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Improved architecture of complementary set of sequences correlation by means of an inverse generation approach

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Abstract: System coding is a growing trend in all fields of engineering. Many different algorithms have been developed and studied for applications in signal processing, radar and multi-emission systems, among others. One of the most interesting algorithms, among these, is the complementary sets of sequences (CSS) given their potential and simplicity. They are characterised by a distinctive correlation and orthogonality properties. Nowadays, sustained efforts are being devoted to reducing the calculations involved in the generation and/or correlation of these sequences by means of recursive algorithms. Some authors have brought forward efficient algorithms that are based on modular architectures made up of adders, multipliers and delays. This work introduces an inverse generation approach to perform the correlation of CSS. This approach allows one to substantially reduce calculations, and enables the simultaneous correlation of M sequences, adopting neither time-multiplexing schemes nor complex parallel implementations. This is theoretically demonstrated by means of generation and correlation algorithms. An analysis of the performance and efficiency is then conducted in a reconfigurable hardware platform. The proposal represents an advance in the practical application of these sequences in the above-mentioned fields.

1 Introduction

Coding systems are used in different fields of engineering to improve signals identification. Those codes with good correlation properties, which allow one to detect signals in noisy environments, are of particular interest. Traditional approaches based on pseudo-random sequences have been widely applied despite their disadvantages, such as, their failure to remove correlation sidelobes. In this case, complementary sets of sequences (CSS) provide a good solution.

Complementary sequences were defined by Golay [1] as a pair of finite sequences of two kinds of elements with the property that the number of pairs of like elements with any given separation in one sequence is equal to the number of pairs of unlike elements with the same separation in the other sequence. The distinctive feature of these sequences is the possibility of signal detection under high-noise level conditions because of their correlation properties. Years later, Tseng and Liu [2] generalised and extended this concept to CSS. The properties of the complementary sets have improved signal detection, making them suitable for applications in which many digital signals are transmitted over the same physical channel. Golay sequences and complementary sets of M-sequences (M-CSS) are currently being applied to signal coding [3], radar [4], wideband radio channel sounders [5], multiemission systems [6], channel estimation [7] and sensor systems [8].

The application of M-CSS in any of the previously mentioned areas involves the use of logical architectures for generation and correlation. With respect to Golay sequences, Budisin [9] and Popovic [10] proposed efficient architectures able to perform both operations. Along these lines, the generation and correlation of M-CSS can be conducted by means of recursive methods that reduce computational load and hardware complexity as compared to the straightforward implementation. Some authors [11–13] introduced efficient architectures to generate and correlate M-CSS. Despite the fact that these modular architectures herald significant progress in the implementation of M-CSS-based systems, the correlation approach by these authors is based on the recursive method proposed by Popovic [10] for pairs of complementary sequences. That correlation algorithm executes the correlation of a single input signal with the two sequences of the complementary pair. Therefore two options for processing a pair of sequences exist: the first one is time multiplexing and the other one is architecture duplication. In the former, the architecture is the same, although the processing time doubles. In the latter, the processing is much faster, although the architecture duplicates (with the consequent hardware consumption). This problem accentuates in M-CSS, in which the possibilities are either time-multiplexing M sequences composing a macro-sequence or multiplying the architectures by M.

The simultaneous correlation of Golay sequences was achieved in a previous work [14], applying an inverse generation approach. According to said approach, two sequences are correlated simultaneously with neither time multiplexing nor correlation architecture duplication. Considering that these pairs are a particular case of complementary sets, this paper introduces a generic correlation algorithm for M-CSS. This approach allows for a regular and modular architecture, which executes the correlation of M inputs on a simultaneous basis, with the consequent calculation reduction. This is theoretically demonstrated by means of generation and correlation algorithms. An analysis of the performance and efficiency is then realised in a reconfigurable hardware platform and compared to those brought forward by other authors.

2 Complementary sequences

Complementary pairs of sequences (also known as Golay sequences [1]) are defined as a pair of sequences, $\{S_{1,N}[k]; S_{2,N}[k]\}$, composed of two binary elements, -1 and $+1$, respectively, which can be generated of length $L = 2^N$ elements ($N \in \{\mathbb{N} - 0\}$). The condition to be met by these sequences is that the addition of their periodic autocorrelation functions is a Kronecker delta of amplitude $2L$ for $k = 0$, and is null for $k \neq 0$. This can be expressed as follows

$$Y[k] = C_1[k] + C_2[k] = 2L\delta[k] \tag{1}$$

were $C_1[k]$ and $C_2[k]$ are the autocorrelations of $S_{1,N}[k]$ and $S_{2,N}[k]$, respectively.

Golay defines rules to generate complementary pairs of different length from shorter ones. These rules can be recursively applied to generate diverse pairs, as put forward by Busidin [9]

$$\begin{aligned} S_{1,n}[k] &= S_{1,n-1}[k] + w_{1,n} S_{2,n-1}[k - D_n] \\ S_{2,n}[k] &= S_{1,n-1}[k] - w_{1,n} S_{2,n-1}[k - D_n] \end{aligned} \tag{2}$$

where $S_{1,n}[k]$, $S_{2,n}[k]$ are the pairs of sequences at nth iteration, $D_n = 2^{n-1}$ and $w_{1,n}$ is a seed coefficient with value ± 1. Applying Z transform, (2) can be expressed as

$$\begin{aligned} \begin{vmatrix} S_{1,n}[z] \\ S_{2,n}[z] \end{vmatrix} = \begin{vmatrix} +1 & +1 \\ +1 & -1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & w_{1,n} \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & z^{-2^{n-1}} \end{vmatrix} \\ \cdot \begin{vmatrix} S_{1,n-1}[z] \\ S_{2,n-1}[z] \end{vmatrix} \end{aligned} \tag{3}$$

or

$$\boldsymbol{S}_n[z] = \boldsymbol{H}_2 \cdot \boldsymbol{W}_{2,n} \cdot \boldsymbol{D}_{2,n} \cdot \boldsymbol{S}_{n-1}[z] \tag{4}$$

where $\boldsymbol{H}_2$ is a Hadamard matrix of order 2; $\boldsymbol{W}_{2,n}$ is a seed matrix of order 2 of the nth iteration and $\boldsymbol{D}_{2,n}$ is a delay matrix of order 2 of the nth iteration.

Golay sequences correlation can also be performed by recursive algorithms, as first advanced by Popovic [10] and later optimised by Donato *et al.* [14]. The correlator of Popovic is a digital matched filter, which simultaneously performs the correlation of one input signal with both sequences of the Golay pair. The correlator by Donato *et al.* [14] is an architecture enabling the inverse generation process, and thereby enabling the simultaneous correlation of two inputs. This recursive algorithm is summed up as follows

$$\begin{aligned} C_{1,n-1}[k] &= C_{1,n}[k - D_n] + C_{2,n}[k - D_n] \\ C_{2,n-1}[k] &= w_{1,n} \cdot C_{1,n}[k] - w_{1,n} \cdot C_{2,n}[k] \end{aligned} \tag{5}$$

where $C_{1,n}[k]$, $C_{2,n}[k]$ are a pair of sequences at the input of the nth stage of the correlator and $C_{1,n-1}[k]$, $C_{2,n-1}[k]$ are the outputs of the same stage. Expressing this algorithm with matrices and Z transform

$$\boldsymbol{C}_{n-1}[z] = \boldsymbol{D}'_{2,n} \cdot \boldsymbol{W}_{2,n} \cdot \boldsymbol{H}_2 \cdot \boldsymbol{C}_n[z] \tag{6}$$

where

$$\boldsymbol{D}'_{2,n} = \begin{vmatrix} z^{-2^{n-1}} & 0 \\ 0 & 1 \end{vmatrix} \tag{7}$$

and the correlator output is

$$Y[k] = C_{1,0}[k] + C_{2,0}[k] = 2L\delta[k - L] \tag{8}$$

where $C_{1,0}[k]$, $C_{2,0}[k]$ are the correlation outputs at the last stage and the output of $Y[k]$ is a Kronecker delta of amplitude $2L$ delayed L samples because of the iterative process.

CSS is a generalisation derived from Golay sequences [2]. An M-CSS is defined as a set of $M = 2^m$ sequences of length $L = M^N$ elements ($m, N \in [\mathbb{N} - 0]$), where each one is a binary element ($+1$ or -1). A set is characterised by the sum of their autocorrelations

$$Y[k] = \sum_{i=1}^{M} C_i[k] = ML\delta[k - L] \tag{9}$$

where C_i is the autocorrelation of the sequence $S_{i,N}[k]$

$$C_i[k] = \sum_{j=1}^{L} S_{i,N}[j] \cdot S_{i,N}[j + k] \tag{10}$$

where $S_{i,N}$ is a sequence of length $L = M^N$. The autocorrelation addition of the M sequences generates a Kronecker delta of amplitude ML, with null sidelobes, just like with Golay sequences.

Recursive algorithms were developed for M-CSS. Álvarez *et al.* [11] proposed an efficient architecture to generate and correlate a 4-CSS. Afterwards, De Marziani *et al.* [12] generalised generation and correlation architectures to M-CSS. This proposal was then improved by Pérez *et al.* [13], who obtained a more efficient architecture, especially for interleaved sequences schemes. Even though these correlation algorithms anticipate significant progress, the correlation approach by these authors is based on the recursive method proposed by Popovic [10] for Golay sequences, in which the correlation is performed between a single input signal and the two sequences of the complementary pair. Then, for all cases, it is necessary to multiply the number of correlators by M, or else, use an interleaved transmission scheme to process the signal.

3 New correlation approach

Just like Golay sequences, the correlation of M-CSS can be performed as an inverse generation process. This section develops and demonstrates this concept for M-CSS of any length.

The algorithm for M-CSS generation can be obtained starting with the generation algorithm by Budisin [9] (3) and applying the properties presented by Tseng and Liu [2]

$$\begin{vmatrix} S_{1,n}[z] \\ \cdots \\ \cdots \\ S_{M,n}[z] \end{vmatrix} = \begin{vmatrix} \boldsymbol{H}_{(M/2)} & \boldsymbol{H}_{(M/2)} \\ \boldsymbol{H}_{(M/2)} & -\boldsymbol{H}_{(M/2)} \end{vmatrix} \cdot \begin{vmatrix} \boldsymbol{W}_{(M/2),n-1} & 0 \\ 0 & w_{r,n} \cdot \boldsymbol{W}_{(M/2),n-1} \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 0 & z^{-M^{n-1}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & z^{-(M-1)M^{n-1}} \end{vmatrix} \cdot \begin{vmatrix} S_{1,n-1}[z] \\ \cdots \\ \cdots \\ S_{M,n-1}[z] \end{vmatrix} \quad (11)$$

or

$$\boldsymbol{S}_n[z] = \boldsymbol{H}_M \cdot \boldsymbol{W}_{M,n} \cdot \boldsymbol{D}_{M,n} \cdot \boldsymbol{S}_{n-1}[z] \quad (12)$$

where

- $\boldsymbol{H}_{(M/2)}$ is a Hadamard matrix of order $(M/2)$, with $\boldsymbol{H}_1 = 1$;
- $\boldsymbol{W}_{(M/2),n-1}$ is a seed matrix of order $M/2$, with $\boldsymbol{W}_{2,0} = \begin{bmatrix} 1 & 0 \\ 0 & w_{1,0} \end{bmatrix}$;
- $w_{r,n}$ is a seed coefficient, with $r = \log_2 M$:
- $z^{-(M-i)M^{n-1}}$ are delays with $i = 1, \ldots, (M-1)$, $i \in \mathbb{N}$.

Fig. 1 illustrates a detailed scheme of a 4-CSS generator stage with the simplified arithmetic architecture. The correspondence between (11) and the elements in the figure can be easily noticed. The delays are grouped at the input of each stage and the product $\boldsymbol{H}_M \cdot \boldsymbol{W}_{M,\,n}$ is represented by the adders and the gains. The length of the M sequences can be extended with the concatenation of several stages following Golay rules [1].

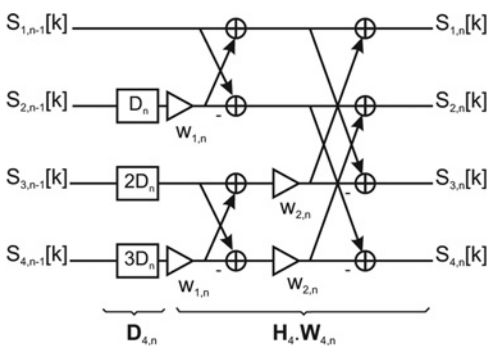

Fig. 1 *4-CSS example of a generation stage*

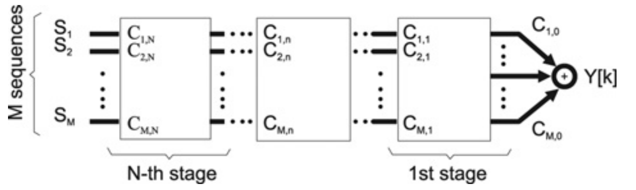

Fig. 2 *M-CSS correlator of N stages*

Once the generation equations are defined, the next step consists in performing the correlation as an inverse of the generation process [14]. A generic recursive architecture is proposed, as shown in Fig. 2, made up of N stages. The underlying idea is to decompose an M-CSS into a shorter one, and to repeat the process iteratively until it is reduced to a single delta. If the correlator inputs are the M sequences of an M-CSS of length $L = M^N$, the sequences length are reduced to M^{N-1} in the first stage and their amplitudes are multiplied by M. In the second stage, in turn, they are reduced to M^{N-2}, and their amplitudes are multiplied by M once again and so on. After N stages, the sequences length is reduced to 1 with an amplitude of M^N.

In order to better understand the process, let us consider a vector $\boldsymbol{S}_n$ as an M-CSS of length L at the input of the correlator. If the product $\boldsymbol{W} \cdot \boldsymbol{H} \cdot \boldsymbol{S}$ is performed, a new M-CSS is obtained, in which each sequence is made up of a sum of complementary sequences of length L/M. For example, given an $\boldsymbol{S}_n$ obtained from (12)

$$\boldsymbol{S}_n = \begin{vmatrix} S_{1,n} \\ S_{2,n} \\ S_{3,n} \\ S_{4,n} \end{vmatrix} = \begin{vmatrix} S_{1,n-1} + S_{2,n-1} \cdot z^{-4^{n-1}} + S_{3,n-1} \cdot z^{-2\cdot 4^{n-1}} + S_{4,n-1} \cdot z^{-3\cdot 4^{n-1}} \\ S_{1,n-1} - S_{2,n-1} \cdot z^{-4^{n-1}} + S_{3,n-1} \cdot z^{-2\cdot 4^{n-1}} - S_{4,n-1} \cdot z^{-3\cdot 4^{n-1}} \\ S_{1,n-1} + S_{2,n-1} \cdot z^{-4^{n-1}} - S_{3,n-1} \cdot z^{-2\cdot 4^{n-1}} - S_{4,n-1} \cdot z^{-3\cdot 4^{n-1}} \\ S_{1,n-1} - S_{2,n-1} \cdot z^{-4^{n-1}} - S_{3,n-1} \cdot z^{-2\cdot 4^{n-1}} + S_{4,n-1} \cdot z^{-3\cdot 4^{n-1}} \end{vmatrix} \quad (13)$$

the result $\boldsymbol{W}_{4,n} \cdot \boldsymbol{H}_4 \cdot \boldsymbol{S}_n$ (considering $\boldsymbol{W}_{4,n} = \mathbb{I}$ for simplicity) is

$$\begin{vmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{vmatrix} \cdot \begin{vmatrix} S_{1,n} \\ S_{2,n} \\ S_{3,n} \\ S_{4,n} \end{vmatrix} = \underbrace{\begin{vmatrix} 4S_{1,n-1} \\ 4S_{2,n-1} \cdot z^{-4^{n-1}} \\ 4S_{3,n-1} \cdot z^{-2\cdot 4^{n-1}} \\ 4S_{4,n-1} \cdot z^{-3\cdot 4^{n-1}} \end{vmatrix}}_{L/4} \quad (14)$$

As a result, four sequences of length $L/4$ are obtained, each one delayed $L/4$ with respect to the previous one. Fig. 3 illustrates an example of this processing for a set of length $L = 64$.

If a proper delay is applied to each sequence, a new M-CSS ($\boldsymbol{C}_{n-1}$) with lower length and larger amplitude is obtained. The correlation algorithm for the nth iteration is

$$\boldsymbol{C}_{n-1} = \boldsymbol{D}'_{M,n} \cdot \boldsymbol{W}_{M,n} \cdot \boldsymbol{H}_M \cdot \boldsymbol{C}_n \quad (15)$$

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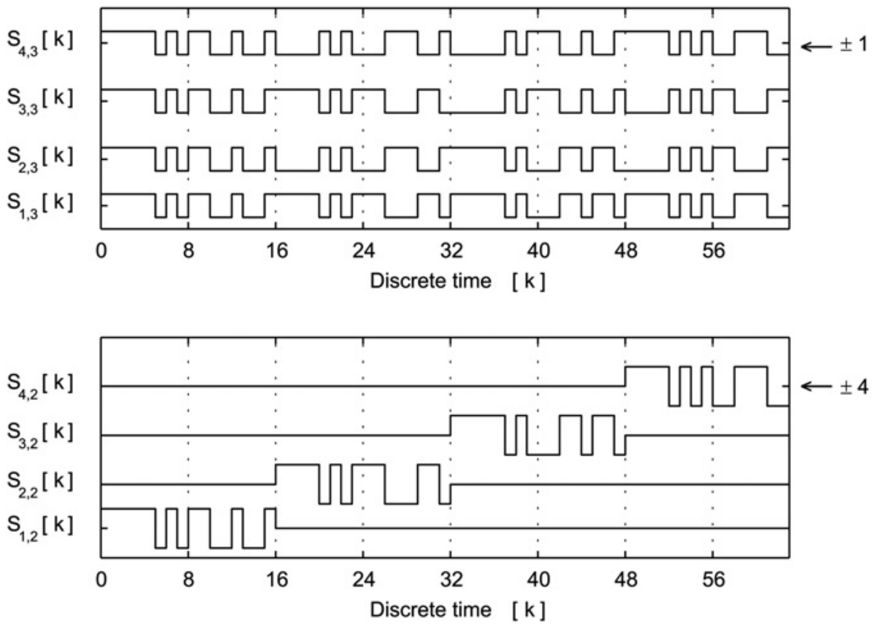

Fig. 3 *Example of the product $W_{4,n}H_4S_n$ with a set of length $L = 64$*

where $\boldsymbol{D}'_{M,n}$ is the delay matrix

$$\boldsymbol{D}'_{M,n} = \begin{vmatrix} z^{-(M-1)M^{n-1}} & 0 & \cdots & 0 \\ 0 & z^{-(M-2)M^{n-1}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{vmatrix} \quad (16)$$

Note that the delays are ordered inversely with respect to (11). Fig. 4 depicts the detail of the proposed correlation stage for a 4-CSS.

Generalising, the formulation for N stages is

$$\boldsymbol{C}_0 = \prod_{n=N}^{1} (\boldsymbol{D}'_{M,n} \cdot \boldsymbol{W}_{M,n} \cdot \boldsymbol{H}_M) \cdot \boldsymbol{C}_n \quad (17)$$

and like with pairs of sequences, the output of the correlator (Y) is the addition of the M correlations outputs

$$Y[k] = \sum_{i=1}^{M} C_{i,0}[k] \quad (18)$$

3.1 Example

Consider an M-CSS generated with (11) denoted as $\boldsymbol{S}_N$, which has to be correlated. The input of the correlator could be assumed to be the generator output, $\boldsymbol{C}_N = \boldsymbol{S}_N$ (Fig. 2). For simplicity reasons, an M-CSS generated with $N = 2$ ($L = M^2$) is assumed. The output of the first correlation stage is (15)

$$\boldsymbol{C}_1 = (\boldsymbol{D}'_{M,2}\boldsymbol{W}_{M,2}\boldsymbol{H}_M) \cdot \boldsymbol{C}_2 = (\boldsymbol{D}'_{M,2}\boldsymbol{W}_{M,2}\boldsymbol{H}_M) \cdot \boldsymbol{S}_2 \quad (19)$$

Using (12)

$$\begin{aligned}\boldsymbol{C}_1 &= (\boldsymbol{D}'_{M,2}\boldsymbol{W}_{M,2}\boldsymbol{H}_M)(\boldsymbol{H}_M\boldsymbol{W}_{M,2}\boldsymbol{D}_{M,2}) \cdot \boldsymbol{S}_1 \\ &= \boldsymbol{D}'_{M,2}\boldsymbol{W}_{M,2}\boldsymbol{H}_M\boldsymbol{H}_M\boldsymbol{W}_{M,2}\boldsymbol{D}_{M,2}\boldsymbol{S}_1 \quad (20)\end{aligned}$$

Computing some associations

$$\begin{aligned}\boldsymbol{C}_1 &= \boldsymbol{D}'_{M,2}\boldsymbol{W}_{M,2}(\boldsymbol{H}_M\boldsymbol{H}_M)\boldsymbol{W}_{M,2}\boldsymbol{D}_{M,2}\boldsymbol{S}_1 \\ &= M \cdot \boldsymbol{D}'_{M,2}\boldsymbol{W}_{M,2}\boldsymbol{W}_{M,2}\boldsymbol{D}_{M,2}\boldsymbol{S}_1 \quad (21)\end{aligned}$$

The product $\boldsymbol{H}_M\boldsymbol{H}_M = M \cdot \mathbb{I}$, where M is a scalar and $\mathbb{I}$ is the identity matrix. In the same way, $\boldsymbol{W}_2 \cdot \boldsymbol{W}_2 = \mathbb{I}$ when the seeds are ± 1. After such simplification the following is obtained

$$\boldsymbol{C}_1 = M(\boldsymbol{D}'_{M,2}\boldsymbol{D}_{M,2})\boldsymbol{S}_1 \quad (22)$$

From (11) and (16), the product $(\boldsymbol{D}'_{M,2}\boldsymbol{D}_{M,2})$ can be simplified by a delay and an identity matrix

$$\boldsymbol{C}_1 = M \cdot (z^{-(M-1)M} \cdot \mathbb{I})\boldsymbol{S}_1 \quad (23)$$

Using (15)

$$\boldsymbol{C}_0 = (\boldsymbol{D}'_{M,1}\boldsymbol{W}_{M,1}\boldsymbol{H}_M) \cdot M(z^{-(M-1)M} \cdot \mathbb{I})\boldsymbol{S}_1 \quad (24)$$

Then, if $\boldsymbol{S}_1 = (\boldsymbol{H}_M\boldsymbol{W}_{M,1}\boldsymbol{D}_{M,1}) \cdot \boldsymbol{S}_0$,

$$\begin{aligned}\boldsymbol{C}_0 = M \cdot z^{-(M-1)M}&(\boldsymbol{D}'_{M,1}\boldsymbol{W}_{M,1}\boldsymbol{H}_M) \\ &\cdot (\boldsymbol{H}_M\boldsymbol{W}_{M,1}\boldsymbol{D}_{M,1})\boldsymbol{S}_0 \quad (25)\end{aligned}$$

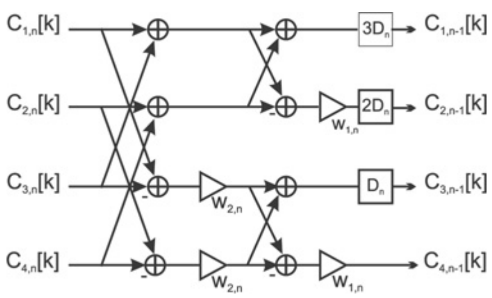

Fig. 4 *4-CSS example of the proposed correlation stage*

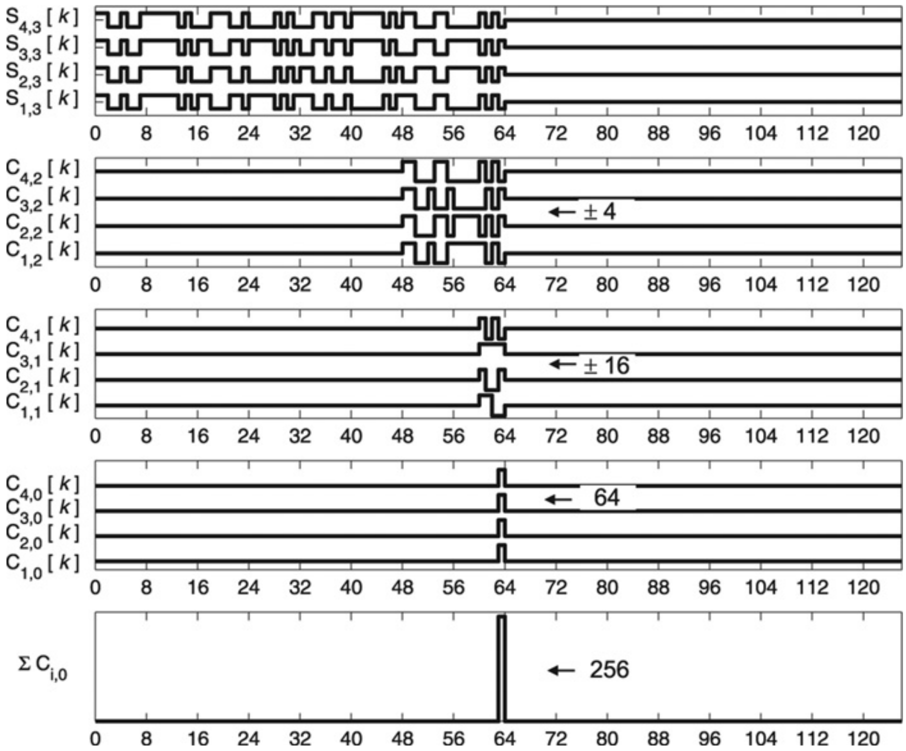

Fig. 5 *4-CSS correlation*

and after iteratively processing (25), the result is

$$\boldsymbol{C}_0 = M^2 \cdot z^{-(M^2-1)} \cdot \mathbb{I} \cdot \boldsymbol{S}_0 \tag{26}$$

The output of the correlator (Y) is the addition of the M correlation outputs. Being $\boldsymbol{C}_0 = [C_{1,0}C_{2,0} \ldots C_{M,0}]^{\mathrm{T}}$,

$$Y = \sum_{i=1}^{M} C_{i,0} = M \cdot M^2 \cdot z^{-(M^2-1)} \tag{27}$$

a Kronecker delta of amplitude $ML = MM^2$ is obtained.

If this is considered a generic case in which N is any natural number, the correlator output is

$$Y = \sum_{i=1}^{M} C_{i,0} = M^N \sum_{i=1}^{M} S_{i,0} \cdot z^{-(M^N-1)} \tag{28}$$

Rewriting (28) as a difference equation yields

$$Y[k] = M^N \sum_{i=1}^{M} S_{i,0}[k - (M^N - 1)] \tag{29}$$

and considering that the generation stimulus was a $S_{i,\,0} = \delta[k]$, then

$$Y[k] = M^N \sum_{i=1}^{M} \delta[k - (M^N - 1)]$$
$$= MM^N \delta[k - (M^N - 1)] \tag{30}$$

From (30), the output of the proposed correlator, $Y[k]$, is a Kronecker delta of amplitude $MM^N = ML$ delayed M^N samples, considering that at $k = 0$ the first element of the M sequences is generated and correlated.

3.2 Simulations

In order to illustrate the operation of the correlator described in this section, a simulation is conducted. Fig. 5 displays the waveforms of the proposed correlator in which a 4-CSS of length $L = 64$ (three stages) is present at the input. As explained above, in each correlation stage, the length of the sequences is divided by M and the amplitude is multiplied by the same factor. So, if the input signals are of $L = 64$ and unitary amplitude, the output at the first correlation stage is a set of sequences of $L = 16$ and amplitude ± 4. In the same way, at the second correlation stage, four sequences of amplitude ± 16 and length $L = 4$ are obtained, and at the third stage $L = 1$ with a 64 amplitude. Finally, the sum of the correlations is a Kronecker delta of amplitude $ML = 256$, as expected.

Table 1 Number of calculations for an M-CSS correlation

Operations	Straightforward	Efficient [12]	Proposed
multiplications	M^{N+2}	$(M^2/2)\ N \log_2(M)$	$(M/2)\ N \log_2(M)$
add/subs.	$M^2(M^N - 1)$	$M^2 N \log_2(M)$	$MN \log_2(M)$
delays	$2M^{N+2}$	$(M^2/2)\ (M^N - 1)$	$(M/2)\cdot(M^N - 1)$

4 Proposed correlator efficiency

The main contribution of the proposed correlator is the reduction of calculation resources. The efficient correlator [12] executes $M\ (M - 1)$ unnecessary correlations, and consequently M correlators are needed to obtain the sum of autocorrelations, just like in the straightforward correlation.

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Table 2 Amounts of operations to perform 4-CSS correlations

Operation	Straightforward				Efficient				Proposed			
N	1	2	3	4	1	2	3	4	1	2	3	4
mult.	64	256	1024	4096	16	32	48	64	4	8	12	16
add/subs.	48	240	1008	4080	32	64	96	128	8	16	34	32
delays	128	512	2048	8192	24	120	504	2040	6	30	126	510

Table 3 Implementation results for 4-CSS and 8-CSS

M	4					8		
N	1	2	3	4	5	1	2	3
FGs	110	208	322	452	598	328	661	1066
FFs	92	344	1364	5456	21 836	372	2920	23 388
clock, [MHz]	149.1	83.3	57.8	44.2	35.8	105.4	57.3	39.4

Table 4 Proposals comparison

$M = 4$, $L = 64$	Alvarez *et al.* [11]	Pérez *et al.* [13]	Proposed
slices	396	349	289
LUTS	not reported	578	446
FFs	not reported	not reported	140
frequency	41.6 MHz	33.8 MHz	57.11 MHz

Conversely, the proposed correlator uses only the resources of one of the so-called efficient correlators. Table 1 summarises the number of calculations to perform $Y[k]$ for an M-CSS of N stages; and Table 2 exemplifies a 4-CSS. The reduction of any of the amounts of operations considered is important especially for larger L. Note that the number of calculations of the efficient correlator and that proposed here are related by a scale factor M (set size) or N (number of stages).

The evaluation of the proposed architecture was performed in a real platform applying Xilinx Spartan 3 development kit (3S1500FG320-5) with Xilinx ISE software. The FPGA applied contains 13 312 slices, each with the following elements in common: two logic function generators (FG), two storage elements (FF), wide-function multiplexers, carry logic and arithmetic gates. Table 3 summarises the implementation results for $M = 4$ and 8, respectively. The parameters evaluated were: the set size M, the number of stages N, and the quantisation of the input sequences $q = 8$. The latter is inherent to any practical application, in which an input signal is acquired with an analogue-to-digital converter. The same kind of seed matrix was used for all the implementations.

Tables 2 and 3 illustrate the relevance of calculation reduction. The number of calculations impacts directly on the hardware regarding resources utilisation. Such utilisation increases as the number of stages does, as well with the set size. Hence, processing speed reduces inversely with these parameters. Table 4 shows some examples of the efficiency achieved in this proposal by comparing it to those reported in the literature. For this comparison, the correlator was implemented in the same FPGA, but with the option of SRL (Luts-based shift registers) synthesis enabled. This option allow flip-flops consumption reduction by synthesising the shift registers as states machines, and it was reported as a synthesis technique in the paper by Pérez *et al.* [13]. As shown in Table 4, the efficiency attained is about 20% with respect to the correlator introduced by Pérez *et al.* [13] and even greater with regard to that of Alvarez *et al.* [11], just comparing the architecture. Also bear in mind that those proposals ([11, 13]) require M correlators to perform the simultaneous correlation of all the sequences of a complementary set.

5 Conclusions

This paper presents an improved correlation algorithm for M-CSS based on an inverse generation process. The approach herein simplifies the correlation algorithm, allowing a recursive regular and modular calculation scheme that simultaneously performs M correlations of M inputs. It also enables a more efficient implementation that requires lower hardware complexity with respect to the straightforward and other previously proposed correlation schemes. The reduction attained is almost M times for all cases. The calculation efficiency encouraged by this contribution represents another essential step forward in the practical application of these sequences in signal coding, radar and multiemission systems, among others.

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