USE OF A CPWL APPROACH FOR OPERATIVITY ANALYSIS: A CASE STUDY

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Abstract

The operating point of a chemical process is usually computed by optimizing an steady-state objective function, e.g., the profit, subject to some characteristics of the plant. Typically, the resulting point lies on the boundary of the operating region. Then, the presence of disturbances can easily cause constraint violations. Thus, it is necessary to move the operating point away from the active constraints into the feasible region (back-off). The magnitude of this back-off has direct economical meaning. The purpose of this paper is to analyze the dynamic operability and performance of a steam generating unit using a Canonical Piecewise Linear approximation. The motivation for this analysis is the large operating cost involved in the operation of this equipment and the need to satisfy specific energy demands.

1. Introduction

The economical impact of disturbances in the operation of a process has been a subject of increasing interest in the latest years. The effects caused by internal and external disturbances require special attention from both, an operating and an economical point of view; this situation is particularly important in systems of high operating costs.

In a well-operated chemical plant, a challenging task is to achieve the optimum of some measure of plant performance within the various limitations placed on plant operation. The requirements for determining the optimum steady-state operating conditions are an appropriate objective function, a process model, and any operational requirements expressed as a set of inequality constraints. Mathematically, this can be written as (Bandoni *et al.*, 1994),

Problem 1 (Steady-State Optimization): Given a constant fixed vector of exogenous inputs ($\tilde{\mathbf{w}}$) we shall compute the set of control input (**u**) such that the following function will be optimized:

$$\min_{u} z_0(\mathbf{x},\mathbf{u}) \quad (1)$$
$$\text{suject to}$$
$$\dot{\mathbf{x}} = f(\mathbf{x},\mathbf{u},\mathbf{w}) = \mathbf{0}$$
$$\mathbf{z}_c = p(\mathbf{x},\mathbf{u},\mathbf{w}) \le 0$$
$$\mathbf{w} = \tilde{\mathbf{w}}$$

where $z_0(\mathbf{x},\mathbf{u})$ is the objective function, f(.,.,.) is the set of system equations, p(.,.,.) is the set of operative constraints, **x** is the system state vector, **u** is the optimization variable vector and **w** is the exogenous vector input, which can represent disturbances.

When this problem is solved, the operating condition is fixed by using the solution vector **u**. Generally, the solution of this problem is at the intersection as many active constraints as the dimension of **u**. Usually, however, it would not be possible to operate the plant right on these constraints, as process disturbances (i.e., a value for **w** different of $\tilde{\mathbf{w}}$) will cause the plant to fluctuate around the nominal optimum point, leading to constraint violations. One way to overcome this problem is to move the original optimal operating point far enough into the feasible region so as to ensure that no constraint violation occurs during plant operation. The extent of this movement of the operating point (called back-off) due to the likely effect of disturbances will quantify the economic penalty for this feasible but non-optimum operation. To compute this magnitude we will suppose that all possible disturbances are in the set W,

$$W = \left\{ \begin{array}{l} w_i(t); i = 1,..,m; \\ \quad w_i(t) = \begin{cases} \tilde{w}_i \ \text{if} \ t < 0 \ \text{with} \ \underline{w}_i \le \tilde{w}_i \le \overline{w}_i \\ w_i \ \text{if} \ t \ge 0 \ \text{with} \ \underline{w}_i \le w_i \le \overline{w}_i \end{cases} \end{array} \right\}$$

where $\tilde{w}_i$ is the nominal value and $\underline{w}_i$ and $\overline{w}_i$ are the lower and the upper bounds over the disturbances.

In order to be sure that the movement is minimum we will formulate the following problem (Figueroa *et al.*, 1994; Perkins and Walsh, 1994)

Problem 2 (Dynamic-State Back-Off): We shall compute the set of control input (**u**) such that the following function will be optimized:

$$\min_{u} z_0(\mathbf{x},\mathbf{u}) \quad (2)$$
$$\text{subject to}$$
$$\left.\begin{array}{l} \dot{\mathbf{x}} - f(\mathbf{x},\mathbf{u},\mathbf{w}) = 0 \\ \mathbf{z}_c = p(\mathbf{x},\mathbf{u},\mathbf{w}) \le 0 \end{array}\right\} \forall \mathbf{w} \in W$$

In this problem it is important to remark that the vector **u** is not depending on the time and the objective function is evaluated at steady-state.

This problem might not have a solution. In practice, this means that it is not possible to operate the plant under some requirements of performance for the disturbance magnitude defined by $\mathbf{w} \in W$. Note the difference between problems 1 and 2. In the former the constraints need to be verified for a specific $\mathbf{w} = \tilde{\mathbf{w}}$; in the latter, the constraints need to be satisfied for all $\mathbf{w} \in W$.

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Our strategy to find an appropriate solution for this problem, due to the computational complexity present in the general non-linear case, consists in finding a Canonical Piecewise Linear Approximation (CPWL) of the system and constraints under consideration (Figueroa and Desages, 1998).

2. CPWL Approach

The general formulation of piecewise linear functions allows us to write a non-linear system as several linear expressions, each of them valid in a certain operating region. To make this approximation, the domain of variables $\aleph$ (involving the domain of the variables **x**, **u** and **w**) is partitioned in to σ non-empty regions, $\aleph^i$, such that $\aleph = \bigcup_{i=1}^{\sigma} \aleph^i$. In each of these regions, $\aleph^k$, the non-linear differential equation, the constraints and the objective function are approximated using CPWL representations in the form

$$\dot{\mathbf{x}} = \xi^k_{xx}.\mathbf{x} + \xi^k_{xu}.\mathbf{u} + \xi^k_{xw}.\mathbf{w} + \eta^k_x \quad (3)$$

$$\mathbf{z}_c = \xi^k_{cx}.\mathbf{x} + \xi\xi^k_{cu}.\mathbf{u} + \xi^k_{cw}.\mathbf{w} + \eta^k_c \quad (4)$$

$$z_o = \xi^k_{ox}.\mathbf{x} + \xi^k_{ou}.\mathbf{u} + \eta^k_o \quad (5)$$

That can be written in an unique expression as (Chua and Deng, 1986)

$$\dot{\mathbf{x}} = \mathbf{a}_x + \mathbf{B}_{xx}\mathbf{x} + \mathbf{B}_{xu}\mathbf{u} + \mathbf{B}_{xw}\mathbf{w} + \sum_{i=1}^{\sigma} c_{xi}|\rho_i|$$

$$\mathbf{z}_c = \mathbf{a}_c + \mathbf{B}_{cx}\mathbf{x} + \mathbf{B}_{cu}\mathbf{u} + \mathbf{B}_{cw}\mathbf{w} + \sum_{i=1}^{\sigma} c_{ci}|\rho_i|$$

$$\mathbf{z}_0 = \mathbf{a}_o + \mathbf{B}_{ox}\mathbf{x} + \mathbf{B}_{ou}\mathbf{u} + \mathbf{B}_{ow}\mathbf{w} + \sum_{i=1}^{\sigma} c_{oi}|\rho_i|$$

where the matrices and vectors have appropriate dimensions with elements in the real field and $\rho_i = \alpha_{xi}\mathbf{x} + \alpha_{ui}\mathbf{u} + \alpha_{wi}\mathbf{w} - \beta_i$. Note that the vector with the i^{th} entry $sign(\rho_i)$ determines in which sector the system is operating.

In the following, we will assume that in each region $\aleph^i$, the inverse of the resulting matrix ξ^k_{xx} exists. Then, for a set of external variables $(\tilde{\mathbf{u}}, \tilde{\mathbf{w}})$, it is possible to compute the steady-state point using the algorithm developed by Figueroa and Desages (1998) (see Appendix A).

Once that the steady-state point is computed, it is possible to obtain an approximate expression for $\mathbf{x}(t)$ when a disturbance is applied. Mathematically, it could be written as (Appendix B),

$$\mathbf{x}(t) = \Phi_{xu}(t)\mathbf{u} + \Phi_{xw}(t)\mathbf{w} + \Phi_x(t, \tilde{\mathbf{w}}) \quad (6)$$

Now, we can compute an expression for the constraints for a generic sector $\aleph^h$,

$$\mathbf{z}_c = \Phi^h_{cu}(t)\mathbf{u} + \Phi^h_{cw}(t)\mathbf{w} + \Phi^h_c(t, \tilde{\mathbf{w}}) \quad (7)$$

where $\Phi^h_{cu}(t) = (\xi^h_{cx}.\Phi_{xu} + \xi^h_{cu})$; $\Phi^h_{cw}(t) = (\xi^h_{cx}.\Phi_{xw} + \xi^h_{cw})$ and $\Phi^h_c(t, \tilde{\mathbf{w}}) = (\xi^h_{cx}.\Phi_x + \xi^h_c)$.

Now, our strategy is to determine the "worst perturbation" $\mathbf{w} \in W$ in the sense of producing the largest value of the entries of $\mathbf{z}_c$. To do so, for the j^{th} entry of vector $\mathbf{z}_c$, a function $\lambda_j(\mathbf{u})$ is defined as

$$\lambda_j(\mathbf{u}) = \max_{\mathbf{w} \in W} \max_{t \in [0,\infty)} [\mathbf{z}_c(\mathbf{x}(t), \mathbf{u}, \mathbf{w})]_j \quad (8)$$

where $\mathbf{x}(t)$ is the solution to $\dot{\mathbf{x}} - f(\mathbf{x}, \mathbf{u}, \mathbf{w}) = 0$ and the subscript j means the j^{th} row of the vector (or matrix). If we consider that the maximum of $\max_{t \in [0,\infty)} [\mathbf{z}_c(\mathbf{x}(t), \mathbf{u}, \mathbf{w})]_j$ is in sector $\aleph^h$ at time $t = t^{max}$, then it is possible to write,

$$\max_{t \in [0,\infty)} [\mathbf{z}_c(t), \mathbf{u}, \mathbf{w}]_j = [\Phi^h_{cw}(t^{max})\mathbf{w} + \Phi^h_{cu}(t^{max})\mathbf{u} + \Phi^h_c(t^{max})]_j \quad (9)$$

Now, given a fixed control input vector u, and assuming that the argument t^{max}, the sectors [$\aleph^0$, $\aleph^1$, $\aleph^2$, ..., $\aleph^h$] and the times [t^0, t^1, t^2, ..., t^h] are not depending on **w**, the solution to the problem (8) is for the disturbance

$$\mathbf{w}^{max} = \left\{ w_j = \begin{cases} \overline{w}_j \text{ if } [\Phi^h_{cw}(t^{max})]_j \geq 0 \\ \underline{w}_j \text{ if } [\Phi^h_{cw}(t^{max})]_j < 0 \end{cases} \right\} \quad (10)$$

However, this situation is unrealistic because when **w** changes, the values of t^0, t^1, t^2, ..., t^h, t^{max}, $\aleph^0$, $\aleph^1$, $\aleph^2$, ..., $\aleph^{h-1}$ and $\aleph^h$ will also change. Then, in order to compute $\mathbf{w}^{max}$ we propose the following algorithm

Algorithm 1: Worst Disturbance Determination

Data: A set of external variables $(\mathbf{u}, \tilde{\mathbf{w}})$, an initial disturbance $\mathbf{w}^0$ and the set W of possible disturbances. Set k=1,

Step 1: Perform the simulation using Algorithm of Appendix B, determine the time for which the j^{th} entry of vector $\mathbf{z}_c$ ($[\mathbf{z}_c]_j$) is at maximum and compute the matrices $\Phi^h_{cw}(t^{max})$, $\Phi^h_{cu}(t^{max})$ and $\Phi^h_c(t^{max})$. Compute

$$\lambda^{k-1}_j = [\Phi^h_{cw}(t^{max}).\mathbf{w}^{k-1} + \Phi^h_{cu}(t^{max}).\mathbf{u} + \Phi^h_c(t^{max})]_j$$

Step 2: Compute the argument, $\mathbf{w}^k = \mathbf{w}^{max}$, that maximizes the expression

$$\max_{\mathbf{w} \in W} \max_{t \in [0,\infty)} [\mathbf{z}_c(t), \mathbf{u}, \mathbf{w}]_j$$

using equation (10), set

$$\lambda^k_j = [\Phi^h_{cw}(t^{max}).\mathbf{w}^k + \Phi^h_{cu}(t^{max}).\mathbf{u} + \Phi^h_c(t^{max})]_j$$

and adapt the limits for the disturbance set as

$$\overline{\mathbf{w}} = \left\{ \overline{w}_j = \begin{cases} \overline{w}_j & if \quad [\Phi^h_{cw}(t^{max})]_j \geq 0 \\ w^{k-1}_j & if \quad [\Phi^h_{cw}(t^{max})]_j < 0 \end{cases} \right\}$$

and

$$\underline{\mathbf{w}} = \left\{ \underline{w}_j = \begin{cases} w^{k-1}_j & if \quad [\Phi^h_{cw}(t^{max})]_j \geq 0 \\ \underline{w}_j & if \quad [\Phi^h_{cw}(t^{max})]_j < 0 \end{cases} \right\}.$$

Step 3: If $\lambda^{k-1}_j \neq \lambda^k_j$, make $k=k+1$ and return to Step 1. Otherwise, $\lambda_j = [\Phi^h_{cw}(t^{max}).\mathbf{w}^k + \Phi^h_{cu}(t^{max}).\mathbf{u} + \Phi^h_c(t^{max})]_j$

and $\hat{\mathbf{w}}^j = \mathbf{w}^k$ is the argument for which it happened; Stop the algorithm.

Note that the iterations in this algorithm are necessary due to the non linear nature of the original problem. If the problem were linear, the convergence of this algorithm will be guaranteed in one iteration. Now, it is possible to group equations (9) for j=1,2,.., n^c, as,

$$\lambda = \mathbf{D}.\mathbf{u} + \mathbf{E} \qquad (11)$$

where $\mathbf{D} = \begin{bmatrix} [\Phi_{cu}]_1 \\ [\Phi_{cu}]_2 \\ \vdots \\ [\Phi_{cu}]_{n_c} \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} [\Phi_{cw}]_1 . \hat{\mathbf{w}}^1 + [\Phi_c]_1 \\ [\Phi_{cw}]_2 . \hat{\mathbf{w}}^2 + [\Phi_c]_2 \\ \vdots \\ [\Phi_{cw}]_{n_c} . \hat{\mathbf{w}}^{n_c} + [\Phi_c]_{n_c} \end{bmatrix}$.

Now, let us analyze the objective function $z_0(\mathbf{x}(0), \mathbf{u})$. If this problem is constrained to the k^{th} sector, the steady-state vector is

$$\mathbf{x} = -\left(\xi^{k}_{xx}\right)^{-1} . \left(\xi^{k}_{xu} . \mathbf{u} + \xi^{k}_{xw} . \tilde{\mathbf{w}} + \eta^{k}_{x}\right). \qquad (12)$$

Introducing this expression in (5), the objective function constrained to the k^{th} sector could be written as

$$\mathbf{x} = \mathbf{A}^{k} . \mathbf{u} + \mathbf{B}^{k} \qquad (13)$$

where $\mathbf{B}^k = \left(-\xi^{k}_{ox}\left(\xi^{k}_{xx}\right)^{-1}\xi^{k}_{xw}\right).\tilde{\mathbf{w}} + \left(\eta^{k}_{o} - \xi^{k}_{ox}.\left(\xi^{k}_{xx}\right)^{-1}\eta^{k}_{x}\right)$, $\mathbf{A}^k = \left(\xi^{k}_{ou} - \xi^{k}_{ox}.\left(\xi^{k}_{xx}\right)^{-1}\xi^{k}_{xu}\right)$, and $\tilde{\mathbf{w}}$ is the disturbance considered as nominal. Note that this expression of the objective function will be valid only for the steady-states solution lying in the k^{th} sector. This condition of restriction to sector it is known as the Sector Belonging Condition for the steady state solution (Figueroa and Desages, 1998) and could be written as the following set of inequalities,

$$\mathbf{z}^{k}_{\aleph} = \mathbf{D}^{k}_{c} . \mathbf{u} + \mathbf{E}^{k}_{c} \leq \mathbf{0} \qquad (14)$$

where $\mathbf{D}^k_c = \left(\xi^{k}_{\aleph u} - \xi^{k}_{\aleph x}\left(\xi^{k}_{xx}\right)^{-1}\xi^{k}_{xu}\right)$ and $\mathbf{E}^k_c = \left(\xi^{k}_{\aleph w} - \xi^{k}_{\aleph x}\left(\xi^{k}_{xx}\right)^{-1}\xi^{k}_{xw}\right)$ with $\left[\xi^{k}_{\aleph x}\right]_i = -\gamma^k_i \alpha_{xi}$, $\left[\xi^{k}_{\aleph u}\right]_i = -\gamma^k_i \alpha_{ui}$, $\left[\xi^{k}_{\aleph w}\right]_i = -\gamma^k_i \alpha_{wi}$, $\left[\eta^{k}_{\aleph}\right]_i = \gamma^k_i \beta_i$, $\gamma^k_i = sign(\alpha_{xi}x + \alpha_{ui}u + \alpha_{wi}w - \beta_i)$ and $[.]_i$ means the i^{th} row in the matrix [.].

Then, the problem (2) constrained to the k^{th} sector, could be expressed as

$$\min_{\mathbf{u}} \quad \mathbf{A}^{k}\mathbf{u} + \mathbf{B}^{k}$$

subject to (15)

$$\mathbf{D}.\mathbf{u} + \mathbf{E} \leq \mathbf{0}$$

$$\mathbf{D}^{k}_{c} . \mathbf{u} + \mathbf{E}^{k}_{c} \leq \mathbf{0}$$

Note that in (15) the first set of constraints comes from the operative constraints in the original system, while the second set comes from the specific sector belonging condition (i.e., $(\mathbf{x}, \mathbf{u}, \tilde{\mathbf{w}}) \in \aleph^k$). In this situation, the optimum will be either on the boundary of the sector $\aleph^k$ or in the ones defined by the operative constraints. If the solution to the problem (15) is on the boundary of sector $\aleph^k$, at least one entry in the vector $\mathbf{D}^k_c\mathbf{u} + \mathbf{E}^k_c$ will be zero. If this occurs, we must change the sign of this entry (the rest remaining invariable) and, in this way, we go on with the optimization in the next sector $\aleph^{k+1}$. If no entry of $\mathbf{D}^k_c\mathbf{u} + \mathbf{E}^k_c$ is zero, we are in the border fixed by the operative constraints.

Also note that the set of inequalities $\mathbf{D}.\mathbf{u} + \mathbf{E} \leq \mathbf{0}$ is not dependent on the k^{th} sector, however, the matrices **D** and **E** are dependent on the vector **u**; then, each time that the vector **u** changes they should be computed again. Then we have solved the problem. In summary, we have the following algorithm:

Algorithm 2 (Dynamic Back-off Computation):

Data: An initial guess for **u** ($\mathbf{u}^0$) and a nominal disturbance $\tilde{\mathbf{w}}$.

Step 0: Compute a steady-state point $(\mathbf{x}^0, \mathbf{u}^0, \tilde{\mathbf{w}}) \in \aleph^0$ and the vector γ^0 which identifies this sector using Algorithm 1. Set k=0.

Step 1: Compute the matrices **D** and **E**, using Algorithm 1 and Equation (11).

Step 2: In the sector $\aleph^k$ compute the control $\mathbf{u}^k$ that solves the following minimization problem

$$\min_{\mathbf{u}} \quad \mathbf{A}^{k}\mathbf{u} + \mathbf{B}^{k}$$

subject to

$$\mathbf{D}.\mathbf{u} + \mathbf{E} \leq \mathbf{0}$$

$$\mathbf{D}^{k}_{c} . \mathbf{u} + \mathbf{E}^{k}_{c} \leq \mathbf{0}$$

Step 3: If any entry in the vector $\mathbf{D}^k_c\mathbf{u} + \mathbf{F}^k_c$ is zero, change the sign of the correspondent entry in the vector γ^k to obtain the new vector γ^{k+1} that identifies the next sector $\aleph^{k+1}$. Set k=k+1 and return to Step 1. Otherwise (i.e. if not entry of $\mathbf{D}^k_c\mathbf{u} + \mathbf{F}^k_c$ is zero), continue.

Step 4: Compute the worst disturbance using Algorithm 3. If any entry in the vector λ is larger than zero, return to Step 1. Otherwise Stop.

In Step 3 there might be a problem of determining which region (close to $\aleph^k$) could be chosen as $\aleph^{k+1}$. Suppose that the optimum in region $\aleph^k$ is at the intersection of l hyperplanes. This means that l entries of $\mathbf{D}^k_c\mathbf{u} + \mathbf{F}^k_c$ are zero. This makes it difficult to continue with the optimization algorithm in the sector $\aleph^{k+1}$, because this new sector may be obtained by changing the sign of either of these entries of γ^k, or any combination of them. This gives $(2^l - 1)$ possibilities. It is obvious that the convergence of the algorithm depends on our choice. Figueroa and Desages (1998) presents three possible search methods to avoid this problem. The convergence of this algorithm towards the global optimum can not always be guaranteed, however this is the typical problem of the nonlinear optimization. The use of the approximation does not produce extra limitations.

In this work this strategy to operativity analysis will be applied to a steam generating unit to emphasize the use of this technique. The motivation for this analysis is the large operating cost involved in the operation of these units and their need to satisfy specific energy demands. Our attention has been focused to determine the feasibility of operation while ensuring no constraint violations, for a set process disturbances. The study has allowed us to establish an economic penalty for the feasible operation.

3. Steam Generating Unit Model

Power and steam systems, in which the boiler is a fundamental part, should be included in this category due to their large operating cost and their need of satisfying specific energy demands. Despite these facts, utility systems have not received the same degree of attention as other process units when dealing with disturbances effects. One reason for this situation has been the uneasy availability of simple reliable mathematical models for boilers in the open literature. The steam generating unit studied in this paper consists of five pulverizers supplying fuel to a 200 MW drum type boiler (Figure 1). The model for this unit has been developed by Ray and Majumder (1983).

3.1. Pulverizer model

Primary air required for this unit is supplied by two P.A. fans and then bifurcated into hot air and cold air flows for pulverizer units. A nonlinear model for a single pulverizer has been developed having as inputs the feeder speed, the hot air damper opening, the cold air damper opening and the P.A. fans speed. The states for each unit are the fuel output of the pulverizer, the hot air flow and the cold air flow. The output variable is the fuel outlet from pulverizers, supplied to the boiler. Simulation of a single pulverizer unit can be performed using the following set of equations

$$\frac{dF^i}{dt} = c_1^i\, u_2^i\, H^i + c_2^i\, u_3^i\, C^i + c_3^i\, F^i + c_4^i\left(H^i + C^i\right) u_7^i$$

$$\frac{dH^i}{dt} = c_5^i\, u_1^i + c_6^i\, u_6^i - H^i\, u_2^i - C^i\, u_3^i$$

$$\frac{dC^i}{dt} = c_7^i\, u_1^i + c_8^i\, u_6^i + c_9^i\, H^i\, u_2^i + c_{10}^i\, C^i\, u_3^i$$

where F is the fuel output of the pulverizer, H is the hot air flow, C is the cold air flow, u_1 and u_6 are the P.A. fan speed (nominally 24.7252 rad/sec), u_2 is the hot air damper opening (0.8), u_3 is the cold air damper opening (0.2) and u_7 is the feeder speed (3 r.p.m.). The parameters are included in Table I.

The rest of the units will be having similar kind of dynamics. In this analysis (for the purposes of algorithm demonstration) we will consider the feeder speed in each pulverizer as a disturbance, because it depends on the coal characteristics.

The manipulated variables are the hot air damper opening and the speed of the two P.A. fan units. Also, we consider that the hot and the cold air damper openings are normalized, then they should verify the following relation $u_3 = 1 - u_2$

TABLE I: PULVERIZER PARAMETERS

i		*2*	*3*	*4*	*5*
c_1^i		9	11	10	11
c_2^i	2	1.8	2.2	2.1	1.9
c_3^i	-.073591	-.07	-.075	-.071	-.072
c_4^i	.057306	.06	.063	.059	.055
c_5^i	.001413	.0013	.0015	.0014	.00125
c_6^i	.001413	.0013	.0015	.0014	.00125
c_7^i	.003	.0027	.0033	.0031	.0028
c_8^i	.003	.0027	.0033	.0031	.0028
c_9^i	-1.903016	-1.88	-1.93	-1.9	-1.89
c_{10}^i	-3.	-2.9	-3.1	-3.1	-3.

3.2. Boiler model

The states of the non-linear drum type coal fired boiler model are the drum pressure (P), the steam flow to the H.P. turbine (S) and the drum level (L). It has four inputs, there are the fuel input from the pulverizers outputs (F_i), the feed water input (w_c, nominally 193 Kg/sec), the feed water temperature (Te nominally 288°C) and the control valve setting (c_v, nominally 0.8). The model is as follows,

$$\frac{dP}{dt} = -0.00193\, S\, P^{1/8} + 0.014524 \sum_{i=1}^{5} F_i - 0.000736\, w_c + 0.00121\, L + 0.000176\, T_e$$

$$\frac{dS}{dt} = 10\, c_v\, P^{1/2} - 0.785716\, S$$

$$\frac{dL}{dt} = 0.00863\, w_c + 0.002 \sum_{i=1}^{5} F_i + 0.463\, c_v - 6\times 10^{-6} P^2 - 0.00914\, L - 8.2\times 10^{-5} L^2 - 0.007328\, S$$

The purpose of this model is to describe the gross behavior of the plant. The control variables for the boiler are the fuel input (from the pulverizers) and the feed water input. The disturbances are the feed water temperature and control valve setting. The last one represents the variation on the steam demand to the service units.

3.3. Control Scheme

The control scheme for this systems is composed for two control structures:

Control system for the boiler control: It involves two SISO loops, controlling the pressure and the drum level by using the fuel and the water feed inputs respectively as manipulated variables. These loops are closed with PI controllers with parameters as in Table II.

TABLE II. PI CONTROLLERS

	Loop 1 (P-F)	Loop 2 (L-w_c)
P	0.75	0.05
I	50000	10

Control system for the pulverizers: In this case, the manipulated variable are the is the hot air damper opening (u_2) and the controlled variables are the fuel outputs of the pulverizer (*F*). It is important to remark that the value of the reference of these loops are computed using a divisor, and each of these controllers are a slave controller which master is the pressure loop of the boiler (i.e. $F^{spi} = 0.2F^{sp} i = 1,..5$. This controllers are proportional with parameter K_p= 100.

In order to study the complete system operation, we consider the five pulverizers supplying fuel to the boiler plus the controllers, so we have a total of 20 non-linear differential equations. There are 16 freed variables considered, these are the hot air damper and the two P.A. fan speeds for each pulverizer and the feed water input at the boiler. The disturbances considered are the feeder speed for each pulverizer, the control valve displacement and the feed water temperature.

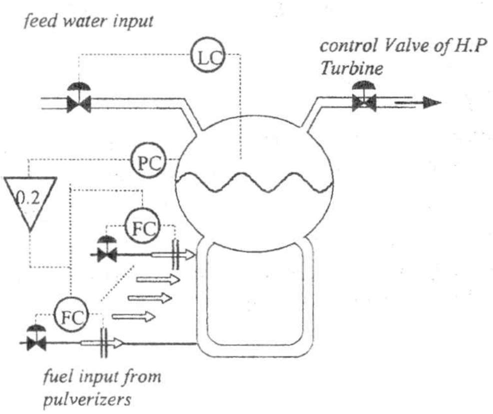

Fig. 1 Steam Generating Unit with control Scheme

There are 15 operative constraints, they are

Minimal Steam flow $S \geq 110$
Bounds on the drum pressure $140 \leq P \leq 200$
Bounds on the drum level $45 \leq L \leq 66$
Bounds on the fuel output on each pulverizer $5 \leq F_i \leq 9$

The objective is to minimize the operation cost (fuel and water). In this point we will assume that all pulverizers have the same operative cost.

$$z_{obj} = 0.25\, w_c + \sum_{i=1}^{5} 10 F_i$$

The set of manipulated variables are allowed to move in the following ranges:

$0.8 \leq u_2 \leq 0.95$ (for each pulverizer)
$22 \leq u_1 = u_6 \leq 25.5$ (for each pulverizer)
$22 \leq u_1 = u_6 \leq 25.5$ (for each pulverizer)
$175.0 \leq w_c \leq 210.0$

and also we allow free value for disturbances between the following limits (with the nominal value between parenthesis):

$280.0 \leq T_e$ (288 °C) ≤ 292.0
$2.9 \leq u_7$ (3 r.p.m) ≤ 3 (for each pulverizer)
$0.8 \leq c_v$ (0.8) ≤ 0.85

In order to apply these algorithms, the variable domain has been divided in 140 regions. To perform this CPWL model a direct not constrained optimization algorithm was used on each individual non linearity of the system, and then, they are joined in a complete CPWL model.

The optimum back-off objective function $z_{obj} = 356.39$ is obtained in two iterations of Algorithm 2. The values of the manipulated variables are $u_2^i = 0.8$ for *i*=1,..,5; $u_1^i = 22.0$ for *i*=1,2,3,5; $u_1^4 = 22.56$; $u_6^i = 22.0$ for *i* = 1,2,4,5; $u_6^3 = 22.25$ and w_c =175

4. Conclusions

An approximation for a detailed pulverizers-boiler system has been developed. It allowed to specifically consider steam generating unit operability in process plant disturbances analysis. This study was conducted by determining an optimal operating point belonging to the permanent feasible region of the utility system. It allowed to quantify the deviation from the optimal nominal condition, providing an economic indicator of the potential value of implementing a control system. The used CPWL-based algorithm needs less than 1/5 of the time for convergence as compared to the non-linear models.

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APPENDIX A: Algorithm for Steady-State Computation

Data: A set of external variables $(\tilde{\mathbf{u}}, \tilde{\mathbf{w}})$ and an initial guess $\mathbf{x}^0$. Set k=0.

Step 1: Compute the positive λ_i^k that makes the ith entry of the following vector zero:

$$\mathbf{z}_{\aleph}^{k} = \xi_{\aleph x}^{k}(\mathbf{x}^{k} + \lambda(\xi_{xx}^{k})^{-1}\mathbf{z}_{x}^{k}) + \xi_{\aleph u}^{k}\tilde{\mathbf{u}} + \xi_{\aleph w}^{k}\tilde{\mathbf{w}} + \eta_{\aleph}^{k}$$

for i=1,..,σ, where $\mathbf{z}_{x}^{k} = \xi_{xx}^{k}\mathbf{x}^{k} + \xi_{xu}^{k}\tilde{\mathbf{u}} + \xi_{xw}^{k}\tilde{\mathbf{w}} + \eta_{x}^{k}$.

Step 2: Compute $\lambda_c = \min_i \lambda_i^k$ and $\mathbf{x}^{k+1} = \mathbf{x}^{k} - \lambda_c(\xi_{xx}^{k})^{-1}\mathbf{z}_{x}^{k}$ with $\mathbf{z}_{x}^{k} = \xi_{xx}^{k}\mathbf{x}^{k} + \xi_{xu}^{k}\tilde{\mathbf{u}} + \xi_{xw}^{k}\tilde{\mathbf{w}} + \eta_{x}^{k}$.

Step 3: If λ_c is smaller than one, set k=k+1 and return to Step 1. Otherwise, $\mathbf{x}^{k+1}$ is the steady-state value. Stop.

For details and the proof of convergence see Figueroa and Desages, 1998.

APPENDIX B: Computation of x(t)

Consider the system at the steady state point $(\mathbf{x}^0, \mathbf{u}, \tilde{\mathbf{w}}) \in \aleph^0$. Then, a disturbance $\mathbf{w}(t) \in W$ is applied to the system. While the system is in sector $\aleph^0$, it is easy to see that the state vector will be

$$\mathbf{x}(t) = e^{\xi_{xx}^{0}t}\mathbf{x}^{0} - \left(\mathbf{I} - e^{\xi_{xx}^{0}t}\right)\left(\xi_{xx}^{0}\right)^{-1}.\left(\xi_{xu}^{0}.\mathbf{u} + \xi_{xw}^{0}.\mathbf{w} + \eta_{x}^{0}\right).$$

This expression will be valid till the time t^0 in which the system reach the next sector. In this new sector, another linear expression for the system is valid Then, it is possible to obtain an algorithm to perform the dynamic simulation as follow,

Algorithm B: Dynamic Simulation

Data: A set of external variables $(\mathbf{u}, \tilde{\mathbf{w}})$, the magnitude of the step in the disturbance **w** and the horizon (T^{max}) to perform the simulation.

Step 0: Compute the steady-state solution for the model (i.e. $\mathbf{x}^0$), Set k=0.

Step 1: Determine in which sector, $\aleph^k$, the point $(\mathbf{x}^k, \mathbf{u}, \mathbf{w})$ lies; and compute the linear model valid in this sector. The state as a function of the time for this sector is

$$\mathbf{x}(t) = e^{\xi_{xx}^{k}t}\mathbf{x}^{k} - \left(\mathbf{I} - e^{\xi_{xx}^{k}t}\right)\left(\xi_{xx}^{k}\right)^{-1}.\left(\xi_{xu}^{k}.\mathbf{u} + \xi_{xw}^{k}.\mathbf{w} + \eta_{x}^{k}\right).$$

Step 2: Compute the time t^{k+1} that first makes zero an entry of the vector,

$$\mathbf{z}_{\aleph}^{k} = \xi_{\aleph x}^{k}\mathbf{x}(t) + \xi_{\aleph u}^{k}\mathbf{u} + \xi_{\aleph w}^{k}\mathbf{w} + \eta_{\aleph}^{k};$$

and determines the state for which it happens as

$$\mathbf{x}(t^{k+1}) = e^{\xi_{xx}^{k}t^{k+1}}\mathbf{x}^{k} - \left(\mathbf{I} - e^{\xi_{xx}^{k}t^{k+1}}\right)\left(\xi_{xx}^{k}\right)^{-1}.\left(\xi_{xu}^{k}.\mathbf{u} + \xi_{xw}^{k}.\mathbf{w} + \eta_{x}^{k}\right).$$

Step 3: If t^{k+1} is smaller that T^{max}, set $\mathbf{x}^{k+1} = \mathbf{x}(t^{k+1})$, make k=k+1 and return to Step 1 to proceed similarly in the next sector. Otherwise, Stop.

Using the results of this algorithm, a generic expression for $\mathbf{x}(t)$ when the system goes through sectors [$\aleph^0$, $\aleph^1$, $\aleph^2$, ..., $\aleph^h$] could be written as

$$\mathbf{x}(t) = \Phi_{xu}(t)\mathbf{u} + \Phi_{xw}(t)\mathbf{w} + \Phi_{x}(t, \tilde{\mathbf{w}})$$

with

$$\Phi_{xu}(t) = -e^{\xi_{xx}^{h}t}\left(\prod_{j=1}^{h-1}e^{\xi_{xx}^{j}t^{j}}\right)\left(\xi_{xx}^{0}\right)^{-1}\xi_{xu}^{0} - e^{\xi_{xx}^{h}t}\sum_{i=1}^{h-1}\left(\prod_{j=i+1}^{h-1}e^{\xi_{xx}^{j}t^{j}}\right)\ldots$$

$$\left(\mathbf{I} - e^{\xi_{xx}^{i}t^{i}}\right)\left(\xi_{xx}^{i}\right)^{-1}\xi_{xu}^{i} - \left(\mathbf{I} - e^{\xi_{xx}^{h}t}\right)\left(\xi_{xx}^{h}\right)^{-1}\xi_{xu}^{h}$$

$$\Phi_{xu}(t) = -e^{\xi_{xx}^{h}t}\sum_{i=0}^{h-1}\left(\prod_{j=i+1}^{h-1}e^{\xi_{xx}^{j}t^{j}}\right)\left(\mathbf{I} - e^{\xi_{xx}^{i}t^{i}}\right)\left(\xi_{xx}^{i}\right)^{-1}\xi_{xw}^{i} - \left(\mathbf{I} - e^{\xi_{xx}^{h}t}\right)\left(\xi_{xx}^{h}\right)^{-1}\xi_{xw}^{h}$$

$$\Phi_{x}(t, \tilde{\mathbf{w}}) = -e^{\xi_{xx}^{h}t}\left(-\left(\prod_{j=0}^{h-1}e^{\xi_{xx}^{j}t^{j}}\right)\left(\xi_{xx}^{0}\right)^{-1}\xi_{xu}^{0}\tilde{\mathbf{w}} + \left(\prod_{j=1}^{h-1}e^{\xi_{xx}^{j}t^{j}}\right)\left(\xi_{xx}^{0}\right)^{-1}\xi_{x}^{0}\ldots\right.$$

$$\left.+\sum_{i=1}^{h-1}\left(\prod_{j=i+1}^{h-1}e^{\xi_{xx}^{j}t^{j}}\right)\left(\mathbf{I} - e^{\xi_{xx}^{i}t^{i}}\right)\left(\xi_{xx}^{i}\right)^{-1}\xi_{x}^{i}\right) - \left(\mathbf{I} - e^{\xi_{xx}^{h}t}\right)\left(\xi_{xx}^{h}\right)^{-1}\xi_{x}^{h}$$

where t^i is the time at which the system leaves sector $\aleph^i$. Note the dependence of the matrices $\Phi_{xu}(t)$, $\Phi_{xw}(t)$ and $\Phi_{x}(t, \tilde{\mathbf{w}})$ on the sectors [$\aleph^0$, $\aleph^1$, $\aleph^2$, ..., $\aleph^h$] and on the times [t^0, t^1, t^2, ..., t^h]. This means that, in general, this expression is not longer valid if any change occurs in the inputs u or w.

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