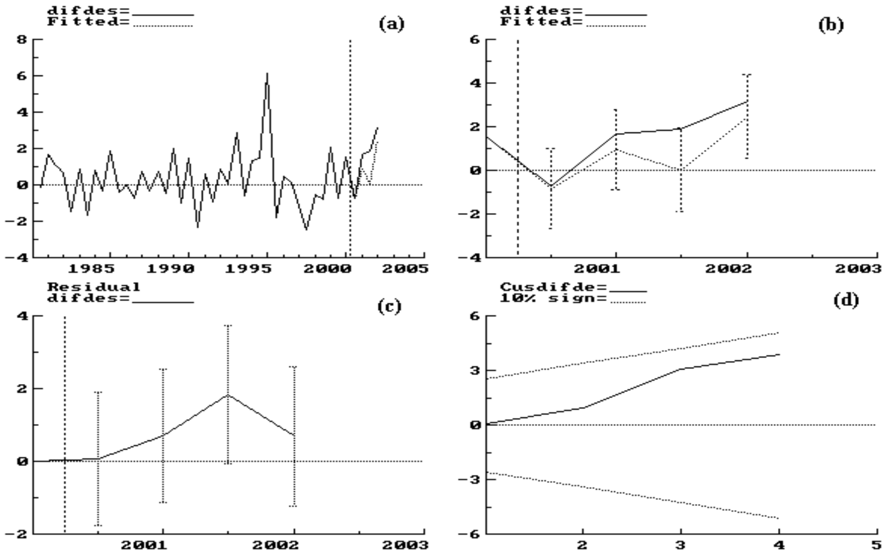

Figure 8: First differences of the unemployment rate (full line) with predicted values (dotted line) (a), predicted values (dotted line) with 95% confidence band (b), prediction residuals with 95% confidence band (c) and CUSUM with 90% confidence band (d)

With a simple transformation of (22) we obtain

$$y_t = -0.161449\,(g_t - \tilde{c}_t) + \tilde{\gamma}_{t/n} + 3.2925\,D_t + \varepsilon_{t/n}\ , \tag{25}$$

where the estimated "normal" growth rate of the product, $\tilde{c}_t$, was defined in (23) and their values are given in Table 1. The other components were defined above.

The variable c_t plays a similar role as the trend in a basic structural model without slope. It can change in time and it can capture the structural breaks that happen in the economy. The other variables, such as the seasonality and the interventions, are included to give the model a higher generality and to let it to have the usual time series econometric model structure.

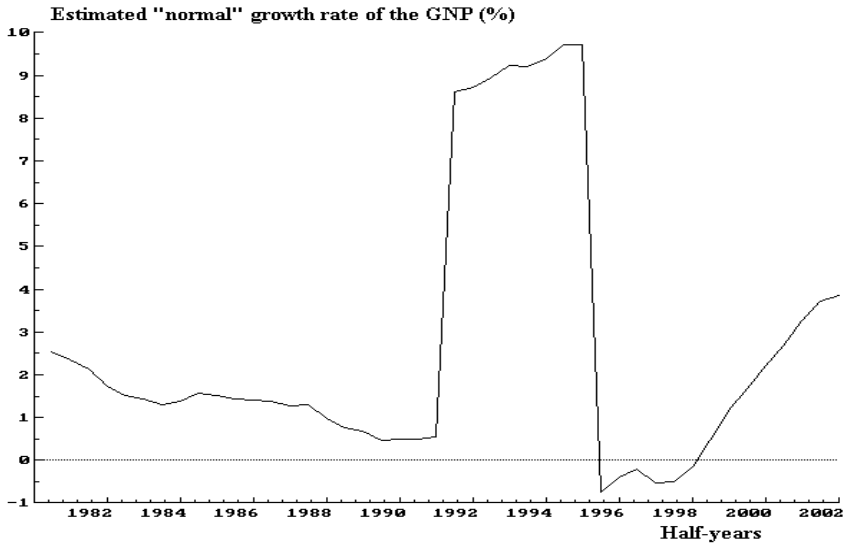

Figure 9: Estimated "normal" growth rate of the GNP of Argentina. 1980-2002

From the analysis of the results, specially from Figure 9 and Table 1, we observe clearly that we can distinguish three periods in the last twenty two years of Argentinian economy. The first period goes from 1980 to the initial part of 1991 and the distinctive feature of it was a high rate of inflation with an hyperinflationary peak during 1989. There, the natural rate of growth of the product was running from 2.53% to 0.49%, with an average of 1.30%. This together with the estimated value of the coefficient ρ, which is $\tilde{\rho} = 0.161449$, means that in this first period, the GNP should had to grow 7.50% in average in order to obtain one point of decrease in the unemployment rate. The end of this first period is coincidental with the beginning of the application of the stabilization plan of 1991.

On the other hand, the second period goes from the final part of 1991 to the initial part of 1995. The main characteristic of this period was a fast stabilization of the inflationary problem and an implementation of an important privatization scheme. The GNP had a tremendous increase as well as the unemployment. The natural rate of growth of the product was running from 8.60% to 9.71%, with an average of 9.19%. This means that in this second period, the GNP should had to grow at the impressive value of 15.35%, in average, in order to obtain one point of decrease in the unemployment rate. Clearly, for any country this is a very ambitious task, and even more for the Argentinian economy. The end of this second period coincided with the Mexican crisis which affected all the Latin American economies. Moreover, we detected an outlier in 1995-1 when this crisis, called the "tequila effect", was at a maximum.

Finally, the last period goes from the last part of 1995 to the initial part of 2002. It has two subperiods, one up to the final part of 1998 and the second up to the end of the period under study. In the first subperiod there was an increasing GNP with a slowly decreasing unemployment rate. Even more, the natural rate of growth of the product was negative in this subperiod (–0.42 in average), meaning that any increase in the GNP would result in a decrease of the unemployment rate, as it happened. The main characteristics of the second subperiod were an important recession with very stable prices and increasing unemployment. The natural rate of growth of the product for this subperiod had a positive slope, and it was running from 0.48% to 3.85%, with an average of 2.41%. This means that in this second subperiod, the GNP should had to grow at the value of 8.60%, in average, in order to obtain one point of decrease in the unemployment rate.

t	$\tilde{c}_t$	t	$\tilde{c}_t$	t	$\tilde{c}_t$
1980-2	2,53	1991-2	8,60	1995-2	-0,75
1981-1	2,36	1992-1	8,71	1996-1	-0,39
1981-2	2,13	1992-2	8,95	1996-2	-0,22
1982-1	1,74	1993-1	9,24	1997-1	-0,54
1982-2	1,51	1993-2	9,22	1997-2	-0,50
1983-1	1,44	1994-1	9,38	1998-1	-0,14
1983-2	1,30	1994-2	9,71	1998-2	0,48
1984-1	1,39	1995-1	9,71	1999-1	1,15
1984-2	1,56			1999-2	1,67
1985-1	1,51			2000-1	2,22
1985-2	1,44			2000-2	2,68
1986-1	1,42			2001-1	3,25
1986-2	1,38			2001-2	3,72
1987-1	1,26			2002-1	3,85
1987-2	1,30				
1988-1	0,98				
1988-2	0,76				
1989-1	0,67				
1989-2	0,45				
1990-1	0,50				
1990-2	0,49				
1991-1	0,55				

Table 1: Estimated “normal” growth rate of the GNP of Argentina.1980-2002

5. CONCLUDING REMARKS

Four points can be noted as conclusions of the work of measuring the Okun Relation for Argentina in the period considered.

Country	Estimated Coefficient (1981-1994)
United State	0.47
Germany	0.42
United Kingdom	0.49
Japan	0.23

Table 2: Estimated value of the coefficient ρ for different countries

1. The estimated coefficient ρ has the expected positive sign, indicating an inverse effect of the growth (actually, of the positive deviation of the growth in relation to the so called "normal" growth) on the unemployment. The estimated value of this coefficient (0.16) tells us that a growth which exceeds a one per cent the value of the rate $\tilde{c}_t$ of our equation, is translated in a descent of the unemployment of 0.16 points. This value is somehow lower in comparison with the corresponding to more developed countries for a period of time similar to the one considered in this work. In fact, for the period 1981-1994, the values given in Table 2 were obtained.
   The value of the coefficient ρ partly depends on decisions of the firms related, in particular, with the form in which they adjust the number of employees as an answer to the temporary deviations of the product. This type of decisions depends in turn on factors of different kinds and amplitudes, such as the form of the internal organization of the companies and the legal framework related with the contracting and dismissal of employees. This explain why the value of this coefficient differs from one country to another, as can be seen in the given Table 2. The relatively low value observed in Japan, for example, has been interpreted as a consequence of the higher labour security that the workers have there. In the USA, on the contrary, the legal and institutional restrictions that the companies face in order to adjust the employment are very much lesser. In the case of Argentina, the estimated value of the coefficient seems to be an indirect indicator of the high inflexibility existing in the labour market.

2. The rate of growth which was called "normal" in our work responds, as we have seen, to the sum of the increments of the labour productivity and of the labour force. In our estimation, the value of this component ($\tilde{c}_t$ in the equation) shows two structural changes. Clearly, the first one appears in the second part of the year 1991. From this moment onwards, the value of $\tilde{c}_t$ is substantially higher, passing from 1.30 to 9.19, in average. Without considering these actual values, the structural change can be taken as a clear evidence of the impressive turn over produced in the Argentinian economy as a result of the stabilization plan implemented in 1991. The substantive increment of the labour productivity and of the labour force explain the

observed deep change in Argentina. This second period ended in the first part of the year 1995, with the second structural change appearing in the second part of this year. The end of the second period coincided with the Mexican crisis which affected all the Latin American economies. The value of $\tilde{c}_t$ in the third period is substantially lower, passing from 9.19 to 1.18, in average. One of the possible reasons for this important decrease in $\tilde{c}_t$ is the almost permanent reduction in the labour productivity after the impact of the Mexican crisis.

3. The estimations in our model allow us to detect an outlier in the first half of 1995 coinciding with the maximum shock of the Mexican crisis which was called the "tequila effect". This outlier appears as a consequence of the 1995's recession, after a constant growth during the period 1991-94.

4. During the period that goes from 1991 up to 1995, the macroeconomic stability in Argentina coincides with a significant increase of the rate of unemployment and, simultaneously, with a high growth only detained, in part, during the crisis of the "tequila effect" previously mentioned. Relating all of this with the high value of our estimation of c_t, we can defend the hypothesis of a significant rise of the natural rate of unemployment as a result of the structural changes observed in the Argentinian economy in this period.

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