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NEW STEIGLITZ-McBRIDE ADAPTIVE LATTICE NOTCH FILTERS

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Abstract— **Two novel adaptive notch filters are presented. The updating algorithms are based on the Steiglitz-McBride error criterion minimization and the basic realizations of the notch filters are all-pass based lattice filters. The proposed realizations represent an extension of a previous ad-hoc scheme for adaptive notch filtering and avoid finding the roots of a high order polynomial to obtain the unknown frequencies of interest. Since the structure is based on the lattice realization, suitable properties with finite length precision realizations can be expected. Computer simulations are included to verify the adaptive filter performance when compared with alternative realizations.**

Keywords— **Frequency estimation, lattice notch filter, Steiglitz-McBride method.**

I. INTRODUCTION

The input signal model $u(n)$ considered in this work is composed of N sinusoids with unknown amplitude p_k and frequency w_k, immersed in additive Gaussian noise $r(n)$, written as

$$u(n) = \sum_{k=1}^{M} p_k \sin(w_k n + \phi_k) + r(n),$$

where ϕ_k is the corresponding phase of sinusoid k.

The problem of estimating the frequencies of multiple sinusoids can be traced back to the *Adaptive Line Enhancer* (Widrow *et al.*, 1975) where MSE minimization using a k-step FIR prediction filter was the basic structure. The computation of the unknown frequencies requires the evaluation of the roots of the associated polynomial. Other higher precision alternative, with considerable higher complexity, were based in the eigenvalue decomposition of input statistics (Ljung and Soderstrom, 1983). General FIR solutions have proven to be inefficient to recover sinusoids in noise, mainly because the modeling of deep notchs requires high order filters.

Due to its natural efficiency IIR based notch filters or their duals, i.e, narrow passband filters with a very selective frequency characteristic, are becoming more attractive for this application. An ideal notch filter transfer function $H(z)$, evaluated on the unit circle, is described by

$$H(e^{jw}) = \begin{cases} 1 & w = w_k \\ 0 & \text{otherwise} \end{cases}.$$

Although it is not possible to obtain an exact solution, nice and efficient approximations can be obtained using IIR notch realizations of adequate order. A general IIR notch model, proposed by Nehorai (1985), contemplates a canonical (minimum number of parameters corresponding to each unknown frequency to estimate) direct-form realization of order $2M$. The zeros of the notch filter are located on the unit circle and the module of the poles (at the same radius but, logically, inside $|z| = 1$) is a user defined parameter. The properties and accuracy of this adaptive notch filter have been extensively studied in the literature (Ng, 1987; Stoica and Nehorai, 1988). Despite that classical estimation properties can be related to this model, no direct availability of the estimated frequencies is obtained except by finding the roots of a high-order ($2M$) polynomial. Alternative realizations using the same model but with a lattice structure were also studied in the past (Cho *et al.,* 1989) with no particular advantages for the multiple sinusoids case. A different notch filter model based on second-order allpass lattice sections, that has interesting numerical properties (Regalia *et al.,* 1988), was presented in Regalia (1991). In this case, individual notch frequencies are independent of the corresponding pole module. This property has shown to be useful to extend the model application, from the single sinusoid recovering, to the multiple sinusoid case of interest.

This work presents a natural extension of the solution in Regalia (1991), that uses either a direct or a factorized allpass lattice realization to deal with the problem of direct availability of the estimated frequencies. In contrast to what was proposed as an ad hoc solution in Regalia (1991), the proposed adaptive notch filter minimizes a well defined criterion, the Steiglitz-McBride (SM) error.

The paper presentation is organized in the following

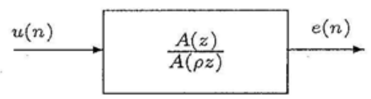

Figure 1: Notch filter representation.

manner. In Section II some available models for adaptive notch filtering and their respective updating algorithms are reviewed. Particularly, direct-form realizations, using both Recursive Prediction Error (RPE) and Steiglitz-McBride (SM) error minimization, are discussed. The novel adaptive lattice-based notch filters are introduced in Section III, where some of their properties are also discussed. A general discussion and comparison of the proposed notch filter with other realizations is presented in Section IV, where some computer simulations to verify the expected properties are included. Finally in Section V some conclusions are presented.

II. RPE AND SM METHODS FOR ADAPTIVE NOTCH FILTERING

As introduced in Nehorai (1985), one of the desirable properties of adaptive notch filters is that their transfer function has zeros on the unit circle in such a way that the corresponding input frequency components can be zeroed. A necessary condition to satisfy this property is that the numerator polynomial of the notch has to be of the form

$$\begin{aligned}A(z) &= 1 + a_1 z^{-1} + \cdots + a_M z^{-M} + \cdots + a_1 z^{-2M+1} + z^{-2M} \\ &= z^{2M} A(z^{-1}),\end{aligned} \tag{1}$$

where z is the complex variable (in the time domain, a unit delay is $z^{-1}u(n) = u(n-1)$) and $2M$ is the number of zeros of $A(z)$. This restriction also limits the number of parameters to a minimum of M. A second requirement for the adaptive notch filter is that its poles have to be on the same radial line as the zeros, but slightly shifted to the origin. This can be obtained using $A(\rho z)$ as denominator, where $\rho < 1$. The notch filter proposed in [Nehorai, 1985] has the form

$$H(z) = \frac{A(z)}{A(\rho z)}. \tag{2}$$

If $u(n)$ is the input signal and $H(z)$ is modeled as in (2) the design of the notch filter can be formulated as the following optimization problem: *find the coefficients a_i of $H(z)$ that minimize the variance of the output signal* $e(n)$, as depicted in Fig. 1. This minimization can be performed using *Recursive Prediction error methods* (RPE), as detailed in Nehorai (1985). Essentially, this method requires the computation of the gradient that is obtained using filtered versions (using the denominator of the notch filter) of the input and output signals. This is a well known model for notch filtering applications, and has many remarkable properties (precision, numeric robustness, stability, fast convergence, etc.).

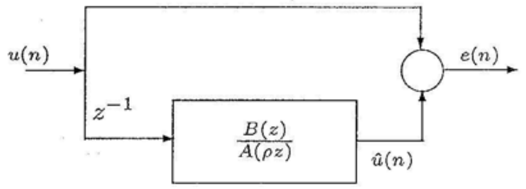

Figure 2: System identification reformulation for notch filter design.

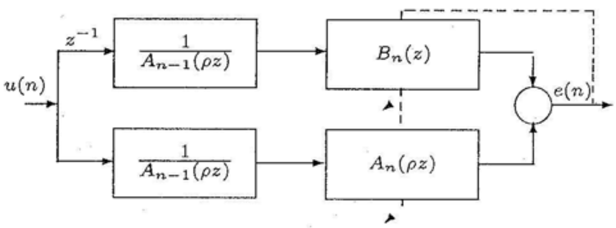

Figure 3: SM method for adaptive notch filtering (SMM-ANF).

This work focuses on overcoming two possible drawbacks of this model: i) in order to obtain the sinusoid frequencies, the roots of a high order polynomial must be found, and ii) direct-form realization does not result in the most practical implementation (if numerical properties are a concern) when compared with other structures, such as the lattice realization for instance.

As a first alternative to the RPE method it is possible to reformulate the previous minimization problem to one of system identification. Using the approach proposed in Cheng and Tsai (1998) consider the following algebraic manipulation

$$\frac{A(z)}{A(\rho z)} = 1 - \frac{A(\rho z) - A(z)}{A(\rho z)} = 1 - z\frac{B(z)}{A(\rho z)}, \tag{3}$$

where $B(z) = z^{-1}[A(\rho z) - A(z)]$. Note that, since $A(z)$ and $A(\rho z)$ are monic polynomials, $B(z)$ is strictly causal. With this reformulation, depicted in Fig. 2, the minimization problem was changed to one of system identification. Many different methodologies can now be applied to the design problem. Most interesting to the purposes of this work is the Steiglitz-McBride method (SM) (Steiglitz and McBride, 1965; Regalia, 1995). Due to its modeling capabilities and simplicity, the SM method has been used in many applications of adaptive IIR filtering. Using this methodology, Cheng and Tsai (1998) obtained an adaptive notch filter (SMM-ANF), different to the approach of [Nehorai, 1985]. The SMM-ANF is illustrated in Fig. 3.

As can be noticed, the SMM-ANF has the same problems of the RPE methodology so a different approach is followed and introduced in the next section.

Another model is the all-pass based notch filter model (Regalia, 1991) given by

$$H(z) = \frac{1}{2}(1 + V(z)), \tag{4}$$

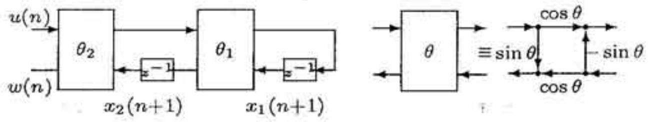

Figure 4: Second-order normalized lattice all-pass filter.

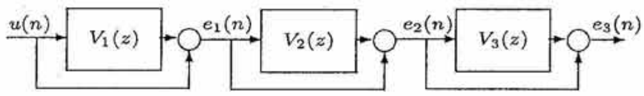

Figure 5: Allpass based notch filter, multiple sinusoids.

where $V(z)$ is a second-order allpass filter. Note that (4) does not necessarily satisfy (2). A suitable structure for the allpass filter is the normalized lattice realization, that has very good properties in terms of stability in a time varying context and also good numerical properties. The parameters are the rotational angles θ_k, $k = 1, 2$ (see Fig. 4). For the unique sinusoid detection case, and if w_0 is the corresponding frequency and B is the 3 dB bandwidth attenuation of $H(e^{jw})$, then (Regalia *et al.*, 1988)

$$\theta_1 = w_0 - \pi/2 \quad w_0 \in (0, \pi), \qquad \theta_2 = \frac{1 - \tan(B/2)}{1 + \tan(B/2)}. \tag{5}$$

The independence of w_0 and B is a very important property if adaptive filtering applications are the main objective. In order to update the parameter θ_1, the following algorithm was proposed (Regalia, 1991)

$$\theta_1(n+1) = \theta_1(n) + \mu(n)e(n)x_1(n) \quad \mu(n) > 0, \tag{6}$$

where $e(n)$ is the notch filter output, while the regressor

$$x_1(n) = \frac{z\cos(\theta_1)\cos(\theta_2)}{1 + \sin(\theta_1)(1 + \sin(\theta_2))z + \sin(\theta_2)z^2} u(n) \tag{7}$$

is available in the lattice realization. B (i.e., θ_2) can be fixed or controlled dynamically. The product $e(n)x_1(n)$ is not the gradient estimation (given by $\partial E[e^2(n)]/\partial\theta_1$), and as a consequence, (6) does not represent a recursive minimization of the output signal variance. In spite of that, convergence of this algorithm can be proved using the differential equation associated to (6) and the methodology of Ljung and Soderstrom (1983). The most remarkable property of this approach is its low computational complexity if compared with the other alternatives.

For the multiple sinusoids detection case, there is not an obvious way to extend the previous approach from second-order $V(z)$ to higher orders, since the regressor $x_1(n)$ is an ad-hoc choice aiming a low complexity realization. Despite of that, [Regalia, 1991] proposed to use in this case a cascade of second-order sections, justifying the proposal in the independence of the each frequency and their corresponding bandwidth. This basic scheme is illustrated in Fig. 5 for the detection of three sinusoids.

Although the application of this approach to the multiple sinusoids case presents reasonable results, it can be expected that the use of a global error, in place of the local errors, can result in a better accuracy in frequency estimation. This can be inferred from the fact that local errors imply the solution of several problems of insufficient order. On the other hand, using a global error a sufficient order problem results. In addition, a way to overcome the problem of root computation must be found. Possible solutions to these problems will be introduced in the proposal of the next section.

III. A NEW ADAPTIVE NOTCH FILTER

In this section the allpass based notch filter is modified to introduce two novel adaptive notch filtering algorithms. A first algorithm uses the allpass based model with a global error minimized using the SM method. The basic limitation of this algorithm is that is not trivial to obtain a minimal parameterization for the particular case of notch filtering applications. This motivates the introduction of a second algorithm, a second-order cascaded lattice notch filter that shows, as illustrated by examples in the next section, a suitable behavior.

A. SM based adaptive lattice notch filter (SM-ALNF)

To introduce the algorithm it is considered a structural interpretation of SM method based on a modified state space representation of the notch filter (Regalia, 1992). The corresponding transfer function is $H(z) = \frac{1}{2}[1 + V(z)]$ where $V(z)$ is an $2M$-order allpass filter. The modified state space representation is given by

$$\begin{bmatrix} \boldsymbol{x}(n+1) \\ w(n) \end{bmatrix} = \boldsymbol{Q}(n) \begin{bmatrix} \boldsymbol{x}(n) \\ u(n) \end{bmatrix}, \tag{8}$$

and

$$y(n) = \boldsymbol{h}^t \begin{bmatrix} \boldsymbol{x}(n+1) \\ w(n) \end{bmatrix}, \tag{9}$$

where $\boldsymbol{x}(n) = [x_1(n), x_2(n), \cdots, x_N(n)]^t$ is the state vector, $w(n)$ is the allpass output, $\boldsymbol{h}(n)$ is the tap vector and $\boldsymbol{Q}(n)$ $(N+1 \times N+1)$ is an orthogonal matrix, that for the lattice realization takes the form $\boldsymbol{Q}(n) = \boldsymbol{U}_1(n) \cdots \boldsymbol{U}_N(n)$, i.e., $\boldsymbol{Q}(n)$ is the product of N givens rotations $\boldsymbol{U}_k(n)$, such that $\boldsymbol{Q}^{-1}(n) = \boldsymbol{Q}^t(n)$. $\{\theta_k(n)\}$, $k = 1, \cdots, N$ determines the elements of $\boldsymbol{Q}(n)$. The m unknown notch frequencies are related to the $2M$-order allpass $V(z)$. Then, $N = 2M$ and $\boldsymbol{h}(n) = [0, 0, 0, \cdots, 0, 1]^t$, such that $y(n) = w(n)$. The structural interpretation of the SM for the lattice

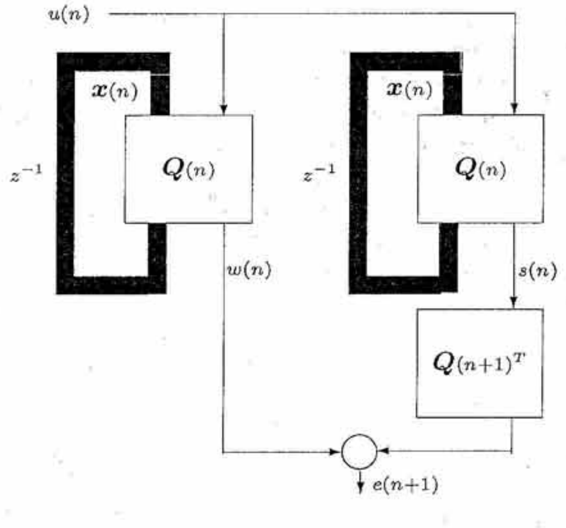

Figure 6: Structural interpretation of the SM method for the notch filter.

notch filter is illustrated in Fig. 6. The a posteriori prediction error is given by

$$e(n+1) = \frac{1}{2}\left\{[0\cdots 0\,1]\mathbf{Q}^t(n+1)\begin{bmatrix} x(n+1) \\ s(n) \end{bmatrix} + w(n)\right\}. \quad (10)$$

Then, defining $\nabla_k = \partial/\partial\theta_k(n+1)$ and $\boldsymbol{q}^t_{N+1}(n+1) = [0\cdots 0\,1]\boldsymbol{Q}^t(n+1)$, results

$$\nabla_k e(n+1) = \frac{1}{2}[\nabla_k \boldsymbol{q}^t_{N+1}(n+1)]\begin{bmatrix} \boldsymbol{x}(n+1) \\ s(n) \end{bmatrix}. \quad (11)$$

Using an a priori version of the prediction error and Lemma 2 of Regalia (1992), after some manipulations, it is possible to obtain

$$\begin{bmatrix} \nabla_1 e(n)/\beta_1(n) \\ \nabla_2 e(n)/\beta_2(n) \\ \vdots \\ \nabla_N e(n)/\beta_N(n) \\ d(n)/2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \boldsymbol{x}(n) \\ u(n) \end{bmatrix}, \quad (12)$$

where $\boldsymbol{x}(n)$ determines the required regressor. Using the fact that $\cos(\theta_k) > 0$ if $\theta_k \in (-\pi/2, \pi/2)$, it is possible to use a simplified version given by

$$\Psi_i(n) = -\nabla_i e(n) \cong -\frac{1}{2}x_i(n), \qquad i = 1, 2, \cdots, N, \quad (13)$$

that results in the following update equation for the *SM based adaptive lattice notch filter* (SM-ALNF)

$$\theta_{2i-1}(n+1) = \theta_{2i-1}(n) + \mu(n)\Psi_i(n)e(n), \quad (14)$$

where $\mu(n)$ is the step size. Note that for the particular case of $M = 1$, the SM-ALNF reduces to that of Regalia (1991), discussed in the previous section. In this way, the algorithm presented allows to give a formal interpretation of the previous ad-hoc result.

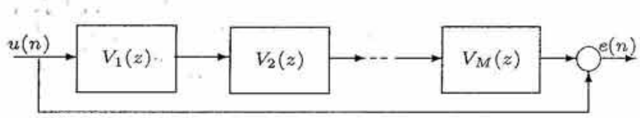

Figure 7: Alternative model for the notch filter, multiple sinusoids.

The main problem with this kind of parameterizations is that the independence between notch frequencies and the corresponding bandwidth is not simple. Indeed, since the relationship between the parameters and the notch frequencies is not linear, the realization is not minimal in the number of parameters. The following Lemma can be used to overcome this problem.
Lemma: Given an all-pass $V(z)$ of order $2M$ obtained using the normalized lattice parameterization, if $\theta_{2i} \to \pi/2$, $i = 1, 2, \cdots, M$, then

$$\theta_{2i-1} = w_i - \pi/2, \quad w_i \in [0, \pi], \quad i = 1, 2, \cdots, M. \quad (15)$$

Proof: using induction (Scoppa, 2002).

With this result at hand, and to avoid the updating of $2M$ parameters, it is possible to update θ_{2i-1}, $i = 1, 2, \cdots, M$ and give to θ_{2i}, $i = 1, 2, \cdots, M$, a variation law that makes $\theta_{2i} \to \pi/2$. In this way, the characteristics of this algorithm can be summarized as: i) linear relationship between the filter parameters and the notch frequencies, ii) utilization of a global error, iii) minimum number of parameters to update. In spite of these nice properties, this algorithm experiments problems in a time varying context, when the poles and zeros of $V(z)$ approach the unit circle.

B. SM based Adaptive Cascade Lattice Notch Filter (SM-ACLNF)

The problem of the previous algorithm can be solved using a second-order cascaded all-pass normalized lattice realization, as illustrated in Fig. 7, where

$$V_i(z) = \frac{\sin(\theta_{2i}) + \sin(\theta_{2i-1})(1+\sin(\theta_{2i}))z + z^2}{1 + \sin(\theta_{2i-1})(1+\sin(\theta_{2i}))z + \sin(\theta_{2i})z^2}. \quad (16)$$

The SM method can also be applied to minimize the prediction error and due to the particular realization used, the independence between notch frequencies and bandwidths for each pole pair is maintained. In this case

$$\theta_{2i-1} = w_i - \pi/2 \quad w_i \in (0, \pi), \qquad \theta_{2i} = \frac{1 - \tan(B_i/2)}{1 + \tan(B_i/2)}. \quad (17)$$

Instead of using the structural interpretation of the SM, a direct approach is used to minimize the prediction error using the SM method. Then, with the help of Fig. 8, it is possible to obtain

$$\begin{aligned} V_i^n(z) &= \frac{\overline{D_i^n}(z)}{D_i^n(z)} \\ &= \frac{\sin(\theta_{2i}) + \sin(\theta_{2i-1})(1+\sin(\theta_{2i}))z + z^2}{1 + \sin(\theta_{2i-1})(1+\sin(\theta_{2i}))z + \sin(\theta_{2i})z^2}. \end{aligned} \quad (18)$$

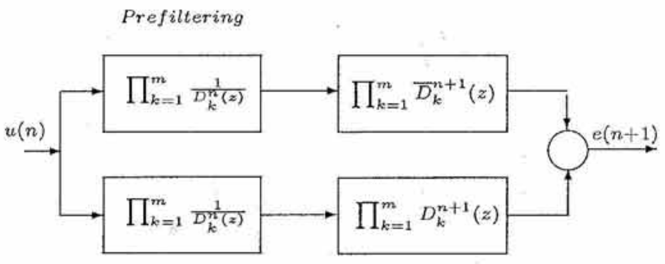

Figure 8: SM method for the SM-ACLNF.

The a posteriori error can be written as

$$e(n+1) = \frac{1}{2}\left[\prod_{k=1}^{m}\frac{\overline{D}_k^{n+1}(z)}{D_k^n(z)} + \prod_{k=1}^{m}\frac{D_k^{n+1}(z)}{D_k^n(z)}\right] u(n). \quad (19)$$

Then, using the a priori error, as usual for an on-line SM version,

$$e(n) = \frac{1}{2}\left[1 + \prod_{k=1}^{m}\frac{\overline{D}_k^n(z)}{D_k^n(z)}\right] u(n) \quad (20)$$

and

$$\frac{\partial e(n)}{\partial \theta_{2i-1}} = -\left[1 + \prod_{\substack{k=1\\k\neq i}}^{m}\frac{\overline{D}_k^n(z)}{D_k^n(z)}\right] \frac{(1+\sin(\theta_{2i}(n))cos(\theta_{2i-1}(n))}{D_i^n(z)}u(n-1). \quad (21)$$

If $\theta_{2i-1} \in (-\pi/2, \pi/2)$, then $cos(\theta_{2i-1})$ is a scale factor that can be introduced in the step size $\mu(n)$. The same can be said if $(1+\sin(\theta_{2i})) > 0$. Finally, using these simplifications, the regressor is given by

$$\Psi_i(n) = -\left[1 + \prod_{\substack{k=1\\k\neq i}}^{m}\frac{\overline{D}_k^n(z)}{D_k^n(z)}\right] \frac{1}{D_i^n(z)}u(n-1) \quad (22)$$

and the following upgrade equation for the *SM based adaptive cascaded lattice notch filter* (SM-ACLNF) results

$$\theta_{2i-1}(n+1) = \theta_{2i-1}(n) + \mu(n)e(n)\Psi_i(n). \quad (23)$$

Some characteristics of the SM-ACLNF are the following: i) due to the lattice realization, its numerical properties are robust. Indeed, there is a linear relation between parameters and notch frequencies (that eliminates the requirement of solving a high order polynomial), ii) the algorithm allows control of the notch frequencies and bandwidths separately; and in this sense it is unique. iii) The complexity required for the computation of the regressor is higher than with the other

	SMM-ANF	SM-ALNF	SM-ACLNF
mean w_1	0.40003	0.39970	0.40000
mean w_2	0.30000	0.30021	0.30002
mean w_3	0.19990	0.19978	0.20001
mean w_4	0.99980	0.10033	0.10002
var w_1	1.00 E-09	2.00 E-08	1.70 E-09
var w_2	1.40 E-09	1.00 E-07	1.20 E-09
var w_3	1.40 E-09	1.00 E-07	9.00 E-10
var w_4	1.30 E-09	3.00 E-08	2.00 E-10
Iterations	500	900	400

Table 1: Simulation 1: stationary context, SNR = 10 dB, ρ fixed.

approaches. Further research is being performed in order to overcome this problem. Also, it is expected that due to the utilization of a global error, a better frequency estimation can be obtained, if compared with the adaptive notch filter of Cheng and Tsai (1998).

IV. DISCUSSION AND COMPARISONS

To compare the algorithms two simulation contexts for the detection of four sinusoidal signals in AWGN are considered: stationary and non stationary. A recursive least square version of the algorithms was implemented, using different predefined variation laws for ρ. The comparisons include the following algorithms: SM-ANF, SM-ALNF and SM-ACLNF.

Simulation 1, Stationary context. The results are summarized in Table 1. As can be observed, the SMM-ANF and the SM-ACLNF have similar performance either in convergence speed or mean and variance of the estimated frequencies, with the advantage in the last case of the direct availability of the estimated parameters. It was observed in general that, in this context, the SMM-ANF and the SM-ACLNF have comparable convergence speed.

Simulation 2. Non stationary context. One of the four sinusoidal signals switches its frequency in cycles of 2000 samples (from 0.35 to 0.25), periodically. In this case since the SMM-ANF uses a similar ρ for all the poles, a poor performance is expected. Figures 9, 10 and 11 illustrate the performance of the SMM-ANF, SM-ALNF and SM-ACLNF, respectively. Due to the linear relationship between the parameters and the frequencies to be estimated, the SM-ALNF and SM-ACLNF have superior performance in a non stationary context, as can be concluded from the figures. Indeed, the algorithm with lower variance in the estimated frequencies was the SM-ACLNF.

V. CONCLUSIONS

Two new algorithms were introduced as alternative to the SM based adaptive notch filter: the SM based adaptive lattice notch filter (SM-ALNF) and the SM based adaptive cascade lattice notch filter (SM-ACLMF). This family of algorithms represents an alternative to the classical RPE based methods for adaptive notch filtering. The new algorithms use a global

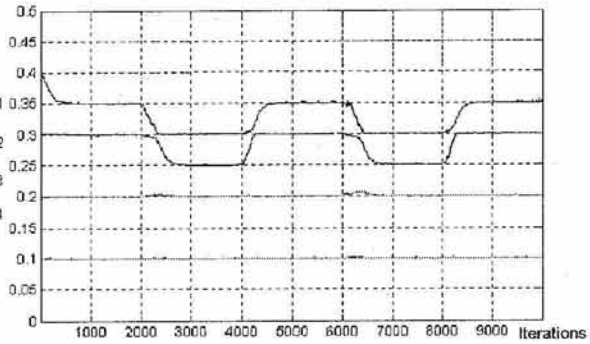

Figure 9: Simulation 2: non stationary context, SMM-ANF, notch frequencies $w_0 = 0.1$, $w_0 = 0.2$, $w_0 = 0.3$, $w_0 = 0.35$. Signal-to-noise ratio: 0 dB.

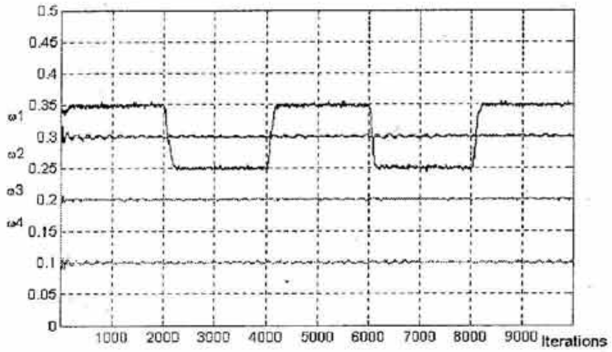

Figure 10: Simulation 2: non stationary context, SM-ALNF, notch frequencies $w_0 = 0.1$, $w_0 = 0.2$, $w_0 = 0.3$, $w_0 = 0.35$. Signal-to-noise ratio: 0 dB.

error and a normalized lattice realization. The simulations presented show the improved performance of the algorithms in non stationary environments.

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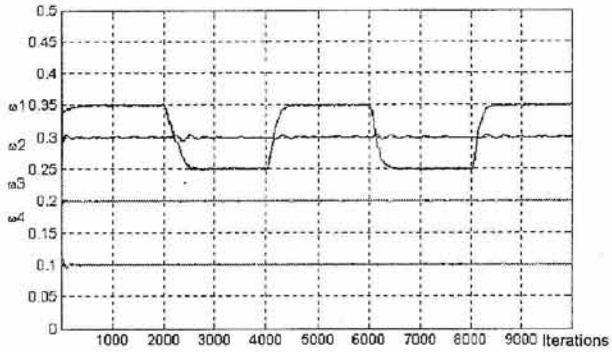

Figure 11: Simulation 2: non stationary context, SM-ACLNF, notch frequencies $w_0 = 0.1$, $w_0 = 0.2$, $w_0 = 0.3$, $w_0 = 0.35$. Signal-to-noise ratio: 0 dB.

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