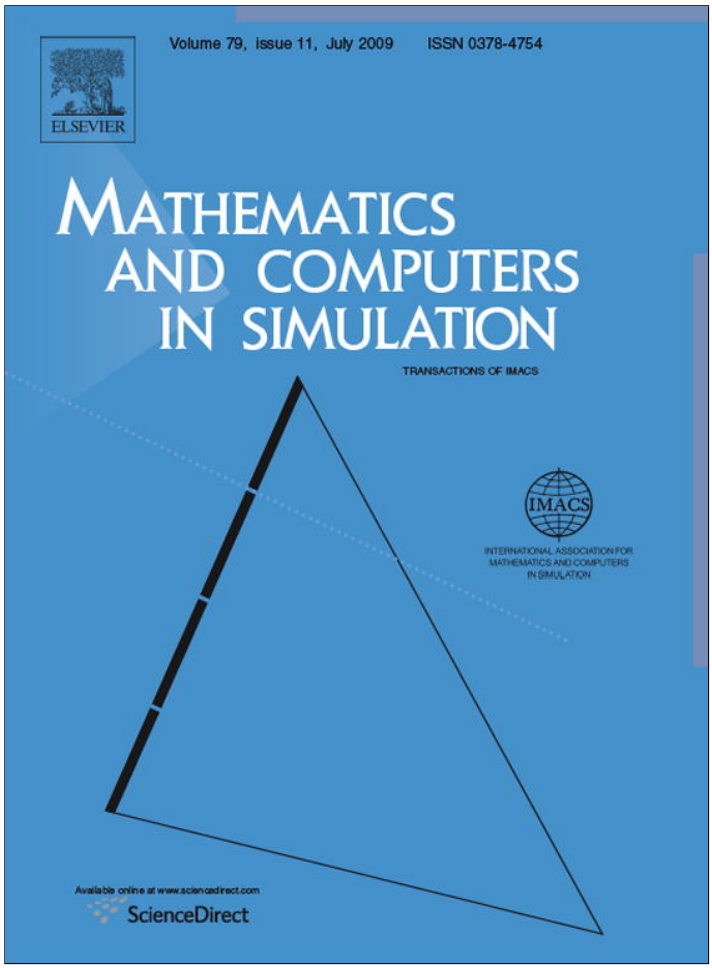

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Wiener and Hammerstein uncertain models identification

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Abstract

Block-oriented models have proved to be useful as simple nonlinear models for a vast number of applications. They are described as a cascade of linear dynamic and nonlinear static blocks. They have emerged as an appealing proposal due to their simplicity and the property of being valid over a larger operating region than a LTI model. In the description of these models, several approaches can be found in the literature to perform the identification process. In this sense, an important improvement is to achieve robust identification of block-oriented models to cope with the presence of uncertainty.

In this article, we focus at two special and widely used types of uncertain block-oriented models: Hammerstein and Wiener models. They are assumed to be represented by a parametric representation. The approach herein followed allows to describe the uncertainty as a set of parameters which is found through the solution of an optimization problem. The identification algorithms are illustrated through a set of simple examples.

Keywords: Wiener models; Hammerstein models; Nonlinear identification; Uncertainty

1. Introduction

A typical problem related to systems theory is the obtainment of an appropriate model to represent a system of interest. This means the model should be capable to capture the observed system dynamics. In this sense, Narendra and Gallman [31] mentioned two tasks concerned with the search of a suitable model. One of them consists in system characterization, which is basically associated to the construction of mathematical models able to represent input–output behavior. The other intrinsical task is the system identification itself, which is concerned with the selection of one particular model among a class of models. From a mathematical point of view, the selected representation must be *equivalent* to the real system of interest. The comparison between the proposed model and the real system is usually performed by analyzing both outputs (the system and the model ones) for an identically designed input. If the system is excited by a sequence $\mathbf{u} = \{u(0), u(1), \ldots, u(K)\}$, it brings the output $\mathbf{y} = \{y(0), y(1), \ldots, y(K)\}$. On the other hand, the output predicted by the identified model is $\hat{\mathbf{y}} = \{\hat{y}(0), \hat{y}(1), \ldots, \hat{y}(K)\}$, and depends on the input excitation $\mathbf{u}$, as well as on the set of the model parameters (θ). These parameters can be computed to minimize the difference between the process and model outputs ($y - \hat{y}$). This can be performed using an optimization algorithm, and this set of parameters are usually referred to as *nominal parameters* θ_N.

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Nomenclature

$B_i(q) : \Re^n \to \Re^n$	a set of any rational basis in $H_2^{n\times n}$
ϵ	prediction error
ϕ	regressor
Φ	regression matrix
γ	regressor
Γ	regression vector
$g_i(\cdot) : \Re^m \to \Re^m$	set of specified basis functions for a nonlinear static mapping
$\mathcal{H}$	set of parameters of the linear block
h_i	the ith parameter of the linear model
$H_2^{m\times n}(T)$	the space of all the $(m \times n)$ transfer matrices that are both stable and causal
K	number of input/output data of the process
$\mathcal{L}$	linear block
M	number of parameters of the linear model
N	number of parameters of the nonlinear model
$\mathcal{N}$	nonlinear block
$\mathcal{N}_\mathcal{H}$	nonlinear Hammerstein block
$\mathcal{N}_\mathcal{W}$	nonlinear Wiener block
$\mathcal{V}$	set of intermediate data
$\mathcal{P}$	set of parameters of the nonlinear block
q	the forward shift operator
$\Re$	field of real numbers
$\Re^{m\times n}$	the field of $m \times n$ matrices with entries in $\Re$
θ	parameters vector
u	input
v	intermediate signal
y	output
$\hat{}$	estimate
$\leq, \geq$	for scalar the usual meaning, for vectors and matrices applied entry by entry
$[\ ]_i$	the ith entry of a vector or a ith dimension of a set
$[\ ]_{i,j}$	the element of a matrix on the ith row and the jth column

When the interest aims at obtaining an uncertainty related with this nominal model, a typical approach is to define a set of possible models to represent all the process behaviors. This is performed by considering a set of model parameters Θ such that when these parameters $\theta \in \Theta$ are used, the whole set of exciting inputs **u** is "mapped" onto an output set which contains the set of the output data **y** (see Fig. 1). In this way, we assume the same format for all the possible models in the uncertain set. This models family is defined in terms of a set of parameters. Based on this, we can have an idea of the conservatism of the model description, by comparing the set $\{\hat{y}|\hat{y} = f(\mathbf{u}, \theta), \theta \in \Theta\}$ with the set **y**.

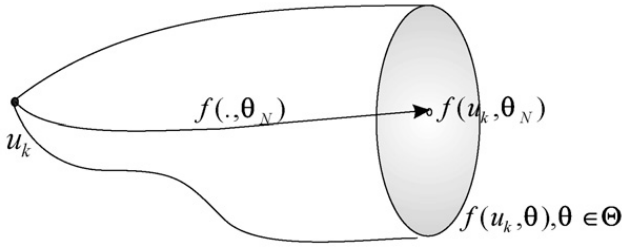

Fig. 1. Model under uncertainties.

The modeling based entirely on fundamental principles leads to mathematical representations with clear physical interpretation. However, these models tend to be highly complex [37] and nonlinear [28], which may not be appropriated for subsequent applications, such as process control. On the other extreme, we found linear models, which are appreciated due to their simplicity and the well developed theory regarding for example its identification and control [1,46]. However, linear descriptions for strong nonlinear behaviors may not be suitable in many cases. In this sense, an appealing alternative can be the block-oriented models. Though, they can be established without knowing the detailed underlying physics and dynamics of the system, they have shown to be useful and flexible for many applications.

Block-oriented models are a special type of nonlinear models [19,35,25]. They consist of a cascade combination of a linear dynamics block ($\mathcal{L}$) with a memoryless nonlinear one ($\mathcal{N}$). A vast type of dynamic processes can be described by such representations consisting of these two simple elements usually referred to as subsystems [9]. The advantage of using such models is twofold: the low computational effort associated to identification and the suitability for control design.

In the literature there are two typical block-oriented models: they are the Hammerstein (where $\mathcal{N}$ is followed by $\mathcal{L}$) and the Wiener (where $\mathcal{N}$ follows $\mathcal{L}$) models.

As regards identification procedures for Hammerstein models, different approaches can be mentioned. A very popular algorithm is the one due to Narendra and Gallman [31] which is a minimum prediction error approach, leading to an iterative procedure. Since then, a vast number of papers have been dealt with the estimation of this type of block-oriented models (see [40] and references therein). A noniterative method was proposed by Chang and Luus [8], they introduced it as an extension of [22], and made a comparison with [31]. Another approach to accomplish the identification are the two-stage methods. In Bai's work [2], a linear parametrization of the nonlinear model is estimated in a first step and, in a second one, the nonlinear model parameters are obtained. In [3], the linear dynamics and the nonlinearity are decoupled to accomplish the identification. An extension of Bai's results can be found in the work by Gómez and Baeyens [14]. Their widely cited method is valid for multiple input–multiple output systems and a more general description for the nonlinear block is considered.

Another category of methods are the stochastic ones. Among them are the works by Billings and Fakhouri, based on correlation analysis [4–6]. Nonparametric regression methods deserve an special mention. This approach is particularly suitable for those systems in which the nonlinearity is not parameterized by a finite set of parameters. In this category are the works by Greblicki [15] and Pawlak [17,18].

As regards Wiener models identification, many different methods have been reported. One approach consists in using one of the methods for Hammerstein models. The underlying idea for this approach is the identification of the inverse of a Wiener-type system, dealing with the problem as a Hammerstein model identification one [16,8,35]. This particular statement of the identification problem has been considered appealing for Wiener-systems control purposes [34]. However, this is quite restrictive, because it demands from the static nonlinear element to be invertible, and from the Wiener model to be minimum phase. A special mention deserve the works by Wigren on Wiener models [43–45]. His recursive identification methods and convergence analysis have been widely cited. In [45] a different representation is proposed for the static nonlinearity, replacing the widely used polynomial description by a piecewise linear model [30]. As well as for Hammerstein models, the correlation method by Billings and Fakhouri [4–6] are applicable for Wiener models.

A relevant fact is that in most papers on Wiener and Hammerstein models identification, the nonlinearity is assumed to have a known structure. Otherwise, a structural estimation problem has to be solved [19].

As regards applicability, both Wiener and Hammerstein structures appear in a wide range of fields. Among others, it can be mentioned the use in the area of communications [23,20,39], of chemical engineering [26,33,12,32,44,47,42], biology [24] and medicine [7]. A detailed review on applications can be read in [25].

The approach followed in this paper is based on the assumption that an uncertain parametric block-oriented model must be identified. Moreover, the source of such uncertainty is assumed unknown. In this work it is considered that the structure of the system under study is Wiener or Hammerstein type, that the static nonlinearity entirely determines the steady state characteristic of the whole model and that some kind of uncertainty is present in the system. The resulting methodology is robust in the sense that the identified set of parameters is such that any of the collected data can be reproduced by at least one of the models in the set. These set of parameters are obtained by solving a simple optimization problem. This approach ensures the existence of a convex set of models that can describe the whole data collected from the process.

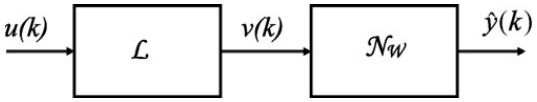

Fig. 2. Wiener model.

The paper is organized as follows. In Section 2, block-oriented nominal model identification is treated. We basically revisit the results reported in [14], that constitute a starting point for the identification approach herein presented. In Section 3 the problem of robust identification of the uncertain block-oriented model is tackled. This is the main contribution of this paper: a mechanism for uncertain model characterization and identification. In Section 4, the results are evaluated on the basis of several simulation examples. Final remarks are addressed in Section 5.

2. Block-oriented models identification

To identify both types of block-oriented models, the following parameterization have been adopted [14,10]:

- **Linear block**:

$$\mathcal{L}(q) = \sum_{i=1}^{M} h_i B_i(q^{-1}), \quad h = [h_1, \ldots, h_M]^T, \tag{1}$$

where $B_i(q^{-1})$ is any rational basis such as Laguerre, Kautz, orthonormal, etc.[1]

- **Nonlinear block**:

$$\mathcal{N}(x) = \sum_{i=1}^{N} p_i g_i(x), \quad p = [p_1, \ldots, p_N]^T, \tag{2}$$

where $g_i(\cdot) : \Re \to \Re$ are a set of specified basis functions such as polynomial, trigonometric, piecewise linear functions.

In the identification of block-oriented models there is a scale factor which can be arbitrarily distributed between the linear block and the memoryless one without affecting the input–output characteristics of the model [36]. In the following we assume $p_1 = 1$, since that any other value of this gain can be included in the linear block. Therefore, the identification task will involve the determination of $M + N - 1$ unknown parameters.

2.1. Nominal identification of Wiener models

Fig. 2 depicts the general structure of Wiener models. It consists of a LTI system $\mathcal{L}$ followed by a static nonlinearity $\mathcal{N}_W$. That is, in the Wiener case, the linear model $\mathcal{L}$ maps the input sequence $\{u(k)\}$ into the intermediate sequence $\{v(k)\}$, and the overall model output is the output of the nonlinear block, i.e., $\hat{y}(k) = \mathcal{N}_W(v(k))$.

The present identification algorithm requires that the static nonlinearity is invertible,[2] i.e., the signal $v(k)$ can be written as

$$v(k) = \mathcal{N}_W^{-1}(\hat{y}(k)) = \sum_{i=1}^{N} p_i g_i(\hat{y}(k)). \tag{3}$$

Note that the nonlinear block has been represented as in Eq. (2), where $\mathcal{N} = \mathcal{N}_W^{-1}$ and $x = \hat{y}(k)$.

[1] Note that q standsfor the forward shift operator.

[2] This restriction is related to the fact that the signal connecting both subsystems is not measured. However, some of the reported techniques in the literature allow noninvertible nonlinearities [29,21].

From Fig. 2, this signal can also be written as the output of the linear block:

$$v(k) = \sum_{i=1}^{M} h_i B_i(q^{-1}) u(k). \tag{4}$$

Equating both sides of these equations (with the inclusion of an error function $\epsilon(k)$ to allow for modeling error) the following equation is obtained:

$$\epsilon(k) = \sum_{i=2}^{N} p_i g_i(y(k)) + g_1(y(k)) - \sum_{i=1}^{M} h_i B_i(q^{-1}) u(k) \tag{5}$$

which is a linear regression in the parameters. Defining θ and ϕ_k as follows:

$$\theta = [p_2, \ldots, p_N, h_1, \ldots, h_M]^T, \tag{6}$$

$$\phi_k = [g_2(y(k)), \ldots, g_N(y(k)), -B_1(q^{-1})u(k), \ldots, -B_M(q^{-1})u(k)]^T, \tag{7}$$

then, Eq. (5) can be written as

$$\epsilon(k) = \theta^T \phi_k + g_1(y(k)). \tag{8}$$

Now, an estimate $\hat{\theta}$ of θ can be computed by minimizing a quadratic criterion on the prediction errors $\epsilon(k)$ (i.e., the least squares estimate) [26,14]. It is well known that this estimate is given by

$$\hat{\theta} = (\Phi_K \Phi_K^T)^{-1} \Phi_K \Gamma, \tag{9}$$

where $\Gamma = [g_1(y(1)), \ldots, g_1(y(K))]^T$ and $\Phi_K = [\phi_1, \ldots, \phi_K]$ use the set of K input/output data available from the process. Note that experimental identifiability condition must be imposed to θ, which implies that Φ is full column rank. Two main advantages of this approach are the low complexity and the uniqueness of the solution.

Now, estimates of the parameters $\hat{p}_i$ $(i = 2, \ldots, N)$ and $\hat{h}_i$ $(i = 1, \ldots, M)$ can be computed by partitioning the estimate $\hat{\theta}$, according to the definition of θ in (6). It is important to remark that we are identifying the inverse of the nonlinearity, which is frequently used in many control applications.

2.2. Nominal identification of Hammerstein models

Fig. 3 depicts the structure of a Hammerstein model. It consists of a static nonlinearity $\mathcal{N}_H$ followed by a LTI system $\mathcal{L}(q)$. That is, in the Hammerstein case, the nonlinear gain maps the input sequence $\{u(k)\}$ into the intermediate sequence $\{v(k)\}$, and it is mapped through the linear model to produce the output $\hat{y}(k) = \mathcal{L}(q)\, v(k) = \mathcal{L}(q)\ \mathcal{N}_H(u(k))$.

Now, using the parametric representation of Eqs. (1) and (2), $y(k)$is

$$\begin{aligned} y(k) &= \left(\sum_{i=1}^{M} h_i B_i(q^{-1})\right) \left(\sum_{j=1}^{N} p_j g_j(u(k))\right) + \varepsilon(k) \\ &= \sum_{i=1}^{M}\sum_{j=1}^{N} h_i p_j B_i(q^{-1}) g_j(u(k)) + \varepsilon(k). \end{aligned} \tag{10}$$

Again, without loss of generality, it will be considered that $p_1 = 1$. Now, following [14] it is possible to write a linear regression in the parameters as follows:

$$y(k) = \theta^T \phi_k + \varepsilon(k), \tag{11}$$

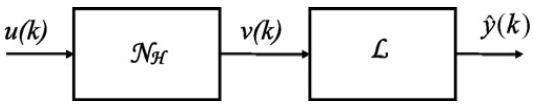

Fig. 3. Hammerstein model.

where

$$\theta = \left[h_1, p_2h_1, \ldots, p_Nh_1, h_2, \ldots, p_Nh_2, \ldots, h_M, \ldots, p_Nh_M\right]^T, \tag{12}$$

$$\phi_k = \Big[B_1(q^{-1})g_1(u(k)), B_1(q^{-1})g_2(u(k)), \ldots, B_1(q^{-1})g_N(u(k)), B_2(q^{-1})g_1(u(k)),$$
$$B_2(q^{-1})g_2(u(k)), \ldots, B_M(q^{-1})g_{N-1}(u(k)), B_M(q^{-1})g_N(u(k))\Big]^T. \tag{13}$$

Then, a least squares estimate $\hat{\theta}$ of θ for the Hammerstein case, can be computed by minimizing a quadratic criterion on the prediction errors $\varepsilon(k) = y(k) - \theta^T\phi_k$ [14]. This solution is similar to Eq. (9), but in this case:

$$\Gamma = [y(1), y(2), \ldots, y(K)]^T, \tag{14}$$

$$\Phi_K = [\phi_1, \ldots, \phi_K]. \tag{15}$$

Note that now, the dimension of the vector $\hat{\theta}$ is $N.M$ against the $N + M$ parameters that were estimated for the Wiener case.

A second problem faced in the case of the Hammerstein model identification is that the parameters $\hat{p}_i$ $(i = 2, \ldots, N)$, and $\hat{h}_i$ $(i = 1, \ldots, M)$ have to be estimated from the estimate $\hat{\theta}$. A typical problem found at this time, is the bilinear relationship between the parameters h_i and p_i. In [14], the authors provide a solution to this problem based on Singular Value Decomposition.

3. Uncertain model identification

In this section it is considered the more general case which is the uncertain block-oriented model. In order to develop a methodology to characterize the uncertainties, we base on the nominal description obtained in Section 2. To perform it, let us introduce a set of parameters $\mathcal{H}$ for the linear dynamic block and a set $\mathcal{P}$ for the parameters of the nonlinear block we want to identify (i.e., $\mathcal{N}_W^{-1}$ for the Wiener model and $\mathcal{N}_H$ for the Hammerstein model):

$$\mathcal{H} = \{h : h_i^l \leq h_i \leq h_i^u, 1 \leq I \leq M\}, \tag{16}$$

$$\mathcal{P} = \{p : p_i^l \leq p_i \leq p_i^u, 1 \leq i \leq N\}, \tag{17}$$

where the subscript i means the ith entry of a vector.

3.1. Uncertain Wiener models

Now, let us consider the following assumption:

Assumption 1. Let us assume that the nonlinear block $\mathcal{N}_W^{-1} = \sum_{i=1}^{N} p_i g_i$ is an invertible function for all $p \in \mathcal{P}$.

This assumption implies that it is possible to obtain the nonlinear block $\mathcal{N}_W$ as the inverse of the nonlinear function obtained from identification, not only for the nominal model, but also for all possible model described by the parameters in $\mathcal{P}$.

Now, to determine the parameters bounds in the Wiener Model, let us first define some sets [11]. Given the input datum u_k, the linear uncertain system defined by $\mathcal{H}$ maps at some specific time k over a set:

$$\mathcal{V}_u = \{v : v(k) = \sum_{i=1}^{M} h_i B_i(q^{-1})\, u(k), h \in \mathcal{H}\}. \tag{18}$$

Given an input $u(k)$, the basis term of order i, i.e., $h_i B_i(q^{-1})\, u(k)$, is a real number and the set $\mathcal{V}_u$ takes the form of $\mathcal{V}_u = \{v : v_l \leq v(k) \leq v_u\}$.

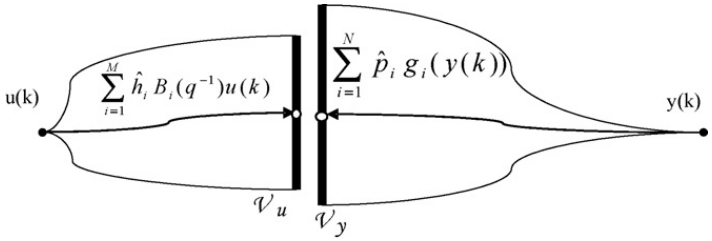

Fig. 4. Uncertainties sets in a Wiener model.

On the other hand, if we consider the uncertain description of the parameters in $\mathcal{P}$, a given output $y(k)$ maps at some specific time k over a set:

$$\mathcal{V}_y = \{v : v(k) = \sum_{i=1}^{N} p_i\, g_i(y(k)),\ p = [p_1 \dots p_N]^T \in \mathcal{P}\}. \tag{19}$$

This situation is illustrated in Fig. 4.

Remember that in order to obtain an uncertain model, every input data $u(k)$ should be mapped through the model to the corresponding $y(k)$. From this picture it is clear that the parameters set will match the uncertainties description of Section 1 if $\mathcal{V}_y \cap \mathcal{V}_u \neq \varnothing$ for all k. In this way, the input $u(k)$ is mapped onto $\mathcal{V}_u$ through the family of models generated by $\mathcal{H}$. Then, since $\mathcal{V}_y \cap \mathcal{V}_u \neq \varnothing$, the intermediate value (i.e., $v(k)$) will result in $y(k)$ through the inverse of $\mathcal{N}_w^{-1}$ defined by Assumption 1.

Now, let us analyze this situation in order to compute the parameters bounds to satisfy this condition. This determination is based on the whole input/output data available.

Note that the linear bases $B_i(q^{-1})$ are a set of real numbers for each input $u(k)$. Let $B(q^{-1})\ u(k)$ be the vector whose ith entry is the linear basis $B_i(q^{-1})\,u(k)$. Since the entries of $B(q^{-1})\,u(k)$ could be positive or negative, it is possible to split the vector. For this purpose, we define $B^+(u(k)) \triangleq \max(B(q^{-1})\,u(k), 0)$ and $B^-(u(k)) \triangleq \min(B(q^{-1})\,u(k), 0)$ and form the new vector $\gamma_B(k) \triangleq [(B^-(u(k)))^T, (B^+(u(k)))^T]^T$. Note that the construction of vector $B^-(u(k)) = \min(B(u(k)), 0)$ involves keeping all the negative elements in the vector and putting zero otherwise. Analogously, $B^+(u(k))$ keeps the positive elements and puts zero otherwise. Therefore, the resultant γ_B is a vector with all negative elements in the first rows and all positive elements in the last ones. In a similar way, since the nonlinear bases $g_i(y(k))$ are real numbers for each output $y(k)$, it is possible to define $\gamma_g(k) \triangleq [(g^-(y(k)))^T, (g^+(y(k)))^T]^T$ where $g^+(y(k)) \triangleq \max(g(y(k)), 0)$ and $g^-(y(k)) \triangleq \min(g(y(k)), 0)$.

Theorem 1. *The bounds of the uncertain parameters h^l, h^u, p^l, p^u can be obtained by solving the following optimization problem*:

$$\min_{h^l, h^u, p^l, p^u} \left(\alpha \sum_{i=1}^{M} (h_i^u - h_i^l) + (1-\alpha) \sum_{i=2}^{N} (p_i^u - p_i^l) \right) \tag{20}$$

subject to

$$[(h^l)^T, (h^u)^T, -(p^u)^T, -(p^l)^T] \begin{bmatrix} \gamma_B(k) \\ \gamma_g(k) \end{bmatrix} \geq 0; \quad k = 1, \dots, K, \tag{21}$$

$$[(h^u)^T, (h^l)^T, -(p^l)^T, -(p^u)^T] \begin{bmatrix} \gamma_B(k) \\ \gamma_g(k) \end{bmatrix} \leq 0; \quad k = 1, \dots, K, \tag{22}$$

where the parameter $\alpha \in (0, 1)$ is a selected factor which allows to distribute the weight of the uncertainty between the linear and the nonlinear blocks. Note that the robust identification requirement $\mathcal{V}_y \cap \mathcal{V}_u \neq \varnothing$ must be ensured $\forall k$.

Proof. In order to demonstrate this theorem it is necessary to see that if the constraints (21) and (22) hold, then the condition $\mathcal{V}_y \cap \mathcal{V}_u \neq \varnothing$ is satisfied. Since $\mathcal{V}_y, \mathcal{V}_u \in \Re$ the identification problem can be interpreted from a graphical point of view, as an intersection problem (see Fig. 5).

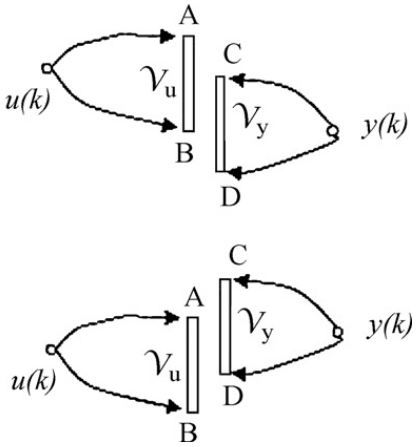

Fig. 5. Graphical representation of the intersection problem.

Let:

$$\mathcal{V}_u = \{v : B \leq v \leq A\}, \tag{23}$$

$$\mathcal{V}_y = \{v : D \leq v \leq C\}. \tag{24}$$

Note that A, B, C, D are all real values. From Fig. 5, it is easy to see that the upper bound of $\mathcal{V}_u$ as function of the parameter set $\mathcal{H}$ is given as $A = [(h^l)^T, (h^u)^T]\gamma_B$. In the same line, it is possible to see that $B = [(h^u)^T, (h^l)^T]\gamma_B$, $C = [(p^l)^T, (p^u)^T]\gamma_g$ and $D = [(p^u)^T, (p^l)^T]\gamma_g$.

Based on these definitions, the condition $\mathcal{V}_y \cap \mathcal{V}_u \neq \varnothing$ will be satisfied if and only if the upper bound of $\mathcal{V}_u$ is greater than the lower bound of $\mathcal{V}_y$ (i.e., $A \geq D$), and if the lower bound of $\mathcal{V}_u$ is lower than the upper bound of $\mathcal{V}_y$ (i.e., $B \leq C$). The constraint $A - D \geq 0$ is the one written in Eq. (21)[41]. Similarly, the mathematical formulation for $C - B \geq 0$ is given in Eq. (22).

In this way, the optimization problem in (20) will produce an uncertain model that minimizes the magnitude of the uncertainty while ensures the description of the behavior of all the collected data. □

Note that the proposed approach for the identification problem, allows to transform it into a Linear Programming problem with convex feasible region. The number of optimization variables is twice the number of model parameters and the number of constrains is twice the number of the process data. Due to the suitable formulation of these problem, its solution is obtained in an efficient way.

3.2. Uncertain Hammerstein models

In order to characterize the uncertainty present in a Hammerstein model, a two step procedure is proposed. First, by considering the linear regression in Eq. (11), a set of uncertain parameters are computed in terms of θ, as follows:

$$\Theta = \{\theta : \theta^l \leq \theta \leq \theta^u\}. \tag{25}$$

Then, in a second step, based on the definition of the parameters vector in Eq. (12), the set Θ is translated to the parametric description given by Eqs. (16) and (17).

To solve the first step, we should state the following problem.

Problem 1. Given a set of data $u(k)$ and $y(k)$ for $k = 1, 2, \ldots, K$, we should compute a set of parameters Θ such that

$$\mathcal{Y} = \{y : y(k) = \theta^T \phi_k, \quad \theta \in \Theta\}, \tag{26}$$

where $y(k) \in \mathcal{Y}$.

Now, considering that the regressor ϕ_k is a real vector for each input $u(k)$, whose entries could be positive or negative, it is possible to split it into ϕ_k^+ and ϕ_k^- by defining $\phi_k^+ \triangleq \max(\phi_k, 0)$ and $\phi_k^- \triangleq \min(\phi_k, 0)$ and form the vector:

$$\Phi_k = [(\phi_k^-)^T, (\phi_k^+)^T]^T. \tag{27}$$

Theorem 2. *The bounds θ^l, θ^u for the uncertain parameters Θ can be computed by solving the following optimization problem*:

$$\min_{\theta^l, \theta^u} \sum_{i=1}^{NM} (\theta_i^u - \theta_i^l) \tag{28}$$

subject to

$$[(\theta^l)^T, (\theta^u)^T]\Phi_k \geq y(k); \quad k = 1, \ldots, K, \tag{29}$$

$$[(\theta^u)^T, (\theta^l)^T]\Phi_k \leq y(k); \quad k = 1, \ldots, K \tag{30}$$

is a solution for the Problem 1.

Proof. It is similar to the proof of Theorem 1. In this case it is easy to see that satisfaction of the constraints in Eq. (29) implies the satisfaction of an upper bound on the data $y(k)$ and the satisfaction of Eq. (30) implies the satisfaction of a lower bound on the data. □

Now, the second problem for the identification of the uncertain Hammerstein model is to obtain the set of parameters $\mathcal{H}$ and $\mathcal{P}$ as defined in Eqs. (16) and (17) from the bounds on the θ parameters (Eq. (25)). This can be posed as Problem 2.

Problem 2. Given the set Θ of parameters θ defined as Eq. (25), determine the set of parameters $\mathcal{H}$ and $\mathcal{P}$ such that

$$\mathcal{P} \otimes \mathcal{H} \supset \Theta, \tag{31}$$

where the set $\mathcal{P} \otimes \mathcal{H}$ is defined as

$$\mathcal{P} \otimes \mathcal{H} = \{\theta : \theta = [p^T h_1, p^T h_2, \ldots, p^T h_M]^T; p \in \mathcal{P}; h \in \mathcal{H}\}. \tag{32}$$

The bilinear relationship between the unknown parameters is typical of Hammerstein models [10,14]. Note that we are looking for a set of parameters $\mathcal{H}$ and $\mathcal{P}$ such that they justify the set Θ. Fig. 6 illustrates this situation. The physical system (which has known structure but unknown parameters) is excited by the input sequence $u(k)$, and gives out $y(k)$. In a first step identification, the set Θ is obtained. As robust identification is performed, the set of outputs generated by using the input process data $u(k)$ and the identified Θ, will include the real collected outputs. In a second step, and based on the set Θ, the set $\mathcal{P} \otimes \mathcal{H}$ is determined. The proposed robust identification method is conservative and, as a result, the output set obtained with $\mathcal{P} \otimes \mathcal{H}$ will contain the previous one (i.e., the one generated by means of Θ).

Now, considering the expression for θ from Eq. (12) and the bounds θ^l and θ^u, it is possible to compute the bounds on the parameters of the Hammerstein models h^l, h^u, p^u and p^l.

To proceed, let us consider the following partition of vector θ:

$$\theta = [\theta_1^T, \theta_2^T, \ldots, \theta_M^T]^T, \tag{33}$$

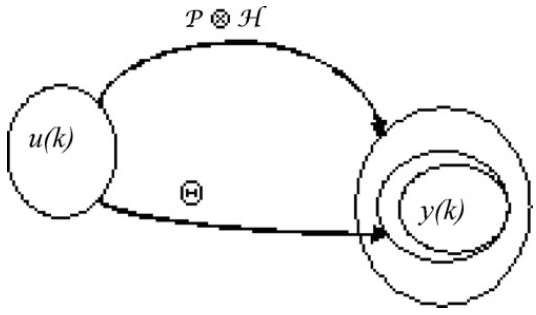

Fig. 6. Output sets generated by $\mathcal{P} \otimes \mathcal{H}$ and Θ.

where

$$\theta_i \triangleq p\, h_i, \quad \text{for } i = 1, \ldots M. \tag{34}$$

Then, a solution to Problem 2 is provided by means of the following theorem.

Theorem 3. *The bounds h^l, h^u, p^l, p^u for the uncertain parameters h and p can be computed by solving the following optimization problem*:

$$\min_{h^l, h^u, p^u, p^l} \left(\alpha \sum_{i=1}^{M} (h_i^u - h_i^l) + (1-\alpha) \sum_{i=2}^{N} (p_i^u - p_i^l) \right) \tag{35}$$

subject to the following (*entry by entry*) *inequalities*:

$$p^l \leq p \leq p^u, \tag{36}$$

$$h^l \leq h \leq h^u, \tag{37}$$

$$\max_{h_i^l, h_i^u, p^u, p^l} (p\ \ h_i) \geq \theta_i^u, \quad i = 1, \ldots, M, \tag{38}$$

$$\min_{h_i^l, h_i^u, p^u, p^l} (p\ \ h_i) \leq \theta_i^l, \quad i = 1, \ldots, M, \tag{39}$$

where the parameter $\alpha \in (0, 1)$ is a selected factor which allows to distribute the weight of the uncertainty between the linear and the nonlinear blocks. The formulation in Eqs. (35)–(39) is the mathematical statement which brings the solution to Problem 2.

Proof. Through the statement and solution to Problem 1 robust identification of parameters θ was accomplished. This gives out the lower and upper bounds θ_i^l and θ_i^u. By definition in Eq. (12) we have $\theta_i = p\ \ h_i$. Based on these partial results, we want now to find the best bounds for the original parameters of the Hammerstein model (i.e., h and p). This involves the optimization stated in Eq. (35). The conservative criterion stated in Eq. (31), gives rise to the following (entry by entry) constraints:

$$\theta_i^u \leq (p\, h_i)^u, \quad i = 1, \ldots, M, \tag{40}$$

$$\theta_i^l \geq (p\, h_i)^l, \quad i = 1, \ldots, M. \tag{41}$$

Due to the bilinearity in $p\, h_i$, and the possible presence of negative elements, the right terms in (40) and (41) can be found in the set generated by the combinatory of the individual bounds on p and h_i. Therefore:

$$(p\, h_i)^u = \max_{h_i^l, h_i^u, p^u, p^l} (p\, h_i), \quad i = 1, \ldots, M, \tag{42}$$

$$(p\, h_i)^l = \min_{h_i^l, h_i^u, p^u, p^l} (p\, h_i), \quad i = 1, \ldots, M \tag{43}$$

which gives Eqs.(38) and (39). □

Consequently, the general formulation of the robust identification problem results in a nonlinear optimization one. In this way, finding the solution to Problem 2 could be burdensome as well as time-consuming.

Therefore, we propose a simplified approach for the identification of the bounds on p and h, which can be useful while applicable under certain conditions.[3] The main advantage is that it transforms the original problem into a Linear Programming one.

Theorem 4. *Let us first define $\pi_i \triangleq 1/h_i$ [10]. The bounds of the uncertain parameters π^l, π^u, p^u and p^l computed by solving the following optimization problem*:

$$\min_{\pi^l, \pi^u, p^u, p^l} \left(\beta \sum_{i=1}^{M} (\pi_i^u - \pi_i^l) + (1-\beta) \sum_{i=1}^{N} (p_i^u - p_i^l) \right), \tag{44}$$

[3] One limitation to this simplified approach appears when any of the parameters h_i changes its sign, i.e., when $\text{sign}(h_i^l) \neq \text{sign}(h_i^u)$.

subject to the following (*entry by entry*) *constraints*:

$$(p^j) - \theta^u \pi_i^j \geq 0 \quad \text{for any } p^j \in \mathcal{P},\ \frac{1}{\pi_i^j} = h_i^j,\ h^j \in \mathcal{H}, \tag{45}$$

$$(p^k) - \theta^l \pi_i^k \leq 0 \quad \text{for any } p^k \in \mathcal{P},\ \frac{1}{\pi_i^k} = h_i^k,\ h^k \in \mathcal{H}, \text{with } j \neq k, \tag{46}$$

$$p_i^l \leq p_i \leq p_i^u, \quad i = 1, \ldots, N, \tag{47}$$

$$\pi_i^l \leq \pi_i \leq \pi_i^u, \quad i = 1, \ldots, M, \tag{48}$$

where the parameter $\beta \in (0, 1)$ *plays the same role as* α *in* Problem 2.

Inequalities (45 and 46) can be interpreted as the constraints to be satisfied for robust identification. This is the optimization formulation for the restriction in Eq. (31) depicted in Fig. 6.

Proof. Performing the same partition for the bounds than the one used for the parameter θ, and considering the objective of representing all possible uncertainty in Θ we have already wrote the conditions of Problem 2 as

$$p\, h_i \geq \theta_i^u, \tag{49}$$

$$p\, h_i \leq \theta_i^l \tag{50}$$

for $i = 1, \ldots M$. Now using the definition $\pi_i = 1/h_i$, and provided $h_i \geq 0$, they can be rewritten as

$$p \geq \theta_i^u \pi_i, \tag{51}$$

$$p \leq \theta_i^l \pi_i, \tag{52}$$

or, in an equivalent form:

$$p - \theta_i^u \pi_i \geq 0, \tag{53}$$

$$p - \theta_i^l \pi_i \leq 0. \tag{54}$$

However, if $h_i < 0$, then the inequalities changed (also in Eqs. (45) and (46)) to

$$p - \theta_i^u \pi_i \leq 0, \tag{55}$$

$$p - \theta_i^l \pi_i \geq 0. \tag{56}$$

Then, the corresponding inequalities hold $\forall p_i, h_i$ such that $p_i^l \leq p_i \leq p_i^u$ $(i = 1, \ldots, N)$ and $\pi_i^l \leq \pi_i \leq \pi_i^u$ $(i = 1, \ldots, M)$.

Note that inequalities in Eqs. 49–56 apply entry by entry. □

3.3. Remarks

Note that the proposed identification method for the uncertain Hammerstein model involves a two-step procedure. In a first stage, nominal parameters are determined by solving a Linear Programming (LP) problem which involves as many restrictions as collected process data. Based on these results, bounds for the original model's parameters are calculated in a second stage. The main advantage of this approach is that the first step allows a substantial reduction of the optimization problem which is accomplished at the time of the LP step. Then, the nonlinear optimization problem, which is solved in a second step, is a reduced one. Otherwise, the NL optimization would include as many constraints as process data. This would be the drawback of formulating the uncertain model identification problem as a one-step nonlinear optimization one.

As regards the identification problem stated in this work, it should be noted that the deterministic problem has been dealt with. If additive output noise is considered, the proposed Wiener model identification approach results in asymptotically biased parameter estimation due to the correlation between some regressor entries and the output noise. An approach to remedy this fact is given by Kalafatis et al. [27]. They connect the bias with the signal-to-noise ratio,

and that propose an iterative least squares approach to gradually remove the bias from the estimates. In this sense, a closely related procedure is the instrumental variable method by Söderström and Stoica [38].

Another relevant item is that, though in the statement of Wiener and Hammerstein models identification it was assumed that $p_1 = 1$, it would be likewise valid to replace the condition for structural identifiability by $h_1 = 1$.

4. Simulation examples

4.1. Example 1

To illustrate the proposed identification approach, we recall the Wiener model in [44] which describes a control valve for fluid flow. The mathematical description is

$$v(k) = \frac{0.1044\,q^{-1} + 0.0883\,q^{-2}}{1 - 1.4138\,q^{-1} + 0.6065\,q^{-2}}\,u(k), \tag{57}$$

$$y(k) = \mathcal{N}(v(k)) = \frac{v(k)}{\sqrt{0.10 + 0.90\,v^2(k)}}, \tag{58}$$

where $u(k)$ is the controller output, i.e., the signal applied to the stem. The stem position is $v(k)$. The resulting flow through the valve is represented by $y(k)$. In order to construct the input, a random zero-mean sequence between -0.5 and 0.5 was first generated. The input $u(k)$ was then constructed by adding a bias of 0.5 and holding each value in the sequence during six sampling intervals.

The Wiener model to accomplish the identification was assumed to be a Laguerre system followed by a polynomial-type nonlinearity. Three Laguerre terms with dominant pole in -0.6 were considered for the linear block. A third order polynomial was proposed for the nonlinear block. Therefore, $M = 3$ and $N = 3$.

The solution of the optimization problem in Eqs. (20)–(22) gives the result shown in Table 1. The structural identifiability constraint $h_1 = 1$ has been specified.

Note that in this example the weighting factor α was taken equal to 0.5. Therefore, the uncertainty was equally weighted between the linear and the nonlinear blocks, i.e., both terms of uncertainty were identically weighted in the optimization cost function. However, from results in Table 1 it arises that the bounds on h_i are closer than the ones on p_i. Note that a prediction on the bounds proximity could not be made a priori. The reason for this is the optimization results strongly depend on the chosen nonlinear structure as well as on the particular selection of the linear model. From these results, it is clear that in this example, the input/output data is better described by uncertainty in the nonlinear block.

Fig. 7 depicts the bounds on the sets $\mathcal{V}_u$ and $\mathcal{V}_y$. As described in Section 3.1, the solution approach requires that $\mathcal{V}_u \in \mathcal{V}_y \neq \varnothing$, and Fig. 7 shows this restriction holds in this example.

4.2. Example 2

The system considered in this example is a similar plant than the one reported by Falugi et al. [10]. They propose this one as an example of a more complex Hammerstein model. We consider the plant consists in a polynomial type nonlinearity followed by a Finite Impulse Response (FIR) system. The "true" model parameters are: $p = [1\ \ 3\ \ 2]^T$, $h = [1\ \ 2\ \ 0.3\ \ 4\ \ 1\ \ 1\ \ 0.5]^T$. Therefore, $N = 3$ and $M = 7$.

Table 1
Bounds on Wiener model's parameters.

Parameter	Bounds
h_2	[0.0115, 0.1272]
h_3	[1.4783, 1.4783]
p_1	[1.2644, 2.6265]
p_2	[1.8124, 1.8124]
p_3	[−2.9801, −0.7056]

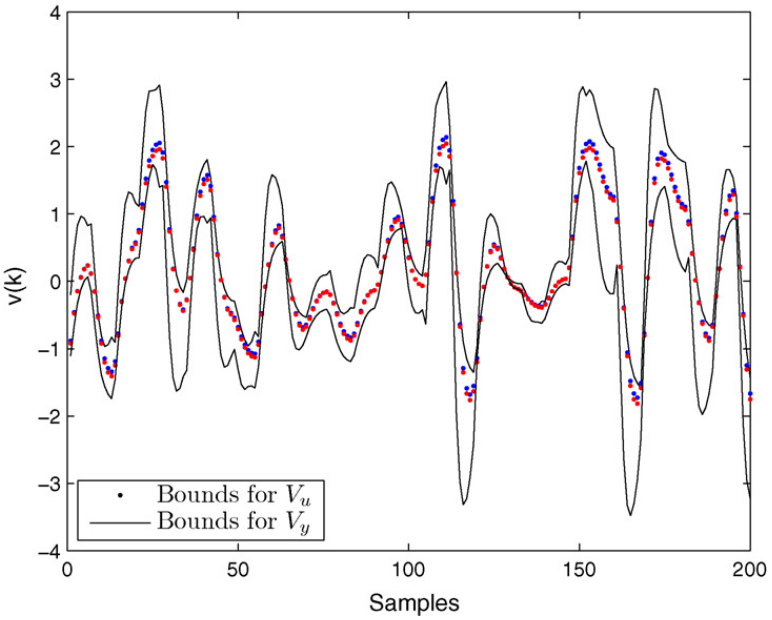

Fig. 7. Bounds on $\mathcal{V}_u$ (dots) and bounds on $\mathcal{V}_y$ (solid line).

In order to collect the experimental data, the Hammerstein system was excited by a zero-mean random signal uniformly distributed between -0.5 and 0.5.

After solving Problem 1, the following bounds on θ were obtained:

$$\begin{aligned}\theta_l &= [1.3222\ \ 2.9843\ \ 1.6597\ \ 2.0068\ \ 5.7604\ \ 3.9391\ \ 0.1322\ \ 0.8124\ \ 0.7763\ \ 3.9319\ \ 11.9125\ \ 8.0932\\ &\quad 0.8725\ \ 3.2395\ \ 2.2254\ \ 1.0569\ \ 3.0748\ \ 1.7546\ \ 0.2850\ \ 1.6632\ \ 0.9446]^T,\\ \theta_u &= [1.3222\ \ 2.9843\ \ 1.6597\ \ 2.3526\ \ 5.7604\ \ 3.9391\ \ 0.2869\ \ 0.8124\ \ 0.7763\ \ 3.9692\ \ 11.9125\\ &\quad 8.0932\ \ 0.9134\ \ 3.2395\ \ 2.2254\ \ 1.3216\ \ 3.0748\ \ 1.7546\ \ 0.6263\ \ 1.6632\ \ 0.9446]^T.\end{aligned} \tag{59}$$

Fig. 8 illustrates that the collected output data of the process are between the upper and lower bounds obtained by means of the proposed robust identification procedure.

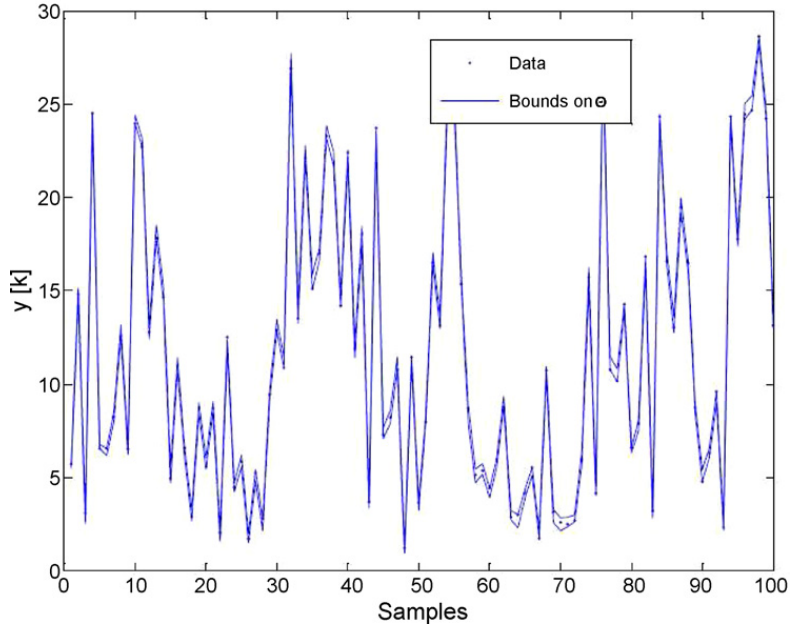

Fig. 8. Output data and bounds obtained from robust identification.

Table 2
Bounds on the parameters of the Hammerstein model.

Parameter	Bounds	Parameter	Bounds
h_1	[0.9600, 1.3222]	h_6	[1.0149, 1.3216]
h_2	[1.9013, 2.3526]	h_7	[0.2850, 0.6263]
h_3	[0.1322, 0.2869]	p_2	[3.0297, 3.0297]
h_4	[3.9319, 3.9692]	p_3	[1.7289, 2.7059]
h_5	[0.8725, 1.0692]		

At this time, solution of Problem 2 must be accomplished. Without loss of generality, one degree of freedom was eliminated by setting $p_1 = 1$. Then, the bounds shown in Table 2 are obtained for parameters h_i and p_i, respectively.

Fig. 9 illustrates the bounds on θ and the extreme values obtained with the identified parameters h_i and p_i. Note that the constraints imposed by Eqs. (38) and (39) are held.

From the inspection of the numerical results, the bounds shown in Table 2 could be interpreted as "worse" than the ones obtained by Falugi et al. However, this fact does not involve the method in [10] is better than the one herein introduced, but just different. While they found the best feasible parameters set based on both an assumption on the measurement noise and an optimization criterion, the present approach obtains the best bounds in order to guarantee the totality of the input/output data can be described by the resultant family of models. Therefore, the essentials of these methods are different.

4.3. Example 3

Now, we recall the simulation example presented in [13]. It is assumed that the "true" nonlinear block is

$$\mathcal{N}(u(k)) = 0.8585\,u(k) + 0.0149\,u^2(k) - 0.5113\,u^3(k) - 0.0263\,u^4(k) \tag{60}$$

and the transfer function in the "true" linear block:

$$\mathcal{L}(z) = \frac{z^2 + 0.7\,z - 1.5}{z^3 + 0.9\,z^2 + 0.15\,z - 0.002}. \tag{61}$$

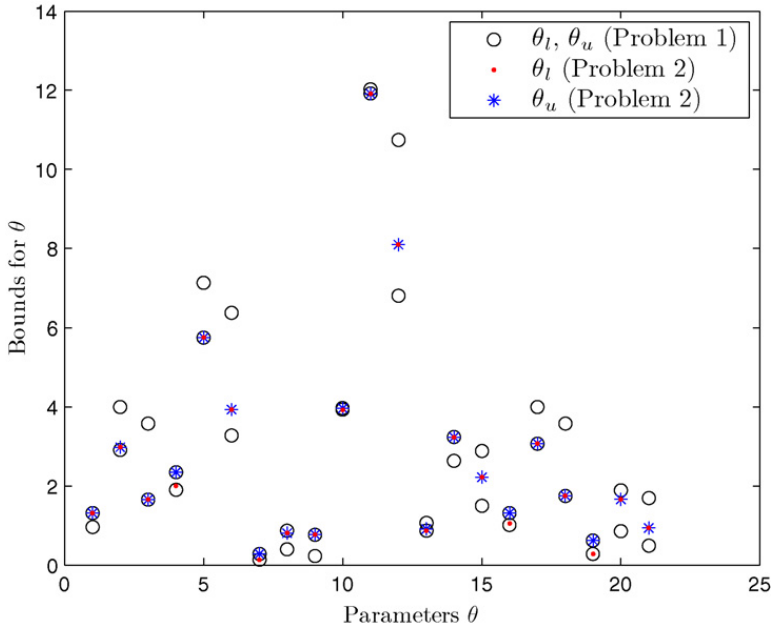

Fig. 9. Bounds on θ obtained from the solution of Problems 1 and 2.

Table 3
Bounds of the model's parameters (nonlinear optimization).

Parameter	Bounds
h_1	[3.6999, 4.8231]
h_2	[−8.4129, −7.9207]
h_3	[3.5180, 4.4062]
p_2	[0.0371, 0.0394]
p_3	[1.1251, 1.1391]

Table 4
Bounds of the model's parameters (linear programming).

Parameter	Bounds
h_1	[3.8091, 4.8231]
h_2	[−8.6612, −8.2553]
h_3	[3.5180, 4.5923]
p_2	[0.0360, 0.0378]
p_3	[1.0929, 1.0929]

As in [13], the following input was considered to excite the system:

$$u(k) = \sin(0.00057\pi k) + 0.5\ \sin(0.0015\pi k) + 0.3\ \sin(0.0025\pi k) + 0.1\ \sin(0.0035\pi k), \tag{62}$$

Provided the real system dynamics is ignored, we assume known the polynomial characteristic for the nonlinearity as well as the ARMA nature for the linear dynamics. Details on each block (such as order, nominal parameters, etc.) are assumed unknown. Therefore, we keep the polynomial description for the static block, i.e.:

$$\mathcal{N}(u(k)) = \sum_{i=1}^{3} p_i\ \ u^i(k). \tag{63}$$

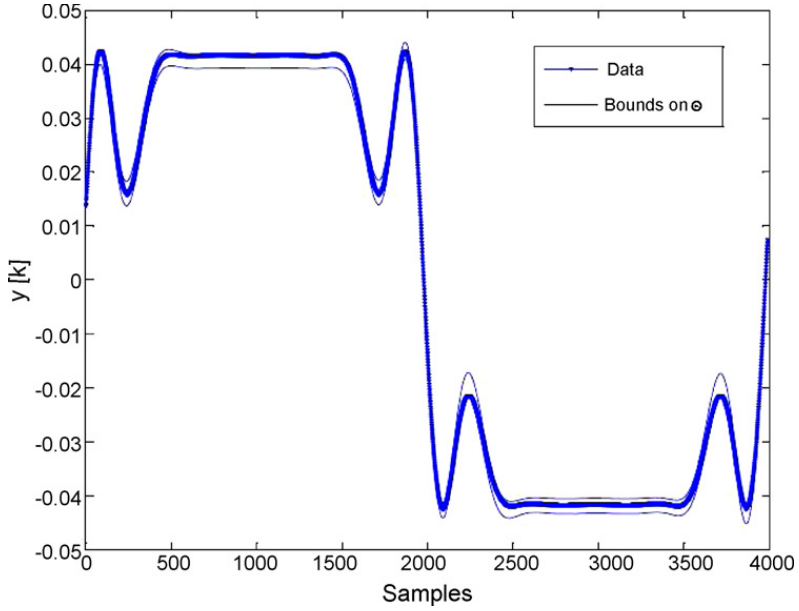

Fig. 10. Output data and bounds obtained from robust identification.

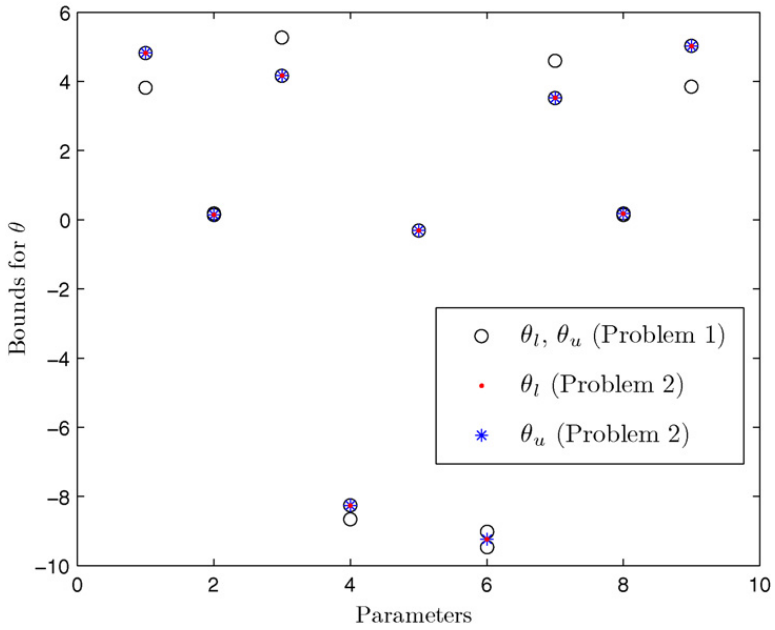

Fig. 11. Bounds on θ (linear programming).

With respect to the linear block we propose a Laguerre model:

$$\mathcal{L}(q) = \sum_{i=1}^{3} h_i \;\; B_i(q^{-1}). \tag{64}$$

This replacement is to satisfy the condition in Eq. (1), which imposes that necessary model structure for the implementation of the identification approach. A dominant pole placed in -0.01 is assumed.

After solving Problem 1, the following bounds on θ were obtained:

$$\begin{aligned} \theta_l &= [4.8191 \;\; 0.1374 \;\; 4.1631 \;\; -8.2553 \;\; -0.3125 \;\; -9.2320 \;\; 3.5180 \;\; 0.1738 \;\; 5.0191]^T, \\ \theta_u &= [4.8231 \;\; 0.1374 \;\; 4.1631 \;\; -8.2553 \;\; -0.3125 \;\; -9.2320 \;\; 3.5180 \;\; 0.1738 \;\; 5.0191]^T. \end{aligned} \tag{65}$$

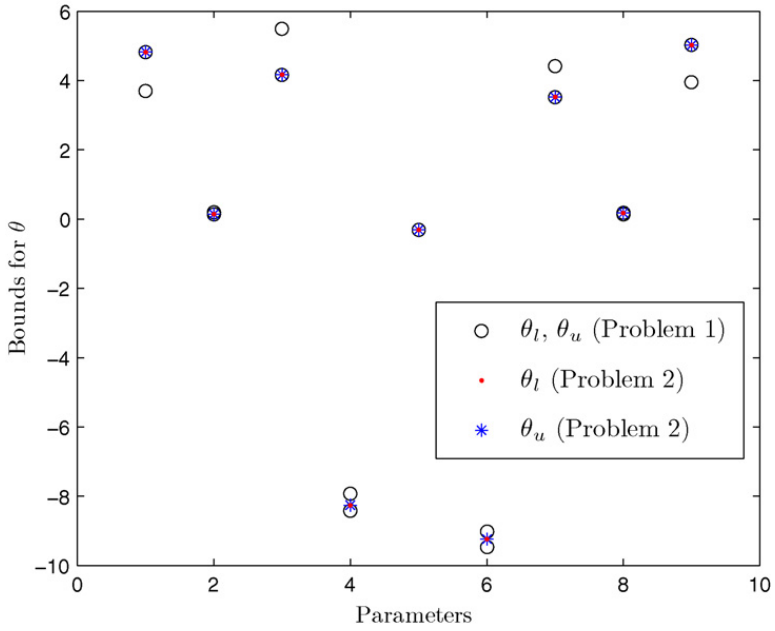

Fig. 12. Bounds on θ (nonlinear optimization).

Then, solution of Problem 2 is accomplished. The structural identifiability restriction $p_1 = 1$ was set. The solution of the nonlinear optimization problem is shown in Table 3.

Additionally, the problem was solved via Linear Programming. The results are reported in Table 4.

Fig. 10 illustrates that the collected output data of the process are between the bounds obtained through robust identification.

Fig. 11 shows the bounds on θ and extreme values for h_i and p_i, when the LP procedure is performed. Again, the constraints imposed by Eqs. (38) and (39) are held. On the other hand, the nonlinear optimization procedure gives the results shown in Fig. 12.

5. Conclusions

Block-oriented models are appealing descriptions for multiple applications in many fields. In particular, identification of Wiener and Hammerstein models has been dealt with in this work due to the wide acceptance and applicability these systems have shown.

Noniterative algorithms for the identification of both Wiener and Hammerstein uncertain models have been presented. A bounding procedure has been proposed for the identification of uncertain Wiener and Hammerstein parametric models. The approach allows robust identification, in the sense that the whole set of data can be reproduced by the family of models which is obtained. The suitability of the proposed identification methods has been illustrated through various simulation examples.

The procedures have been developed for SISO systems, however, the methodology could be extended to MIMO systems. This is a topic worthy of future research.

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