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Economic performance of variable structure control: a case study

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Abstract

The operating point of a chemical process is usually computed by optimizing a steady-state objective function, e.g. the profit, subject to the steady-state characteristics of the plant. However, the resulting point typically lies in the boundary of the operating region. The presence of disturbances can easily cause constraint violations in the transient. Thus, it is necessary to move the operating point away from the active constraints into the feasible region. The magnitude of this 'back-off' has a direct influence on the economic side. The purpose of this paper is to study the effect of the combination of a state-observer and controller designed using structure variable techniques at the economical level of the process control.

Keywords: Optimization; Process control; Observers; Variable structure control

1. Problem formulation

Consider the following system

$$\dot{x} = f(x) + g^r(x)r + g^u(x)u + g^w(x)w \quad (1)$$

$$y = h(x) + l^r(x)r + l^u(x)u + l^w(x)w \quad (2)$$

where $x \in \Re^{n_x}$ is the system state, $y \in \Re^{n_y}$ is the system output, $r \in \Re^l$ is the optimization variable that will be considered constant all the time, $u \in \Re^k$ is the control input and $w \in \Re^m$ is the disturbance input belonging to the set $W = \left\{ w : \begin{cases} \hat{w}, \forall t \geq 0 \\ 0, \forall t < 0 \end{cases} \text{with } \|\hat{w}\|_\infty \leq 1 \right\}$.

In order to complete the description of our system, consider now a set of inequalities, that should be satisfied at any time,

$$z_c(x,r,u,w) = p(x) + q^r(x)r + q^u(x)u + q^w(x)w \leq 0 \quad (3)$$

where $z_c(x,r,u,w) \in \Re^{n_z}$. Normally, this set of inequalities follows from some process variable constraints (e.g. product quality, safety and environmental regulations, etc.).

The vector fields $f:\Re^{n_x} \to \Re^{n_x}$, $g^r:\Re^l \to \Re^{n_x}$, $g^u:\Re^k \to \Re^{n_x}$, $g^w:\Re^m \to \Re^{n_x}$, $h:\Re^{n_x} \to \Re^{n_y}$, $l^r:\Re^l \to \Re^{n_y}$, $l^u:\Re^k \to \Re^{n_y}$, $l^w:\Re^m \to \Re^{n_y}$, $p:\Re^{n_x} \to \Re^{n_z}$, $q^r:\Re^l \to \Re^{n_z}$, $q^u:\Re^k \to \Re^{n_c}$ and $q^w:\Re^m \to \Re^{n_c}$ are C^∞. Let us assume, that given r, u and w the steady state is unique (i.e. the equation $\dot{x} = z_x = 0$ has a unique solution of x).

In this system (for given values of u and w), the operating point is usually designed to optimize some objective function (z_0) at the steady state. This optimization can be posed as,

1.1. Optimization problem

$$\min_r \; z_0(x, r)$$
$$\text{s.t.} \; \dot{x} = f(x) + g^r(x)r + g^u(x)u + g^w(x)w = 0$$
$$z_c(x, r, u, w) = h(x) + l^r(x)r + l^u(x)u + l^w(x)w \leq 0 \quad (4)$$

Then, the operating condition is fixed by setting the vector r. Generally, the solution of this problem is at the intersection of many constraints (l') as the dimension of vector r (l). The effect of the disturbances at this point is to prevent the plant from working at this desired operating point. Under these perturbed conditions, the process operation may become unfeasible. To maintain feasibility, a move may be required in the operating point away from that determined at the optimization level. The magnitude of this movement

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due to the likely effect of the disturbances is referred to as 'back-off'. The possibility of minimizing the necessary back-off is an appealing idea, as it promises the minimizing of the effect of the disturbances in the process economy and it has been studied by several authors (Bandoni, Romagnoli & Barton, 1994; Perkins & Walsh, 1994). Figueroa, Bahri, Bandoni and Romagnoli (1996), deal with this problem for the case of dynamic closed loop system, solving the following problem.

1.2. Back-off problem

$$\min_{r} \; z_o(x_o, r, u_o)$$

s.t.

$$\left.\begin{aligned} \dot{x} &= f(x) + g^r(x)r + g^u(x)u + g^w(x)w \\ z_c(x, r, u, w) &= p(x) + q^r(x)r + q^u(x)u + q^w(x)w \le 0 \\ y &= h(x) + l^r(x)r + l^u(x)u + l^w(x)w \\ u &= \kappa(x, y) \end{aligned}\right\} \; \forall w \in W, \; \forall t \ge 0 \tag{5}$$

where the subscript o in the objective function means the values of x and u at the steady state, and $\kappa(x, y)$ is a controller expression, which can be dynamic or static. A typical selection of the control scheme is made by using the concept of constrained control (Maarleveld & Rijnsdorp, 1970), where the regulatory control objectives are chosen as the set of active constraints that define the optimal operating point. The operative condition obtained for the solution of Problem 5 is called back-off operative point.

Obviously, the back-off magnitude depends on the selected control scheme. It should present good disturbance rejection properties to maintain the plant operation as close as possible to the steady-state optimum point. In this context, it is known that variable structure controllers could be designed to present good disturbance rejection properties (Hung, Gao & Hung, 1993). Variable structure control with sliding mode is a special type of control technique that is capable of making a highly robust control system with respect to system parameter variations and external disturbances. In addition, this technique provides an easy way to design the control law for linear or nonlinear plants.

However, the application of this kind of controller requires the complete knowledge of all the system variables and states. This is a very strong limitation for the industrial application of these techniques. To solve this problem, in this paper, we proposed the use of a non linear observer by means of a second order sliding mode. Then, the combined observer-controller is analyzed in an economical context.

The paper is organized as follows. The next section describes the variable structure control and in Section 3 a concept of observer is introduced. An illustrative example, in terms of a flowsheet system, is studied in Section 4 and some general conclusions are presented in Section 5.

2. Variable structure control

The variable structure control (VSC) and their related sliding modes have been widely studied in the last 30 years (see e.g. Emelyarov, 1985; De Carlo, Zak & Mattheews, 1988; Stolite & Sastry, 1988).

Let us assume that, to apply the VSC control, the state vector x is fully measurable and the system has a strong relative vectorial degree in an open set $D \in \Re^{n_x}$. Let us define an auxiliary function $s: \Re^{n_x} \to S \subset \Re^n$ with entries $s_i(x)$ for $i = 1, \ldots, n$. This function $s(x)$, called commutation function, divides the state space into regions in which $s_i(x)$ has different signs. The number of such functions is considered equal to the number of control inputs ($n = k$).

The control strategy in the VSC system, consists of an adequate set of non-continuous functions of states, disturbances and set point signals. These control actions are usually defined as

$$u_i(t) = \begin{cases} u_i^+(x, t) & \text{if } s_i(x) > 0 \\ u_i^-(x, t) & \text{if } s_i(x) < 0 \end{cases} \tag{6}$$

The set of equations $s_i(x) = 0$ are called switching functions. They describe the surfaces of discontinuity in the control action u. Then, a control strategy, valid in each region, will drive the system state at least to some scalar switching function in a finite time. This procedure is usually known as a reaching mode. Once the system has reached this surface, another function takes control of the process, repeating the procedure.

The control objective in our case is to regulate the system on the optimal operating condition fixed by r^*, despite the presence of the disturbances $w(t)$. Consider that x^* is the steady state at this operative point. Then, an appropriate choice for the switching functions is,

$$s_i = |x_i - x_i^*| - \varepsilon_i \tag{7}$$

for a small scalar $\varepsilon_i > 0$. However, the number of such functions should be equal to the number of control variables (i.e. k). Applying the concept of constrained control, the l' states related to the active constraints plus the $k-l'$ states for which the distance between the optimal and the back-off operative point (i.e. $|x_i^{bo} - x_i^*|$) is the largest[1], will be selected simultaneously as switching functions.

The control action vector for this application is defined as

[1] This choice is equivalent to considering the states with steady-state which is the most sensitive to the disturbances.

$$u = u_0 + \Delta_u \quad (8)$$

where u_0 is the constant manipulated variable necessary to retain the operating point at its steady-state value free from disturbances and Δ_u is the varying part of the control action. Now, Δ_u is defined as,

$$\Delta_u(t) = \begin{cases} u_i^+(x, t) \text{to force } s_i \text{ to decrease if } s_i(x) > 0 \\ u_i^-(x, t) \quad \text{if } s_i(x) < 0 \end{cases} \quad (9)$$

It is interesting to note the dependence of u_i^+ in terms of $\text{sign}\{x_i - x_i^*\}$. This means that the control action will be different depending either on $x_i > x_i^*$ or on $x_i < x_i^*$, regardless of $s_i(x) > 0$ in both cases. This policy will force the process state to go into the region defined by $s_i = |x_i - x_i^*| - \varepsilon_i < 0$. Then, the system will go to the steady state in an autonomous form, since x^* is a stable equilibrium point.

From the above definition, it is clear that if $s_i > 0$, then u_i^+ should verify

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = \frac{\mathrm{d}s_i}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = \text{sign}\{x_i - x_i^*\} \times (f(x) + g^r(x)r + g^u(x)(u_0 + u_i^+(t)) + g^w(x)w) < 0 \quad (10)$$

for $i = 1, \ldots, k$ in order to reduce s.

This reaching mode is known as direct switching function. There are two aspects to consider, based on this control action. Firstly, the discontinuity in $s = 0$ could produce some problems in the plant. It is possible to reduce this fact by smoothing the control action. Secondly, if the magnitude of the control action is too large, some problems of saturation could affect the process. At this point some bounds are usually applied to the values of u. In our particular application, this limit can be determined by using the constraints information from expression (Eq. (3)).

3. Variable structure observer

In the previous section a VSC structure had been proposed under the assumption that the state vector x was fully measurable. As a result of this assumption, the control action Δ_u in Eq. (9) depends on the states and on the disturbances. However, some of these variables are typically unmeasured. In order to solve this problem, we propose the use of a second order sliding mode technique in a nonlinear estimation framework (Chiacchiarini, Desages, Romagnoli & Palazoglu, 1995; Colantonio, Desages, Romagnoli & Palazoglu, 1995).

Let us consider the following system for the observer

$$\dot{\hat{x}} = f(\hat{x}) + g^r(\hat{x})r + g^u(\hat{x})u + g^w(\hat{x})v \quad (11)$$

$$y_m = h(\hat{x}) + l^r(\hat{x})r + l^u(\hat{x})u + l^w(\hat{x})v \quad (12)$$

where $\hat{x} \in \Re^{n_x}$ is the observer state; $y_m \in \Re^{n_y}$ is the observer output and v is the disturbance vector. Note the similarity between the plant (Eqs. (1) and (2)) and its observer (Eqs. (11) and (12)), the state $\hat{x}$ and the output signal y_m in the observer directly correspond to the state and output in the physical system. Moreover, on the right side of Eqs. (11) and (12) it is possible to consider $\hat{x}_i = x_i$ and $w_i = v_i$ for the entries related to the measured states and disturbances, respectively. Then, $y \neq y_m$ only if $w_i \neq v_i$ for unmeasured disturbances. In this context, the observer objective will be obtain to $w = v$ in order to have $y_m = y$.

An appropriate choice for the switching functions for the observer are

$$s_i^o = y_i - y_{mi} \quad (13)$$

Then, let us propose the following Lyapunov function

$$L_i = \alpha(s_i^o)^2 + \psi(\dot{s}_i^o)^2 \quad \alpha, \psi > 0 \quad (14)$$

It is clear that the solution of $L_i = 0$ is given by $(s_i^o, \dot{s}_i^o) = (0, 0)$. To make sure that this point is asintotically reached, it is enough to verify that $\dot{L}_i < 0$, then we should prove,

$$\alpha s_i^o + \psi \ddot{s}_i^o < 0 \quad (15)$$

From the Lyapunov stability point of view, Eq. (15) it is a necessary and sufficient condition to ensure that the system will reach the surface $s_i^o = 0$ with zero derivative. To obtain also a specified dynamic for the system, we can modify the right side of Eq. (15) to obtain

$$\alpha s_i^o + \psi \ddot{s}_i^o < -\eta \ \text{sign}(s_i^o) \quad (16)$$

for a small scalar $\eta > 0$. This sliding mode is called second order sliding mode (Chiacchiarini et al., 1995).

Note that the number of the switching functions defined by Eq. (13) should be equal to the number of non measurable disturbances. If we consider the completed system described by the equations Eqs. (1), (2), (11) and (12), we note that the variables v_i are unknown signals that represent the unmeasured disturbances w_i which we want to estimate. Therefore, from the above mentioned, our objective will be to modify the value of v_i satisfying Eq. (16) in order to obtain $y_m = y$. In the next section, this idea will be clarified in an illustrative example.

4. Example

The case study considered in this section consists of two continuous stirred tank reactors (CSTR) in series, with an intermediate mixer introducing a second feed (de Hennin & Perkins, 1993), as shown in Fig. 1. A single irreversible, exothermic, first order reaction A → B takes place in both reactors. The dynamic model of these reactions is

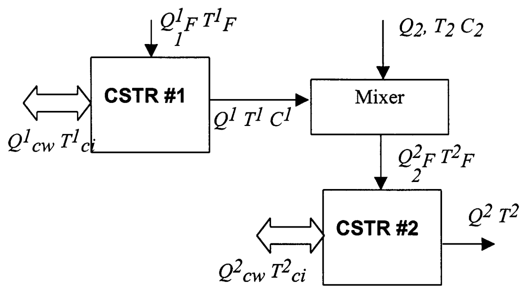

Fig. 1. Flowsheet example.

$$V^1\frac{d(C^1)}{dt} = -k_o e^{-E/RT^1}C^1V^1 + Q^1_F(C^1_F - C^1) \quad (17)$$

$$V^1\frac{d(T^1)}{dt} = D_h k_o e^{-E/RT^1}C^1V^1 + Q^1_F(T^1_F - T^1) + \text{Cool}^1 \quad (18)$$

$$V^2\frac{d(C^2)}{dt} = -k_o e^{-E/RT^2}C^2V^2 + Q^2_F(C^2_F - C^2) \quad (19)$$

$$V^2\frac{d(T^2)}{dt} = D_h k_o e^{-E/RT^2}C^2V^2 + Q^2_F(T^2_F - T^2) + \text{Cool}^2 \quad (20)$$

In order to model the mixer its dynamics are neglected. Therefore, the balances around the mixer are as follows,

$$C^2_F = \frac{(Q^1_F C^1 + Q_2 C_2)}{Q^2_F} \quad T^2_F = \frac{(Q^1_F T^1 + Q_2 T_2)}{Q^2_F}$$

$$Q^2_F = Q^1_F + Q_2$$

The reactors are cooled by cooling jackets surrounding each reactor volume. The amount of heat transferred between reactor and coolant is,

$$\text{Cool}^1 = \frac{U_a Q^1_{cw} c_p}{U_a + Q^1_{cw} c_p}(T^1_{ci} - T^1)$$

$$\text{Cool}^2 = \frac{U_a Q^2_{cw} c_p}{U_a + Q^2_{cw} c_p}(T^2_{ci} - T^2)$$

The temperatures and compositions of the two feeds are considered as disturbances (i.e. $w = [T^1_F \quad C^1_F \quad T_2 \quad C_2]^T$). The range of these disturbances are shown in Table 1.

The objective function of the process is related to the net profitability of flowsheet,

$$z_o = 10(Q^1_F C^1_F + Q_2 C_2 - Q^2 0.3) - 0.01 \ \text{Cool}^1$$

Table 1
Lower and upper bounds for process disturbances

Disturbance	Lower bound	Upper bound
T^1_F	295 K	320 K
C^1_F	19.5 mol/m^3	22 mol/m^3
T_2	295 K	320 K
C_2	19.5 mol/m^3	22 mol/m^3

Table 2
Summary of the results

Case	Q^1_F	Q_2	z_O ($)
No disturbances	0.3552	0.2062	90.352
Ziegler–Nichols[a]	0.2724	0.1875	72.300
Fully measurable	0.3224	0.2199	86.600
Observer	0.3537	0.1863	86.586

[a] Disturbances are considered only for first reactor.

$$-1.0\ \text{Cool}^2 - 0.1Q^1_F - 0.1Q_2$$

expressed in [$/h]. The optimization variables are the first and second feed flowrates (Q^1_F, Q_2). There are eight operational constraints placed on the process (c^i; $i = 1, \ldots, 8$),

$$c^1{:}T^1 \le 350\ \text{K} \qquad c^2{:}T^2 \le 350\ \text{K}$$
$$c^3{:}Q^1_F + Q_2 \le 0.8\ \text{m}^3/\text{s} \qquad c^4{:}\text{cool}^1 \le 30.0\ \text{W}$$
$$c^5{:}\text{cool}^2 \le 20.0\ \text{W} \qquad c^6{:}Q^1_F \ge 0.05\ \text{m}^3/\text{s}$$
$$c^7{:}Q_2 \ge 0.05\ \text{m}^3/\text{s} \qquad c^8{:}C^2 \le 0.3\ \text{mol/m}^3$$

For the nominal value of disturbances, the optimum operating point is at the intersection of constraints c^1 and c^5 with an objective function of $z_o = 90.3522$ $/h for the optimizing variables $Q^1_F = 0.3552$ m^3/s and $Q_2 = 0.2062$ m^3/s.

Using the back-off concept let us analyze the performance of the VSC control. It is compared with a classical multiloop strategy. For the VSC structure, two cases are considered. First, the ideal case of completely measurable states is assumed; then, an observer is included in the scheme. In both cases, the manipulated variable is the cooling flowrate for the reactors (Q^1_{cw}, Q^2_{cw}). The economical profit is compared by solving Problem 5.

The multiloop strategy consists of two loops controlling the temperature in each reactor with the respective cooling flowrate. The PI controllers are tuned using the Ziegler–Nichols technique. The economical performance of this loop is shown in Table 2.

4.1. Fully measurable problem

In this section, we suppose that the state vector is fully measurable. The switching surfaces are chosen as,

$$s_i = |T^i - T^{*i}| - 0.1 \qquad i = 1,\ 2.$$

Note that s_1 is defined in terms of an active constraint (c^1) while s_2 is defined in terms of a variable related to the other active constraint (c^5). The use of Cool^2 is not recommendable because this variable cannot the obtained directly by measurement. The manipulated variable for each surface is defined as

$$Q^i_{cw} = Q^*_{cw} + \Delta Q^i_{cw}$$

where $Q^*_{cw} = A^*_i U_a / c_p(U_a - A^*_i)$ with $A^*_i = -(-D_h k_o e^{-E/RT^{*i}} C^{*i} V^i + Q^*_F(T^*_F - T^{*i}))/V^i(T^i_{ci} - T^{*i})$. This is the value necessary to keep the operating point at its optimum, free from disturbances.

The varying part of the control action ($\Delta Q^i_{cw}(t)$) is defined as

$$\Delta Q^i_{cw}(t) = \begin{cases} \Delta Q^{+i}_{cw} = \dfrac{A_i U_a}{c_p(U_a - A_i)} - Q^*_{cw} + s_i \operatorname{sign}\{T^i - T^{*i}\} & \text{if } s_i(x) \geq 0 \\ \Delta Q^{-i}_{cw} = 0 & \text{if } s_i(x) < 0 \end{cases}$$

with $A_i = -(-D_h k_o e^{-E/RT^i} C^i V^i + Q^i_F(T^i_F - T^i))/V^i(T^i_{ci} - T^i)$ in order to verify inequality (Eq. (10)).

To avoid saturation and physical limits, lower and upper bounds are imposed over each manipulated variable. First, it is clear that Q^i_{cw} should be larger than 0. The upper bounds are established by using the maximum available cooling (Constraints c^4 and c^5),

$$Q^1_{cw} \leq \frac{30 U_a}{c_p(U_a(T^1_{ci} - T^1) - 30)}$$

$$Q^2_{cw} \leq \frac{20 U_a}{c_p(U_a(T^2_{ci} - T^2) - 20)}$$

4.2. Non fully measurable problem

Now, we will consider the more realistic assumption related to the impossibility of measuring the states C^1 and C^2 and disturbance inputs C^1_F and C_2. Note that the control action in the previous section will not be applied now, because A_i depends on these variables. Then, to avoid this problem, let us consider the following model for the observer:

$$V^1 \frac{d(C^1_m)}{dt} = -k_o e^{-E/RT^1} C^1_m V^1 + Q^1_F(C^1_{Fm} - C^1_m) \quad (21)$$

$$V^1 \frac{d(T^1_m)}{dt} = D_h k_o e^{-E/RT^1} C^1_m V^1 + Q^1_F(T^1_F - T^1) + \text{Cool}^1 \quad (22)$$

$$V^2 \frac{d(C^2_m)}{dt} = -k_o e^{-E/RT^2} C^2_m V^2 + Q^2_F(C^2_{Fm} - C^2_m) \quad (23)$$

$$V^2 \frac{d(T^2_m)}{dt} = D_h k_o e^{-E/RT^2} C^2_m V^2 + Q^2_F(T^2_F - T^2) + \text{Cool}^2 \quad (24)$$

$$C^2_{Fm} = \frac{(Q^1_F C^1_m + Q_2 C_{2m})}{Q^2_F}$$

where C^1_{Fm} and C_{2m} are unknown variables that represent the feeds' composition C^1_F and C_2, that should be estimated. Note that the states in the observer Eqs. (21)–(24) will differ from the states in the process Eqs. (17)–(20) only if $C^1_{Fm} \neq C^1_F$ and $C_{2m} \neq C_2$. Then, our objective will be to make $C^1_{Fm} = C^1_F$ and $C_{2m} = C_2$ in order to obtain $C^1_m = C^1$ and $C^2_m = C^2$. In this case, the control action described in the last section will be applicable again.

Due to the fact that only T^1 and T^2 are measurable, the following sliding surfaces are defined,

$$s^o_i = T^i - T^i_m \quad i = 1, 2 \quad (25)$$

By using them, a second order sliding mode as the one defined in Eq. (16) will be constructed. To do this we should compute

$$\dot{s}^o_i = \dot{T}^i - \dot{T}^i_m = D_h k_o e^{-E/RT^i}(C^i - C^i_m) \qquad i = 1, 2. \quad (26)$$

$$\ddot{s}^o_1 = D_h k_o e^{-E/RT^1}\left\{(C^1 - C^1_m) \times \left(-k_o e^{-E/RT^1} - \frac{Q^1_F}{V^1} + e^{-E/RT^1}\frac{E/R}{(T^1)^2}\dot{T}^1\right) + \frac{Q^1_F}{V^1}(C^1_F - C^1_{Fm})\right\}$$

and

$$\ddot{s}^o_2 = D_h k_o e^{-E/RT^2}\left\{(C^2 - C^2_m) \times \left(-k_o e^{-E/RT^2} - \frac{Q^2_F}{V^2} + e^{-E/RT^2}\frac{E/R}{(T^2)^2}\dot{T}^2\right) + \frac{Q^1_F}{V^2}(C^1 - C^1_m) + \frac{Q_2}{V^2}(C_2 - C_{2m})\right\}$$

By replacing these expressions in Eq. (16), we obtain

$$\alpha(T^1 - T^1_m) + \psi D_h k_o e^{-E/RT^1}\left\{(C^1 - C^1_m) \times \left(-k_o e^{-E/RT^1} - \frac{Q^1_F}{V^1} + e^{-E/RT^1}\frac{E/R}{(T^1)^2}\dot{T}^1\right) + \frac{Q^1_F}{V^1}(C^1_F - C^1_{Fm})\right\} \leq -\eta \operatorname{sign}(T^1 - T^1_m) \quad (27)$$

and

$$\alpha(T^2 - T^2_m) + \psi D_h k_o e^{-E/RT^2}\left\{(C^2 - C^2_m) \times \left(-k_o e^{-E/RT^2} - \frac{Q^2_F}{V^2} + e^{-E/RT^2}\frac{E/R}{(T^2)^2}\dot{T}^2\right) + \frac{Q^1_F}{V^2}(C^1 - C^1_m) + \frac{Q_2}{V^2}(C_2 - C_{2m})\right\} \leq -\eta \operatorname{sign}(T^2 - T^2_m) \quad (28)$$

Now, let us define $\Delta C^1_F = C^1_{Fm} - C^1_F$ and $\Delta C_2 = C_{2m} - C_2$, then from Eqs. (27) and (28), we reach

$$\Delta C^1_F \leq \frac{1}{\psi D_h k_o e^{-E/RT^1}(Q^1_F/V^1)}\{-\alpha(T^1 - T^1_m) - \eta \operatorname{sign}(T^1 - T^1_m)\} + \frac{V^1}{Q^1_F}(C^1 - C^1_m) \times \left(-k_o e^{-E/RT^1} - \frac{Q^1_F}{V^1} + e^{-E/RT^1}\frac{E/R}{(T^1)^2}\dot{T}^1\right) \quad (29)$$

and

$$\Delta C_2 \leq \frac{1}{\psi D_h k_o e^{-E/RT^2}(Q_2/V^2)}\{-\alpha(T^2 - T_m^2) - \eta\,\mathrm{sign}(T^2 - T_m^2)\} - \frac{Q_F^1}{Q_2}\Delta C_F^1 + \frac{V^2}{Q_2}(C^2 - C_m^2) \times \left(-k_o e^{-E/RT^2} - \frac{Q_F^2}{V^2} + e^{-E/RT^2}\frac{E/R}{(T^2)^2}\dot{T}^2\right) \quad (30)$$

where C^1 and C^2 are obtained using Eq. (26). In these expressions, $\dot{T}^1$ and $\dot{T}^2$ are evaluated by using finite differences. These expressions allow us to write the following algorithm for the estimation of C_{Fm}^1 and C_{2m}.

Estimation algorithm step 1, set $k = 0$, $C_{Fm}^1(0) = 20$ and $C_{2m}(0) = 20$; step 2, compute $\Delta C_F^1(k)$ using Eq. (29) and then compute $C_{Fm}^1(k+1) = \Delta C_F^1(k) + C_{Fm}^1(k)$; step 3, compute $\Delta C_2(k)$ using Eq. (30) and then compute $C_{2m}^1(k+1) = \Delta C_2(k) + C_{2m}(k)$; and step 4, $k = k+1$, return to step 2.

In this way, when $C_{Fm}^1 \to C_F^1$ and $C_{2m} \to C_2$, then $\Delta C_F^1 \to 0$ and $C_{2m} \to C_2$. Thus, we obtain the values necessary to implement the controller of Section 4.1. The performance of this estimator could be appreciated in Fig. 2, where C^1 and their estimation C_m^1 are shown, when a step is applied to the composition in the first feed.

The comparison between both control schemes is performed in terms of dollars. The back-off problem is solved for these controllers using the algorithm described in Figueroa et al. (1996). The set of disturbances considered are described in Table 1, some of the critical perturbations detected are related to the upper limit of the feed composition and temperatures. The results are summarized in Table 2. The operating points shown in this table allow optimal and feasible operation for all possible disturbances.

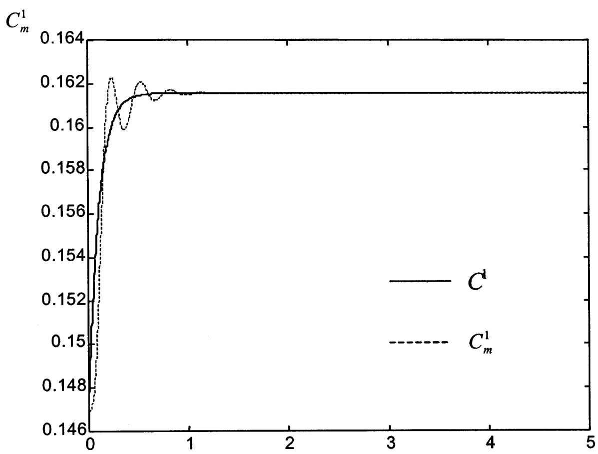

Fig. 2. Time simulation for the estimator.

5. Conclusions

In this paper, the capabilities of the VSC schemes in the economical context of the process control have been studied. The controller structure is used to bring the actual operating point as close as it is possible to the optimum (in terms of dollars) in the presence of disturbances. The use of estimators to compute the non measurable disturbances does not produce any additional loss in the economic performance of the process and their implementation only implies software modifications.

6. Nomenclature

x	system state vector
r	optimization variables vector
u	control input vector
w	disturbance input vector
y	system output vector
W	set of possible disturbances
z_c	vector of constraints
s	vector of commutation functions
z_o	objective function for the optimization problem
$\hat{x}$	observer state vector
y_m	observer output vector
v	observer disturbance vector
n_x	dimension of the state vector
l	number of optimization variables
k	number of control inputs
m	number of disturbance inputs
n_y	number of system outputs
n_c	number of constraints
l'	number of active constraints
n	number of commutation functions
f, g^r, g^u, g^w, h, l^r, l^u, l^w, p, q^r, q^u, q^w	vector fields that describe the model
$\kappa(x, y)$	generic expression for the controller
$u_i^{+(-)}$	control action applied when $S_i(x) > 0$ (< 0)
u_0	constant value of the manipulated variable
Δ_u	varying value of the manipulated variable
L	Lyapunov functions

CSTR's example

V^i	volume of CSTR # i ($V^1 = V^2 = 5.0$ m^3)
C^i	concentration of A in CSTR # i
T^i	temperature in CSTR # i

k_0	Arrhenius Constant ($2.7 \cdot 10^8$ s^{-1})
C_F^1	concentration in feed #1 (20 mol/m^3)
C_F^2	feed Concentration in CSTR #2
C_2	concentration in feed #2 (20 mol/m^3)
T_F^1	temperature in feed #1 (300 K)
T_F^2	feed temperature in CSTR #2 (300 K)
T_2	temperature in feed #2 (300K)
T_{ci}^1	coolant inlet temperature in CSTR #1 (250 K)
T_{ci}^2	coolant inlet temperature in CSTR #2 (250 K)
Q_{cw}^1	coolant flowrate in CSTR #1 (at steady-state 0.35 m^3/s)
Q_{cw}^2	coolant flowrate in CSTR #2 (at steady-state 0.8 m^3/s)
C_p	coolant heat capacity (1 J/kg K)
U_a	overall heat transfer coefficient (0.35 W/°C)
E/R	activation energy (6000 K)
D_h	heat of reaction (5.0 K m^3/mol)
Cool^1	heat transferred between cooling jacket and reactor in CSTR #1 (W)
Cool^2	heat transferred between cooling jacket and reactor in CSTR #2 (W)
Q_F^1	feed flowrate in feed #1
Q_F^2	feed flowrate in CSTR #2
Q_2	feed flowrate in feed #2
C^i	constraints on the CSTR's system
z_o	process profit
Δ	variable part
C_{Fm}^1	unknown composition in feed #1
C_{2m}	unknown composition in feed #2
C_m^i	Concentration of A in CSTR #i in the observer
T_m^I	temperature in CSTR #i in the observer

Subscripts

0	values at steady-state
i	ith entry of a vector

Superscripts

*	optimal value
bo	back-off point
o	observer

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