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Adaptive control of a Wiener type system: application of a pH Neutralization reactor

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In this paper the problem of on-line identification and adaptive control of a Wiener type non-linear system is studied. First, a Wiener model is defined whose linear and non-linear parts are described using Laguerre and piecewise linear basis functions, respectively. Then, an adaptive identification algorithm for this model is presented. A local convergence analysis for the adaptive identification is performed. The model obtained is used to adapt the parameters of a controller designed for the specific structure of the model. The complete scheme is applied to a simulation of a pH neutralization reactor subject to several perturbations. The results show the improved behaviour of the proposed scheme compared with other approaches found in the literature.

1. Introduction

In the last decades, many contributions for controller design have been based on the assumption of a linear model of the system. However, in some cases it is difficult to represent a given process using a linear model. This is the case when the system is highly non-linear and the operating point changes within a wide region, or when the process is non-stationary, i.e., the characteristics change with time.

In these cases, the controller design can be performed using special techniques, such as exact linearization, non-linear model predictive control, or other special-purpose procedures (Ogunnaike and Ray 1994).

One of the solutions to control such a system is adaptive control, where the parameters of a linear controller are adjusted to follow the variations of the process behaviour. Several control schemes assume a model structure whose parameters are identified on-line using an adaptive identification algorithm. The identified model parameters are then used for updating the controller parameters.

It is well known that some systems can be described by a linear dynamic model followed by a static non-linearity, referred to as Wiener systems (Pearson and Potman 2000, Pearson 2003). Pajunen (1987) proposes two adaptive control schemes for the control of Wiener systems. However, in the identification algorithms of these schemes, the Wiener structure is lost, resulting in a large number of parameters for the process model.

The first step in the construction of an adaptive control algorithm is to obtain an efficient adaptive identification scheme. In the literature we can find four main approaches for the identification of Wiener models. The first one is the traditional iterative algorithm proposed by Narendra and Gallman (1966) for the identification of the Hammerstein model. In this algorithm, an appropriate parameterization of the system allows the prediction error to be separately linear in each set of parameters characterizing the linear and the non-linear parts. The estimation is carried out by minimizing alternatively a quadratic criterion on the prediction errors with respect to each set of parameters.

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An analytical counterexample by Stoica (1981) showed that the original algorithm could diverge in some particular cases.

The second approach, based on correlation techniques, was introduced by Billings and Fakhouri (1978) and Billings (1980). The method relies on a separation principle, but with the rather restrictive requirement on the input to be white noise. A similar approach was introduced by Greblicki (1994).

The third approach is a more recent technique for the identification of Wiener models introduced by Kalafakis *et al.* (1995, 1997) and extended to general block-oriented models by Bai (1998). In particular, Bai's algorithm is based on least-squares estimation (LSE) and singular value decomposition (SVD). The approach is useful only for the single input/single output (SISO) cases due to the particular parameterization used. Gomez and Baeyens (2004) extended the results of Bai to include a more general parameterization that enables the use of multiple input/multiple output systems (MIMO).

The fourth approach introduced by Wigren (1993, 1994) is a recursive identification scheme that uses a non-linear Wiener model with a fixed (known) piecewise linear (PWL) non-linearity. In particular, parameter convergence can be verified using an IIR model in the linear part if the non-linearity is strictly monotonic (i.e., invertible). In addition, output error convergence for an FIR linear part can be obtained if the known non-linearity is monotonic (Wigren 1998). Figueroa *et al.* (2004) presented an algorithm for adaptive filtering using a Wiener structure with an FIR filter to describe the linear block and a PWL non-linear function to describe the non-linear gain. The same non-linear description is used in the present work.

In this paper we propose a new approach for adaptive control of Wiener type systems. Using a particular parametrization of the model, an adaptive identification scheme is proposed extending the idea of Gomez and Baeyens (2004). Using this approach, the inverse of the non-linearity is directly identified, avoiding the inversion problem. After that, the structure of the Wiener model is fully exploited to obtain all possible advantages.

The performance of the proposed algorithm is tested in the control of a neutralization reactor. It is well known that the control of pH processes is particularly difficult. The main reason is the strong non-linearity involved. The slope of a chemical system's titration curve can vary several orders of magnitude over a modest range of pH values, causing the overall process gain to change accordingly. Several non-linear schemes have been proposed for the control of this kind of processes (Norquay *et al.* 1998, 1999, Gerkšič *et al.* 2000, Lussón Cervantes *et al.* 2003a, Biagiola *et al.* 2004, Åkesson *et al.* 2005). The basic assumptions of these schemes are a fixed Wiener structure of the model and a non-linear controller. Several control strategies were used, including gain scheduling, model predictive control, H_∞, etc.

However, when perturbations are applied to the process, a fixed Wiener model cannot represent the process adequately. For example, the titration curve changes drastically (Kalafatis *et al.* 2005a). Some authors have applied robustness ideas in the controller design (Lussón Cervantes *et al.* 2003b), by using feedforward controllers (Kalafatis *et al.* 2005a) or adaptive identification algorithm (Kalafatis *et al.* 2005b). Pajunen (1992) proposes an adaptive algorithm to adapt the controller parameters when the process is varying. In this paper we want to propose a more efficient algorithm to solve the adaptive control.

The paper is organized as follows. In § 2, the model structure and the adaptive identification procedure are presented. The control design for fixed parameters is discussed in § 3. Section 4 describes the adaptive control algorithm and discusses some implementation details. A simulation example describing the application to control a pH neutralization reactor is detailed in § 5. Finally, § 6 draws the conclusions.

2. Adaptive identification scheme

2.1 *Model description*

In this paper, we propose a special description for the process where the linear dynamic model is described by a Laguerre basis series and the non-linear static block is modelled as a PWL model. This model is illustrated in figure 1.

The Laguerre basis allows the use of prior knowledge about the dominant poles (Wahlberg 1991, 1994, Lindskog 1996). This model describes the linear model with the following basis function expansion

$$H(z) = \sum_{i=0}^{M} h_i L_i(z, a), \tag{1}$$

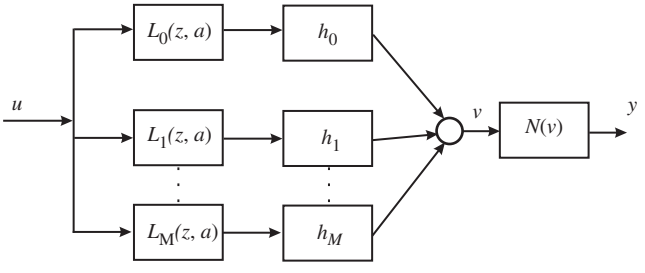

Figure 1. The Laguerre-PWL Wiener model.

where

$$L_i(z,a) = \frac{\sqrt{1-a^2}}{z-a}\left(\frac{1-az}{z-a}\right)^{i-1} \tag{2}$$

and h_i are the parameters of the model, $a \in \Re$ is a filter coefficient chosen *a priori* (Wahlberg 1991).

For the representation of the static non-linear gain, $N(\cdot)$, we use a PWL description. In general $N(\cdot)$ is real-valued function of one variable, i.e., $y = N(v)$: $\Re^1 \to \Re^1$, which we assume to be invertible. (This is a common assumption is the literature on Wiener type systems that simplifies the identification and controller design (Lussón *et al*. 2003a). However, it is not necessary considering the non-invertible static non-linearity used in the algorithms of Wigren (1990, 1993).) For the specific control algorithm that we use, we prefer to describe the inverse of this non-linearity, i.e., $v = N^{-1}(y)$.

The PWL function have proved to be a very powerful tool in the modeling and analysis of non-linear systems (Chua and Ying 1983, Julián *et al*. 1999). These functions allow a systematic and accurate treatment of approximating functions. The PWL functions enable the representation of a non-linear system through a set of linear expressions, each of them valid in a certain operating region. To make this approximation, the domain of variables $\aleph^i$ is partitioned into a set of σ non-empty regions $\aleph^i$, such that $\aleph = \bigcup_{i=1}^{\sigma} \aleph^i$. In each of these regions, $\aleph^i$, the non-linear function is approximated using a linear (affine) representation.

It can be proved (Julián *et al*. 1999) that any continuous non-linear function $f(v)$: $\Re^m \to \Re^1$ can be uniquely represented using PWL functions. Lussón *et al*. (2003) use these functions in the context of a model predictive control algorithm for Wiener models. Moreover, Figueroa *et al*. (2004) use this representation in the context of an adaptive filter algorithm.

To express the nonlinear function (see figure 1), we will use a function expansion with the basis functions and parameters (Julián *et al*. 1999)

$$v = N^{-1}(y) = \mathbf{c}^T \mathbf{\Lambda}(y). \tag{3}$$

We will consider the case where the basis functions that are included in the vector $\mathbf{\Lambda}$ have been predetermined and the values of vector $\mathbf{c}$ are the parameters to be estimated. In this paper we use the orthogonal basis description proposed by Julián *et al*. (2000). The elements of the basis depend on the partitioning of the domain ($\aleph^i$) and on the variable y.

Since the basis functions $\mathbf{\Lambda}$ are fixed, the output is a linear function of the parameters. This allows us to use linear regression to estimate the parameters. The two basic advantages of this approach are that it is fast and gives a unique estimate.

We note that in Kalafakis *et al*. (1995, 1997) a similar parameterization for the model is used. In that paper, the linear block is modelled as a frequency-sampling filter and the nonlinear block as a polynomial basis. In particular, the choice of PWL functions allows a more efficient realization of the static non-linearity (Figueroa *et al*. 2004).

2.2 *Adaptive implementation*

Let us define an adaptive algorithm for the identification of the Wiener model described in previous section. As shown figure 1, the signal $v(k)$ is given by

$$v(k) = \sum_{i=0}^{M} h_i L_i[u(k)] \tag{4}$$

and also as

$$v(k) = N^{-1}[y(k)] = \mathbf{c}^T \mathbf{\Lambda}[y(k)] = \sum_{i=0}^{N} c_i \mathbf{\Lambda}_i[y(k)]. \tag{5}$$

By equalling both sides of (4) and (5) (including $\epsilon(k)$ to allow for a modelling error), and fixing the parameter $h_0 = 1$ to overcome the well-known gain ambiguity in Wiener models, the following equation is obtained

$$\sum_{i=0}^{N} c_i \Lambda_i[y(k)] = L_0[u(k)] + \sum_{i=1}^{M} h_i L_i[u(k)] + \epsilon(k) \tag{6}$$

or rewritten as a linear regression

$$\begin{aligned}\epsilon(k) &= \sum_{i=0}^{N} c_i \Lambda_i[y(k)] - L_0[u(k)] - \sum_{i=1}^{M} h_i L_i[u(k)] \\ &= \boldsymbol{\theta}^T(k)\boldsymbol{\phi}(k) - L_0[u(k)],\end{aligned} \tag{7}$$

where vectors $\boldsymbol{\theta}(k)$ and $\boldsymbol{\phi}(k)$ are defined as

$$\boldsymbol{\theta}(k) = [c_0, c_1, c_2, \ldots, c_N, h_1, h_2, \ldots, h_M]^T \tag{8}$$

$$\begin{aligned}\boldsymbol{\phi}(k) = [&\Lambda_0[y(k)], \Lambda_1[y(k)], \ldots, \Lambda_N[y(k)], \\ &-L_1[u(k)], -L_2[u(k)], \ldots, -L_M[u(k)]]^T.\end{aligned} \tag{9}$$

Next we consider an LMS (or stochastic gradient) algorithm that allows us to recursively estimate the parameters of the model $\boldsymbol{\theta}$. For this purpose, we use as an objective function J_θ the instantaneous squared error

$$J_{\boldsymbol{\theta}}[\epsilon(k)] = \epsilon^2(k) = \left\{\boldsymbol{\theta}^T(k)\boldsymbol{\phi}(k) - L_0[u(k)]\right\}^2. \tag{10}$$

The recursion of the LMS based algorithm that minimizes the above objective function is given by

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \mu\boldsymbol{\phi}(k)\epsilon(k), \tag{11}$$

where μ is the step size controlling the convergence and final error of $\boldsymbol{\theta}$.

In order to ensure convergence, the step size μ in equation (11) is chosen in the range

$$0 < \mu < \frac{1}{\lambda}, \tag{12}$$

where λ is the maximum eigenvalue of the matrix $E(\phi\phi^T)$.

2.3 *Local convergence analysis*

Local convergence analysis of the proposed algorithm can be performed using the ordinary differential equation (ODE) method (Ljung 1977a, 1977b, Ljung and Söderstrom 1983). The general ODE approach comes from the field of stochastic approximation theory, and enables us to convert the study of convergence of a stochastic non-linear equation into the study of the stability of the solutions of a deterministic differential equation.

Two different kinds of algorithms can be studied in this form: vanishing-gain algorithms (i.e., with the step size $\mu \to 0$), mostly oriented to estimation in a stationary environment, and constant-gain algorithms (of interest here), where the step size μ is kept constant in order to consider tracking situations. For the latter case the ODE method guarantees that the adaptation algorithm converges in probability (not with probability one as with vanishing gain algorithms).

Consider a generic stochastic gradient algorithm of the form

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \mu\epsilon[\boldsymbol{\theta}(k)]\nabla[\boldsymbol{\theta}(k)] \tag{13}$$

where $\boldsymbol{\theta}(k)$ is the parameter vector, $\epsilon(k)$ is the error to be minimized and $\nabla(k)$ is the estimated gradient of $\epsilon(k)$. The associated ODE has the form

$$\frac{\partial \boldsymbol{\theta}_D(t)}{\partial t} = \boldsymbol{f}(\boldsymbol{\theta}_D) \tag{14}$$

where $\boldsymbol{f}(\boldsymbol{\theta}_D)$ depends on the specific algorithm used. In order to guarantee the ODE association some conditions much be verified.

Following the ODE method, the input $u(k)$ and the output $y(k)$ signals are considered stochastic stationary processes and the filters that generate the signals $\epsilon(k)$ and $\nabla(k)$ are considered exponentially stable. Then, $\boldsymbol{f}(\cdot)$ represents an average adaptation direction that can be used to study the convergence properties of the algorithm. For example, stationary points $\boldsymbol{\theta}^*$ of the generic algorithm can be obtained by means of the solutions of $\boldsymbol{f}(\boldsymbol{\theta}^*) = 0$. In particular, if local convergence is addressed, we can use the indirect method of Lyapunov (i.e., linearization around a stationary point) to perform the study. Then, based on a Taylor series expansion of (14), we construct a related linear differential equation of the form

$$\frac{\partial \boldsymbol{\theta}_D(t)}{\partial t} = -\left[\frac{\partial \boldsymbol{f}[\boldsymbol{\theta}_D(t)]}{\partial \boldsymbol{\theta}_D}\right]_{\boldsymbol{\theta}_D = \boldsymbol{\theta}_D^*} [\boldsymbol{\theta}_D(t) - \boldsymbol{\theta}_D^*]. \tag{15}$$

The point $\boldsymbol{\theta}_D^*$ is a stable stationary point of (14) if and only if $\boldsymbol{\theta}_D = 0$ is an exponentially stable stationary point of (15). This is equivalent to showing that the eigenvalues of the stability matrix $-[\partial \boldsymbol{f}(\boldsymbol{\theta}_D(t))/\partial \boldsymbol{\theta}_D]_{\boldsymbol{\theta}_D^*}$ is a stability matrix if it is positive definite.

Now, the ODE associated with our problem is

$$\frac{\partial \boldsymbol{\theta}}{\partial t} = E[\phi\epsilon(k)] \tag{16}$$

Assuming that the true non-linear model is described by the Laguerre-CPWL filter except for a bounded zero-mean measurement noise $r(k)$, it can be verified that a stationary point $\boldsymbol{\theta}^*$ of the proposed algorithm corresponds to the solution of $E[\phi\epsilon(k)] = 0$. This rather restrictive assumption is usually made (Wigren 1993).

With local convergence properties in mind, a linearization of (16) in a neighbourhood of the stationary point $\boldsymbol{\theta}^*$ leads to the following expression

$$\frac{\partial \boldsymbol{\theta}}{\partial t} \cong -\left.\frac{E[\phi\epsilon(k)]}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}^*} (\boldsymbol{\theta} - \boldsymbol{\theta}^*). \tag{17}$$

Using the basic definitions from equations (10) and (11) we obtain

$$\boldsymbol{P}(\boldsymbol{\theta}^*) = \left.\frac{E[\phi\epsilon(k)]}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}^*} = E[\phi\phi^T]_{\boldsymbol{\theta}^*}. \tag{18}$$

Local convergence can be guaranteed if the eigenvalues of $\boldsymbol{P}^T(\boldsymbol{\theta}^*)$ have positive real part. Note that matrix $\boldsymbol{P}(\boldsymbol{\theta}^*)$ is symmetric and positive semidefinite. Therefore, local convergence is assured if matrix $\boldsymbol{P}(\boldsymbol{\theta}^*)$ is positive definite.

With ϕ defined in (9), equation (18) becomes

$$\boldsymbol{P}(\boldsymbol{\theta}^*) = \left[\begin{bmatrix}\boldsymbol{\Lambda} \\ \boldsymbol{Ł}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Lambda} \\ \boldsymbol{Ł}\end{bmatrix}^T\right]_{\boldsymbol{\theta}^*}, \tag{19}$$

where $\boldsymbol{\Lambda} = [\boldsymbol{\Lambda}_0[y(k)], \ldots, \Lambda_N[y(k)]]^T$ and $\boldsymbol{Ł} = [-L_1[u(k)], \ldots, -L_M[u(k)]]$. Note that matrix $\boldsymbol{P}(\boldsymbol{\theta}^*)$ is independent of the parameter vector $\boldsymbol{\theta}$.

The local convergence results related to the proposed adaptive nonlinear identification algorithm are summarized in the following theorem.

Theorem 1: *Consider the algorithm described in table* 1. *Assume that*

1. $u(k)$ *is a bounded, zero-mean, stationary stochastic signal persistently exciting of order* M.

2. *The true process is described by a Wiener-CPWL model except for a measurement noise* $r(k)$, *which is a stationary stochastic process not correlated with input* $u(k)$.
3. $y(k)$ *is such that the probability density function* (*pdf*), $h_v(v)$, *of* $v(k|\boldsymbol{\theta}^*)$ *fulfills* $h_v(v) \geq \delta > 0$ *in at least one partition of the PWL function.*

Then, the proposed algorithm is locally convergent to a stationary point $\boldsymbol{\theta}^*$.

Proof: For the proof we rewrite equation (19) as

$$\boldsymbol{P}(\boldsymbol{\theta}^*) = E\begin{bmatrix} \boldsymbol{\Lambda}\boldsymbol{\Lambda}^T & \boldsymbol{\Lambda}\boldsymbol{Ł}^T \\ \boldsymbol{Ł}\boldsymbol{\Lambda}^T & \boldsymbol{Ł}\boldsymbol{Ł}^T \end{bmatrix} = E\begin{bmatrix} 0 & \boldsymbol{\Lambda}\boldsymbol{Ł}^T \\ \boldsymbol{Ł}\boldsymbol{\Lambda}^T & \boldsymbol{Ł}\boldsymbol{Ł}^T \end{bmatrix} + E\begin{bmatrix} \boldsymbol{\Lambda}\boldsymbol{\Lambda}^T & 0 \\ 0 & 0 \end{bmatrix}, \tag{20}$$

where the first term on the left-hand side of equation (20) is positive definite by construction, i.e.,

$$E\begin{bmatrix} 0 & \boldsymbol{\Lambda}\boldsymbol{Ł}^T \\ \boldsymbol{Ł}\boldsymbol{\Lambda}^T & \boldsymbol{Ł}\boldsymbol{Ł}^T \end{bmatrix} \geq 0. \tag{21}$$

Because the entries of $\boldsymbol{\Lambda}$ and $\boldsymbol{Ł}$ from two orthogonal bases we have

$$E[\boldsymbol{\Lambda}\boldsymbol{\Lambda}^T] > 0 \tag{22}$$

and

$$E[\boldsymbol{Ł}\boldsymbol{Ł}^T] > 0. \tag{23}$$

By invoking Lemma 5.1 in Wigren (1990), we can conclude that $\mathbf{P}(\boldsymbol{\theta}^*)$ is positive definite. □

The conditions of Theorem 1 are usual assumption for this kind of results: all the dynamics of the model should be excited, the system should be perfectly by the model and the pdf needs to be non-zero in at least one non-zero interval in each piecewise interval of the PWL function.

With respect to the third condition, it expresses the fact that we need energy in each interval where the static non-linearity should be adapted (Wigren 2003). Moreover, the signal energy could be concentrated in the intervals where a high accuracy of the identified model is required. Because the PWL description is continuous in the adjacent intervals (due to the continuity of the PWL functions (Julián *et al.* 1999), once the value at the extreme of an interval is fixed, it is the same as the value of the adjacent interval), it is possible to relax the energy condition to be concentrated in every second interval (Wigren 2003).

Table 1. The proposed adaptive non-linear identification algorithm.

Parameters
- M = number of $\boldsymbol{h}$ coefficients
- N = number of $\boldsymbol{c}$ coefficients
- μ = step size

Data
- $u(k)$ input signal at time k
- $y(k)$ output signal at time k

Initialization
- $\boldsymbol{h}(0) = 0$
- $\boldsymbol{c}(0) = [-1 \quad 1 \quad 0 \quad \ldots \quad 0 \quad -1]^T$

For each k
- $\boldsymbol{\theta}(k) = [c_0, \ldots, c_N, h_1, \ldots, h_M]^T$
- $\phi(k) = [\Lambda_0[y(k)], \ldots, \Lambda_N[y(k)], -L_1[u(k)], \ldots, -L_M[u(k)]]^T$
- $\epsilon(k) = \boldsymbol{\theta}^T(k)\phi(k) - L_0[u(k)]$
- $\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \mu\phi(k)\epsilon(k)$

3. Controller design

In the context of adaptive control, the essential idea is to identify a process and, based on the obtained model, adjust the controller parameters to improve the closed-loop performance. For tuning the control parameters any classical strategy could be used, like minimum ITAE, retain constant loop gain, Ziegler–Nichols, Cohen–Coon, internal model control, etc. (Ogunnaike and Ray 1994). In our application, we use a direct synthesis approach, which was modified to be applicable to a Wiener model.

3.1 *Controller design for known process parameters*

In order to design the controller, we will follow the principle of the non-linear regulator as presented by Wigren (1990). Consider the closed-loop system of figure 2. The process is assumed to be represented by a Wiener model with a non-linear gain $N(\cdot)$ that is invertible. We can use the inverse of this block to extract the nonlinearities outside the closed loop. In this way, a linear controller K should be designed to compensate the behaviour of the linear dynamic block of the process model.

In our case, the inverse of the non-linearity is obtained directly from the identification process. To design the

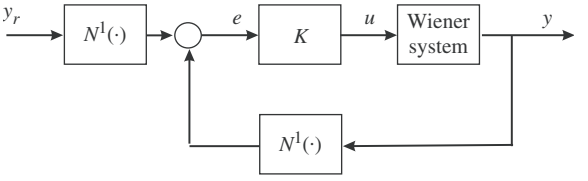

Figure 2. The closed-loop scheme for a known Wiener model.

linear controller, we adopt the direct synthesis approach (Ogunnaike and Ray 1994), applied to the Laguerre model of equation (4). The controller specification is to obtain a closed-loop pole in a_c without offset when the set point is changed in the form of steps. If the discrete transfer function of the linear model is called $H(z)$ and $H^{-1}(z)$ is stable and causal, then the controller can be defined as

$$K(z) = \frac{a_c}{z-1}\frac{1}{H(z)}. \tag{24}$$

A similar approach for the control of pH neutralization was presented in Kalafakis *et al*. (2005a) under the name of Linearizing control. In that paper, the linear controller is also designed by the direct synthesis method, but in the state variable form. An approach to the adaptive control was also proposed employing a feedforward approach for the on-line updating of the non-linear block.

3.2 *Adaptive controller*

Let us now consider the problem of controlling the process when the parameters of the Wiener model are unknown and varying along the operation. We can use the adaptive identification algorithm of § 2 to obtain the parameters of the model and use them to adapt the controller coefficients.

The complete adaptive scheme is shown in figure 3. The dotted lines denote the parameter information flow from the identification scheme to the compensator. In this formulation, in order to ensure the persistence of excitation, it may be necessary to add a random signal to the manipulated variable.

This formulation presents some theoretical limitations. Below we consider some ''practical'' remarks with respect to this scheme.

- The adaptation algorithm for identification (§ 2) is based on open-loop data. This is not the case in the present use. Moreover, the presence of measurement noise could make the process input to be correlated with the output in such a way that the first hypothesis of Theorem 1 is not fulfilled in practice. (A possible solution to this fact is to consider in the adaptive algorithm concepts related to closed-loop identification for Wiener models (Chou and Verhaegen 1999, Bloemen *et al*. 2001). This problem is currently under investigation.) However, extensive computer simulations verify that the controller integral action filters the measurement noise, which reduces the correlation effects of the measurement noise on the control action.
- The time constants of the processes (identification and control) are different. The identification should perform well even when changes of the operation point and the controller happen at any sample time. Then, there is a practical decorrelation of both problems (identification and control) as is discussed in the paper. This fact is confirmed by simulation results.
- The conditions on the identification algorithm in an adaptive control scheme need to be modified. In particular, since the control loop will act to maintain the operative point at the set point, the identification algorithm will act to adapt the model on a limited part of the whole operative region. This will obviously relax the conditions for the persistent excitation of Theorem 1.
- Regarding the on-line change of the controller parameters from the implementation point of view, the tuning of these parameters should be robust enough to ensure stability. This is reasonable due to the different time constants mentioned above.

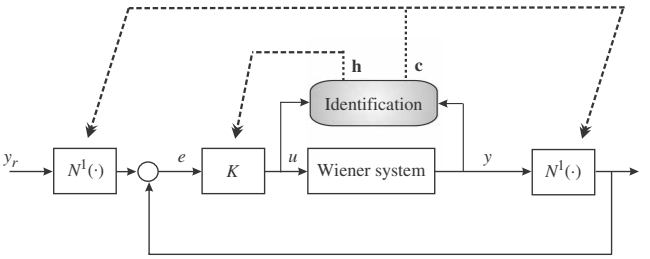

Figure 3. The control adaptation scheme.

In the next section, this scheme is applied to the control of a pH neutralization reactor.

4. Example: pH neutralization

4.1 *Process description*

A chemical process with a strong non-linearity was selected. The example considers a neutralization reaction between a strong acid (HA) and a strong base (BOH) in the presence of a buffer agent (BX), as described by Galán (2000). The neutralization takes place in a continuously stirred tank reactor (CSTR) with a constant volume V.

It is well known that the control of pH processes is particularly difficult (Norquay *et al*. 1999, Biagiola *et al*. 2004, Åkesson *et al*. 2005). The main reason is the strong non-linearity involved. The slope of the titration curve of the chemical system can vary several orders of magnitude over a modest range of pH values, causing the overall process gain to change accordingly. The regions of high and low slope on the titration curve correspond to conditions of high and low gain for a pH control loop, respectively.

In the continuous pH neutralization reactor, an acidic solution with a time-varying volumetric flow $q_A(t)$ of a composition $x_{1i}(t)$ is neutralized using an alkaline solution with volumetric flow $q_B(t)$ of known composition made up of base x_{2i} and buffer agent x_{3i}. Due to the high reaction rates of the acid-base neutralization, chemical equilibrium conditions are instantaneously achieved. Moreover, assuming that the acid, the base and the buffer are strong enough, the total dissociation of the three compounds takes place.

The process dynamics model can be obtained by considering the electroneutrality condition (which is always preserved) and through mass balances of equivalent chemical species (known as chemical invariants) that were introduced by Gustafsson and Waller (1983). For this specific case, under the previous assumptions, the dynamic behaviour of the process can be described considering the following state variables:

$$x_1 = [A^-] \tag{25}$$

$$x_2 = [B^+] \tag{26}$$

$$x_3 = [X^-]. \tag{27}$$

Therefore, the mathematical model of the process can be written in the following way (Galán 2000):

$$\dot{x}_1 = \frac{1}{\eta}(x_{1i} - x_1) - \frac{1}{V}x_1 q_B \tag{28}$$

$$\dot{x}_2 = -\frac{1}{\eta}x_2 + \frac{1}{V}(x_{2i} - x_2)q_B \tag{29}$$

$$\dot{x}_3 = -\frac{1}{\eta}x_3 + \frac{1}{V}(x_{3i} - x_3)q_B \tag{30}$$

$$F(x,\xi) \equiv \xi + x_2 + x_3 - x_1 - K_w/\xi - x_3/[1 + (K_x\xi/K_w)] = 0, \tag{31}$$

where $\xi = 10^{-pH}$ and $\theta = V/q_A$. K_w and K_x are the dissociation constants of the buffer and the water, respectively. The parameters of the system represented by equations (28)–(31) are addressed in table 2. Equation (31) was derived by McAvoy (1972), and takes the standard form of the widely used implicit expression which connects the pH value with the states of the process.

Noted that the pH neutralization process does not follow a specific Wiener model. However, under certain reasonable assumptions, it can be adequately represented by a Wiener model structure (Kalafatis *et al.* 2005a, b). For example, this is the case if the flow rate of the control reagent is much smaller that the feed flow rate. Moreover, the representation of the pH process has proven very convenient for the development of various pH identification and control strategies.

Table 2. Neutralization parameters.

Parameter	Value
x_{1i}	0.0012 mol HCL/l
x_{2i}	0.0020 mol NaOH/l
x_{3i}	0.0025 mol $NaHCO_3$/l
K_x	10^{-7} mol/l
K_w	10^{-14} mol^2/l^2
q_A	1 l/m
V	2.5 l

4.2 *Wiener model*

A Wiener model describing this process has been presented for several control applications using q_B (manipulated variable) to control the pH (controlled variable), see, e.g., Lussón *et al.* (2003a), and Biagiola and Figueroa (2004). However, when perturbations are present in the process ($q_A(t)$ and $x_{1i}(t)$), a single Wiener model cannot provide an adequate representation of the plant, because the parameters of the linear and non-linear block can vary significantly (Biagiola *et al.* 2005).

The chosen parameters for our model are a third-order Laguerre basis with a pole at $a = 0.7$ to represent the linear dynamic model, i.e. $M = 2$. To represent the inverse of the non-linear gain, the domain of the pH, the range [3, 9.5], is divided in 10 regions, i.e. $N = 11$. The adaptation step size is $\mu = 0.015$, and was chosen to satisfy the bound in equation (12).

4.3 *Linear compensator*

In this particular application, the linear controller takes the form

$$K(z) = \frac{a_c(z^3 - 3az^2 + 3a^2z - a^3)}{\sqrt{1-a^2}((1 - ah_2 + a^2h_3)z^2 + (h_2 - 2ah_3 + a^2 - 2a)z + (h_3 - ah_2 + a^2))(z-1)}, \tag{32}$$

where the closed-loop pole is fixed at $a_c = 0.8$. In the adaptive adjustment of this controller, it is important to check the stability at every iteration. If a pole is outside the unit circle, it should be replaced by its stable reciprocal.

4.4 *Simulation results*

In this section the results of the proposed adaptive controller are presented, and the performance is

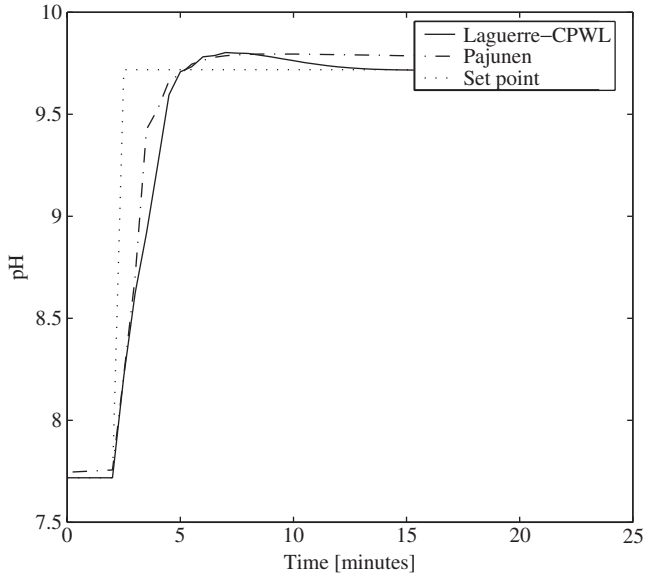

Figure 4. Controlled variable $\mathrm{pH} = -\log_{10}\xi$ for a single set point change.

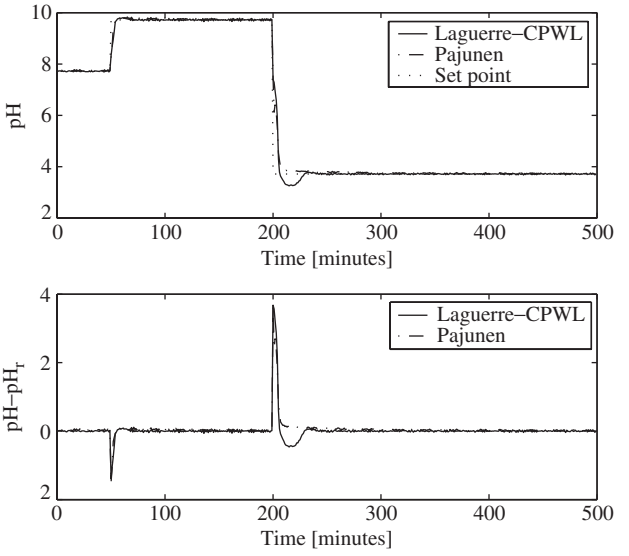

Figure 5. Controlled variable $\mathrm{pH} = -\log_{10}\xi$ without perturbations (top), and difference from reference signal (bottom).

compared with that of the model reference adaptive technique proposed by Pajunen (1992).

Figure 4 shows the performance of both controllers for a single set point change from $\mathrm{pH}_1 = 7.7182$ to $\mathrm{pH}_2 = 9.7182$ at $t = 2$ mm. From this plot it is clear that both the controllers present similar time responses, but the proposed control strategy shows a faster convergence to the set point.

Next simulations involve a set point change from $\mathrm{pH}_1 = 7.7182$ to $\mathrm{pH}_2 = 9.7182$ at $t_2 = 50$ min and to $\mathrm{pH}_3 = 5.7182$ at $t_3 = 200$ min. In all simulations white noise with uniform amplitude distribution in the range ± 0.05 is considered.

Figure 5 illustrates the pH behaviour of the controllers when no perturbations are applied to the process. This plot also includes the difference between the pH and the reference signal. The performance of both the controllers is seen to be acceptable. Moreover, at $t = 220$ min the proposed Laguerre-SCPWL Wiener adaptive controller a small overshot, while Pajunen's scheme does not show this behaviour. These results are in accordance with the fact that the process under these conditions is perfectly described by a Wiener model.

The simulations are reproduced, considering that perturbations are applied in q_A (which is increased from 1 to 1.2 at $t = 72$ min and then reduced to 0.8 at $t = 226$ min) and in x_{1i} (which increases 10 percent from the original value at $t = 126$ min and is then reduced 20 percent at $t = 276$ min).

Figure 6 illustrates the pH behaviour of the controllers and the difference between the pH value and

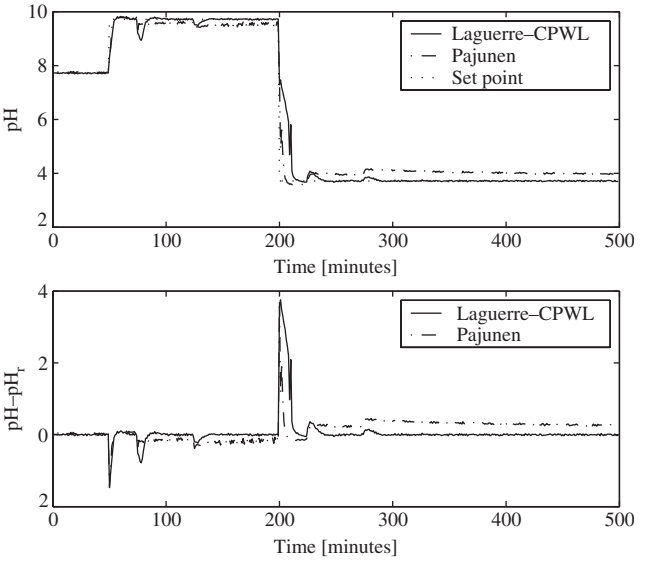

Figure 6. Controlled variable $\mathrm{pH} = -\log_{10}\xi$ with perturbations (top), and difference from reference signal (bottom).

the reference signal. It is clear that the performance of the proposed Laguerre–SCPWL Wiener adaptive controller is better than that of the one proposed by Pajunen (1992). Note the effect of the perturbations (the pH value is moved from the set point when a perturbation appears). In our method the effects of these perturbations are completely suppressed. For Pajunen's scheme, an offset is present for a long time. In this case, the process cannot be adequately described by a Wiener model.

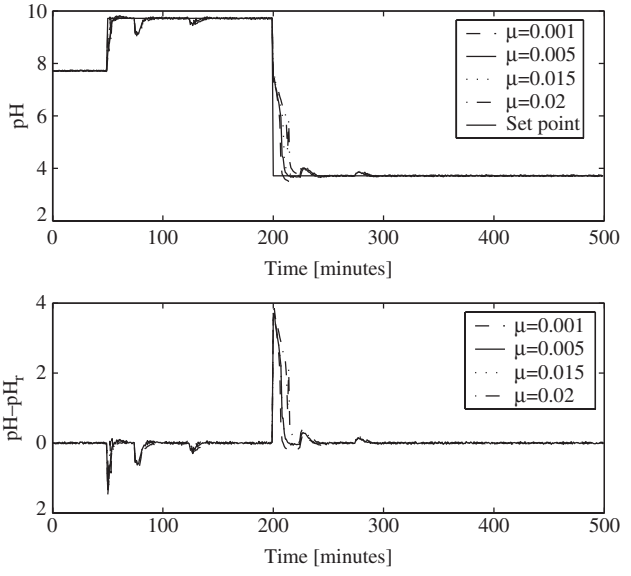

Figure 7. Controlled variable $\mathrm{pH} = -\log_{10}\xi$ with perturbations (top), and difference from reference signal (bottom) for $\mu = 0.001$, 0.002, 0.015 and 0.02.

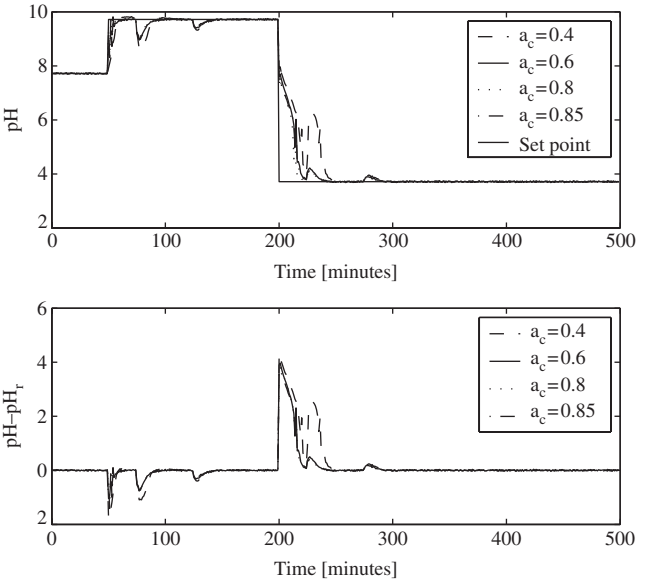

Figure 8. Controlled variable $\mathrm{pH} = -\log_{10}\xi$ with perturbations (top), and difference from reference signal (bottom) for $a_c = 0.4$, 0.6, 0.8 and 0.85.

An interesting point is to compare the number of the parameters involved in both approaches. While the identification of the Laguerre-CPWL involves 13 parameters, Pajunen's approach involves 23 parameters. This is because the model reference adaptive technique cannot take advantage of the Wiener structure of the model.

Figures 7 and 8 show the pH behaviour of the controllers and the difference between the pH and the reference signal closed-loop response for several values of step size in the identification algorithm (figure 7) and closed-loop pole (figure 8). These results verify the robustness of the algorithm with respect to design parameters.

5. Conclusions

The problem of on-line identification and adaptive control of a Wiener type non-linear system is studied. The linear and non-linear parts are modeled using a Laguerre and piecewise linear basis functions, respectively. An LMS based adaptive identification algorithm is presented. The information of the identified model is used to adjust on-line the parameters of a controller, designed for the specific structure of the model. The complete scheme is successfully applied to a simulation of a pH neutralization reactor subject to several perturbations.

References

B.M. Åkesson, H.T. Toivonen, J.B. Waller and R.H. Nyström, "Neural network approximation of a nonlinear model predictive controller applied to a pH neutralization process", *Computers and Chemical Engineering*, 29, pp. 323–335, 2005.

E. Bai, "An optimal two-stage identification algorithm for Hammerstein-Wiener non-linear systems", *Automatica*, 34, pp. 333–338, 1998.

S.I. Biagiola, O.E. Agamennoni and J.L. Figueroa, "H_∞ control of a Wiener type system", *Int. J. Cont.*, 77, pp. 572–583, 2004.

S.I. Biagiola, O.E. Agamennoni and J.L. Figueroa, "Robust control of Wiener systems: A case study", *Computers and Chemical Engineering*, 2005 (Submitted).

S. Billings and S. Fakhouri, "Identification of nonlinear systems using the Wiener model", *Electronics Letters*, 13, pp. 502–504, 1978.

S. Billings, "Identification of nonlinear systems – A survey," in *Proceedings IEE*, Part D, 127, 1980, pp. 272–285.

H.H.J. Bloemen, C.T. Chou, T.J.J. van de Boom, V. Verdult, M. Verhaegen and T.C. Backx, "Wiener model identification and predictive control for dual composition control of a distillation column", *J. Proc. Cont.*, 11, pp. 601–620, 2001.

C.T. Chou and M. Verhaegen, "An indirect approach to closed-loop identification of Wiener models", *Proc. on American Control Conference*, San Diego, QA, USA, pp. 3451–3455, 1999.

L.O. Chua and L.P. Ying, "Canonical piecewise-linear analysis", *IEEE Trans. Circuits and Sys.*, 30, pp. 125–140, 1983.

J.L. Figueroa, J.E. Cousseau and R.J.P. de Figueiredo, "A simplicial canonical piecewise linear adaptive filter", *Circuits, System and Signal Processing*, 23, pp. 365–386, 2004.

O. Galán, "Robust multi-linear model-based control for nonlinear plants", PhD thesis, University of Sydney, Australia (2000).

S. Gerkšič, D. Juričić, S. Strmčnik and D. Matko, "Wiener model based nonlinear predictive control", *Int. J. Sys. Sci.*, 31, pp. 189–202, 2000.

J.C. Gomez and E. Baeyens, "Identification of block-oriented nonlinear systems using orthogonormal bases", *J. Proc. Cont.*, 14, pp. 685–697, 2004.

W. Greblicki, ''Nonparametric identification of Wiener systems by orthogonal series'', *IEEE Trans. Auto. Cont*., 39, pp. 2077–2086, 1994.
T. Gustafsson and K. Waller, ''Dynamic modelling and reaction invariant control of pH'', *Chem. Eng. Sci*., 38, pp. 389–398, 1983.
P. Julián, A.C. Desages and O.E. Agamennoni, ''High level canonical piecewise linear representation using a simplicial partition'', *IEEE Trans. Circuits and Sys*., 46, pp. 463–480, 1999.
P. Julián, A. Desages and M.B. D'Amico, ''Orthonormal high-level canonical PWL functions with applications to model reduction'', *IEEE Trans. Circuits and Syst. I*, 47, pp. 281–288, 2000.
A. Kalafatis, N. Arifin, L. Wang and W.R. Cluett, ''A new approach to the identification of pH processes based on the Wiener model'', *Chem. Eng. Sci*., 50, pp. 3693–3701, 1995.
A.D. Kalafatis, L. Wang and W.R. Cluett, ''Identification of Wiener-type nonlinear systems in a noisy environment'', *Int. J. Control*, 66, pp. 923–941, 1997.
A.D. Kalafatis, L. Wang and W.R. Cluett, ''Linearizing feedforward-feedback control of pH processes based on the Wiener model'', *J. Proc. Cont*., 15, pp. 103–112, 2005a.
A.D. Kalafatis, L. Wang and W.R. Cluett, ''Identification of time-varing pH processes using sinusoidal signals'', *Automatica*, 41, pp. 385–691, 2005b.
P. Lindskog, ''Methods, algorithms and tools for system identification based on prior Knowledge'', PhD Thesis, Linköping Universitet, Sweden (1996).
L. Ljung, ''On positive real transfer functions and the convergence of some recursive schemes'', *IEEE Trans Automat. Contr*., AC-22, pp. 539–551, 1977a.
L. Ljung, ''Analysis of recursive stochastic algorithms'', *IEEE Trans. Automat. Contr*., AC-22, pp. 551–575, 1977b.
L. Ljung and T. Soderstrom, *Theory and Practice of Recursive Identification*, Cambridge, MA: MIT Press, 1983.
A. Lussón Cervantes, O.E. Agamennoni and J.L. Figueroa, ''A nonlinear model predictive control scheme based on Wiener piecewise linear models'', *J. Proc. Cont*., 13, pp. 655–666, 2003a.
A. Lussón Cervantes, O.E. Agamennoni and J.L. Figueroa, ''Robust Identification of PWL-Wiener models: use in model predictive control'', *Latin Amer. Appl. Res*., 33, pp. 435–442, 2003b.
T. McAvoy, E. Hsu and S. Lowenthal, ''Dynamics of pH in controlled stirred tank reactor'', *Indust. Eng. Chem. Proc. Design Dev*., 11, pp. 68–70, 1972.
K.S. Narendra and P.G. Gallman, ''An iterative method for the identification of nonlinear systems using a Hammerstein model'', *IEEE Trans. Auto. Cont*, AC-11, pp. 546–550, 1966.
S.J. Norquay, A. Palazoglu and J.A. Romagnoli, ''Model predictive control based on Wiener models'', *Chem. Eng. Sci*., 53, pp. 75–84, 1998.
S. Norquay, A. Palazoglu and J.A. Romagnoli, ''Application of Wiener model predictive control (WMPC) to pH neutralization experiment'', *IEEE Trans. Cont. Sys. Tech*., 7, pp. 437–445, 1999.
B.A. Ogunnaike and W.R. Ray, *Proc. Dyn., Model. Cont*., Oxford University, 1994.
G.A. Pajunen, ''Comparison of linear and nonlinear adaptive control of a pH-process,'', *IEEE Cont. Sys. Mag*., 28, pp. 781–785, 1987.
G.A. Pajunen, ''Adaptive control of Winer type nonlinear systems'', *Automatica*, 28, pp. 781–785, 1992.
R.K. Pearson and M. Potman, ''Gray-box identification of block-oriented nonlinear models'', *J. Proc. Cont*., 10, pp. 301–315, 2000.
R.K. Pearson, ''Selecting nonlinear model structures for computer control'', *J. Proc. Cont*., 13, pp. 1–26, 2003.
P. Stoica, ''On the convergence of an iterative algorithm used for Hammerstein system identification'', *IEEE Trans. Aut. Cont*., AC-26, pp. 967–969, 1981.
B. Wahlberg, ''System identification using Laguere models'', *IEEE Trans. Aut. Cont*., 36, pp. 551–562, 1991.
B. Wahlberg, ''System identification using Kautz models'', *IEEE Trans. Aut. Cont*., 39, pp. 1276–1282, 1994.
T. Wigren, ''Recursive identification based on the nonlinear Wiener model'', PhD Dissertation, Uppsala, Sweden (1990).
T. Wigren, ''Recursive prediction error identification using the nonlinear Wiener model'', *Automatica*, 29, pp. 1011–1025, 1993.
T. Wigren, ''Convergence analysis of recursive identification algorithms based on the nonlinear Wiener model'', *IEEE Trans. Aut. Cont*., 38, pp. 2191–2206, 1994.
T. Wigren, ''Output error convergence of adaptive filters with compensation for output nonlinearities'', *IEEE Trans. Aut. Cont*., 43, pp. 975–978, 1998.
T. Wigren, ''User choices and model validation in system, identification using nonlinear Wiener'', *Proc. SYSID 2003*, the Netherlands: Rotterdam, pp. 863–868, 2003.